Chapter : 32. BINOMIAL DISTRIBUTION

Exercise : 32

Question: 1

A coin is tossed

Solution:

As the coin is tossed 6 times the total number of outcomes will be 2^6

And we know that the favourable outcomes of getting at least 3 heads will be ${}^6c_3$ + ${}^6c_4$ + ${}^6c_5$ + ${}^6c_6$

Thus, the probability of getting at least 3 heads will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$\Rightarrow = \frac{\binom{6}{3}+\binom{6}{4}+\binom{6}{5}+\binom{6}{6}}{2^6}$

$\Rightarrow = \frac{21}{32}$

Question: 2

A coin is tossed

Solution:

As the coin is tossed 5 times the total number of outcomes will be $2^5 = 32$.

And we know that the favourable outcomes of a head appearing even number of times will be,

That either the head appears 0, 2 or 4 times so,

The respective probabilities will be:- ${}^5C_0$ + ${}^5C_2$ + ${}^5C_4$ = 16

Thus, the probability

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$\Rightarrow = \frac{16}{32} = \frac{1}{2}$

Hence, the probability is $_{1/2}$.

Question: 3

7 coins are tosse

Solution:

As 7 coins are tossed simultaneously the total number of outcomes are 2^7=128.

The favourable number of outcomes that a tail appears an odd number of times will be, ${}^7C_1$ + ${}^7C_3$ + ${}^7C_5$ + ${}^7C_7$ = 64.

Thus, the probability

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$= \frac{64}{128}$

$= \frac{1}{2}$

Hence, the probability is $1/2$.

Question: 4

A coin is tossed

Solution:

(i) As the coin is tossed 6 times the total number of outcomes will be 2^6= 64

And we know that the favourable outcomes of getting exactly 4 heads will be ${}^6C_4$ = 15

Thus, the probability of getting exactly 4 heads will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

⇒ 15/64

(ii) As the coin is tossed 6 times the total number of outcomes will be 2^6= 64

And we know that the favourable outcomes of getting at least 1 heads will be ${}^6C_1$ + ${}^6C_2$ + ${}^6C_3$ + ${}^6C_4$ + ${}^6C_5$ + ${}^6C_6$ = 63

Thus, the probability of getting at least 1 head will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

⇒ 63/64

(iii) As the coin is tossed 6 times the total number of outcomes will be 2^6= 64

And we know that the favourable outcomes of getting at most 4 heads will be ${}^6C_0$ + ${}^6C_1$ + ${}^6C_2$ + ${}^6C_3$ + ${}^6C_4$ = 57

Thus, the probability of getting at most 4 heads will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

⇒ 57/64

Question: 5

10 coins are toss

Solution:

(i) As 10 coins are tossed simultaneously the total number of outcomes are 2^{10}=1024.

the favourable outcomes of getting exactly 3 heads will be

${}^{10}C_3$ = 120

Thus, the probability

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$= \frac{120}{1024}$

$= \frac{15}{128}$

Hence, the probability is $\frac{15}{128}$.

(ii) As 10 coins are tossed simultaneously the total number of outcomes are 2^{10}=1024.

the favourable outcomes of getting not more than 4 heads will be

$^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 386$

Thus, the probability

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$= \frac{386}{1024}$

$\Rightarrow \frac{193}{512}$

Hence, the probability is $\frac{193}{512}$.

(iii) As 10 coins are tossed simultaneously the total number of outcomes are 2^{10}=1024.

the favourable outcomes of getting at least 4 heads will be

$^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} = 848$

Thus, the probability

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$= \frac{848}{1024}$

$\Rightarrow \frac{53}{64}$

Hence, the probability is $\frac{53}{64}$.

Question: 6

A die is thrown 6

Solution:

(i) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 6 times the total number of outcomes will be 6^6.

And we know that the favourable outcomes of getting exactly 5 successes will be, either getting 2, 4 or 6 i.e., 1/6 probability of each, total, $\frac{3}{6}$ probability, p = $\frac{1}{2}$,q = $\frac{1}{2}$

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Thus, the probability of getting exactly 5 successes will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$\Rightarrow {}^6C_5\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}$

$\Rightarrow {}^6C_5.\frac{1}{64}$

$\Rightarrow \frac{3}{32}$

(ii) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 6 times the total number of outcomes will be 6^6.

And we know that the favourable outcomes of getting at least 5 successes will be, either getting 2, 4 or 6 i.e, 1/6 probability of each, total, $\frac{3}{6}$ probability, p = $\frac{3}{6}$, q = $\frac{3}{6}$

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Thus, the probability of getting at least 5 successes will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$\Rightarrow (^6C_5 + {}^6C_6)\ \frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}$

$\Rightarrow (^6C_5 + {}^6C_6).\frac{1}{64}$

$\Rightarrow \frac{7}{64}$

(iii) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 6 times the total number of outcomes will be 6^6.

And we know that the favourable outcomes of getting at most 5 successes will be, either getting 2, 4 or 6 i.e, 1/6 probability of each, total, $\frac{3}{6}$ probability of success .

