CHAPTER 12 Factorisation

Learning Objectives

In this chapter, you will learn:

- *About factors of algebraic expressions.*
- *How to make factors.*
- *About different methods of factorisation.*

12.1 Introduction

In earlier classes, we have learnt about factors of composite numbers, let us consider a natural number, (say)

$42 = 1\times42$ Here 1 and 42 are factors of 42.

Or $42 = 1\times2\times21$ Here 1, 2 and 21 are factors of 42.

Or $42 = 1\times2\times3\times7$ Here 1, 2, 3 & 7 are factors of 42.

As we know that 1 is neither composite nor prime number.

Therefore 2, 3, 7 are prime factors of 42. Similarly the prime factors of 30 are 2, 3, & 5 and the prime factors of 70 are 2, 5, & 7.

Similarly algebraic expressions can also be expressed as the product of their factors.

12.2 Factors of Algebraic Expression:

In Class VII, we have learnt that terms of algebraic expressions are formed as product of factors. For example, in algebraic expressions 3xy, the term 3xy is formed by factors 3, x and y i.e. $3xy = 3\times x\times y$ here 3, x and y are factors of 3xy.

We observe that these factors 3x & y of 3xy which further cannot be expressed as a product of factors, called '**Irreducible factors**'.

Note that $3\times(xy)$ is not irreducible form of 3xy. Since the factor xy can be further expressed as a product of x and y i.e. $xy = x\times y$

> Since 3xy can also be written as $1\times3\times x\times y$. Note 1 is a factor of 3xy. Infact 1 is a factor of every term. But we do not show 1 as a separate factor of any term, unless it is specially required.

Let us consider the expression $5x(x+3)$.

It can be written as a product of factors 5, x and (x+3) i.e. $5x(x+3) = 5\times x\times(x+3)$

Similarly Irreducible factors of $12x(y+3)(z+5)$ are 2, 2, 3, x, (y+3) and (z+5)

Also, the product of 2x+3 and $2x-3 = (2x+3)(2x-3) = 4x^2-9$

We can say that 2x+3 and 2x–3 are factors of $4x^2-9$. we can write it as $4x^2-9 = (2x+3)(2x-3)$

12.3 What is Factorisation

As we have learnt, when an algebraic expression can be written as product of two or more expressions then each of these expression is called a factor of the given expression. To find factors of a given expression means to obtain two or more expressions whose product is the given expression.

The process of writing an algebraic expression as the product of two or more algebraic expressions is called factorisation. Thus, factorisation is the reverse process of multiplication.

For Example

Product of 5xy and (3xy–7) is $5xy(3xy-7) = 15x^2y^2 - 35xy$ and factors of $15x^2y^2 - 35xy$ are 5xy and (3xy–7)

Similarly product of (2a+3b) and (2a–3b) is $(2a+3b)(2a-3b) = 4a^2 - 9b^2$ and factors of $4a^2 - 9b^2$ are 2a + 3b and 2a – 3b.

Now we shall learn the factorisation of expressions by different methods.

12.3.1 Method of finding Common Factors

In this section, we shall learn about how to take out the common factor (s) and use the Distributive property

$$ab \pm ac = a \times b \pm a \times c$$
$$= a \times (b \pm c)$$
$$= a(b \pm c)$$

Example 12.1. Factorise : 2x+6

Solution- We shall write each term as a product of irreducible factors.

Here $2x = 2 \times x$ and $6 = 2 \times 3$

Hence $2x + 6 = 2 \times x + 2 \times 3$

Observe that both terms have '2' as common factor.

$\therefore 2x + 6 = 2 \times (x + 3) = 2(x + 3)$

Example 12.2. Factorise : $7a^2 + 14a$

Solution:- Here $7a^2 = 7 \times a \times a$ and $14a = 2 \times 7 \times a$

Observe that both terms have '7' and 'a' as common factors

Hence $7a^2 + 14a = 7 \times a \times a + 2 \times 7 \times a$

$= 7 \times a \times (a + 2)$

$= 7a(a + 2)$

Example 12.3. Factorise : $5x^2y - 15xy^2$

Solution : We know $5x^2y = 5 \times x \times x \times y$ and $15xy^2 = 3 \times 5 \times x \times y \times y$

Both terms have 5, x and y as common factors.

Hence $5x^2y - 15xy^2 = 5 \times x \times x \times y - 3 \times 5 \times x \times y \times y$

$= 5 \times x \times y(x - 3y)$

$= 5xy(x - 3y)$

Example 12.4 **Factorise : $14x^2y^2 + 10x^2y + 8xy^2$**

Solution : We know $14x^2y^2 = 2 \times 7 \times x \times x \times y \times y$

$10x^2y = 2 \times 5 \times x \times x \times y$

$8xy^2 = 2 \times 2 \times 2 \times x \times y \times y$

Therefore, $14x^2y^2 + 10x^2y + 8xy^2 = 2 \times 7 \times x \times x \times y \times y + 2 \times 5 \times x \times x \times y + 2 \times 2 \times 2 \times x \times y \times y$

$= 2 \times x \times y \times (7 \times x \times y + 5 \times x + 2 \times 2 \times y)$

Here three terms have 2, x and y as common irreducible factors.

$= 2xy\,(7xy + 5x + 4y)$

Example 12.5 **Factorise : $4x^2 + 9x + 18$**

Solution : Here $4x^2 = 2 \times 2 \times x \times x$, $9x = 3 \times 3 \times x$ and $18 = 2 \times 3 \times 3$

We observe that there is no common term in the given three terms:

In such cases, 1 is the common factor and the given expression will be written as it is.

Hence $4x^2 + 9x + 18 = 2 \times 2 \times x \times x + 3 \times 3 \times x + 2 \times 3 \times 3$

$= 1\,(2 \times 2 \times x \times x + 3 \times 3 \times x + 2 \times 3 \times 3)$

$= 4x^2 + 9x + 18$

Note: We cannot factorise such algebraic expression if there is no common factor among the terms.

