CBSE Class 11 Mathematics

Important Questions

Chapter 8

Binomial Theorem

1 Marks Questions

1. What is The middle term in the expansion of $(1+x)^{2n+1}$

Ans. Since $(2n+1)$ is odd there is two middle term

$i.e\ {}^{2n+1}C_{n}\, x^{n+1}$ and ${}^{2n+1}C_{n+1}\, x^{n}$

2. When n **is a positive integer, the no. of terms in the expansion of** $(x+a)^{n}$ **is**

Ans. The no. of terms in the expansion of $(x+a)^{n}$ is one more than the index $n.\quad i.e\quad (n+1).$

3. Write the general term $(x^{2}-y)^{6}$

Ans. $T^{r+1} = {}^{6}C_{r}\left(x^{2}\right)^{6-r}.(-y)^{r}$

$= {}^{6}C_{r}(x)^{12-2r}.(-1)^{r}.(y)^{r}$

4. In the expansion of $\left(x+\frac{1}{x}\right)^{6}$, **find the 3**$^{\text{rd}}$ **term from the end**

Ans. 3$^{\text{rd}}$ term form end $=(6-3+2)^{th}$ term from beginning i.e $T_5 = {}^{6}C_{4}(x)^{6-4}.\left(\frac{1}{x}\right)^{4}$

$= {}^{6}C_{4}x^{2}.x^{-4}$

$=15^{x-2}$

$=\frac{15}{x^{2}}$

5. Expand $(1+x)^{n}$

Ans. $(1+x)^{n} =1+{}^{n}C_{1}(x)^{1}+{}^{n}C_{2}(x)^{2}+{}^{n}C_{3}(x)^{3}+......x^{n}$

6. The middle term in the expansion of $(1+x)^{2n}$ **is**

Ans. ${}^{2n}C_{n}.x^{n}$

7. Find the no. of terms in the expansions of $(1-2x+x^{2})^{7}$

Ans. $(1-2x+x^{2})^{7}$

$=(x^{2}-2x+1)^{7}$

$=\left[(x-1)^{2}\right]^{7}$

$=(x-1)^{14}$

No. of term is 15

8. Find the coeff of x^{5} **in** $(x+3)^{9}$

Ans. $T_{r+1} = {}^{9}C_{r}(x)^{9-r}.(3)^{r}$

Put $9 - r = 5$

$r = 4$

$T_5 = {}^{9}C_{4}(x)^{5}.(3)^{4}$

Coeff of x^5 is ${}^{9}C_{4}(3)^{4}$

9. Find the term independent of x $\left(x + \frac{1}{x}\right)^{10}$

Ans. $T_{r+1} = {}^{10}C_{r}(x)^{10-r}.\left(\frac{1}{x}\right)^{r}$

$= {}^{10}C_{r}(x)^{10-r}.(x)^{-r}$

$= {}^{10}C_{r}(x)^{10-2r}$

Put $10 - 2r = 0$

$r = 5$

Independent term is ${}^{10}C_{5}$

10. Expand $(a+b)^{n}$

Ans. $(a+b)^{n} = {}^{n}C_{0}a^{n} + {}^{n}C_{1}a^{n-1}b + {}^{n}C_{2}a^{n-2}b^{2} + \ldots\ldots + {}^{n}C_{n}b^{n}$

CBSE Class 12 Mathematics

Important Questions

Chapter 8

Binomial Theorem

4 Marks Questions

1. Which is larger $(1.01)^{10,00000}$ **or** $10,000$

Ans. $(1.01)^{10,00000} = (1+0.01)^{10,00000}$

$= {}^{10,00000}C_0 + {}^{10,00000}C_1(0.01) + \text{other positive term}$

$= 1 + 10,00000 \times 0.01 + \text{other positive term}$

$= 1 + 10,000$

$= 10,001$

Hence $(1.01)^{10,00000} > 10,000$

2. Prove that $\sum_{r=0}^{n} 3^r \; {}^nC_r = 4^n$

Ans.