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Thus, the probability of getting at most 5 successes will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$\Rightarrow (^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5).\ \frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}$

$\Rightarrow (^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5).\frac{1}{64}$

$\Rightarrow \frac{63}{64}$

Question: 7

A die is thrown 4

Solution:

Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

We know that the favourable outcomes of getting exactly 3 successes will be, either getting 1 or a 6 i.e, total, $\frac{2}{6}$ probability

The probability of success is $\frac{2}{6}$ and of failure is $\frac{4}{6}$.

Thus, the probability of getting exactly 3 successes will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$\Rightarrow ({}^4C_3)\frac{2}{6}.\frac{2}{6}.\frac{2}{6}.\frac{4}{6}$

$\Rightarrow ({}^4C_3)\frac{2}{81}$

$\Rightarrow \frac{8}{81}$

(ii) Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

We know that the favourable outcomes of getting at least 2 successes will be, either getting 1 or a 6 i.e, total, $\frac{2}{6}$ probability

The probability of success is $\frac{2}{6}$ and of failure is $\frac{4}{6}$.

Thus, the probability of getting at least 2 successes will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$\Rightarrow ({}^4C_2)\ )\frac{2}{6}.\frac{2}{6}.\frac{4}{6}.\frac{4}{6} + ({}^4C_3)\ )\frac{2}{6}.\frac{2}{6}.\frac{2}{6}.\frac{4}{6} + ({}^4C_4)\frac{2}{6}.\frac{2}{6}.\frac{2}{6}.\frac{2}{6}$

$\Rightarrow \frac{33}{81}$

$\Rightarrow \frac{11}{27}$

(iii) Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

We know that the favourable outcomes of getting at most 2 successes will be, either getting 1 or a 6 i.e, total, $\frac{2}{6}$ probability

The probability of success is $\frac{2}{6}$ and of failure is $\frac{4}{6}$.

Thus, the probability of getting at most 2 successes will be

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

$\Rightarrow ({}^4C_0)\frac{4}{6}.\frac{4}{6}.\frac{4}{6}.\frac{4}{6} + ({}^4C_1)\frac{2}{6}.\frac{4}{6}.\frac{4}{6}.\frac{4}{6} + ({}^4C_2)\frac{2}{6}.\frac{2}{6}.\frac{4}{6}.\frac{4}{6}$

$\Rightarrow \frac{72}{81}$

$\Rightarrow \frac{8}{9}$

Question: 8

Find the probabil

Solution:

The total outcomes = 36,

The favourable outcomes are (1,4), (2,4), (3,4), (4,4), (5,4), (6,4), (4,1), (4,2), (4,3) ,(4,5), (4,6)

Thus, the probability = favourable outcomes/total outcomes

$\Rightarrow \frac{11}{36}$

Question: 9

A pair of dice is

Solution:

As the pair of die is thrown 4 times,

The total number of outcomes = 36

Using Bernoulli's Trial P(Success=x) = $^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of 2 successes = $^{4}C_{2}.(\frac{1}{6})^{2}(\frac{5}{6})^{2}$

$\Rightarrow \frac{25}{216}$

Question: 10

A pair of dice is

Solution:

(i) Using Bernoulli's Trial P(Success=x) = $^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of no success = $^{7}C_{0}.(\frac{1}{6})^{0}(\frac{5}{6})^{7}$

$\Rightarrow (\frac{5}{6})^{7}$

(ii) Using Bernoulli's Trial P(Success=x) = $^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of exactly 6 successes = $^{7}C_{6}.(\frac{1}{6})^{6}(\frac{5}{6})^{1}$

$\Rightarrow 35.(\frac{1}{6})^{7}$

(iii) Using Bernoulli's Trial P(Success=x) = $^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of at least 6 successes =

$^7C_6.(\frac{1}{6})^6(\frac{5}{6})^1 + {}^7C_7.(\frac{1}{6})^7(\frac{5}{6})^0$

$\Rightarrow 36.(\frac{1}{6})^7$

$\Rightarrow (\frac{1}{6})^5$

(iv) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of at least 6 successes =

$^7C_0.(\frac{1}{6})^0(\frac{5}{6})^7 + {}^7C_1.(\frac{1}{6})^1(\frac{5}{6})^6 + {}^7C_2.(\frac{1}{6})^2(\frac{5}{6})^5 + {}^7C_3.(\frac{1}{6})^3(\frac{5}{6})^4 + {}^7C_4.(\frac{1}{6})^4(\frac{5}{6})^3 + {}^7C_5.(\frac{1}{6})^5(\frac{5}{6})^2 + {}^7C_6.(\frac{1}{6})^6(\frac{5}{6})^1$

$\Rightarrow (1- (\frac{1}{6})^7)$

Question: 11

There are 6% defe

Solution:

Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =8

The probability of success, i.e. the bulb is defective = p = $\frac{6}{100} = \frac{6}{100}$

q = $1 - \frac{6}{100} = \frac{94}{100}$

probability of that there is not more than one defective piece=

P(0 defective items) + P(1 defective item) =

$^8C_0.(\frac{6}{100})^0(\frac{94}{100})^8 + {}^8C_1.(\frac{6}{100})^1(\frac{94}{100})^7$

$\Rightarrow ((\frac{47}{50})^7 \times (\frac{71}{50}))$

Question: 12

In a box containi

Solution:

(i) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

The probability of success, i.e. the bulb is defective = p = $\frac{6}{60} = \frac{1}{10}$

$q = 1 - \frac{1}{10} = \frac{9}{10}$

probability of that no bulb is defective piece=

P(0 defective items) =

$^5C_0.(\frac{1}{10})^0(\frac{9}{10})^5$

$\Rightarrow ((\frac{9}{10})^5)$

(ii) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

The probability of success, i.e. the bulb is defective = $p = \frac{6}{60} = \frac{1}{10}$

$q = 1 - \frac{1}{10} = \frac{9}{10}$

probability of that there are exactly 2 defective pieces=

P(2 defective items) =

$^5C_2.(\frac{1}{10})^2(\frac{9}{10})^3$

$\Rightarrow ((\frac{729}{10000}))$

Question: 13

The probability t

Solution:

(i) The probability that the bulb will fuse = 0.05 = p

The probability that the bulb will not fuse = 1-0.05 = 0.95 = q

Using Bernoulli's we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

Probability that none will fuse =

$^5C_0.(0.05)^0(0.95)^5$

$\Rightarrow (0.95)^5$

(ii) The probability that the bulb will fuse = 0.05 = p

The probability that the bulb will not fuse = 1-0.05 = 0.95 = q

Using Bernoulli's we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

Probability that at least one will fuse = P(1) + P(2) + P(3) + P(4) + P(5)

$^5C_1.(0.05)^1(0.95)^4 + {^5C_2}.(0.05)^2(0.95)^3 + {^5C_3}.(0.05)^3(0.95)^2 + {^5C_4}.(0.05)^4(0.95)^1 + {^5C_5}.(0.05)^5(0.95)^0$

$\Rightarrow (1-(0.95)^5)$

(iii) The probability that the bulb will fuse = 0.05 = p

The probability that the bulb will not fuse = 1-0.05 = 0.95 = q

Using Bernoulli's we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

Probability that not more than one will fuse = P(0) + P(1)

$^5C_0.(0.05)^0(0.95)^5 + {}^5C_1.(0.05)^1(0.95)^4$

$\Rightarrow (1.20).(0.95)^5$

Question: 14

In the items prod

Solution:

(i) The probability that the item is defective = $\frac{1}{10}$ = p

The probability that the bulb will not fuse = $1-\frac{1}{10}=\frac{9}{10}$ = q

Using Bernoulli's we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability that exactly 2 defective items are,

$\Rightarrow {}^6C_2.(\frac{1}{10})^2(\frac{9}{10})^4$

$$\Rightarrow \frac{3}{20}\times\left(\frac{9}{10}\right)^4$$

(ii) The probability that the item is defective = $\frac{1}{10}$ = p

The probability that the bulb will not fuse = $1-\frac{1}{10}=\frac{9}{10}$ = q

Using Bernoulli's we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability that not more than 2 defective items are,

$\Rightarrow {}^6C_0.(\frac{1}{10})^0(\frac{9}{10})^6 + {}^6C_1.(\frac{1}{10})^1(\frac{9}{10})^5 + {}^6C_2.(\frac{1}{10})^2(\frac{9}{10})^4$

$$\Rightarrow \left(\frac{81+54+15}{10^6}\right).(9^4) = \frac{150\times 9^4}{10^6}$$

(iii) The probability that the item is defective = $\frac{1}{10}$ = p

The probability that the bulb will not fuse = $1-\frac{1}{10}=\frac{9}{10}$ = q

Using Bernoulli's we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability of at least 3 defective items are,

P(3) + P(4) + P(5) + P(6)

$\Rightarrow {}^6C_3.(\frac{1}{10})^3(\frac{9}{10})^3 + {}^6C_4.(\frac{1}{10})^4(\frac{9}{10})^2 + {}^6C_5.(\frac{1}{10})^5(\frac{9}{10})^1 + {}^6C_6.(\frac{1}{10})^6(\frac{9}{10})^0$

$\Rightarrow \frac{15850}{10^6}$

Question: 15

Assume that on an

Solution:

The probability that the called number is busy is $\frac{1}{15}$

Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability that at least three of them will be busy is:-

P(0) + P(1) + P(2) + P(3)

$\Rightarrow {}^6C_0(\frac{1}{15})^0(\frac{14}{15})^6 + {}^6C_1(\frac{1}{15})^1(\frac{14}{15})^5 + {}^6C_2(\frac{1}{15})^2(\frac{14}{15})^4 + {}^6C_3(\frac{1}{15})^3(\frac{14}{15})^3$

$\Rightarrow 1 - (\frac{14}{15})^4.(\frac{59}{45})$

Question: 16

Three cars partic

Solution:

The probability that any one of them has an accident is 0.1.

