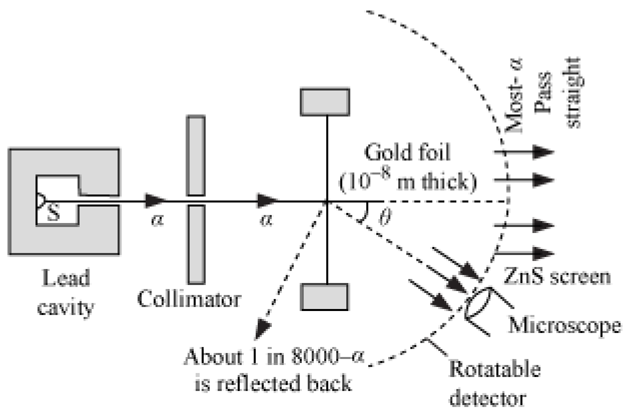
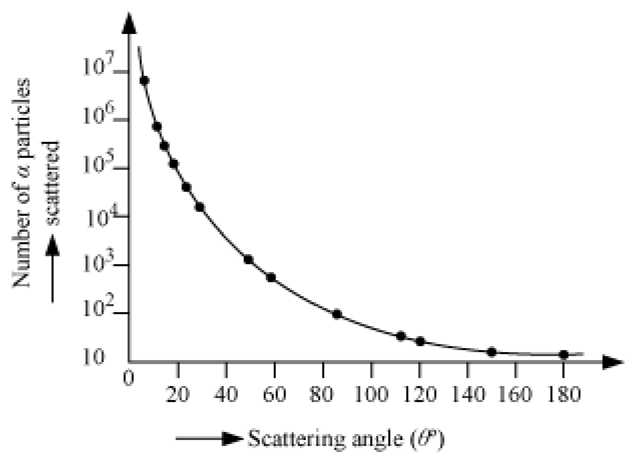
Atoms

Alpha Particle Scattering and Rutherford's Nuclear Model of an Atom



The experimental set up used by Rutherford and his collaborators, Geiger and Marsden, is shown in the above figure.

Observations − A graph is plotted between the scattering angle θ and the number of α-particles $N(\theta)$, scattered at $\angle\theta$ for a very large number of α-particles.



Conclusions:

- Most of the alpha particles pass straight through the gold foil.
- Only about 0.14% of incident α-particles scatter by more than 1^0.
- About one α-particle in every 8000 α-particles deflects by more than 90°.

Explanation

- In Rutherford's model, the entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away.
- The electrons would be moving in orbitals about the nucleus just as the planets do around the sun.
- The size of the nucleus comes out to be 10^{-15} m to 10^{-14} m. From kinetic theory, the size of an atom was known to be 10^{-10} m, about 10000 to 100,000 times larger than the size of the nucleus. Thus, most of an atom is empty space.
- The trajectory of an alpha particle can be computed employing Newton's second law of motion and Coulomb's law for electrostatic force of repulsion between the alpha particle and the positively charged nucleus.

  The magnitude of this force is

  $$F = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{r^2}$$

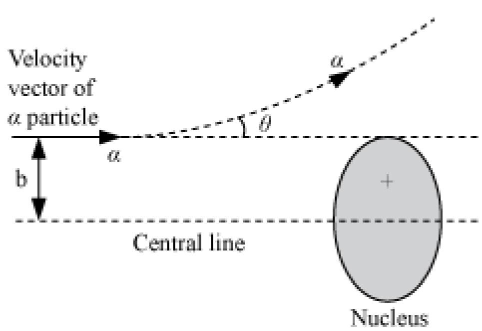
  Where,

  Ze – Charge of gold nucleus

  2*e* – Charge on alpha particle

  r – Distance between α-particle and the nucleus
- Alpha particle trajectory

  Trajectory traced by an *α*-particle depends on the impact parameter *b* of collision. The impact parameter is the perpendicular distance of the initial velocity vector of the *α*-particle from the centre of the nucleus.



- For large impact parameters, force experienced by the alpha particle is weak because $F \propto \frac{1}{(\text{Distance})^2}$. Hence, the alpha particle will deviate through a much smaller angle.

When impact parameter is small, force experienced is large and hence, the alpha particle will scatter through a large angle.