$\sum_{r=0}^{n} 3^r \; {}^nC_r = \sum_{r=0}^{n} {}^nC_r . 3^r$

$= {}^nC_0 + {}^nC_1 .3 + {}^nC_2 .3^2 + ... + {}^nC_n 3^n$

$\left[\because (1+a)^n = 1 + {}^nC_1 .a + {}^nC_2 a^2 + {}^nC_3 a^3 + ... + a^n\right]$

$= (1+3)^n$

$= (4)^n$

$H.P$

3. Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.

Ans. Let $6^n = (1+5)^n$

$= 1 + {}^nC_1 5^1 + {}^nC_2 5^2 + {}^nC_3 5^3 + ... + 5^n$

$= 1 + 5n + 5^2 \left({}^nC_2 + {}^nC_3 .5 + ... + 5^{n-2} \right)$

$6^n - 5n = 1 + 25 \left({}^nc_2 + {}^nc_3 .5 + ... + 5^{n-2} \right)$

$= 1 + 25k \left[\text{where } k = {}^nc_2 + {}^nc_3 .5 + ... 5^{n-2} \right]$

$= 25k + 1$

$H.P$

4.Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}} \right)^{18}$, $x \neq 0$

Ans. The general term in the expansion of

$\left(9x - \frac{1}{3\sqrt{x}} \right)^{18}$ is

$T_{r+1} = {}^{18}C_r (9x)^{18-r} \left(-\frac{1}{3\sqrt{x}} \right)^r$

For 13th term, $r + 1 = 13$

$r = 12$

$= {}^{18}C_{12}(9x)^6\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$= {}^{18}C_{12}(3)^{12}.x^6\left(-\frac{1}{3}\right)^{12}.(x)^{-6}$

$= {}^{18}C_{12}(3)^{12}.(-1)^{12}.(3)^{-12}$

$= {}^{18}C_{12}$

$= 18564$

5. Find the term independent of x in the expansion of $\left(\sqrt[3]{x}+\frac{1}{2\sqrt[3]{x}}\right)^{18}, x > 0$

Ans.

$T_{r+1} = {}^{18}C_r\left(\sqrt[3]{x}\right)^{18-r}\left(\frac{1}{2\sqrt[3]{x}}\right)^r$

$= {}^{18}C_r(x)^{\frac{18-r}{3}}.\left(\frac{1}{2}\right)^r.x^{\frac{-r}{3}}$

$= {}^{18}C_r(x)^{\frac{18-r-r}{3}}.\left(\frac{1}{2}\right)^r$

For independent term $\frac{18-2r}{3} = 0$

$r = 9$

The req. term is ${}^{18}C_9\left(\frac{1}{2}\right)^9$

6. Find the coefficient of x^5 in the expansion of the product $(1+2x)^5(1-x)^7$

Ans.

$$(1+2x)^6(1-x)^7 = \left(1+{}^6C_1(2x)+{}^6C_2.(2x)^2+{}^6C_3(2x)^3+{}^6C_4(2x)^4+{}^6C_5(2x)^5+{}^6C_6(2x)^6\right)$$

$$\left(1-{}^7C_1x+{}^7C_2(x)^2-{}^7C_3(x)^3+{}^7C_4(x)^4-{}^7C_5(x)^5+{}^7C_6(x)^6-{}^7C_7(x)^7\right)$$

$$=\left(1+12x+60x^2+160x^3+240x^4+192x^5+64x^6\right).\left(1-7x+21x^2-35x^3+35x^4-21x^5+7x^6-x^7\right)$$

Coeff of x^5 is

$$=1\times(-21)+12\times35+60(-35)+160\times21+240\times(-7)+192\times1$$
$$=171$$