The probability any car reaches safely is 0.9.

The probability that all the cars reach the finishing line without any accident is = (0.9)(0.9)(0.9) = 0.729

Question: 17

Past records show

Solution:

The probability that the operations performed are successful is = 0.8

The probability that at least three operations are successful is = P(3) + P(4)

$\Rightarrow {}^4C_3(0.8)^3(0.2)^1 + {}^4C_4(0.8)^4(0.2)^0$

$\Rightarrow \frac{512}{625}$

Question: 18

The probability o

Solution:

Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

p = � q = �

The probability of hitting the target at least twice is = P(2) + P(3) + P(4) + P(5) + P(6) + P(7)

$\Rightarrow 1-(P(0) + P(1))$

$\Rightarrow 1 - ({}^7C_0(\frac{1}{4})^0(\frac{3}{4})^7 + {}^7C_1(\frac{1}{4})^1(\frac{3}{4})^6)$

$\Rightarrow 1 - (\frac{10}{4})(\ (\frac{3}{4})^6)$

$\Rightarrow \frac{4547}{8192}$

Question: 19

In a hurdles race

Solution:

The probability that the hurdle will be cleared is 5/6

Using Bernoulli's Trial we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =10

p = 5/6 q = 1/6

Probability that he will knock down fewer than 2 hurdles is =

P(0) + P(1)

$\Rightarrow {}^{10}C_0(\frac{1}{6})^0(\frac{5}{6})^{10} + {}^{10}C_1(\frac{1}{6})^1(\frac{5}{6})^9$

$\Rightarrow \frac{5^{10}}{2\times6^9}$

Question: 20

A man can hit a b

Solution:

The probability that the bird will be shot, is 1/3

Using Bernoulli's Trial we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =3

p = 1/3 q =2/3

Probability that he will hit at least one bird is =

P(1) + P(2) + P(3)

$\Rightarrow {}^3C_1(\frac{1}{3})^1(\frac{2}{3})^2 + {}^3C_2(\frac{1}{3})^2(\frac{2}{3})^1 + {}^3C_3(\frac{1}{3})^3(\frac{2}{3})^0$

$\Rightarrow \frac{19}{27}$

Question: 21

If the probabilit

Solution:

The probability that a man aged 60 will live to be 70 is 0.65

Using Bernoulli's Trial we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =8

p = 0.65 q =0.35

Probability that out of 10 men, now 60, at least 8 will live to be 70 is: P(8) + P(9) + P(10)

${}^{10}C_8(0.65)^8(0.35)^2 + {}^{10}C_9(0.65)^9(0.35)^1 + {}^{10}C_{10}(0.65)^{10}(0.35)^0$

$\Rightarrow 0.2615$

Question: 22

A bag contains 5

Solution:

(i) Balls are drawn at random,

So, the probability that none is white is,

In a trial the probability of selecting a non-white ball is $\frac{15}{20}$

So, in 4 trials it will be,

$\Rightarrow (\frac{15}{20})(\frac{15}{20})(\frac{15}{20})(\frac{15}{20}) = \frac{81}{256}$

(ii) Balls are drawn at random,

So, the probability that all are white is,

In a trial the probability of selecting a white ball is $\frac{5}{20}$

So, in 4 trials it will be,

$\Rightarrow (\frac{5}{20})(\frac{5}{20})(\frac{5}{20})(\frac{5}{20}) = \frac{1}{256}$

(iii) Balls are drawn at random,

So, the probability that at least one is white is,

In a trial the probability of selecting a white ball is $\frac{5}{20}$

So, in 4 trials the probability that at least one is white is,

Selecting a white and then choosing from the rest,

$\Rightarrow 1 - \frac{81}{256}$ that no ball is white

is $\frac{175}{256}$.

Question: 23

A policeman fires

Solution:

The probability that the burglar will be hit by a bullet is 0.6.

Using Bernoulli's Trial we have,

$P(\text{Success}=x) = {}^{n}C_{x}.p^{x}.q^{(n-x)}$

$x=0, 1, 2, \ldots\ldots\ldots n$ and $q = (1-p)$, $n = 6$

$p = 0.6$ $q = 0.4$

The probability that the burglar is unhurt is,

${}^{6}C_{0}(0.6)^{0}(0.4)^{6}$

$\Rightarrow 0.004096$

Question: 24

A die is tossed t

Solution:

Using Bernoulli's Trial we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =3

p = 2/6 = 1/3, q = 4/6 = 2/3

P(x = 0) = P (no success) = P (all failures) = $(\frac{2}{3})(\frac{2}{3})(\frac{2}{3}) = \frac{8}{27}$

P(x = 1) = P (1 success and 2 failures) = $^3C_1(\frac{1}{3})^1(\frac{2}{3})^2 = \frac{12}{27}$