- Electron orbits

Let

F_c – Centripetal force required to keep a revolving electron in orbit

F_e – Electrostatic force of attraction between the revolving electron and the nucleus

Then, for a dynamically stable orbit in a hydrogen atom,

$F_c = F_e$

$$\frac{mv^2}{r} = \frac{(e)(e)}{4\pi\varepsilon_0 r^2} \qquad \ldots(\mathrm{i})$$

$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2} \qquad \ldots(\mathrm{ii})$$

K.E. of electron in the orbit,

$$K = \frac{1}{2}mv^2$$

From equation (i),

$$K = \frac{e^2}{8\pi\varepsilon_0 r}$$

Potential energy of electron in orbit,

$$\mathrm{U} = \frac{(e)(-e)}{4\pi\varepsilon_0 r} = \frac{-e^2}{4\pi\varepsilon_0 r}$$

Negative sign indicates that revolving electron is bound to the positive nucleus.

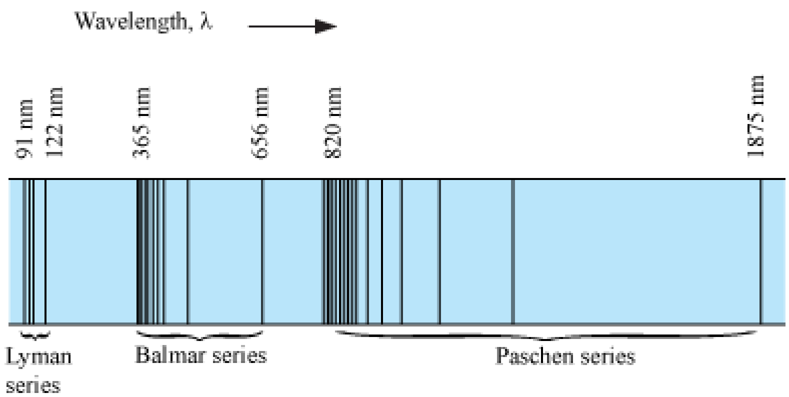
∴ Total energy of electron in hydrogen atom

$$E = k + U = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\boxed{E = -\frac{e^2}{8\pi\varepsilon_0 r}}$$

Atomic Spectra

- Each element emits a characteristic spectrum of radiation.
- In the excited state, the atoms emit radiations of a spectrum, which contains certain specific wavelengths only. This spectrum is termed as emission line spectrum and it consists of bright lines on a dark background.
- The spectrum emitted by atomic hydrogen is shown in the figure below.



- **Spectral Series**

  When the electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength. It is called a spectral line.

  In H-atom, when an electron jumps from the orbit n_i to orbit n_f, the wavelength of the emitted radiation is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

  Where,

  $R \rightarrow$ Rydberg's constant = 1.09678×10^7 m^{-1}

For transition of the electron between two different energy levels, the spectral lines of different wavelengths are obtained. These spectral lines are found to fall into a number of spectral series as discussed below.

1. **Lyman series**

   For Lyman series, $n_f = 1$ and $n_i = 2, 3, 4, \ldots$

   $$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$$

   These spectral lines lie in ultraviolet region.

2. **Balmer series**

   For Balmer series, $n_f = 2$ and $n_i = 3, 4, 5, \ldots$

   $$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

   Where, $n_i = 3, 4, 5, \ldots$

   These spectral lines lie in the visible region.

3. **Paschan series**

   For Paschan series, $n_f = 3$ and $n_i = 4, 5, 6, \ldots$

   $$\therefore \frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$$

   These spectral lines lie in the IR – region.

4. **Brackett series**

   For Brackett series, $n_f = 4$ and $n_i = 5, 6, 7, \ldots$

   $$\frac{1}{\lambda} = R_H\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$$

   The spectral lines of this series lie in IR – region.

5. **Pfund series**

For Pfund series, $n_f = 5$ and $n_i = 6, 7, 8, \ldots$

$$\frac{1}{\lambda} = R_H\left(\frac{1}{5^2} - \frac{1}{n_i^2}\right)$$

These series lie in the far infrared region.