7. Compute $(98)^5$

Ans. $(98)^5=(100-2)^5$

$$={}^5C_0(100)^5-{}^5C_1(100)^4.2+{}^5C_2(100)^3.2^2$$

$$-{}^5C_3(100)^2.(2)^3+{}^5C_4(100)(2)^4-{}^5C_5(2)^5$$

$$=10000000000-5\times100000000\times2+10\times1000000\times4$$
$$-10\times10000\times8+5\times100\times16-32$$

$$=100\ 4\ 000\ 8000-1000\ 8000\ 32=9039207968$$

8. Expand $\left(x+\dfrac{1}{x}\right)^6$

Ans.

$$\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x^5)\left(\frac{1}{x}\right) + {}^6C_2(x^4)\left(\frac{1}{x}\right)^2 +$$

$${}^6C_3(x^3)\left(\frac{1}{x}\right)^3 + {}^6C_4(x^2)\left(\frac{1}{x}\right)^4 + {}^6C_5(x)^1\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6x^5\left(\frac{1}{x}\right) + 15x^4\left(\frac{1}{x^2}\right) + 20x^3.\left(\frac{1}{x^3}\right) + 15x^2.\frac{1}{x^4} + 6x.\frac{1}{x^5} + \frac{1}{x^6}$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

9. Find the fourth term from the end in the expansion of $\left(\frac{x}{x^2} - \frac{x^3}{3}\right)^9$.

Ans. Fourth term from the end would be equal to $(9-4+2)^{th}$ term from the beginning

$$T_7 = T_{6+1} = {}^9C_6\left(\frac{3}{x^2}\right)^{9-6}.\left(\frac{-x^3}{3}\right)^6$$

$$= {}^9C_6(3)^3.(x)^{-6}.(x)^{18}.(3)^{-6}$$

$$= \frac{9!}{6!3!}.(3)^{-3}.(x)^{12}$$

$$= \frac{28}{9}x^{12}$$

10. Find the middle term of $\left(2x - \frac{x^2}{4}\right)^9$.

Ans. $n = 9$ so there are two middle term i.e $\left(\frac{9+1}{2}\right)^{th}$ term and $\left(\frac{9+1}{2}+1\right)^{th}$ term

$T_5 = T_{4+1} = {}^9C_4(2x)^{9-4}.\left(\frac{-x^2}{4}\right)^4$

$= \frac{63}{4}x^{13}$

$T_6 = T_{5+1} = {}^9C_5(2x)^{9-5}\left(\frac{-x^2}{4}\right)^5$

$= -{}^9C_4(2)^4.x^4\ \frac{(x)^{10}}{(4)^5}$

$= \frac{-63}{32}\ x^{14}$

11. Find the coefficient of a^5b^7 in $(a-2b)^{12}$.

Ans.

$T_{r+1} = {}^{12}C_r(a)^{12-r}.(-2b)^r$

Put $12-r=5$

$r=7$

$T_8 = {}^{12}C_7(a)^5.(-2b)^7$

$= {}^{12}C_7(a)^5.(-2)^7.b^7$ s

coeff. Of a^5 b^7 is ${}^{12}C_7\ (-2)^7$

12. Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6.

Ans. $T_{r+1} = {}^{m}C_{r}(1)^{m-r}.(x)^r$

$= {}^{m}C_{r}(x)^r$

Put $r = 2$

ATQ ${}^{m}C_{2} = 6$

$$\frac{m!}{2!(m-2)!} = \frac{6}{1}$$

$$\frac{m(m-1)\cancel{(m-2)!}}{2\times1\times\cancel{(m-2)!}} = \frac{6}{1}$$

$m^2 - m = 12$

$m^2 - m - 12 = 0$

$m(m-4) = 3(m-4) = 0$

$(m-4)(m-3) = 0$

$m = 4$

$m = -3(\text{neglect})$

13. Show that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$.

Ans. As $2n$ is even so the expansion $(1+x)^{2n}$ has only one middle term which is