P(x = 2) = P (2 success and 1 failure) =$^3C_2(\frac{1}{3})^2(\frac{2}{3})^1 = \frac{6}{27}$

P(x = 3) = P (all 3 success) = $^3C_3(\frac{1}{3})^3(\frac{2}{3})^0 = \frac{1}{27}$

∴ The probability distribution of the random variable x is -

x : 0 1 2 3

P(x) : $\frac{8}{27}$ $\frac{12}{27}$ $\frac{6}{27}$ $\frac{1}{27}$

x_1 p_1 p_1x_1 $p_1x_1^2$

0 $\frac{8}{27}$ 0 0

1 $\frac{12}{27}$ $\frac{12}{27}$ $\frac{12}{27}$

2 $\frac{6}{27}$ $\frac{12}{27}$ $\frac{24}{27}$

3 $\frac{1}{27}$ $\frac{3}{27}$ $\frac{9}{27}$

1 $\frac{45}{27}$

Mean μ = Σp_1x_1 = 1

Variance = $\sigma^2 = \Sigma p_1x_1^2$ - μ

⇒ 5/3 - 1/1

⇒ 2/3

Question: 25

A die is thrown 1

Solution:

Probability of getting an even number is = 3/6 = 1/2

Probability of getting an odd number is = 3/6 = 1/2

Variance = npq

⇒ $100\times\frac{1}{2}\times\frac{1}{2}$

⇒ 25

Question: 26

Determine the bin

Solution:

Mean = np = 9

Variance = npq = 6

$\Rightarrow q = \frac{6}{9} = \frac{2}{3}$

$\Rightarrow p = 1 - \frac{6}{9} = \frac{1}{3}$

$\Rightarrow n = 27$

Binomial distribution

${}^{27}C_r . \left(\frac{1}{3}\right)^r . \left(\frac{2}{3}\right)^{(27-r)}$ where r = 0, 1, 2, 3, ………, 27

Question: 27

Find the binomial

Solution:

Mean = np = 5

Variance = npq = 2.5

$\Rightarrow q = \frac{2.5}{5} = \frac{1}{2}$

$\Rightarrow p = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow n = 10$

Probability distribution is:-

${}^{10}C_r . \left(\frac{1}{2}\right)^r . \left(\frac{1}{2}\right)^{(10-r)}, 0 \le r \le 10$

Question: 28

The mean and vari

Solution:

Mean = np = 4

Variance = npq = 4/3

$\Rightarrow q = \frac{1}{3}$

$\Rightarrow p = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow n = 6$

The probability ($X \geq 1$) is

${}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5$

$= \frac{728}{729}$

Question: 29

For a binomial di

Solution:

Mean = np = 6

Variance = npq = 2

$\Rightarrow q = \frac{1}{3}$

$\Rightarrow$ p = 1 - $\frac{1}{3} = \frac{2}{3}$

$\Rightarrow$ n = 9

The probability of getting 5 successes,

$^9C_5(\frac{2}{3})^5(\frac{1}{3})^4$

Question: 30

In a binomial dis

Solution:

Mean + Variance = np + npq = np(1 + q) = 25/3

Variance = n^2p^2q = n2 = 50/3 ...(i)

$n^2p^2(1 + q)^2$ = 625/9 ...(ii)

Dividing (i) by (ii), we get,

$$\frac{q}{(q + 1)^2} = \frac{\frac{50}{3}}{\frac{625}{9}} = \frac{6}{25}$$

$\Rightarrow$ 6q2-13q + 6 = 0

$\Rightarrow$ q = 2/3 or 3/2

$\Rightarrow$ But as q can not be greater than 1 thus, q = 2/3.

$\Rightarrow$ p = 1/3

$\Rightarrow$ n = 15

Binomial distribution,

$$^{15}C_r \cdot \left(\frac{1}{3}\right)^r \cdot \left(\frac{2}{3}\right)^{(15-r)}$$

Question: 31

Obtain the binomi

Solution:

Mean is 10,

Standard deviation is $2\sqrt{2}$

So, variance is σ^2 i.e. 8

Thus,

Mean = np = 10

Variance = npq = 8

$\Rightarrow$ q = $\frac{4}{5}$

$\Rightarrow$ p = 1 - $\frac{4}{5} = \frac{1}{5}$

$\Rightarrow$ n = 50

Thus, the binomial distribution is

$$\cdot \ ^{50}C_r \cdot \left(\frac{1}{5}\right)^r \cdot \left(\frac{4}{5}\right)^{(50-r)}, 0 \le r \le 50$$

Question: 32

Bring out the fal

Solution:

Variance can not be greater than mean as then, q wll be greater than 1, which is not possible.

