CBSE Board
Class IX Mathematics
Sample Paper 10

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

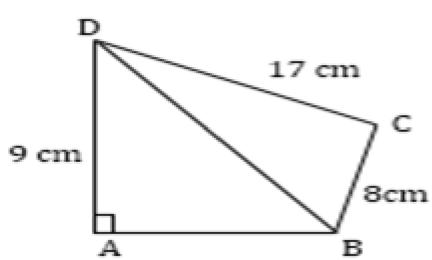
1. If $\left(\sqrt{5}+\sqrt{6}\right)^2$ = a + b $\sqrt{30}$, Find the respective values of a and b.

2. If for one of the solutions of the equation ax + by + c = 0, x is negative and y is positive, then a portion of the above line definitely lies in which Quadrant?

OR

Write two solutions for the equation 2x + y = 1.

3. In the given figure, find the length of AB, if area of □ ABCD is 122 cm^2 and the area of ΔBCD is 68 cm^2.



4. $7x^3 - 2x^2 + 3\sqrt{x} - 4$, Is given expression is a Polynomial?

5. The ages of ten students of a group are given below. The ages have been recorded in years and months:

8 – 6, 9 – 0, 8 – 4, 9 – 3, 7 – 8, 8 – 11, 8 – 7, 9 – 2, 7 – 10, 8 – 8

Determine the range?

OR

In a frequency distribution, the mid-value of a class is 10 and the width of the class is 8. Find the lower limits of the class.

6. Can we say every Rectangle is a Square?

Section B

(Questions 7 to 12 carry 2 marks each)

7. If a = 2 + $\sqrt{3}$, find the value of a + $\frac{1}{a}$.

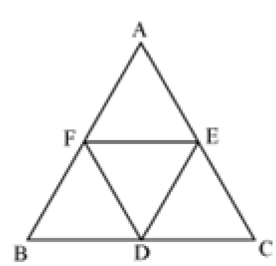
8. Without actually calculating the cubes, find the value of $75^3 - 25^3 - 50^3$.

OR

If $x + \sqrt{2}$ is a factor of the polynomial $x^2 + mx - 8$, then find the value of *m*.

9. Find the value of k, if x = 1, y = 1 is a solution of the equation 9kx + 12ky = 63.

10. D, E and F are respectively the mid-points of the sides BC, CA and AB of ΔABC. Show that BDEF is a parallelogram.



11. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?

OR

How many litres of water flow out through a pipe having 5 cm^2 area of cross section in one minute, if the speed of water in the pipe is 30 cm/sec?

12. Write two solutions for $\pi x + y = 9$.

Section C

(Questions 13 to 22 carry 3 marks each)

13. Represent $\sqrt{3.2}$ on the number line.

OR

If $\frac{2\sqrt{7}+3\sqrt{5}}{\sqrt{7}+\sqrt{5}} = P\sqrt{35}+Q$, then what is the value of 2P + Q?

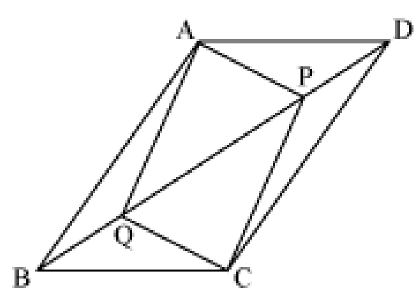
14. Factorise: $b^2 + c^2 + 2(ab + bc + ca)$.

OR

Factorise: $x^3 - 23x^2 + 142x - 120$.

15. If $x = (3+\sqrt{8})$, find the value of $\left(x^2 + \frac{1}{x^2}\right)$

16. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see the given figure). Show that (i) $\Delta APD \cong \Delta CQB$ (ii) AP = CQ



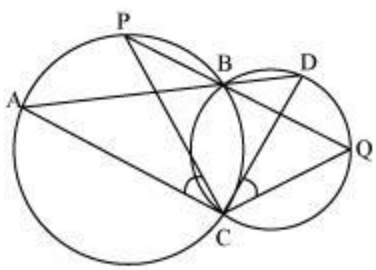
17. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

18. A survey was conducted by a group of students as a part of their Environment Awareness Programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

No of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
No of houses	1	2	1	5	6	2	3

19. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.

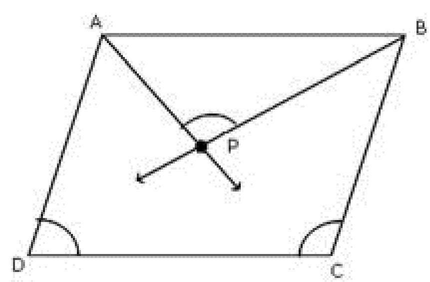
Prove that $\angle ACP = \angle QCD$.



20. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

OR

AP and BQ are the bisectors of angles A and B of a quadrilateral ABCD. Prove that
$2\angle APB = \angle C + \angle D$



21. 1500 families with 2 children were selected randomly and the following data was recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

i. 2 girls ii. 1 girl iii. No girl

22. A storehouse measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the storehouse.

OR

A cubical box with an edge of 10 cm and another cuboidal box having dimensions 12.5 cm long, 10 cm wide and 8 cm high, are to be compared. Which box has the greater lateral surface area and by how much?

Section D

(Questions 23 to 30 carry 4 marks each)

23. Simplify:

$$\left(\frac{16}{9}\right)^{-\frac{1}{2}} \div \left[\left(\frac{256}{81}\right)^{-\frac{1}{4}} + \frac{\sqrt{3}}{\sqrt{27}}\right]$$

OR

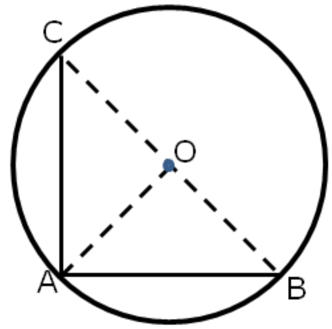
(a)Simplify: $\left\{5\left(8^{1/3} + 27^{1/3}\right)^3\right\}^{1/4}$

(b) Represent $\sqrt{7}$ on the number line.

24. Find the value of $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

25. Without actual division, prove that $2x^4 + x^3 - 14x^2 - 19x - 6$ is exactly divisible by $x^2 + 3x + 2$.

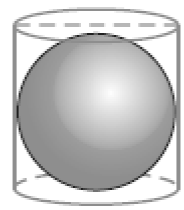
26. In the given figure, AB and AC are two equal chords of a circle with centre O. Show that O lies on the bisectors of $\angle BAC$.



OR

If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

27. A right circular cylinder just encloses a sphere of radius r.



Find the

i. Surface area of the sphere,

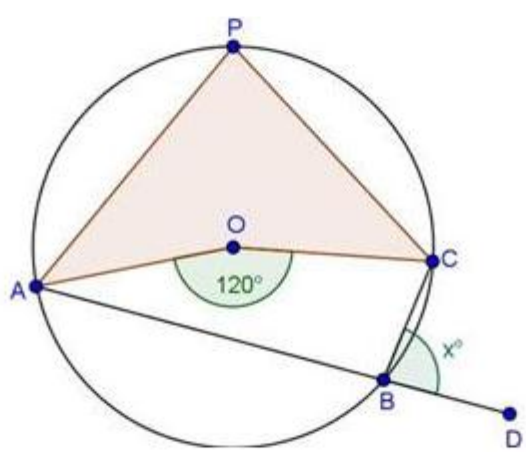
ii. Curved surface area of the cylinder,

iii. Ratio of the areas obtained in i. and ii.

OR

A spherical ball 28 cm in diameter is melted and recast into a right circular cone mould whose base is 35 cm in diameter. Find the height of the cone.

28. If O is the centre of the circle, find the value of x in the following figure.



29. Construct a right triangle in which one side is of length 4 cm and the difference between the hypotenuse and the other side is 2 cm.

30. Draw the graph of the linear equation $x + 2y = 8$. From the graph, check whether $(-1, -2)$ is a solution of this equation.

Class IX Mathematics

Sample Paper 10 – Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. $\left(\sqrt{5}+\sqrt{6}\right)^2=5+6+2\sqrt{30}=11+2\sqrt{30}$

   On comparing $a+b\sqrt{30}$ and $11+2\sqrt{30}$, we get $a=11$ and $b=2$.

2. Since, In the II Quadrant x-axis contains positive numbers and y-axis contains negative numbers.

   If for one of the solutions of the equation ax + by + c = 0, x is negative and y is positive, then a portion of the above line definitely lies in the **II Quadrant**.

   OR

   The co-ordinates (0, 1) and (1/2, 0) satisfy the equation 2x + y = 1. So the solutions are (0, 1) and (1/2, 0).

3. Here, Area of □ ABCD = Area of ΔABD + Area of ΔBCD

   $\therefore$ Area of ΔABD = 122 – 68 = 54

   Area of ΔABD = $\frac{1}{2}\times AB\times AD$

   $\therefore\ 54=\frac{1}{2}\times AB\times 9\Rightarrow AB=\frac{54\times 2}{9}$ = 12 cm

4. $7x^3-2x^2+3\sqrt{x}-4$ is not a Polynomial as the exponent of x in $3\sqrt{x}$ is not a positive integer.