$\left(\frac{2n}{2}+1\right)^{th}$ term i.e $(n+1)^{th}$ term

$T_{r+1} = {}^{2n}C_r \; (1)^{2n-r} \, .(x)^r$

Coeff. of x^n is ${}^{2n}C_r$

And $(2n-1)$ is odd so two middle term

$\left(\frac{2n-1+1}{2}\right)^{th}$ and $\left(\frac{2n-1+1}{2}+1\right)^{th}$

i.e n^{th} and $(n+1)^{th}$ term

The coefficients of these terms are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$

Now ATQ

$${}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n \quad \left[\because {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r\right]$$

$H.P$

14. Find a if the coeff. of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal

Ans. $T_{r+1} = {}^9C_r\,(3)^{9-r} \, .(ax)^r$

ATQ

$${}^9C_2(3)^7 \, a^2 = {}^9C_3(3)^6 \, . \, a^3$$

$${}^9C_2(3)^1 = {}^9C_3 \, a$$

$$\frac{9!}{2!7!}\times 3=\frac{9!}{3!6!}a$$

$$\frac{3!6!\times 3}{2!7!}=a$$

$$\frac{3\times 2\times 1\times 6!\times 3}{2\times 1\times 7\times 6!}=a$$

$$\frac{9}{7}=a$$

15. Find $(a+b)^4-(a-b)^4$. **Hence evaluate** $\left(\sqrt{3}+\sqrt{2}\right)^4+\left(\sqrt{3}-\sqrt{2}\right)^4$.

Ans. $(a+b)^4-(a-b)^4=\left({}^4C_0a^4+{}^4C_1a^3b+{}^4C_2a^2b^2+{}^4C_3ab^3+{}^4C_4b^4\right)$

$$-\left(a^4-{}^4C_1a^3b+{}^4C_2a^2b^2-{}^4C_3ab^2+{}^4C_4b^4\right)$$

$$=2\left({}^4C_1a^3b+{}^4C_3ab^3\right)$$

$$=2\left(4a^3b+4ab^3\right)$$

$$=8ab\left(a^2+b^2\right)$$

Put $a=\sqrt{3}$, $\quad b=\sqrt{2}$

$$\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4=8\sqrt{3}.\sqrt{2}\,(3+2)$$

$$=40\sqrt{6}$$

16. Show that $9^{n+1}-8n-9$ **is divisible by 64, whenever n is positive integer.**

Ans. $(9)^{n+1}=(1+8)^{n+1}$

$$= 1 + {}^{n+1}C_1 8^1 + {}^{n+1}C_2 .8^2 + {}^{n+1}C_3 8^3 + ... + {}^{n+1}C_{n+1} 8^{n+1}$$

$$= 1 + (n+1).8 + 8^2 \left[{}^{n+1}C_2 + {}^{n+1}C_3 .8 + ... + 8^{n-1} \right]$$

$$9^{n+1} - 8n - 9 = 64\left[{}^{n+1}C_2 + {}^{n+1}C_3 . + ... + 8^{n-1} \right]$$

$$9^{n+1} - 8n - 9 = 64k, \text{ where } k = \left[{}^{n+1}C_2 + {}^{n+1}C_3 .8 + ... + 8^{n+1} \right]$$