Bohr's Model of Hydrogen Atom

Bohr suggested a new model for the atom as Rutherford's atom model was unstable. He introduced the concept of stationary orbits.

Postulates of Bohr's Atom Model

- In a hydrogen atom, the negatively charged electron revolves in a circular orbit around the heavy positively charged nucleus. These are the stationary (orbits) states of the atom.
- The electrons revolve around the nucleus only in those orbits for which the angular momentum is the integral multiple of

  $$\frac{h}{2\pi}$$
  $$L = \frac{nh}{2\pi}$$

- Electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final state.

  $h\nu = E_i - E_f$

  $E_i > E_f,$

  Angular momentum is given by,

  $L = mvr$

  According to Bohr's 2^{nd} postulate,

  $$L_n = mv_n r_n = \frac{nh}{2\pi}$$

  $n \rightarrow$ Principle quantum

$v_n \rightarrow$ Speed of moving electron in the n^{th} orbit

$r_n \rightarrow$ Radius of n^{th}orbit

$$v_n = \frac{e}{\sqrt{4\pi \epsilon_0 m r_n}}$$

$$\therefore v_n = \frac{1}{n} \frac{e^2}{4\pi \epsilon_0} \frac{1}{\left(\frac{h}{2\pi}\right)}$$

$$\therefore r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi \epsilon_0}{e^2}$$

For $n = 1$ (innermost orbit),

$$r_1 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

This is called Bohr radius, represented by the symbol a_0.

Total energy,

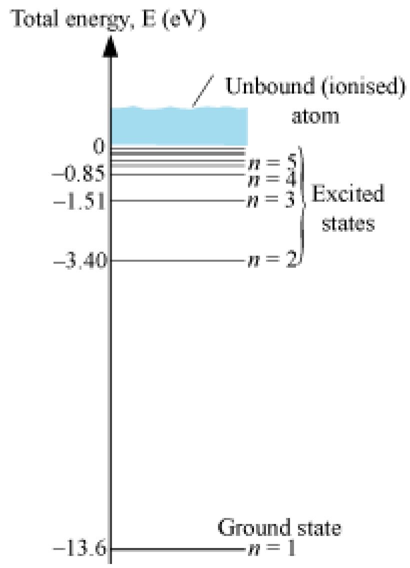
$$E_n = -\frac{e^2}{8\pi e_0}\left(\frac{m}{n^2}\right)\left(\frac{2\pi}{h}\right)^2\left(\frac{e^2}{4\pi \epsilon_0}\right)$$

$$\Rightarrow E_n = -\frac{me^4}{8\pi^2 \epsilon_0^2 h^2}$$

$$\therefore E_n = -\frac{2.18\times10^{-18}}{n^2} \text{J}$$

$$E_n = -\frac{13.6}{n^2} \text{eV}$$

Energy level diagram for a hydrogen atom:



Line Spectra of Hydrogen Atom

When an electron in a hydrogen atom jumps from the higher level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength.

$$h\nu = E_i - E_f$$

$$h\nu = -\left(\frac{1}{4\pi \epsilon_0}\right)^2 \frac{2\pi^2 me^4}{n_i^2 h^2} - \left[-\left(\frac{1}{4\pi \epsilon_0}\right)^2 \frac{2\pi^2 me^4}{n_f^2 h^2}\right]$$

$$\nu = \left(\frac{1}{4\pi \epsilon_0}\right)^2 \frac{2\pi^2 me^4}{h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$c = \nu\lambda$$

$$\therefore \frac{1}{\lambda} = \left(\frac{1}{4\pi \epsilon_0}\right)^2 \frac{2\pi^2 me^4}{ch^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\therefore \frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Here, $R_H = \left(\frac{1}{4\pi \epsilon_0}\right)^2 \frac{2\pi^2 me^4}{ch^3}$

It is called Rydberg's constant and its value is $1.09678 \times 10^7\ m^{-1}$.

The different spectral series are as follows:

- Lyman series:

  $$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$$, where n_i = 2, 3, 4 …

  They lie in the ultraviolet region.

- Balmer series

  $$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$, where n_i = 3, 4, 5 …

  It lies in the visible region.

- Paschen series

  $$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$$, where n_i = 4, 5, 6 …

  It lies in the infra-red region.