5. Range = Highest age – Lowest age

   = (9 years, 3 months) – (7 years, 8 months)

   = (12 × 9 + 3) months – (7 × 12 + 8) months.........(1 year = 12 months)

   = (111 – 92) months

   = 19 months

   = (12 × 1 + 7) months

   = 1 year, 7 months

   OR

Let a be the lower limit of the class.
Hence a + 8 is the upper limit of the class.
Also,

$$\frac{\text{Upper limit} + \text{lower limit}}{2} = \text{Class Mark}$$

$$\Rightarrow \frac{a+(a+8)}{2} = 10$$

$$\Rightarrow a+(a+8) = 20$$

$$\Rightarrow 2a = 20-8$$

$$\Rightarrow 2a = 12$$

$$\Rightarrow a = 6$$

The lower limit of the class is 6.

6. No, Every Rectangle is not a Square.

Because in Square all sides are equal but in Rectangle opposite sides equal.

Section B

7. a = 2 + $\sqrt{3}$

$$\Rightarrow \frac{1}{a} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2} = 2 - \sqrt{3}$$

So, $a + \frac{1}{a} = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$

8. Let x = 75, y = –25, z = –50

x + y + z = 75 – 25 – 50 = 0

We know, if x + y + z = 0 then $x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow 75^3 - 25^3 - 50^3 = 3(75)(-25)(-50) = 281250$

OR

By factor theorem, we know that if x + a is a factor of a polynomial p(x), then p(-a) = 0

Given, $x+\sqrt{2}$ is a factor of $p(x) = x^2 + mx - 8$

$\Rightarrow \left(-\sqrt{2}\right)^2 - m\sqrt{2} - 8 = 0$

$\Rightarrow 2 - m\sqrt{2} - 8 = 0$

$\Rightarrow m\sqrt{2} = -6$

$\Rightarrow m = \frac{-6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-6\sqrt{2}}{2} = -3\sqrt{2}$

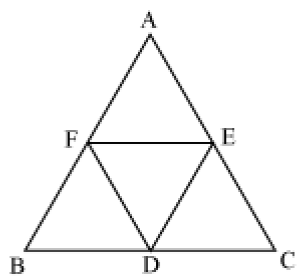
9. Since x = 1, y = 1 is the solution of 9kx + 12ky = 1, it will satisfy the equation.

$\therefore$ 9k(1) + 12k(1) = 63

$\therefore$ 9k + 12k = 63

$\therefore$ 21k = 63 $\Rightarrow$ k = 3

10. In ΔABC, E and F are midpoints of side AC and AB respectively.



By using the mid-point theorem we get,

EF || CB and EF $= \frac{1}{2}$(CB)

As D is the midpoint of CB $\Rightarrow$ BD $= \frac{1}{2}$(CB)

So, BD = EF

The line segments BF and DE join two parallel lines EF and BD of the same length. Hence the line segments BF and DE will also be parallel to each other and also equal in length. Therefore BDEF is a parallelogram.

11. Height (h) of cylindrical tank is 1 m

Base radius (r) of cylindrical tank = $\left(\frac{140}{2}\right)\text{cm} = 70 \text{ cm}$

Base radius (r) of cylindrical tank = 0.7 m

Surface area of cylinder $= 2\pi r[h + r] = 2 \times \frac{22}{7} \times 0.7[1 + 0.7] = 7.48 \text{ m}^2$

Therefore, it will require 7.48 m^2 of sheet.

OR

Area of cross section of pipe = 5 cm^2
Speed of water flowing out of the pipe = 30 cm/sec
Volume of water that flows out in 1 sec = 5 × 30 = 150 cm^3
Volume of water that flows out in 1 minute = 150 × 60 = 9000 cm^3 = 9 litres.

12. For x = 0,

$\pi(0) + y = 9 \Rightarrow y = 9$

So (0, 9) is a solution of this equation.