Example 12.6 **Factorise the following:**

(i) $3a\,(x + y) - 5b\,(x + y)$ **(ii) $2(x - y)^2 + 5\,(x - y)$**

(iii) $6x\,(2a - 3b) - 5y\,(3b - 2a)$

Solution : (i) We have, $3a\,(x + y) - 5b\,(x + y)$

Observe that, we have two terms $3a\,(x + y)$ and $5b\,(x + y)$

Clearly, $(x + y)$ is a common factor among them

Hence, $3a\,(x + y) - 5b\,(x + y) = (x + y)\,(3a - 5b)$

(ii) We have, $2(x - y)^2 + 5\,(x - y)$

Here, we have two terms $2(x - y)^2$ and $5\,(x - y)$.

Clearly $(x - y)$ is a common factor among them

Hence, $2\,(x - y)^2 + 5\,(x - y) = (x - y)\,[2\,(x - y) + 5] = (x - y)\,(2x - 2y + 5)$

(iii) We have, $6x\,(2a - 3b) - 5y\,(3b - 2a)$

Rewrite second term $-5y\,(3b - 2a) = -5y\,[-(2a - 3b)]$

$= 5y\,(2a - 3b)$

Hence, $6x\,(2a - 3b) - 5y\,(3b - 2a) = 6x\,(2a - 3b) + 5y\,(2a - 3b)$

$= (2a - 3b)\,(6x + 5y)$. (Taking common $(2a - 3b)$)

12.3.2 Factorisation by Regrouping Terms

In last section, we have discussed the method of finding common factors in which given algebraic expressions have common factors. But sometimes, we have such algebraic expressions which cannot be factorised directly that can be factorised by regrouping terms. We shall discuss such factorisation as follows:

Consider the expression $3x + 3 + 4xy + 4y$

Here you can notice that no factor is common to all the terms but first two terms have common factor 3 and the last two terms have common factors 4 and y. So in this type of sums, we regroup the tems on follows:

In this case, we write $(3x + 3)$ and $(4xy + 4y)$

$$3x + 3 = 3 \times x + 3 \times 1$$
$$= 3 \times (x + 1)$$
$$\text{and } 4xy + 4y = 4 \times x \times y + 4 \times y$$
$$= 4 \times x \times y + 4 \times y \times 1$$
$$= 4y (x + 1)$$

Hence $3x + 3 + 4xy + 4y = (3x + 3) + (4xy + 4y) = 3 (x+1) + 4y (x+1)$

Now here we have two terms

Observe, we have common factor $(x+1)$ in both terms. Combining these two terms.

$3x + 3 + 4xy + 4y = 3 (x + 1) + 4y (x + 1) = (x + 1) (3 + 4y)$

Thus, The algebraic expression $3x + 3 + 4xy + 4y$ is now in the form of a product of two irreducible factors.

Hence $(x+1)$ and $(3+4y)$ are the factors of $3x + 3 + 4xy + 4y$.

Suppose that the above expression was given as $3x + 4y + 4xy + 3$; Then it may not be easy to factorise directly. For that we have to rearrange the terms after observing.

Rearranging the expression as $3x + 3 + 4xy +4y$ allows us to form groups $(3x+3)$ and $(4xy +4y)$ leading to factorisation. This process is called regrouping. Regrouping may be possible in more than one way. Suppose, we can regroup the expression as $3x + 4xy + 3 + 4y$.

Now $3x + 4xy + 3 + 4y = (3x + 4xy) + (3+4y)$
$= x (3 + 4y) + 1\times(3 + 4y)$
$= (3 + 4y) (x + 1)$

The factors are the same, although they appear in different order.

Example 12.7. **Factorise the following expressions.**

(i) $5xy +7y - 5x^2 - 7x$ **(ii)** $ax - ay + bx - by$

(iii) $5p^2 - 8pq - 10p + 16q$

Solution : (i) Step I. Check if there is a common factor among all terms. Here, we have no common factor.

Step II. Observe that the first two terms have a common factor y;

i.e. $5xy + 7y = y(5x+7)$ ....(1)

Now, observe that in last two terms, there is '$-x$' as a common factor.

$$-5x^2 - 7x = -x\,(5x+7) \quad ....(2)$$

putting (1) & (2) together, we have

$$5xy + 7y - 5x^2 - 7x = y(5x + 7) - x\,(5x + 7)$$

$$= (5x + 7)\,(y - x) \quad \text{[Taking } (5x + 7) \text{ common]}$$

(ii) Given that

$ax - ay + bx - by$

$= a\,(x - y) + b\,(x - y)$

$= (x - y)\,(a + b)$ [Taking $(x - y)$ common]

Here taking common a from first two terms and b from next two terms

(iii) Given that

$5p^2 - 8pq - 10p + 16q$

$= p(5p - 8q) - 2\,(5p - 8q)$

$= (5p - 8q)\,(p - 2)$ [Taking common $(5p - 8q)$]

Taking common 'p' from first two terms and '-2' from next two terms

Exercise 12.1

1. Find the common factors of the given terms.

(i) $15x, 25$ (ii) $3y, 33xy$ (iii) $7pq, 28p^2q^2$

(iv) $2x, 3x^2, 5$ (v) $4abc, 24ab^2, 12a^2b$ (vi) $12x^3, -6x^2, 36x$

(vii) $4xy^3, 10x^3y^2, 8x^2y^2z$ (viii) $3x^2, 5x, 9$

2. Factorise the following expressions:

(i) $6x-48$ (ii) $7p-14q$ (iii) $-24z+30z^2$

(iv) $18\ell^2m+27a\,\ell m$ (v) $25x^2y^2z - 15x^2yz^2$ (vi) $a^2bc + ab^2c + abc^2$

(vii) $px^2y + qxy^2 + rxyz$ (viii) $10pq-15qr+20rp$

3. Factorise:

(i) $3a(2p-3q) - 5b(2p - 3q)$ (ii) $15a\,(x^2+y^2) - 10b\,(x^2+y^2)$

(iii) $4\,(x + y)^2 + 2\,(x + y)$ (iv) $(2a - 5b)^2 + 10b - 4a$

(v) $(5\ell + 3m)^2 - 5l - 3m$

4. Factorise:

(i) $x^2 + xy + 6x + 6y$ (ii) $y^2-yz - 3y + 3z$ (iii) $12xy - 8x + 3y-2$

(iv) $a^2b - ab^2 + 4a - 4b$ (v) $x^3 - 6x^2 + x - 6$

(vi) $a^2 + ab\,(1+b) + b^3$ (Hint: First multiply middle term) (vii) $3px - 6py - 8qy + 4qx$