17. Find the general term in the expansion of $\left(x^2 - yx\right)^{12}$

Ans. $T_{r+1} = {}^{12}C_r \left(x^2\right)^{12-r} .(-yx)^r$

$$= {}^{12}C_r (x)^{24-r} .(-1)^r .y^r .x^r$$

$$= {}^{12}C_r (-1)^r .y^r .(x)^{24-2r}$$

18. In the expansion of $(1+a)^{m+n}$, **prove that coefficients of** a^m **and** a^n **are equal.**

Ans. $T_{r+1} = {}^{m+n}C_r (1)^{m+n-r} .(a)^r$

$$T_{r+1} = {}^{m+n}C_r (a)^r ....(i)$$

Put $r = m$ and $r = n$ respectively

$$T_{m+1} = {}^{m+n}C_m a^m$$

Coeff of a^m is ${}^{m+n}C_m \Rightarrow \dfrac{(m+n)!}{m!n!}$

Coeff of a^n is ${}^{m+n}C_m \Rightarrow \frac{(m+n)!}{n!m!}$ H.P

19. Expand $\left(1-x+x^2\right)^4$

Ans. $\left(1-x+x^2\right)^4=\left[(1-x)+x^2\right]^4$

$= {}^4C_0(1-x)^4+{}^4C_1(1-x)^3.\left(x^2\right)+{}^4C_2(1-x)^2.\left(x^2\right)^2+{}^4C_3(1-x)^1\left(x^2\right)^3+{}^4C_4\left(x^2\right)^4$

$=(1-x)^4+4(1-x)^3.x^2+6(1-x)^2.x^4+4(1-x).x^6+1.x^8$

$=\left(1-4x+6x^2-4x^3+x^4\right)+4\left(1-3x+3x^2-x^3\right)x^2+6\left(1-2x+x^2\right)\left(x^4\right)+4(1-x).x^6+x^8$

$=1-4x+10x^2-16x^3+19x^4-16x^5+10x^6-4x^7+x^8$

20. Find the sixth term of the expansion $\left(y^{\frac{1}{2}}+x^{\frac{1}{3}}\right)^n$, **if the binomial coefficient of the third term from the end is 45.**

Ans. The binomial coeff of the third term from end = binomial coeff of the third term from beginning = ${}^nC_2$

$${}^nC_2=45$$

$$\frac{n(n-1)}{1.2}=45$$

$$n^2-n-90=0$$
$$n=10$$

$$T_{r+1}={}^{10}C_r\left(y^{\frac{1}{2}}\right)^{10-r}.\left(x^{\frac{1}{3}}\right)^r$$

$$r=5$$

$$T_6 = {}^{10}C_5 \left(y^{\frac{1}{2}} \right)^5 . \left(x^{\frac{1}{3}} \right)^5$$

$$= 252 y^{\frac{5}{2}} . x^{\frac{5}{3}}$$

21. Find a if the 17th and 18th terms of the expansion $(2+a)^{50}$ are equal.

Ans. $T_{r+1} = {}^{50}C_r (2)^{50-r} . (a)^r$

ATQ put $r = 16$ and 17

$$\Rightarrow {}^{50}C_{16} (2)^{34} . a^{16} = {}^{50}C_{17} (2)^{33} . a^{17}$$

$$a = \frac{{}^{50}C_{16} \times 2}{{}^{50}C_{17}}$$

$$a = 1$$

22. Find the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^6$

Ans. $T_{r+1} = {}^6C_r \left(\frac{3}{2}x^2 \right)^{6-r} . \left(\frac{-1}{3x} \right)^r$

$$= {}^6C_r \left(\frac{3}{2} \right)^{6-r} . (x)^{12-2r} . \left(\frac{-1}{3} \right)^r x^{-r}$$

$$= {}^6C_r \left(\frac{3}{2} \right)^{6-r} . \left(\frac{-1}{3} \right)^r . (x)^{12-3r}$$

Put $12 - 3r = 0$

$r = 4$

$= {}^6C_4\left(\frac{3}{2}\right)^2 \cdot \left(\frac{-1}{3}\right)^4 \cdot$

$= \frac{5}{12}$

23. If the coeff of $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal find r

Ans. $T_{r+1} = {}^{34}C_r(1)^{34-r} \cdot (x)^r$

$T_{r+1} = {}^{34}C_r(x)^r \text{.......}(i)$

Coeff are

${}^{34}C_{r-6}$ and ${}^{34}C_{2r-2}$

ATQ ${}^{34}C_{r-6} = {}^{34}C_{2r-2}$

$r - 6 = 2r - 2$
$r = -4 \ (\text{neglect})$

$$r - 6 = 34 - (2r - 2) \ \left[\begin{array}{l} \because {}^nC_r = {}^nC_p \\ r = p \ or \ n = r + p \end{array}\right]$$