For x = 1,

$\pi(1) + y = 9 \Rightarrow y = 9 - \pi$

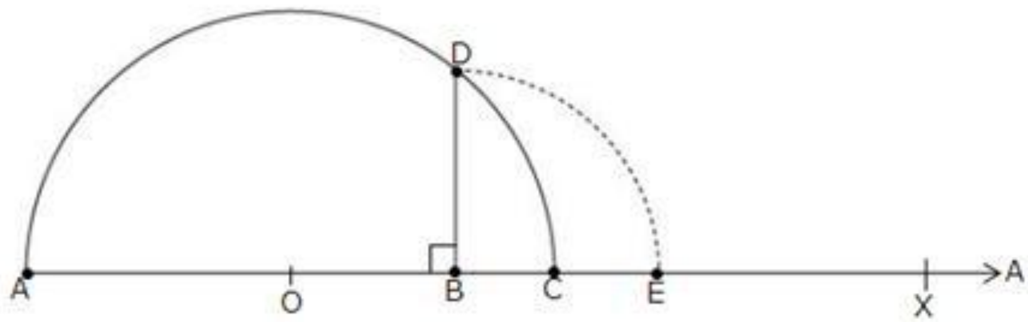
So, (1, 9 – π) is a solution of this equation.

Section C

13. Steps of construction:

1. Draw a line segment AB = 3.2 units and extend it to C such that BC = 1 units.
2. Find the midpoint O of AC. With O as centre and OA as radius, draw a semicircle.
3. Now, draw BD $\perp$ AC, intersecting the semicircle at D. Then, BD = $\sqrt{3.2}$ units.
4. With B as centre and BD as radius, draw an arc meeting AC produced at E.

   Then, BE = BD = $\sqrt{3.2}$ units.



OR

$$\frac{2\sqrt{7}+3\sqrt{5}}{\sqrt{7}+\sqrt{5}}$$
$$= \frac{2\sqrt{7}+3\sqrt{5}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$
$$= \frac{14-2\sqrt{35}+3\sqrt{35}-15}{\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2} \qquad \left[\because (a+b)(a-b)=a^2-b^2\right]$$
$$= \frac{\sqrt{35}-1}{7-5}$$
$$= \frac{\sqrt{35}-1}{2}$$
$$= \frac{1}{2}\sqrt{35}-\frac{1}{2}$$

14. $b^2 + c^2 + 2(ab + bc + ca)$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2$ [Adding and subtracting a^2]

$= [a^2 + b^2 + c^2 + 2ab + 2bc + 2ca] - a^2$

$= (a + b + c)^2 - (a)^2$ [Using $x^2 + y^2 + 2xy + 2yz + 2zx = (x + y + z)^2$]

$= (a + b + c + a)(a + b + c - a)$ [Because $a^2 - b^2 = (a+ b) (a - b)$]

$= (2a + b + c)(b + c)$

OR

Let $p(x) = x^3 - 23x^2 + 142x - 120$

Then $p(1) = (1)^3 - 23(1)^2 + 142(1) - 120 = 1 - 23 + 142 - 120 = 0$

Thus $(x - 1)$ is a factor of $p(x)$.

Now by long division

$$
\begin{array}{r}
x^2 - 22x + 120 \\
x-1 \overline{\smash{)} x^3 - 23x^2 + 142x - 120} \\
x^3 - x^2 \\
\hline
-22x^2 + 142x - 120 \\
-22x^2 + 22x \\
\hline
120x - 120 \\
120x - 120 \\
\hline
0
\end{array}
$$

Write the + and – signs appropriately.

Thus, $x^3 - 23x^2 + 142x - 120 = (x - 1)(x^2 - 22x + 120)$

$= (x - 1)(x^2 - 12x - 10x + 120)$

$= (x - 1)(x(x - 12) - 10(x - 12))$

$= (x - 1)(x - 12)(x - 10)$

15. $x = \left(3+\sqrt{8}\right) \Rightarrow \frac{1}{x} = \frac{1}{\left(3+\sqrt{8}\right)}$

$$\frac{1}{x} = \frac{1}{\left(3+\sqrt{8}\right)} \times \frac{\left(3-\sqrt{8}\right)}{\left(3-\sqrt{8}\right)} = \frac{\left(3-\sqrt{8}\right)}{\left(3^2-\left(\sqrt{8}\right)^2\right)} = \frac{\left(3-\sqrt{8}\right)}{(9-8)} = \left(3-\sqrt{8}\right)$$

$$x + \frac{1}{x} = \left(3+\sqrt{8}\right) + \left(3-\sqrt{8}\right) = 6$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 6^2 = 36$$

$$\therefore\ x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2x.\frac{1}{x} = \left(x + \frac{1}{x}\right)^2 - 2 = 36 - 2 = 34$$

16. i. In ΔAPD and ΔCQB

$\angle ADP = \angle CBQ$ (alternate interior angles for BC || AD)

AD = CB (opposite sides of parallelogram ABCD)

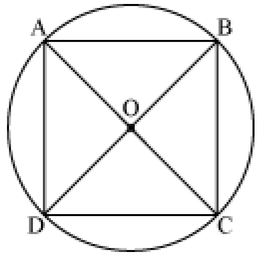
DP = BQ (given)

