Quadratic Equations

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1. **Quadratic Equation:** An equation of the form $ax^2 + bx + c = 0$, where a, b, c real numbers and $(a \neq 0)$, is called a quadratic equation in variable x or if p(x) is a quadratic polynomial then $p(x) = 0$ is called a quadratic equation.

2. **Roots of a Quadratic Equation:** A real number is called a root of the quadratic equation $ax^2 + bx + c = 0\ (a \neq 0)$ if $a\alpha^2 + b\alpha + c = 0$

Note: If α ***is a root of the quadratic equation*** $\mathbf{ax^2 + bx + c = 0}$, ***then*** α ***is called a zero of the polynomial*** $\mathbf{ax^2 + bx + c}$**.**

3. **Solving a Quadratic Equation:** Solving a quadratic equation means finding its roots.

4. **Discriminant:** If $ax^2 + bx + c = 0,\ (a \neq 0)$ is a quadratic equation then the expression $b^2 - 4ac$ is called the discriminant. It is denoted by D.

5. **Nature of the Roots of a Quadratic Equation**

Value of D	Nature of Roots	Real Roots
$D > 0$	two distinct and real roots	$\alpha = \frac{-b\sqrt{D}}{2a}$ and $\beta = \frac{-b-\sqrt{D}}{2a}$
$D = 0$	two equal real root	$each\ root = \frac{-b}{2a}$
$D < 0$	No real root	None

6. **Relation between Roots and Coefficients of a Quadratic Equation:** If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then $(\alpha + \beta) = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

7. **To form a Quadratic Equation with given roots** α **and** β : A quadratic equation with roots α and β is given by: $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e., $x^2 - (sum\ of\ roots)\ x + (product\ of\ roots) = 0$

8. If one root of $ax^2 + bx + c = 0$ is $(p + \sqrt{q})$, then the other root is $(p - \sqrt{q})$.

9. If we can factorize $ax^2 + bx + c$, $(a \neq 0)$ into a product of two linear factors, then the roots of the quadratic equation can be found by equating each factor to zero

10. $ax^2 + bx + c$ is factorizable only when $b^2 - 4ac \geq 0$.

Snap Test

1. For what value of k, does the quadratic equation $9x^2 + 8kx + 16 = 0$

(a) $k = \pm 5$ (b) $k = \pm 8$

(c) $k = \pm 3$ (d) $k = \pm 6$

(e) None of these

Ans. (c)

Explanation: For equal roots discriminant D must be equal to 0.

In the given equation $9x^2 + 8kx + 16 = 0$, we have:

$D = (8k)^2 - (4 \times 9 \times 16)$

Now, $D = 0 \Rightarrow (8k)^2 - (4 \times 9 \times 16) = 0$ $[\therefore D = b^2 - 4ac]$

$\Rightarrow 64\{k^2 - 9\} = 0 \Rightarrow k^2 - 9 = 0 \Rightarrow (k-3)(k+3) = 0$

$\Rightarrow (k-3) = 0$ or $(k+3) = 0 \Rightarrow k = 3$ or $k = -3$.

2. If $x = \frac{2}{3}$ and $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$, find the value of a and b.

(a) $a = 3, b = -6$ (b) $a = 3, b = +6$

(c) $a = 2, b = -5$ (d) $a = 3, b = -2$

(e) None of these

Ans. (a)

Explanation: We have $x = \frac{2}{3}$ for the equation $ax^2 + 7x + b = 0$, therefore,

$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = \Rightarrow 4a + 42 + 9b = 0 \Rightarrow 4a + 9b = -42$ .... (i)

Since $x = -3$ is a root of the equation $ax^2 + 7x + b = 0$, we have

$a.(-3)^2 + 7(-3) + b = 0 \Rightarrow 9a - 21 + b = 0 \Rightarrow 9a + b = 21$ .... (ii)

Multiplying (ii) by 9 and subtracting (i) from it, we get:

$77a = 231 \Rightarrow a = 3.$

Substituting $a = 3$ in (i) we get: $12 + 9b = -42 \Rightarrow 9b = -54 \Rightarrow b = -6$

3. Rewrite the following as a quadratic equation in x and then solve for x:

$$\frac{4}{x} - 3 = \frac{5}{2x+3}, \; x \neq 0, \frac{-3}{2}$$

(a) $x^2 - x + 1 = 0, \{-1, 1\}$ (b) $x^2 - x + 2 = 0, \{-2, 1\}$

(c) $x^2 - x + 4 = 0, \{-1, 2\}$ (d) $x + x + 2 = 0, \{-2, 1\}$

(e) None of these

Ans. (b)

Explanation: $\frac{4}{x} - 3 = \frac{5}{2x+3}, \Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$

$\Rightarrow (4-3x)(2x+3) = 5x \Rightarrow -6x^2 - x + 12 = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0 \Rightarrow 6(x^2 + x - 2) = 0$

$\Rightarrow \quad x^2 + x - 2 = 0$, which is the required quadratic equation.

Now, $x^2 + x - 2 = 0 \Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow \quad x(x+2) - 1(x+2) = 0 \Rightarrow (x+2)(x-1) = 0$

$\Rightarrow \quad (x+2) = 0$ *or* $(x-1) = 0 \Rightarrow x = -2$ *or* $x = 1$

4. A two-digit number is four times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

(a) 45 (b) 36

(c) 12 (d) 24

(e) None of these

Ans. (d)

Explanation: Let the tens digit be a and the units digit be b.

Now, we have the required number $= 10a + b$

According to the question; $10a + b = 4(a + b) \Rightarrow 6a = 3b \Rightarrow b = 2a$

And $10a + b + 18 = 10b + a$

$\Rightarrow \quad 9a - 9b + 18 = 0$

$\Rightarrow \quad 9a - 18a + 18 = 0$

$\Rightarrow \quad a = 2$, thus $b = 4$

5. A two-digit number is even is seven times the sum of its digits and is also equal to 12 less than three times the product of its digits. Find the number.

(a) 84 (b) 36

(c) 34 (d) 93

(e) None of these

Ans. (a)

Explanation: Let tens digit be a and the units digit be b.

Now the required number $= 10a + b$

$\Rightarrow \quad 10a + b = 7(a + b) \Rightarrow 3a = 6b \Rightarrow a = 2b$

And $10a + b = 3ab - 12 \Rightarrow 20b + b = 6b^2 - 12$

$\Rightarrow \quad 6b^2 - 21b - 12 = 0 \Rightarrow 3(2b^2 - 7b - 4) = 0 \Rightarrow 2b^2 - 7b - 4 = 0$

$\Rightarrow \quad 2b^2 - 8b + b - 4 = 0$

$\Rightarrow \quad 2b(b - 4) + 1(b - 4) = 0 \Rightarrow (b - 4)(2b + 1) = 0$

$\Rightarrow \quad b = 4$ or $b = \frac{-1}{2}$

When $b = 4$, then $a = 8$