(viii) $r-7 + 7pq - pqr$

5. **Multiple choice Questions:**

(i) Common factor of 10xy and 12y is

(a) 10x (b) 2xy (c) 2y (d) 2x

(ii) Common factor of $5a^2b$ and $9xy^2$ is

(a) 1 (b) 0 (c) ab xy (d) ax

(iii) $8p^2 - 20pq + 28\ p^2q$

(a) $4p\ (2p + 5q - 7pq)$ (b) $4p\ (2p-5q + 7p^2q)$

(c) $4q\ (2p - 5q + 7q)$ (d) $4p\ (2p - 5q + 7pq)$

(iv) $3(2l - m)^2 + (2l - m) =$

(a) $(2l - m)\ (6l - 3m + 1)$ (b) $(2l - m)\ (6l - 2m)$

(c) $3\ (2l - m)\ (2l - m + 1)$ (d) $(2l - m)\ (3 + 2l - m)$

(v) $p^2 - pq + pr - qr =$

(a) $(p - r)\ (p + q)$ (b) $(p + r)\ (q - p)$

(c) $(p + r)\ (p - q)$ (d) $(p - q)\ (r - p)$

12.3.3 Factorisation using Algebraic Identities

In last section, we have learnt about the factorisation of algebraic expressions using common factors method and regrouping method. In this section, we shall discuss the factorisation using algebraic identities.

We know that

(i) $(a + b)^2 = a^2 + 2ab + b^2$............(I) (ii) $(a - b)^2 = a^2 - 2ab + b^2$............(II)

(iii) $(a + b)\ (a - b) = a^2 - b^2$ ....(III)

When the given expression is in the form of $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$ or $a^2 - b^2$ then it can be converted in the form of $(a + b)^2$, $(a - b)^2$ or $(a - b)\ (a + b)$ respectively. then it can be factorised by using above identity. The following examples illustrate it.

Example 12.8. Factorise :

(i) $x^2 + 10x + 25$ (ii) $y^2 - 6y + 9$ (iii) $25m^2 + 30m + 9$

(iv) $9p^2 - 24p + 16$ (v) $p^4 + 2p^2q^2 + q^4$

Solution : Given expression has three terms. therefore it does not fit identity (iii). Here two terms i.e. first and third terms are perfect squares with positive sign so it is of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

(i) We have , $x^2 + 10x + 25 = x^2 + 10x + 5^2$

First and third terms are perfect squares in form of a^2 and b^2 (where $a = x$, $b = 5$)

and middle term is in the form of $2ab = 2\ (x)\ (5)$

$\therefore\ x^2 + 10x + 25 = x^2 + 2(x)(5) + 5^2$

Thus $x^2 + 10x + 5 = (x + 5)^2$ $[\because a^2 + 2ab + b^2 = (a+b)^2]$

(ii) We have, $y^2 - 6y + 9 = y^2 - 6y + 3^2$

Here, First and third terms are perfect squares in form of a^2 and b^2 where $a = y$, $b = 3$ and middle term is in form of $2ab = 2(y)(3)$

$y^2 - 6y + 3^2 = y^2 - 2(y)(3) + 3^2$

Thus $y^2 - 6y + 9 = (y - 3)^2$ $[\because a^2 - 2ab + b^2 = (a - b)^2]$

(iii) $25m^2 + 30m + 9 = 5^2m^2 + 30m + 3^2$

$= (5m)^2 + 30m + (3)^2$ [Here $a = 5m$, $b = 3$]

$= (5m)^2 + 2(5m)(3) + (3)^2 = (5m+3)^2$

Thus, $25m^2 + 30m + 9 = (5m + 3)^2$

(iv) $9p^2 - 24p + 16 = 3^2p^2 - 24p + 4^2$

$= (3p)^2 - 24p + 4^2$ [Here $a = 3p$, $b = 4$]

$= (3p)^2 - 2(3p)(4) + 4^2 = (3p - 4)^2$

Thus, $9p^2 - 24p + 16 = (3p - 4)^2$

(v) We have, $p^4 + 2p^2q^2 + q^4 = (p^2)^2 + 2p^2q^2 + (q^2)^2$

$= (p^2)^2 + 2(p)^2(q^2) + (q^2)^2$

$= (p^2 + q^2)$

Thus, $p^4 + 2p^2q^2 + q^4 = (p^2 + q^2)^2$

Example 12.9. Factorise the following :

(i) a^2-25 **(ii) $4x^2-9$** **(iii) $49x^2-36y^2$** **(iv) $\frac{9}{16}x^2y^2 - \frac{16}{25}z^2$**

(v) $16x^5 - 144x^3$

Solution : Given expressions has two terms, both are perfect squares with second term is negative. So these are of form $a^2 - b^2 = (a - b)(a + b)$

(i) $a^2-25 = a^2-5^2 = (a - 5)(a + 5)$

(ii) $4x^2 - 9 = 2^2x^2 - 3^2 = (2x)^2 - 3^2 = (2x - 3)(2x + 3)$

(iii) $49x^2 - 36y^2 = 7^2x^2 - 6^2y^2 = (7x)^2 - (6y)^2 = (7x - 6y)(7x + 6y)$

(iv) $\frac{9}{16}x^2y^2 - \frac{16}{25}z^2 = \frac{3^2}{4^2}x^2y^2 - \frac{4^2}{5^2}z^2 = \left(\frac{3}{4}xy\right)^2 - \left(\frac{4}{5}z\right)^2$

$= \left(\frac{3}{4}xy - \frac{4}{5}z\right)\left(\frac{3}{4}xy + \frac{4}{5}z\right)$

(v) $16x^5 - 144x^3 = 16x^3(x^2-9)$ [Taking $16x^3$ common]