$\therefore \Delta APD \cong \Delta CQB$ (using SAS congruence rule)

ii. As observed $\Delta APD \cong \Delta CQ$

$\therefore$ AP = CQ (CPCT)

17. Let ABCD be a cyclic quadrilateral having diagonals BD and AC, intersecting each other at point O.



$m\angle BAD = \frac{1}{2} m\angle BOD = \frac{180^\circ}{2} = 90^\circ$ (Consider BD as a chord)

$m\angle BCD + m\angle BAD = 180^\circ$ (Cyclic quadrilateral)

$m\angle BCD = 180^\circ - 90^\circ = 90^\circ$

$m\angle ADC = \frac{1}{2} m\angle AOC = \frac{1}{2}(180^\circ) = 90^\circ$ (Considering AC as a chord)

$m\angle ADC + m\angle ABC = 180^\circ$ (Cyclic quadrilateral)

$90^\circ + m\angle ABC = 180^\circ$

$m\angle ABC = 90^\circ$

Here, each interior angle of cyclic quadrilateral is of 90°. Hence it is a rectangle.

18. Let us find the class marks x_i of each class by taking the average of the upper class limit and lower class limit and put them in a table.

We can use the Direct Method because numerical values of x_i and f_i are small.

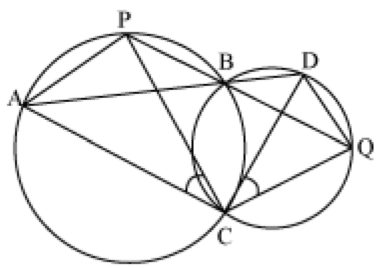
Class interval	No. of houses (f_i)	Class marks (x_i)	$f_i x_i$
0-2	1	1	1
2-4	2	3	6
4-6	1	5	5
6-8	5	7	35
8-10	6	9	54
10-12	2	11	22
12-14	3	13	39
Total	$\sum f_i = 20$		$\sum f_i x_i = 162$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$$

Thus, the mean number of plants per house is 8.1 plants

19. Given, two circles intersect at two points B and C.

Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.



To Prove: ∠ACP = ∠QCD.

Construction: Join chords AP and DQ

Proof: Consider chord AP,

∠PBA = ∠ACP (angles in the same segment) ... (1)

Consider chord DQ,

∠DBQ =∠QCD (angles in the same segment) ... (2)

ABD and PBQ are line segments intersecting at B.

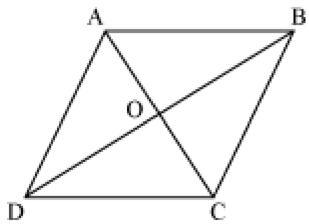
∴ ∠PBA = ∠DBQ (vertically opposite angles) ... (3)

From equations (1), (2), and (3), we obtain

∠ACP = ∠QCD

20. Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at right angle i.e. OA = OC, OB = OD and m∠AOB = m∠BOC = m∠COD = m∠AOD = 90°

To prove that ABCD is a rhombus, we need to prove that ABCD is a parallelogram and all sides of ABCD are equal.



Now, in ΔAOD and ΔCOD,

OA = OC (Diagonal bisects each other)

∠AOD = ∠COD (given)

OD = OD (common)

∴ ΔAOD ≅ ΔCOD (by SAS congruence rule)

∴ AD = CD (1)

Similarly we can prove that

AD = AB and CD = BC (2)

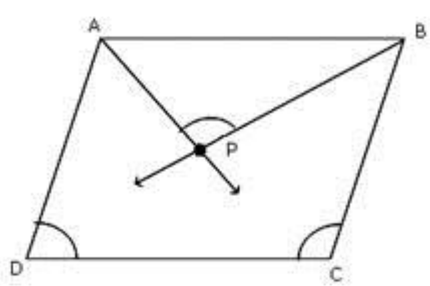
From equations (1) and (2), we can say that

AB = BC = CD = AD

Since opposite sides of quadrilateral ABCD are equal, so, we can say that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, so we can say that ABCD is a rhombus.

OR

Consider ΔAPB,



∠PAB + ∠PBA + ∠APB = 180° (angle sum property for a triangle)

⇒ 2∠PAB + 2∠PBA + 2∠APB = 360°

⇒∠A+ ∠B +2∠APB=360° … (i)

But in quadrilateral ABCD,

∠A + ∠B + ∠C + ∠D = 360°... (ii) (sum of the angles of a quadrilateral is 360 degrees)

From (i) and (ii) we get,

∠C + ∠D =2∠APB. Hence proved.