$r = 14$

24. Show that the coeff of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coeff of two middle terms in the expansion of $(1+x)^{2n-1}$

Ans. As $2n$ is even so the expansion $(1+x)^{2n}$ has only one middle term which is

$\left(\frac{2n}{2}+1\right)^{th}$ $i.e\left(n+1\right)^{th}$ term

Coeff of x^n is ${}^{2n}C_n$

Similarly $(2n-1)$ being odd the other expansion has two middle term i.e

$\left(\frac{2n-1+1}{2}\right)^{th}$ and $\left(\frac{2n-1+1}{2}+1\right)^{th}$ term

i.e n^{th} and $(n+1)^{th}$

The coeff are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$

$${}^{2n-1}C_{n-1}+{}^{2n-1}C_n={}^{2n}C_n\left[\because {}^nC_{r-1}+{}^nC_r={}^{n+1}C_r\right]$$

25. Find the value of r, if the coeff of $(2r+4)^{th}$ and $(r-2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal.

Ans. $T_{r+1}={}^{18}C_r(1)^{18-r}.(x)^r$

$T_{r+1}={}^{18}C_r x^r$

Put $r=r-3$

And $2r+3$

ATQ ${}^{18}C_{2r+3}={}^{18}C_{r-3}$

$18=2r+\cancel{3}+r-\cancel{3}$

$r=6$

26. Find the 13th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$

Ans. $T_{r+1} = {}^{18}C_{r}(9x)^{18-r}.\left(\frac{-1}{3\sqrt{x}}\right)^{r}$

Put $r = 12$

$T_{13} = {}^{18}C_{12}(9x)^{18-12}.\left(\frac{-1}{3}\right)^{12}.(x)^{\frac{-12}{2}}$

$= {}^{18}C_{12}(3)^{12}.x^{6}.(3)^{-12}.(-1)^{12}.x^{-6}$

$= {}^{18}C_{12}$

CBSE Class 12 Mathematics

Important Questions

Chapter 8

Binomial Theorem

6 Marks Questions

1.Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$

Ans. Fifth term from the beginning in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is

$$T_{4+1}={}^nC_4\left(\sqrt[4]{2}\right)^{n-4}.\left(\frac{1}{\sqrt[4]{3}}\right)^4$$

$$T_5={}^nC_4(2)^{\frac{n-4}{4}}.(3)^{-1}\ldots\ldots(i)$$

How fifth term from the end would be equal to $(n-5+2)$ in term from the beginning

$$T_{(n-4)+1}={}^nC_{n-4}\left(\sqrt[4]{2}\right)^{n-(n-4)}.\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$$

$$={}^nC_{n-4}(2)^1(3)^{\frac{n-4}{4}}\ldots\ldots..(ii)$$

ATQ $\dfrac{{}^nC_4.(2)^{\frac{n-4}{4}}(3)^{-1}}{{}^nC_{n-4}(2)^1(3)^{\frac{n-4}{-4}}}=\dfrac{\sqrt{6}}{1}$

$$\frac{(2)^{\frac{n-8}{4}}}{(3)^{\frac{-(n-8)}{4}}} = (6)^{\frac{1}{2}}$$

$$(6)^{\frac{n-8}{4}} = (6)^{\frac{1}{2}}$$

$$\frac{n-8}{4} = \frac{1}{2}$$

$$\Rightarrow 2n-16=4$$

$$n=10$$

2.The coefficients of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio 1:7:42. Find n

Ans. Let three consecutive terms in the expansion of $(1+a)^n$ are $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ term

$$T_{r+1} = {}^nC_r (1)^{n-r} . (a)^r$$

$$T_{r+1} = {}^nC_r \ (a)^r$$

Coefficients are

${}^nC_{r-2}$, ${}^nC_{r-1}$ and ${}^nC_r$ respectively

ATQ $\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{7}$

$$\Rightarrow n-8r+9=0 ....(i)$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{7}{42}$$

$$\Rightarrow n - 7r + 1 = 0......(ii)$$

On solving eq. (i) and (ii) we get $n = 55$

3. The second, third and fourth terms in the binomial expansion $(x+a)^n$ are 240, 720 and 1080 respectively. Find x, a and n.