As, np = 6 and npq = 9

q = 3/2 ...(not possible)

Exercise : OBJECTIVE QUESTIONS

Question: 1

Mark (√) against

Solution:

If A and B are mutually exclusive events then,

P(A) = 0.4, P(B) = X

And P(A ∪ B) = P(A)+P(B) = 0.5 = 0.4 + P(B)

⇒ P(B) = 0.1

Question: 2

Mark (√) against

Solution:

As A and B are independent events such that P(A) = 0.4, P(B) = x

So, P(A ∩ B) = P(A)P(B)

And P(A ∪ B) = P(A)+P(B)+P(A ∩ B)

P(A ∪ B) = 0.4 + X - 0.4X = 0.5

⇒ 0.4 + 0.6X = 0.5

⇒ X = 1/6

Question: 3

Mark (√) against

Solution:

P(B/A) = $\frac{P(A \cap B)}{P(A)}$

⇒ And P(A) = 0.8,

⇒ P(A ∩ B) = 0.32

So, P(A/B) = $\frac{P(A \cap B)}{P(B)}$

⇒ P(A/B) = $\frac{0.32}{0.5}$ = 0.64

⇒ Hence, the answer is b.

Question: 4

Mark (√) against

Solution:

P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$ and P(A ∪ B) = $\frac{7}{11}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A \cap B)$

$\Rightarrow P(A \cap B) = \frac{4}{11}$

$P(A/B) = P(A \cap B)/P(B)$

$\Rightarrow P(A/B) = \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$

Question: 5

Mark (√) against

Solution:

We are having two events A and B such that

$P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(A' \cup B') = \frac{1}{4}$,

$P(A' \cup B') = P'(A \cap B) = 1 - P(A \cap B) = \frac{1}{4}$

$\Rightarrow P(A \cap B) = \frac{3}{4}$

$\Rightarrow$ As $P(A \cap B) \neq P(A).P(B)$ … thus, they are not independent,

$\Rightarrow$ And as $P(A \cup B) \neq P(A) + P(B)$ … thus, they are not mutually exclusive.

Hence, the answer is option d.

Question: 6

Mark (√) against

Solution:

P(A) = probability that A can solve the problem

= 3/5

And P(B) = probability that B can solve the problem = 2/3

P(A U B) = P(A) + P(B), As the events are independent

$\Rightarrow P(A \cap B) = P(A).P(B)$

Thus,

$\Rightarrow$ P(A) + P(B) = 3/5 +2/3 – 2/5 = 13/15

Question: 7

Mark (√) against

Solution:

The probability that the problem is solved = $P(A \cup B \cup C) = P(A)+P(B)+P(C)-P(A \cap B)-P(B \cap C)-P(C \cap A) + 3P(A \cap B \cap C)$

Considering independent events, $P(A \cap B) = P(A).P(B)$,

$P(B \cap C) = P(B).P(C)$, $P(C \cap A) = P(C).P(A)$,

$P(A \cap B \cap C) = P(A).P(B).P(C)$,

Thus, $P(A \cup B \cup C)$ is,

$\Rightarrow \frac{1}{6} + \frac{1}{5} + \frac{1}{3} - \frac{1}{30} - \frac{1}{15} - \frac{1}{18} + 3\left(\frac{1}{90}\right) = \frac{5}{9}$

Question: 8

Mark (√) against

Solution:

P(A) = $\frac{4}{5}$ P(B) = $\frac{3}{4}$ P(C) = $\frac{2}{3}$

P(B ∩ C ∩ A') = P(B ∩ C) – P(B ∩ C ∩ A)

As the events are independent, So, P(B ∩ C) = P(B).P(C) = $\frac{3}{4} \times \frac{2}{3}$

And P(B ∩ C ∩ A) = P(B).P(C).P(A) = $\frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$

P(B ∩ C ∩ A') = $\frac{1}{10}$

Question: 9

Mark (√) against

Solution:

The probability of failure of the first component = 0.2 =P(A)

The probability of failure of second component = 0.3 = P(B)

The probability of failure of third component = 0.5 = P(C)

As the events are independent,

The machine will operate only when all the components work, i.e.,

(1-0.2)(1-0.3)(1-0.5) = P(A')P(B')P(C')

In rest of the cases, it won't work,

So P(AUBUC) = 1 – P(A'∩B'∩C') = 1 – (0.8).(0.7).(0.5)

⇒ 1 – 0.28 = 0.72

Question: 10

Mark (√) against

Solution:

The probability that the outcome which is either, 1, 3 or 5 is prime is

$= \frac{Favorable\ outcomes}{Total\ outcomes}$

Favourable outcomes = 3 or 5

Total outcomes = 1, 3, and 5

Thus, probability=

$\Rightarrow \frac{2}{3}$

Question: 11

Mark (√) against

Solution:

P(A) = 0.3, P(B) = 0.2 and P(A ∩ B) = 0.1

P($\overline{A}$ ∩ B) = P(B) – P(A ∩ B) = 0.2 – 0.1 = 0.1

Question: 12

Mark (√) against

Solution:

P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{3}$ and P(A ∩ B) = $\frac{1}{5}$,