21. Total number of families = 475 + 814 + 211 = 1500

i. Number of families with 2 girls = 475

$$\therefore \text{Required probability} = \frac{\text{Number of families with 2 girls}}{\text{Total number of families}} = \frac{475}{1500} = \frac{19}{60}$$

ii. Number of families with 1 girl = 814

$$\therefore \text{Required probability} = \frac{\text{Number of families with 1 girl}}{\text{Total number of families}} = \frac{814}{1500} = \frac{407}{750}$$

iii. Number of families with no girl = 211

$$\therefore \text{Required probability} = \frac{\text{Number of families with no girls}}{\text{Total number of families}} = \frac{211}{1500}$$

22. Length (l_1) of the storehouse = 40 m

Breadth (b_1) of the storehouse = 25 m

Height (h_1) of the storehouse = 10 m

Volume of storehouse = $l_1 \times b_1 \times h_1 = (40 \times 25 \times 10)\ m^3 = 10000\ m^3$

Length (l_2) of a wooden crate = 1.5 m

Breadth (b_2) of a wooden crate = 1.25 m

Height (h_2) of a wooden crate = 0.5 m

Volume of a wooden crate = $l_2 \times b_2 \times h_2 = (1.5 \times 1.25 \times 0.5)\ m^3 = 0.9375 m^3$

Let n wooden crates be stored in the storehouse.

Volume of n wooden crates = volume of storehouse

0.9375 × n = 10000

$$\therefore\ n = \frac{10000}{0.9375} = 10666.66$$

Since, the number of crates cannot be a decimal number, we take the whole number part.

Thus, 10666 numbers of wooden crates can be stored in the storehouse.

OR

Edge of cube = 10 cm

Length (l) = 12.5 cm, Breadth (b) = 10 cm, Height (h) = 8 cm

Lateral surface area of cubical box $= 4(\text{edge})^2$

$= 4(10 \text{ cm})^2$

$= 400 \text{ cm}^2$

Lateral surface area of cuboidal box $= 2[lh + bh]$

$= [2(12.5 \times 8 + 10 \times 8)] \text{ cm}^2$

$= (2 \times 180) \text{ cm}^2$

$= 360 \text{ cm}^2$

Clearly, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box.

Lateral surface area of cubical box − Lateral surface area of cuboidal box

$= 400 \text{ cm}^2 - 360 \text{ cm}^2$

$= 40 \text{ cm}^2$

Thus, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box by 40 cm^2.

Section D

23.

$$\left(\frac{16}{9}\right)^{-\frac{1}{2}} \div \left[\left(\frac{256}{81}\right)^{-\frac{1}{4}} + \frac{\sqrt{3}}{\sqrt{27}}\right]$$

$$= \left(\frac{4^2}{3^2}\right)^{-\frac{1}{2}} \div \left[\left(\frac{4^4}{3^4}\right)^{-\frac{1}{4}} + \sqrt{\frac{3}{27}}\right]$$

$$= \left[\left(\frac{4}{3}\right)^2\right]^{-\frac{1}{2}} \div \left[\left\{\left(\frac{4}{3}\right)^4\right\}^{-\frac{1}{4}} + \sqrt{\frac{1}{9}}\right]$$

$$= \left(\frac{4}{3}\right)^{2\times\left(-\frac{1}{2}\right)} \div \left[\left(\frac{4}{3}\right)^{4\times\left(-\frac{1}{4}\right)} + \frac{1}{3}\right] \qquad \left[\left(a^x\right)^y = a^{xy}\right]$$

$$= \left(\frac{4}{3}\right)^{-1} \div \left[\left(\frac{4}{3}\right)^{-1} + \frac{1}{3}\right]$$

$$= \frac{3}{4} \div \left(\frac{3}{4} + \frac{1}{3}\right) \qquad \left[a^{-1} = \frac{1}{a}\right]$$

$$= \frac{3}{4} \div \frac{13}{12}$$

$$= \frac{3}{4} \times \frac{12}{13}$$

$$= \frac{9}{13}$$

OR

a)

$$\left\{5\left(2^{3\times\frac{1}{3}} + 3^{3\times\frac{1}{3}}\right)^3\right\}^{\frac{1}{4}}$$

$$= \left[5\left(2+3\right)^3\right]^{\frac{1}{4}}$$

$$= \left(5\times 5^3\right)^{\frac{1}{4}}$$

$$= 5^{4\times\frac{1}{4}}$$

$$= 5$$

b) In order to represent $\sqrt{7}$ on number line, we follow the steps given below:

Step 1: Draw a line and mark a point A on it.