Ans. $T_2 = 240$

$${}^nC_1 x^{n-1}.a = 240........(i)$$

$${}^nC_2 x^{n-2}.a^2 = 720.......(ii)$$

$${}^nC_3 x^{n-3}.a^3 = 1080........(iii)$$

Divide (ii) by (i) and (iii) by (ii)

We get

$\frac{a}{x} = \frac{6}{n-1}$ and $\frac{a}{x} = \frac{9}{2(n-2)}$

$$\Rightarrow n = 5$$

On solving we get

$$x = 2$$
$$a = 3$$

4.If a and b are distinct integers, prove that a-b is a factor of $a^n - b^n$, whenever n is positive.

Ans. Let $a^n = (a-b+b)^n$

$a^n = (b+a-b)^n$

$= {}^nC_0 b^n + {}^nC_1 b^{n-1}(a-b) + {}^nC_2 b^{n-2}.(a-b)^2 + {}^nC_3 b^{n-3}.(a-b)^3 + ... + {}^nC_n (a-b)^n$

$a^n = b^n + (a-b)\left[{}^nC_0 b^n + {}^nC_1 b^{n-1}(a-b) + {}^nC_2 b^{n-2}.(a-b)^2 + {}^nC_3 b^{n-3}.(a-b)^3 + ... + {}^nC_n (a-b)^n\right]$

$a^n - b^n = (a-b)k$

Where

${}^nC_1 b^{n-1} + {}^nC_2 b^{n-2}(a-b) + ... + (a-b)^{n-1} = k$

$H.P$

5. The sum of the coeff. 0f the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^m$ m being natural no. is 559. Find the term of expansion containing x^3

Ans. The coeff. Of the first three terms of $\left(x - \frac{3}{x^2}\right)^m$ are ${}^mC_0, (-3)\,{}^mC_1$ and $9\ \ {}^mC_2$.

Therefore, by the given condition

${}^mC_0 - 3\,{}^mC_1 + 9\,{}^mC_2 = 559$

$1 - 3m + \dfrac{9m(m-1)}{2} = 559$

On solving we get $m = 12$

$T_{r+1} = {}^{12}C_r (x)^{12-r}\left(\dfrac{-3}{x^2}\right)^r$

$= {}^{12}C_r (x)^{12-r} . (-3)^r . (x)^{-2r}$

$= {}^{12}C_r (x)^{12-3r} . (-3)^r$

$12 - 3r = 3 \Rightarrow r = 3,\ req.$ term is $-5940\, x^3$

6.Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.....(2n-1)}{n!}2^n.x^n$.

Ans. As $2n$ is even, the middle term of the expansion $(1+x)^{2n}$ is $(n+1)^{th}$ term

$T_{n+1} = {}^{2n}C_n (1)^{2n-n} .x^n$

$= {}^{2n}C_n x^n$

$= \frac{(2n)!}{n!n!}.x^n$

$= \frac{(2n)(2n-1)(2n-2)......4.3.2.1}{n!n!}x^n$

$= \frac{1.2.3.4......(2n-2)(2n-1)(2n)}{n!n!}x^n$

$= \frac{[1.3.5.....(2n-1)][2.4.6.....(2n)]}{n!n!}x^n$

$= \frac{[1.3.5....(2n-1).2^n.(1.2.3.....n)]}{n!n!}x^n$

$= \frac{[1.3.5....(2n-1)].2^n.\cancel{n!}}{\cancel{n!}n!}.x^n$

$$= \frac{[1.3.5...(2n-1)].2^n.x^n}{n!}$$

7. In the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ **, the ratio of 7th term from the beginning to the 7th term the end is 1:6 find** n