$= 16x^3(x^2-3^2) = 16x^3(x-3)(x+3)$

Example 12.10. **Factorise the following :**

(i) $a^4 - b^4$ **(ii)** $p^4 - 81$ **(iii)** $16x^4 - 1$

Solution : (i) $a^4 - b^4 = (a^2)^2 - (b^2)^2$

$= (a^2-b^2)(a^2+b^2)$ [Applying identity $a^2-b^2 = (a-b)(a+b)$]

$= (a-b)(a+b)(a^2+b^2)$ [Applying identity $a^2-b^2 = (a-b)(a+b)$]

(ii) $p^4 - 81 = (p^2)^2 - 9^2$

$= (p^2 - 9)(p^2 + 9)$

$= (p^2 - 3^2)(p^2 + 9)$ [Applying Identity $a^2-b^2 = (a-b)(a+b)$]

$= (p-3)(p+3)(p^2+9)$ [Applying Identity $a^2-b^2 = (a-b)(a+b)$]

(iii) $16x^4 - 1 = 4^2(x^2)^2 - 1^2$

$= (4x^2)^2 - (1)^2 = (4x^2 - 1)(4x^2+1)$

$= (2x^2 - 1^2)(4x^2 + 1)$

$= [(2x)^2 - 1^2](4x^2 + 1)$

$= (2x - 1)(2x + 1)(4x^2 + 1)$

Example 12.11. **Factorise the following :**

(i) $x^2-2xy+y^2-z^2$ **(ii)** $25a^2 - 4b^2 + 28bc - 49c^2$

(iii) $x^4 - (x - 2)^4$

Solution : (i) We have, $x^2-2xy + y^2 - z^2$

$= (x^2 - 2xy + y^2) - z^2$

$= (x-y)^2 - z^2$ [$\because a^2-2ab + b^2 = (a-b)^2$]

$= (x - y - z)(x - y + z)$ [$\because a^2-b^2 = (a-b)(a+b)$]

(ii) $25a^2 - 4b^2 + 28bc - 49c^2$

$= 25a^2 - (4b^2 - 28bc + 49c^2)$

$= 25a^2 - [(2b)^2 - 2 \times (2b) \times (7c) + (7c)^2]$

$= 25a^2 - (2b-7c)^2 = (5a)^2 - (2b - 7c)^2$ [$\because a^2-2ab + b^2 = (a-b)^2$]

$= [5a - (2b - 7c)][5a + (2b - 7c)]$ [$\because a^2-b^2 = (a-b)(a+b)$]

$= [5a - 2b + 7c)](5a + 2b - 7c)$

(iii) $x^4 - (x - 2)^4$

$= (x^2)^2 - [(x - 2)^2]^2$

$= [x^2 - (x - 2)^2][x^2 + (x-2)^2]$

$= [x - (x - 2)(x + x - 2)][(x^2 + (x - 2)^2]$

$= (x - x + 2)(x + x - 2)[x^2 + (x - 2)^2]$

$= 2(2x - 2)[x^2 + (x-2)^2]$

12.3.4 FACTORS OF THE FORM (x+a) (x+b)

In last section, we have learnt the factorization of algebraic expressions using algebraic identities. In this section, we shall discuss such type of algebraic expression which are in the form of $x^2 + \ell x + m$ i.e. which does not contain two perfect square terms. Let us discuss how we factorise such expressions.

> For factorising an algebraic expression of the type $x^2 + \ell x + m$, we use identity $x^2 + (a + b)x + ab = (x + a)(x + b)$. For this we find two factors a and b of m (i.e. the constant term) such that
>
> $ab = m$ and $a + b = \ell$
>
> i.e. Sum of both factors = Coefficient of x and Product of both factors = Constant term

Example 12.12. Factorise : $x^2 + 14x + 33$

Solution : Step I. Find the two numbers whose product is constant term (i.e. 33) and sum is the coefficient of x (i.e. 14)

Step II. Since the product is positive. Therefore both factors of 33 will be either positive or negative.

Product = 33	
1×33	$(-1) \times (-33)$
3×11	$(-3) \times (-11)$

Step III. But the sum is positive so both factors of 33 will be positive.

Step IV. Factors of 33 are 1×33, 3×11 one pair from above factors is taken whose sum is 14 i.e. 3 & 11.

Therefore the required numbers are 3 & 11

$\therefore \quad x^2 + 14x + 33 = x^2 + (3 + 11)x + 33$

$= (x + 3)(x + 11)$ [By using identity $x^2 + (a+b)x + ab = (x + a)(x + b)$]

Example 12.13. Factorise : $x^2 - 5x + 6$

Solution : We want to find two numbers (integers) whose sum is -5 and product is 6. Here sum is negative, therefore, both factors of 6 will be negative. i.e. $6 = (-1) \times (-6)$ or $(-2) \times (-3)$ required factor of 6 are (-2) and (-3).

Therefore $x^2 - 5x + 6 = x^2 + \{(-2) + (-3)\}x + (-2)(-3)$

$= (x - 2)(x - 3)$

Product = 6	
1×6	$(-1) \times (-6)$
2×3	$(-2) \times (-3)$

Example 12.14 Factorise : $p^2 + 4p - 12$

Solution : We find two numbers whose product is (-12) and sum is 4.

Since product is negative, one number will be positive and the other number will be negative.

and the sum is positive, so numerically greater of two numbers will be positive. So the required factors are 6 and (-2).

Therefore $\quad p^2 + 4p - 12 = p^2 + \{6 + (-2)\}p - 12$

$= p^2 + 6p - 2p - 12$

$= p(p + 6) - 2(p + 6)$

$= (p + 6)(p - 2)$

Product = -12	
$(-1) \times 12$	$1 \times (-12)$
$(-2) \times 6$	$2 \times (-6)$
$(-3) \times 4$	$3 \times (-4)$

Example 12.15 Factorise : $y^2 - 4y - 45$

Solution : We want to find two numbers whose product is (–45) and sum is (–4)

Since the product is negative therefore one of numbers is positive and the other number is negative.

Since sum is negative, therefore, numerically greater of two numbers is negative.