P$(\overline{B}/\overline{A})$ = $\frac{P(\bar{A}\cap\bar{B})}{P(\bar{A})} = \frac{1-P(A\cup B)}{1-P(A)} = \frac{1-\left(\frac{1}{4}+\frac{1}{3}-\frac{1}{5}\right)}{1-\frac{1}{4}}$

⇒ P$(\overline{B}/\overline{A})$ = $\frac{23}{60}$

Question: 13

Mark (√) against

Solution:

P(A) = 0.4, P(B) = 0.8 and

P(B/A) = 0.6,

P(B/A) = $\frac{P(A\cap B)}{P(A)} = 0.6$

P(A ∩ B) = 0.24

⇒ P(A/B) = $\frac{P(A\cap B)}{P(B)} = 0.3$

Question: 14

Mark (√) against

Solution:

P$\left(\overline{A}/\overline{B}\right)$ = $\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \frac{P(\bar{A})P(\bar{B})}{1-P(B)} = 1 - P(A)$

Question: 15

Mark (√) against

Solution:

Given,

P(A ∪ B) = $\left(\frac{5}{6}\right)$, P(A ∩ B) = $\left(\frac{1}{3}\right)$ and

P$(\overline{B})$ = $\left(\frac{1}{2}\right)$, P(B) = 1- P$(\overline{B})$ = 1- $\frac{1}{2} = \frac{1}{2}$

⇒ P(B) = $\frac{1}{2}$

⇒ P(A) = $\frac{2}{3}$

⇒ Hence, these are independent.

Question: 16

Mark (√) against

Solution:

The die is thrown twice,

So the favourable outcomes that the sum appears to be 7 are

(1,6), (2,5), (3,4), (4,3), (5,2) and (6,1)

Out of these 2 appears twice,

So the probability that 2 appears at least once is:

$= \frac{Favorable\ outcomes}{Total\ outcomes}$

$\Rightarrow \frac{2}{6} = \frac{1}{3}$

Question: 17

Mark (√) against

Solution:

The sum will be even when; both numbers are either even or odd,

i.e. for both numbers to be even, the total cases $^5C_1 X ^4C_1$ (Both the numbers are odd)+$^4C_1 X ^3C_1$ (Both the numbers are even)= 32

The favourable number of cases will be,

Both odd, i.e. selecting numbers from 1, 3, 5, 7, or 9, i.e.

$^5C_1 X ^4C_1 = 20$

Thus, the probability that both numbers are odd will be =

$= \frac{Favorable\ outcomes}{Total\ outcomes}$

$\Rightarrow \frac{20}{32} = \frac{5}{8}$

Question: 18

Mark (√) against

Solution:

Given:

60% of the students read mathematics, 25% biology and 15% both mathematics and biology

That means,

Let the event A implies students reading mathematics,

Let the event B implies students reading biology,

Then, P(A) = 0.6

P(B) = 0.25

P(A∩B) = 0.15

We, need to find P(A/B) = P(A∩B)/ P(B)

$\Rightarrow \frac{0.15}{0.25} = \frac{3}{5}$

Question: 19

Mark (√) against

Solution:

The couple has two children and one is known to be boy,

The probability that the other is boy will be =

$\frac{Favorable\ outcomes}{Total\ outcomes}$

Total outcomes are 3,

The first child is a boy, the second girl

The first child is a girl, the second boy

The first child is a boy, second boy

The favourable outcome is one,

Thus, the probability that the other is boy will be

⇒ 1/3

Question: 20

Mark (√) against

Solution:

A die is tossed twice,

The probability of getting a 4, 5 or 6 in the first trial is 3/6 = P(A)

The probability of getting a 1, 2, 3 or 4 in the second trial is 4/6 =P(B)

As the events are independent, the probability of these two events together will be, P(A).P(B) = 1/3.

Question: 21

Mark (√) against

Solution:

Using Bernoulli's Trial P(Success=x) = $^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the coin is thrown 6 times the total number of outcomes will be 2^6.

And we know that the favourable outcomes of getting at least 3 successes will be, getting a head

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

$^{6}C_{3}(\frac{1}{2})^{3}(\frac{1}{2})^{3} + ^{6}C_{4}(\frac{1}{2})^{4}(\frac{1}{2})^{2} + ^{6}C_{5}(\frac{1}{2})^{5}(\frac{1}{2})^{1} + ^{6}C_{6}(\frac{1}{2})^{6}(\frac{1}{2})^{0}$

$\Rightarrow \frac{21}{32}$

Question: 22

Mark (√) against

Solution:

Using Bernoulli's Trial P(Success=x) = $^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the coin is tossed 5 times the total number of outcomes will be 2^5.

And we know that the favourable outcomes of getting the odd tail number of times ,successes will be, getting a tail

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

$^{5}C_{1}(\frac{1}{2})^{1}(\frac{1}{2})^{4} + ^{5}C_{3}(\frac{1}{2})^{3}(\frac{1}{2})^{2} + ^{5}C_{5}(\frac{1}{2})^{5}(\frac{1}{2})^{0}$

$\Rightarrow \frac{16}{32} = \frac{1}{2}$

Question: 23

Mark (√) against

Solution:

Using Bernoulli's Trial P(Success=x) = $^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the coin is tossed 5 times the total number of outcomes will be 2^5.