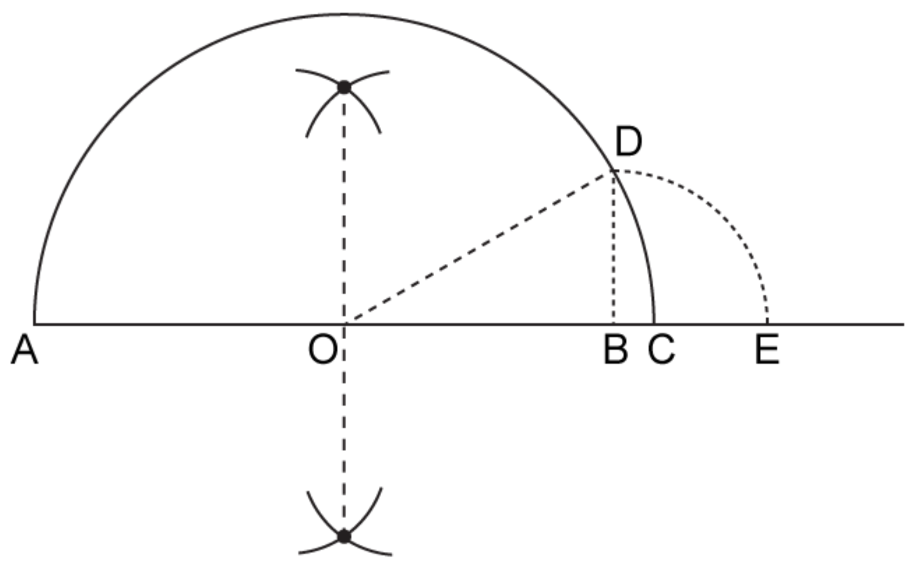
Step 2: Mark a point B on the line drawn in step 1 such that AB = 7 cm.

Step 3: Mark a point C on AB produced such that BC = 1 unit.

Step 4: Find mid-point of AC. Let the mid-point be O.

Step 5: Taking O as the centre and OC = OA as radius draw a semicircle. Then, draw a line passing through B perpendicular to OB. Let the perpendicular cut the semicircle at D.

Step6: Taking B as the centre and radius BD draw an arc cutting OC produced at E. Point E so obtained represents $\sqrt{7}$.



24.

$$\frac{1}{3-\sqrt{8}}=\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}=\frac{3+\sqrt{8}}{9-8}=3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}}=\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}=\frac{\sqrt{8}+\sqrt{7}}{8-7}=\sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}}=\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{\sqrt{6}+\sqrt{5}}{6-5}=\sqrt{6}+\sqrt{5}$$

$$\frac{1}{\sqrt{5}-2}=\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}\times 2}{\sqrt{5}+2}=\frac{\sqrt{5}+2}{5-4}=\sqrt{5}+2$$

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

$$=3+\sqrt{8}-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$

$$=5$$

25. Let $p(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ and $q(x) = x^2 + 3x + 2$

$q(x) = x^2 + 3x + 2 = (x + 1)(x + 2)$

Now, $p(-1) = 2(-1)^4 + (-1)^3 - 14(-1)^2 - 19(-1) - 6 = 2 - 1 - 14 + 19 - 6 = 21 - 21 = 0$

And, $p(-2) = 2(-2)^4 + (-2)^3 - 14(-2)^2 - 19(-2) - 6 = 32 - 8 - 56 + 38 - 6 = 70 - 70 = 0$

Therefore, $(x + 1)$ and $(x + 2)$ are the factors of $p(x)$, so $p(x)$ is divisible by $(x + 1)$ and $(x + 2)$.

Hence, $p(x)$ is divisible by $x^2 + 3x + 2$.

26. In $\triangle AOB$ and $\triangle AOC$,

OA = OA (common side)

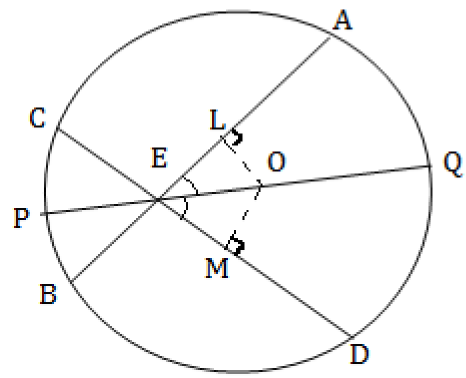
OB = OC (radius of the circle)

AB = AC (given)

$\therefore \triangle AOB \cong \triangle AOC$

Hence, $\angle OAC = \angle OAB$

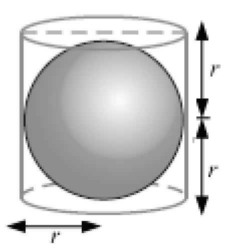
OR



Given that AB and CD are two chords of a circle with centre O, intersecting at a point E. PQ is the diameter through E, such that ∠AEQ = ∠DEQ.