So, the required number are (–9) and 5.

$$\therefore \quad y^2 - 4y - 45 = y^2 + (-9 + 5)\,y - 45$$
$$= y^2 - 9y + 5y - 45$$
$$= y\,(y - 9) + 5\,(y - 9)$$
$$= (y - 9)\,(y + 5)$$

Product = –45

$(-1) \times 45$	$1 \times (-45)$
$(-3) \times 15$	$3 \times (-15)$
$(-5) \times 9$	$5 \times (-9)$

Exercise 12.2

1. Factorise the following expressions:
   (i) $x^2 + 10x + 25$ (ii) $y^2 - 8y + 16$ (iii) $25p^2 + 30p + 9$
   (iv) $49a^2 + 84ab + 36b^2$ (v) $100x^2 - 80xy + 16y^2$
   (vi) $(p+q)^2 - 4pq$ (Hint expand $(p+q)^2$ first)
   (vii) $\ell^4 + 2\ell^2 m^2 + m^4$ (viii) $4x^2 - 8x + 4$ (Hint :First take common 4 from each term)
2. Factorise the following expressions:
   (i) $25a^2 - 64b^2$ (ii) $49x^2 - 36$ (iii) $28x^2 - 63y^2$
   (iv) $\frac{4}{25}x^2 - \frac{9}{49}y^2$ (v) $8x^5 - 72x^3$ (Hint : taking x common first)
   (vi) $(p+q)^2 - (p-q)^2$ (vii) $16a^2b^2 - 25$
   (viii) $(x^2 - 2xy + y^2) - z^2$ (Hint : First use identity $a^2 - 2ab + b^2 = (a-b)^2$ then another)
3. Factorise :
   (i) $x^4 - y^4$ (ii) $a^4 - 81$ (iii) $m^4 - 256$
   (iv) $p^4 - (q+r)^4$ (v) $a^4 - 2a^2b^2 + b^4$
4. Factorise the following:
   (i) $a^2 + 2ab + b^2 - c^2$ (ii) $1 - 9l^2 + 24lm - 16m^2$ (iii) $25p^2 - 40pq + 16q^2 - 49r^2$
5. Factorise the following expressions.
   (i) $x^2 + 7x + 12$ (ii) $y^2 - 10y + 21$ (iii) $a^2 + 3a - 18$
   (iv) $3p^2 + 18p - 48$ (Hint: taking common 3 from each term)
   (v) $q^2 - q - 6$ (vi) $x^2 - 11x - 42$ (vii) $5x^2 + 25x + 30$
   (viii) $3y^2 - 21y + 36$
6. **Multiple choice Questions :**
   (i) $4p^2 - 20pq + 25q^2$
   (a) $(4p-5q)^2$ (b) $(2p-5q)^2$ (c) $(2q-5p)^2$ (d) $(4q-25p)^2$
   (ii) $4x^3 - 9x =$
   (a) $x^2\,(4x-9)\,(4x+9)$ (b) $x\,(2x-3)\,(2x+3)$
   (c) $x^3\,(2x-3)\,(2x+3)$ (d) $x^2\,(2x-3)\,(2x+3)$

(iii) $(a+b)^2 - (a-b)^2$

(a) $-4ab$ (b) $2a + 2b$ (c) $2a-2b$ (d) $4ab$

(iv) $m^2 - 14m - 32 =$

(a) $(m + 16)(m - 2)$ (b) $(m-16)(m-2)$

(c) $(m-16)(m+2)$ (d) $(m+16)(m+2)$

(v) $p^3 - p$

(a) $p(p^2+1)$ (b) $(p^2-1)(p+1)$ (c) $p^2(p-1)$ (d) $p(p-1)(p+1)$

12.4 Division of Algebraic Expressions

We have learnt about addition, subtraction and multiplication of algebraic expressions. Now, we will learn how to divide one algebraic expression by another. We know that division is the inverse operation of multiplication. Thus $7 \times 5 = 35$,

gives $35 \div 5 = 7$ or $35 \div 7 = 5$

Similarly (i) $4x \times 3x^2 = 12x^3$

Therefore $12x^3 \div 4x = 3x^2$

Or $12x^3 \div 3x^2 = 4x$

(ii) $3x(x + 2) = 3x^2 + 6x$

Therefore $(3x^2 + 6x) \div 3x = x + 2$

Or $(3x^2 + 6x) \div (x + 2) = 3x$

Now we shall learn how the division of one expression can be done by another expression.

12.4.1 Division of a monomial by another Monomial :

In this section, we shall learn the division of a monomial by another monomial.

Consider $12x^3 \div 3x$

We may write $3x$ and $12x^3$ in irreducible factors.

$3x = 3 \times x$

and $12x^3 = 2 \times 2 \times 3 \times x \times x \times x$

Or $12x^3 = 3 \times x \times 2 \times 2 \times x \times x$

$= (3x) \times (4x^2)$

(Here seperate 3x from factors of $12x^3$)

Therefore, $12x^3 \div 3x = 4x^2$

This process is tedious and time consuming, so there is a shorter way by cancellation of common factors like we do in division of numbers.

i.e. $$35 \div 5 = \frac{35}{5} = \frac{7 \times \cancel{5}}{\cancel{5}} = 7$$

Similarly $$12x^3 \div 3x = \frac{12x^3}{3x} = \frac{2 \times 2 \times \cancel{3} \times \cancel{x} \times x \times x}{\cancel{3} \times \cancel{x}} = 2 \times 2 \times x \times x = 4x^2$$

ALTERNATIVELY

Division of a monomial by a monomial

We have,

Quotient of two monomials = (Quotient of their coefficient)

× (Quotient of their variables in two monomials)

For Example: $$12x^3 \div 3x = \frac{12x^3}{3x} = \left(\frac{12}{3}\right)\left(\frac{x^3}{x}\right) = 4x^{3-1} = 4x^2 \text{ (Using } a^m \div a^n = a^{m-n})$$

Example 12.16. Divide : (i) $12x^5$ by $-4x^3$ (ii) $(-18y^3)$ by $3y^2$ (iii) $-4x^2y^3$ by $12x^3y$