And we know that the favourable outcomes of getting the head even number of times ,successes will be, getting a head,

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

the probability that head appears an even number of times =

P(0)+P(2)+P(4)

$= {}^5C_2(\frac{1}{2})^2(\frac{1}{2})^3 + {}^5C_3(\frac{1}{2})^3(\frac{1}{2})^2 + {}^5C_5\ (\frac{1}{2})^5(\frac{1}{2})^0$

$\Rightarrow \frac{16}{32} = \frac{1}{2}$

Question: 24

Mark (√) against

Solution:

Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the coin is tossed 8 times the total number of outcomes will be 2^8.

And we know that the favourable outcomes of getting at least 6 heads are,successes will be, getting a head,

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

the probability of getting at least 6 heads is =

P(6) +P(7) +P(8)

$= {}^8C_6(\frac{1}{2})^6(\frac{1}{2})^2 + {}^8C_7(\frac{1}{2})^7(\frac{1}{2})^1 + {}^8C_8(\frac{1}{2})^8(\frac{1}{2})^0$

$\Rightarrow \frac{28+8+1}{256} = \frac{37}{256}$

Question: 25

Mark (√) against

Solution:

Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 5 times the total number of outcomes will be 6^5.

And we know that the favourable outcomes of getting at least 4 successes will be, either getting 1, 3 or 5 i.e., 1/6 probability of each, total, $\frac{3}{6}$ probability, p = $\frac{1}{2}$,q = $\frac{1}{2}$

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

the probability of getting at least 4 successes =

P(4)+P(5)

$\Rightarrow {}^5C_4(\frac{1}{2})^4(\frac{1}{2})^1 + {}^5C_5(\frac{1}{2})^5(\frac{1}{2})^0$

$\Rightarrow \frac{3}{16}$

Question: 26

Mark (√) against

Solution:

Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As we know that the favourable outcomes of getting at least doublets twice are, successes will be, getting a doublet, i.e.,

, $p = \frac{1}{6}, q = \frac{5}{6}$

The probability of success is $\frac{1}{6}$ and of failure is also $\frac{5}{6}$

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

the probability of getting at least 2 successes =

P(2)+P(3)+P(4)

$\Rightarrow {}^4C_2(\frac{1}{6})^2(\frac{5}{6})^2 + {}^4C_3(\frac{1}{6})^3(\frac{5}{6})^1 + {}^4C_4(\frac{1}{6})^4(\frac{5}{6})^0$

$\Rightarrow \frac{19}{144}$

Question: 27

Mark (√) against

Solution:

Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), here n = 7

As we know that the favourable outcomes of getting at most 6 success are, successes will be, getting a total of 7 is success, i.e.,

We can get 7 by, (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

, $p = \frac{6}{36}, q = \frac{30}{36}$

The probability of success is $\frac{1}{6}$ and of failure is also $\frac{5}{6}$

$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$

the probability of getting at most 6 successes =

P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6) = 1-P(7)

$\Rightarrow 1 - {}^7C_7(\frac{1}{6})^7(\frac{5}{6})^0$

$\Rightarrow 1 - (\frac{1}{6})^7$

Question: 28

Mark (√) against

Solution:

The probability that the man hits the target is 3/4

Using Bernoulli's Trial we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ………n and q = (1-p), n =5

p = �, q = �

Probability that he will hit at least 3 times is =

P(3)+P(4)+P(5)

$\Rightarrow {}^5C_3(\frac{3}{4})^3(\frac{1}{4})^2+{}^5C_4(\frac{3}{4})^4(\frac{1}{4})^1+{}^5C_5(\frac{3}{4})^5(\frac{1}{4})^0$

$\Rightarrow \frac{459}{512}$

Question: 29

Mark (√) against

Solution:

The probability of safe arrival of the ship is 1/5

Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ………n and q = (1-p), n =5

p = 1/5 , q = 4/5

Probability of safe arrival of at least 3 ships is =

P(3)+P(4)+P(5)

$\Rightarrow {}^5C_3(\frac{1}{5})^3(\frac{4}{5})^2+{}^5C_4(\frac{1}{5})^4(\frac{4}{5})^1+{}^5C_5(\frac{1}{5})^5(\frac{4}{5})^0$

$\Rightarrow \frac{181}{3125}$

Question: 30

Mark (√) against

Solution:

The probability of occurrence of an event E in one trial is 0.4

Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ………n and q = (1-p), n =3

p = 0.4 , q = 0.6

The probability that E occurs at least once is,

P(1)+P(2)+P(3)

$\Rightarrow {}^3C_1(\frac{2}{5})^1(\frac{3}{5})^2+{}^3C_2(\frac{2}{5})^2(\frac{3}{5})^1+{}^3C_3(\frac{2}{5})^3(\frac{3}{5})^0$

$\Rightarrow \frac{98}{125} = 0.784$