16.COMPLEX NUMBER

1. The complex number system

$z = a + ib$, then $a - ib$ is called congugate of z and is denoted by $\bar{z}$.

2. Equality In Complex Number:

$z_1 = z_2 \Rightarrow Re(z_1) = Re(z_2)$ and $I_m(z_1) = I_m(z_2)$.

3. Representation Of A Complex Number:

4. Properties of arguments

(i) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2m\pi$ for some integer m.

(ii) $\arg(z_1/z_2) = \arg(z_1) - \arg(z_2) + 2m\pi$ for some integer m.

(iii) $\arg(z^2) = 2\arg(z) + 2m\pi$ for some integer m.

(iv) $\arg(z) = 0 \Leftrightarrow$ z is a positive real number

(v) $\arg(z) = \pm \pi/2 \Leftrightarrow$ z is purely imaginary and $z \neq 0$

5. Properties of conjugate

(i) $|z| = |\bar{z}|$ (ii) $z\bar{z} = |z|^2$ (iii) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(iv) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$ (v) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$ (vi) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$ $(z_2 \neq 0)$

(vii) $|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)} = |z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + \bar{z}_1 z_2$

(viii) $\overline{(\bar{z}_1)} = z$ (ix) If $w = f(z)$, then $\bar{w} = f(\bar{z})$

(x) $\arg(z) + \arg(\bar{z})$

6. Rotation theorem

If $P(z_1)$, $Q(z_2)$ and $R(z_3)$ are three complex numbers and $\angle PQR = \theta$, then $\left(\frac{z_3 - z_2}{z_1 - z_2}\right) = \left|\frac{z_3 - z_2}{z_1 - z_2}\right| e^{i\theta}$

7. Demoivre's Theorem :

Case I : If n is any integer then

(i) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(ii) $(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_3)\ldots\ldots(\cos\theta_n + i\sin\theta_n)$
$= \cos(\theta_1 + \theta_2 + \theta_3 + \ldots\ldots\ldots\theta_n) + i\sin(\theta_1 + \theta_2 + \theta_3 + \ldots\ldots\ldots + \theta_n)$

Case II : If $p, q \in Z$ and $q \neq 0$ then $(\cos\theta + i\sin\theta)^{p/q} = \cos\left(\frac{2k\pi + p\theta}{q}\right) + i\sin\left(\frac{2k\pi + p\theta}{q}\right)$

where $k = 0, 1, 2, 3, \ldots\ldots, q - 1$

8. Cube Root Of Unity :

(i) The cube roots of unity are 1, $\frac{-1 + i\sqrt{3}}{2}$, $\frac{-1 - i\sqrt{3}}{2}$.

(ii) If ω is one of the imaginary cube roots of unity then $1 + \omega + \omega^2 = 0$. In general $1 + \omega^r + \omega^{2r} = 0$; where $r \in I$ but is not the multiple of 3.

9. Logarithm Of A Complex Quantity :

(i) $\text{Log}_e(\alpha + i\beta) = \frac{1}{2}\text{Log}_e(\alpha^2 + \beta^2) + i\left(2n\pi + \tan^{-1}\frac{\beta}{\alpha}\right)$ where $n \in I$.