Solution : (i) $12x^5 = 2 \times 2 \times 3 \times x \times x \times x \times x \times x$

and $-4x^3 = -2 \times 2 \times x \times x \times x$

Therefore $12x^5 \div (-4x^3) = \dfrac{\cancel{2}\times\cancel{2}\times 3\times\cancel{x}\times\cancel{x}\times\cancel{x}\times x\times x}{-\cancel{2}\times\cancel{2}\times\cancel{x}\times\cancel{x}\times\cancel{x}} = -3 \times x \times x = -3x^2$

ALITER

$$12x^5 \div (-4x^3) = \frac{12x^5}{-4x^3} = \left(\frac{12}{-4}\right)\left(\frac{x^5}{x^3}\right) = -3 \times x^{5-3} = -3x^2 \text{ (Using } a^m \div a^n = a^{m-n})$$

(ii) $-18y^3 = -2 \times 3 \times 3 \times y \times y \times y$

and $3y^2 = 3 \times y \times y$

Therefore $-18y^3 \div 3y^2 = \dfrac{-2\times\cancel{3}\times 3\times\cancel{y}\times\cancel{y}\times y}{3\times\cancel{y}\times\cancel{y}} = -2 \times 3 \times y = -6y$

ALITER

$$-18y^3 \div 3y^2 = \frac{-18y^3}{3y^2} = \left(\frac{-18}{3}\right)\left(\frac{y^3}{y^2}\right)$$

$$= -6 \times y^{3-2} \text{ (Using } a^m \div a^n = a^{m-n})$$

$$= -6y^1 = -6y$$

(iii) $$(-4x^2y^3) \div 12x^3y = \frac{-4x^2y^3}{12x^3y} = \left(\frac{-4}{12}\right)\left(\frac{x^2}{x^3}\right)\left(\frac{y^3}{y}\right)$$

$$= \left(\frac{-1}{3}\right)\left(\frac{1}{x}\right)\left(y^2\right) = \frac{-y^2}{3x}$$

12.4.2 Division of a polynomial by a monomial

In last section, we have learnt the division of a monomial by another monomial. In this section we shall discuss the division of polynomial by a monomial.

Consider the division of a polynomial $6a^3 + 8a^2 + 4a$ by a monomial $2a$. We shall factorise first

i.e. $6a^3 + 8a^2 + 4a = 2a\,(3a^2 + 4a + 2)$

$$(6a^3 + 8a^2 + 4a) \div 2a = \frac{2a\left(3a^2 + 4a + 2\right)}{2a}$$

$$= 3a^2 + 4a + 2$$

We can divide it without making factors i.e. directly as follows:

$$(6a^3 + 8a^2 + 4a) \div 2a = \frac{6a^3 + 8a^2 + 4a}{2a}$$

(Divide each term by the given monomial)

$$= \frac{6a^3}{2a} + \frac{8a^2}{2a} + \frac{4a}{2a}$$

$$= 3a^2 + 4a + 2$$

Let us explain the above division by taking more examples.

Example 12.17. Divide: (i) $6x^4 + 24x^3 - 5x^2$ by $3x^2$ (ii) $(5y^8 - 10y^5 + 3y^2) \div (-5y^2)$

Solution : (i) We have $(6x^4 + 24x^3 - 5x^2) \div 3x^2 = \dfrac{6x^4 + 24x^3 - 5x^3}{3x^2}$

$$= \frac{6x^4}{3x^2} + \frac{24x^3}{3x^2} - \frac{5x^2}{3x^2}$$ **(Dividing each term by the given monomial)**

$$= 2x^2 + 8x - \frac{5}{3}$$

(ii) We have $(5y^8 - 10y^5 + 3y^2) \div (-5y^2) = \dfrac{5y^8 - 10y^5 + 3y^2}{-5y^2}$

$$= \frac{5y^8}{-5y^2} - \left(\frac{10y^5}{-5y^2}\right) + \left(\frac{3y^2}{-5y^2}\right) = -y^6 + 2y^3 - \frac{3}{5}$$

12.4.3 Division of a Polynomial by another Polynomial (Binomial)

In last section, we have learnt the division of a monomial or polynomial by a monomial. In this section we shall discuss the division of a polynomial by a binomial.

Here, we discuss the cases with zero remainder and methods for division of polynomial by binomial with the help of examples.

Example 12.18. Divide as directed :

(i) $5(2x+1)(3x+5) \div (2x+1)$ (ii) $20(y+4)(y^2+5y+3) \div 5(y+4)$

Solution : (i) We have $5(2x+1)(3x+5) \div (2x+1) = \dfrac{5\cancel{(2x+1)}(3x+5)}{\cancel{(2x+1)}}$

$$= 5(3x+5)$$

(ii) We have $20(y+4)(y^2+5y+3) \div 5(y+4) = \dfrac{\cancel{20}^{4}\cancel{(y+4)}(y^2+5y+3)}{\cancel{5}_{1}\cancel{(y+4)}}$

$$= 4(y^2+5y+3)$$

Example 12.19. Divide: $y^2 + 7y + 10$ by $y + 5$

Solution : First, factorise $(y^2 + 7y + 10)$

Therefore $y^2 + 7y + 10 = y^2 + (5+2)y + 5 \times 2$

$= (y+5)(y+2)$ (Using identity ...(iv))

Now $(y^2+7y+10) \div (y+5)$

$$= \frac{y^2+7y+10}{y+5} = \frac{\cancel{(y+5)}(y+2)}{\cancel{y+5}}$$

[Cancelling the common factor (y + 5) from numerator & denominator]

$$= y + 2$$

Example 12.20. **Divide as directed :**

(i) $(5p^2 - 25p + 20) \div (p-1)$ (ii) $12xy\,(9x^2 - 16y^2) \div 4xy\,(3x + 4y)$

Solution : (i) First factorise $(5p^2 - 25p + 20)$, We get

$$5p^2 - 25p + 20 = 5(p^2 - 5p + 4) = 5\,(p^2 - 4p - p + 4)$$
$$= 5\,[p\,(p-4) - 1\,(p-4)] = 5\,(p-4)\,(p-1)$$

Now, $$(5p^2-25+20) \div (p-1) = \frac{5p^2 - 25p + 20}{p-1} = \frac{5(p-4)\cancel{(p-1)}}{\cancel{p-1}}$$

$$= 5\,(p-4)$$

(ii) First Factorise $(9x^2 - 16y^2)$, We get

$$9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x-4y)\,(3x+4y)$$

$$\text{Now, } 12xy\,(9x^2 - 16y^2) \div 4xy\,(3x+4y) = \frac{12xy\left(9x^2 - 16y^2\right)}{4xy(3x+4y)}$$

$$= \frac{{}^{3}\cancel{12xy}\,(3x-4y)\cancel{(3x+4y)}}{{}_{1}\cancel{4xy}\,\cancel{(3x+4y)}} = 3\,(3x-4y)$$

Let us learn another method of division of polynoments.