Ans. $T_7 = {}^nC_6\left(\sqrt[3]{2}\right)^{n-6}.\left(\frac{1}{\sqrt[3]{3}}\right)^6$

$$= {}^nC_6(2)^{\frac{n-6}{3}}.(3)^{-2} ....(i)$$

7th term from end $=(n-7+2)$ term from beginning

$$T_{n-6+1} = {}^nC_{n-6}\left(\sqrt[3]{2}\right)^{n-n+6}.\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}$$

$$= {}^nC_{n-6}(2)^2.(3)^{\frac{n-6}{-3}} ......(ii)$$

ATQ

$$\frac{{}^nC_6(2)^{\frac{n-6}{2}}.(3)^{-2}}{{}^nC_{n-6}(2)^2(3)^{\frac{6-n}{3}}} = \frac{1}{6}$$

$$\frac{(2)^{\frac{n-12}{3}}}{(3)^{\frac{12-n}{3}}} = \frac{1}{6}$$

$$(6)^{\frac{n-12}{3}} = (6)^{-1}$$

$$\frac{n-12}{3} = \frac{-1}{1}$$

$n-12=-3$

$n=9$

8.If the coeff. Of 5th 6th and 7th terms in the expansion of $(1+x)^n$ are in A.P, then find the value of n.

Ans. $T_{r+1}={}^nC_r(1)^{n-r}.(x)^r$

$T_{r+1}={}^nC_r x^r$ ....(i)

Coeff of 5th , 6th, 7th terms in the expansion of $(1+x)^n$ are ${}^nC_4$, ${}^nC_5$, and ${}^nC_6$

ATQ $2.{}^nC_5={}^nC_4+{}^nC_6$

$$2.\frac{n!}{5!(n-5)!}=\frac{n!}{4!(n-4)!}+\frac{n!}{6!(n-6)!}$$

$n=7,14$

9. If P be the sum of odd terms and Q that of even terms in the expansion of $(x+a)^n$ prove that

$(i)\quad P^2-Q^2=\left(x^2-a^2\right)^n$

$(ii)\quad 4PQ=(x+a)^{2n}-(x-a)^{2n}$

$(iii)\quad 2\left(P^2+Q^2\right)=\left[(x+a)^{2n}+(x-a)^{2n}\right]$

Ans. $(x+a)^n={}^nC_0x^n+{}^nC_1x^{n-1}a+{}^nC_2x^{n-2}a^2+.....+{}^nC_na^n$

$=t_1+t_2+t_3+....+t_n+t_{n+1}$

$= (t_1 + t_3 + t_5 + ...) + (t_2 + t_4 + t_6 + ...)$

$= P + Q ....(i)$

$(x-a)^n = (t_1 - t_2 + t_3 - t_4 + ...)$

$= (t_1 + t_3 + t_5) - (t_2 + t_4 + t_6 .....)$

$= P - Q ....(ii)$

$(i) \times (ii)$

$P^2 - Q^2 = (x^2 - a^2)^n$

Sq. (i) and (ii) and subt.

$\left[(x+a)^{2n} - (x-a)^{2n}\right] = 4PQ$

Sq. and adding we get

$\left[(x+a)^{2n} + (x-a)^{2n}\right] = 2(P^2 + Q^2)$

10.If three successive coeff. In the expansion of $(1+x)^n$ are 220,495 and 792 then find n

Ans. Let coeff are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

ATQ ${}^nC_{r-1} = 220......(i)$

${}^nC_r = 495......(ii)$

${}^nC_{r+1} = 792........(iii)$

Dividing (ii) by (i)

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{495}{220}$$

$$\frac{n-r+1}{r} = \frac{9}{4}$$

$$4n - 13r + 4 = 0 ......(iv)$$

Dividing (iii) by (ii)

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{792}{495}$$

$$\frac{n-r}{r+1} = \frac{8}{5}$$

$$5n - 13r - 8 = 0 ......(v)$$

On solving (iv) and (v) we get $n = 12$