- Brackett series

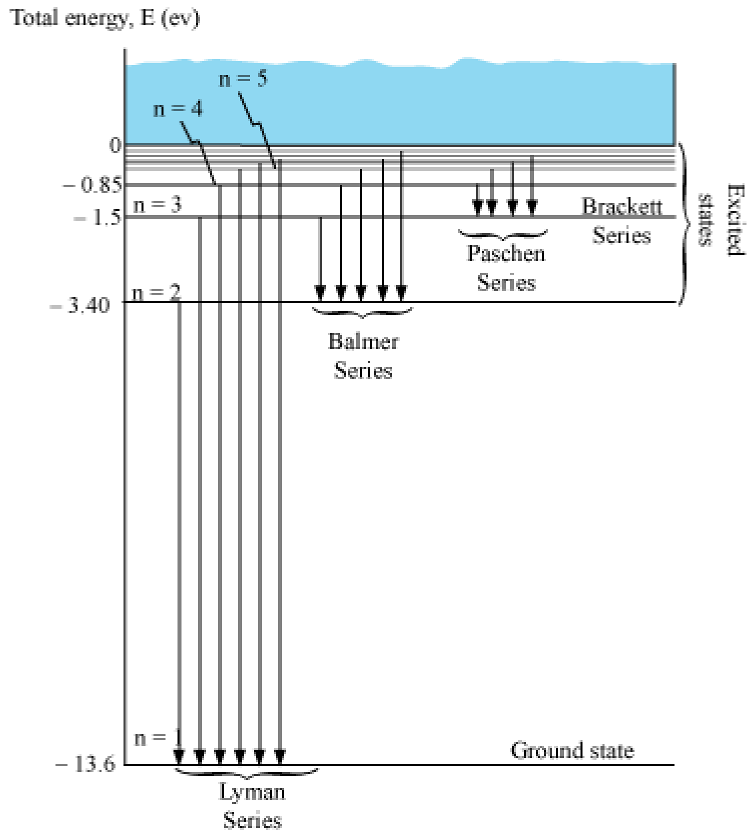
  $$\frac{1}{\lambda} = R_H\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$$, where n_i = 5, 6, 7 …

  It lies in the infra-red region.

- Pfund series

  $$\frac{1}{\lambda} = R_H\left(\frac{1}{5^2} - \frac{1}{n_i^2}\right)$$, where n_i = 6, 7, 8 …

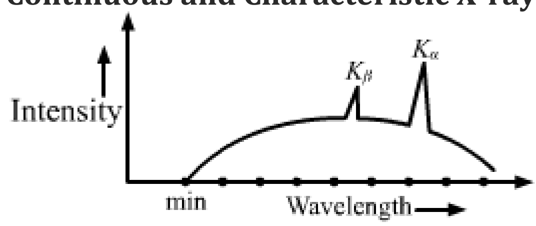
  It lies in the far infra-red region.



de Broglie's Explanation of Bohr's Second Postulate of Quantisation

- de Broglie's hypothesis, which shows that wavelength of an electron is $\lambda = h/mv$, gave an explanation for Bohr's quantised orbits by bringing in the wave particle duality.
- Orbits correspond to circular standing waves in which the circumferences of the orbits are equal to whole numbers of wavelengths.
- Bohr's model is applicable only to hydrogenic (single electron) atoms and cannot be extended to even two-electron atoms.

Continuous and Characteristic X-rays:



X-rays are divided into two categories:

- Characteristic X-rays:

- The intensity of the X-rays is very high at certain sharply defined frequencies. These X-rays are known as characteristic X-rays.
- The graph shows K_α and K_β wavelengths for which the intensity of the X-rays is very large.
- The wavelengths for characteristic X-rays may be used to identify the element from which they originate, since wavelengths have definite values for a particular material.
- Continuous X-rays
- It can be seen from the graph that at all the wavelengths other than those corresponding to K_α and $K_{\beta,}$ the intensity varies gradually. These X-rays are known as continuous X-rays.
- The origin of continuos X-rays and cutoff wavelength can be explained by using the relation $\lambda = hc/E = -hc/eV$. Thus, the wavelength depends upon the accelerating voltage V, and not on the material of the target.