Example 12.21. **Divide $(3 - 11x + 6x^2)$ by $(-1 + 3x)$**

Solution : Write the dividend & divisor in decreasing order of powers of variable.

Step 1. Dividend: $6x^2 - 11x + 3$

Divisor : $3x - 1$

Step 2. Divide the first term of dividend i.e. $6x^2$ by the first term of divisor $3x$, we get $\frac{6x^2}{3x} = 2x$

$$\begin{array}{r} 2x \\ 3x-1 \overline{)\,6x^2 - 11x + 3} \end{array}$$

i.e. We get first term of quotient as $2x$.

Step 3. Multiply the divisor $(3x-1)$ by $2x$ **(Resulting expression of step 2)**

We get $2x\,(3x-1) = 6x^2 - 2x$

Subtract this from dividend $(6x^2 - 11x + 3)$

to get remainder $(6x^2 - 11x + 3) - (6x^2 - 2x) = -9x + 3$

$$\begin{array}{r} 2x \\ 3x-1 \overline{)\,6x^2 - 11x + 3} \\ \underset{-}{6x^2} \underset{+}{-} 2x \\ \hline -9x + 3 \end{array}$$

Step 4. Now consider this remainder $-9x + 3$ as new dividend. Divide the first term of new dividend $(-9x)$ by first term of divisor $3x$.

We get $\frac{-9x}{3x} = -3$ (As in step 2)

i.e. 2nd term of quotient.

$$\begin{array}{r} 2x - 3 \\ 3x-1 \overline{)\,6x^2 - 11x + 3} \\ \underset{-}{6x^2} \underset{+}{-} 2x \\ \hline -9x + 3 \end{array}$$

Step 5. Multiply the divisor $(3x-1)$ by -3 **(Resulting expression of step 4)**

We get $-3\,(3x-1) = -9x + 3$.

Subtract this from new dividend to get remainder

$[(-9x+3) - (-9x+3)]$

$= -9x + 3 + 9x - 3$

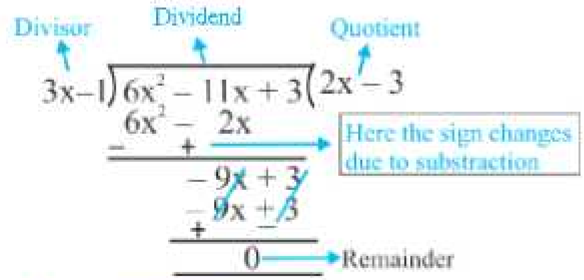
$= 0$

$$\begin{array}{r} 2x - 3 \\ 3x-1 \overline{)\,6x^2 - 11x + 3} \\ \underset{-}{6x^2} \underset{+}{-} 2x \\ \hline -9x + 3 \\ \underset{+}{-}9x \underset{-}{+} 3 \\ \hline 0 \end{array}$$

The remainder is zero and quotient is $2x-3$

Hence, $(6x^2 - 11x + 3) \div (3x-1) = 2x - 3$

The above procedure is called long division method. It is shown below.



Exercise 12.3

1. **Carry out the following divisions.**

(i) $20x^4 \div 10x^2$ (ii) $(-35y^4)$ by $(-7y^3)$ (iii) $16a^4$ by $-6a^2$

(iv) $7x^2y^2z^2 \div 21xyz$ (v) $24p^8q^8 \div (-8\ p^6q^4)$ (vi) $(-15x^2y^3z^2) \div 10x^2y\ z^2$

(vii) $8l^2m^3 \div (-16l^4m^2)$ (viii) $(-12x^2y) \div 20xy^2z$

2. **Divide the given polynomial by given monomial**

(i) $(3x^2 - 4x) \div 7x$ (ii) $(-12a + 22a^2 - 16a^3 + 4) \div 2a$

(iii) $(-8y^3 + 16y^2 + 14y + 1) \div 4y$

(iv) $(ax^8 - bx^6 + cx^4) \div x^4$ (v) $(15x^2y^3 - 10x^3y^2 + 2xy) \div (-5xy^2)$

3. **Divide as directed:**

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$

(ii) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

(iii) $9a^2b^2(3c - 24) \div 27ab(c-8)$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $(x^3y^6 - x^6y^3) \div x^3y^3$

(vi) $48xyz(3x-12)(5y-30) \div 72(x - 4)(y - 6)$

4. **Using factor method, divide the following polynomial by a binomial.**

(i) $(x^2 + 6x + 8)$ by $(x + 2)$ (ii) $(x^2 - x - 42)$ by $(x + 6$ $)$

(iii) $(p^2 - 6p - 27)$ by $(p - 9)$ (iv) $(7x^2 + 14x)$ by $(x + 2)$

(v) $(a^2 - 7a + 12)$ by $(a - 3)$ (vi) $(x^4 + 3x^2 - 10)$ by $(x^2 + 5)$ (Hint put $x^2 = y$)

5. **Divide the following polynomial by a binomial using long division method.**

(i) $(p^2 + 12p + 35)$ by $(p + 7)$ (ii) $(9y^2 - 6y - 8)$ by $(3y - 4)$

6. Divide :

(i) $z(5z^2 - 80)$ by $5z(z+4)$

(ii) $10pq(p^2-q^2)$ by $2p(p+q)$

(iii) $15ab(16a^2 - 25)$ by $10ab(4a+5)$

(iv) $44(x^4 - 5x^3 - 24x^2)$ by $11(x^2 - 8x)$

(v) $39x^3(50x^2 - 98)$ by $26x^2(5x + 7)$

7. Multiple Choice Questions :

(i) $(4x^2-8x) \div (-4x^2) =$

(a) $-1 + 2x$ (b) $\frac{2}{x}$ (c) $-1+\frac{2}{x}$ (d) $2x$

(ii) $(x^2yz + xy^2z + xyz^2) \div xyz =$

(a) xyz (b) $x+y+z$ (c) $x^2 + y^2 + z^2$ (d) $\frac{xy}{z}$

(iii) $2x^2(x+1)(x+3) \div 4x(x+3) =$

(a) $2x(x+1)$ (b) $2x^2(x+1)$ (c) $\frac{x^2(x+1)}{2}$ (d) $\frac{x(x+1)}{2}$

(iv) $(72x^2 - 50) \div (6x - 5) =$

(a) $2(6x + 5)$ (b) $12x + 5$ (c) $12x^2 + 5$ (d) $2(12x + 5)$

(v) $(x^2-8x-20) \div (x-10) =$

(a) $(x - 2)$ (b) $(x + 2)$ (c) $x - 3$ (d) $x + 4$

Learning Outcomes

After completion of this chapter, students are now able to:

- *Apply different methods of finding the factors of an algebraic expressions.*
- *Do division of algebraic expression and division of polynmials by a polynomial.*
- *Apply identities $(a+b)^2 = a^2+2ab+b^2$, $(a-b)^2 = a^2-2ab+b^2$ or $(a-b)(a+b) = a^2-b^2$ etc.*

Answers

Exercise 12.1

1. (i) 5 (ii) $3y$ (iii) $7pq$ (iv) 1 (v) $4ab$ (vi) $6x$ (vii) $2xy^2$
   (viii) 1
2. (i) $6(x-8)$ (ii) $7(p-2q)$ (iii) $-6z(4-5z)$ or $6z(5z-4)$
   (iv) $9\ell m(2\ell+3a)$ (v) $5x^2yz(5y-3z)$ (vi) $abc(a+b+c)$
   (vii) $xy(px+qy+rz)$ (viii) $5(2pq-3qr+4rp)$
3. (i) $(2p-3q)(3a-5b)$ (ii) $5(x^2+y^2)(3a-2b)$ (iii) $2(x+y)(2x+2y+1)$
   (iv) $(2a-5b)(2a-5b-2)$ (v) $(5\ell+3m)(5\ell+3m-1)$
4. (i) $(x+y)(x+6)$ (ii) $(y-z)(y-3)$ (iii) $(3y-2)(4x+1)$
   (iv) $(a-b)(ab+4)$ (v) $(x-6)(x^2+1)$ (vi) $(a+b)(a+b^2)$
   (vii) $(x-2y)(3p+4q)$ (viii) $(r-7)(1-pq)$
5. (i) c (ii) a (iii) d (iv) a (v) c

Exercise 12.2

1. (i) $(x+5)^2$ (ii) $(y-4)^2$ (iii) $(5p+3)^2$ (iv) $(7a+6b)^2$
   (v) $4(5x-2y)^2$ (vi) $(p-q)^2$ (vii) $(\ell^2+m^2)^2$ (viii) $4(x-1)^2$
2. (i) $(5a+8b)(5a-8b)$ (ii) $(7x+6)(7x-6)$
   (iii) $7(2x-3y)(2x+3y)$ (iv) $\left(\frac{2}{5}x+\frac{3}{7}y\right)\left(\frac{2}{5}x-\frac{3}{7}y\right)$
   (v) $8x^3(x+3)(x-3)$ (vi) $4pq$
   (vii) $(4ab+5)(4ab-5)$ (viii) $(x-y+z)(x-y-z)$
3. (i) $(x+y)(x-y)(x^2+y^2)$ (ii) $(a+3)(a-3)(a^2+9)$
   (iii) $(m-4)(m+4)(m^2+16)$ (iv) $(p+q+r)(p-q-r)[p^2+(q+r)^2]$
   (v) $(a+b)^2(a-b)^2$
4. (i) $(a+b-c)(a+b+c)$ (ii) $(1-3\ell+4m)(1+3\ell-4m)$
   (iii) $(5p-4q-7r)(5p-4q+7r)$

5. (i) $(x + 3)(x + 4)$ (ii) $(y - 3)(y - 7)$ (iii) $(a - 3)(a + 6)$

(iv) $3(p + 8)(p - 2)$ (v) $(q - 3)(q + 2)$ (vi) $(x - 14)(x + 3)$

(vii) $5(x + 2)(x + 3)$ (viii) $3(y-3)(y-4)$

6. (i) b (ii) b (iii) d (iv) c (v) d

Exercise 12.3

1. (i) $2x^2$ (ii) $5y$ (iii) $\frac{-8}{3}a^2$ (iv) $\frac{1}{3}xyz$ (v) $-3p^2q^4$ (vi) $\frac{-3}{2}y^2$

(vii) $\frac{-m}{2\ell^2}$ (viii) $\frac{-3x}{5yz}$

2. (i) $\frac{3x}{7} - \frac{4}{7}$ (ii) $-6+11a-8a^2+\frac{2}{a}$ (iii) $-2y^2+4y+\frac{7}{2}+\frac{1}{4y}$

(iv) $ax^4 - bx^2 + c$ (v) $-3xy + 2x^2 - \frac{2}{5y}$

3. (i) $5(3x + 5)$ (ii) $(x + 2)(x + 3)$ (iii) ab (iv) $2z(z - 2)$ (v) $y^3 - x^3$

(vi) $10xyz$

4. (i) $x + 4$ (ii) $x - 7$ (iii) $p + 3$ (iv) $7x$ (v) $a - 4$

(vi) $x^2 - 2$

5. (i) $p+5$ (ii) $3y + 2$

6. (i) $(z - 4)$ (ii) $5q(p - q)$ (iii) $\frac{3}{2}(4a - 5)$ (iv) $4x(x + 3)$

(v) $3x(5x - 7)$

7. (i) c (ii) b (iii) d (iv) a (v) b