To prove that AB = CD.
Draw perpendiculars OL and OM on chords AB and CD respectively.
Now, m ∠LOE = 180° – 90° – m ∠LEO ... [Angle sum property of a triangle]
= 90° – m ∠LEO
⇒ m ∠LOE = 90° – m ∠AEQ
⇒ m ∠LOE = 90° – m ∠DEQ
⇒ m ∠LOE = 90° – m ∠MEQ
⇒ ∠LOE = ∠MOE
In ΔOLE and ΔOME,
∠LEO = ∠MEO
∠LOE = ∠MOE
EO = EO
ΔOLE ≅ ΔOME
OL = OM
Therefore, cords AB and CD are equidistant from the centre.
Hence AB = CD.

27. A right circular cylinder just encloses a sphere of radius r then,



i. Surface area of sphere = $4\pi r^2$

ii. Height of cylinder = r + r = 2r

Radius of cylinder = r

C.S.A. of cylinder = $2\pi rh = 2\pi r\,(2r) = 4\pi r^2$

iii. Required ratio $= \dfrac{\text{Surface area of sphere}}{\text{CSA of cylinder}} = \dfrac{4\pi r^2}{4\pi r^2} = \dfrac{1}{1}$

OR

Diameter of a sphere = 28 cm

$\therefore$ Radius of a sphere, $R = \dfrac{28}{2} = 14$ cm

Diameter of a cone = 35 cm

$\therefore$ Radius of a cone, $r = \dfrac{35}{2} = 17.5$ cm

Now, volume of sphere = volume of cone

$$\therefore \frac{4}{3} \times \pi \times R^3 = \frac{1}{3} \times \pi \times r^2 \times h$$

$$\therefore \frac{4}{3} \times 14^3 = \frac{1}{3} \times \left(\frac{35}{2}\right)^2 \times h$$

$$\therefore \frac{4}{3} \times 14 \times 14 \times 14 = \frac{1}{3} \times \frac{35}{2} \times \frac{35}{2} \times h$$

$$\therefore h = \frac{4 \times 14 \times 14 \times 14 \times 3 \times 2 \times 2}{35 \times 35 \times 3} = \frac{896}{25} = 35.84 \text{ cm}$$

28. We have m∠AOC = 120°

By the degree measure theorem,

m∠AOC = 2m∠APC

$\therefore$ 120° = 2m∠APC

$\therefore$ m∠APC = 60°

Now, m∠APC + m∠ABC = 180° ....(Opposite angles of a cyclic quadrilateral)

$\therefore$ 60° + m∠ABC = 180°

$\therefore$ m∠ABC = 180° – 60° = 120°

m∠ABC + m∠DBC = 180° ....(Linear pair of angles)

$\therefore$ 120° + x° = 180°

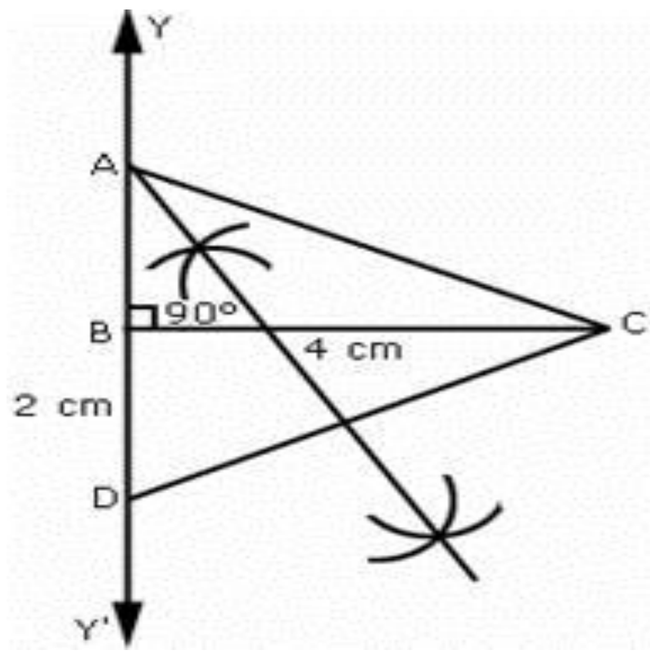
$\therefore$ x = 180° – 120° = 60°

29. Given: In ΔABC BC = 4 cm, m∠ABC = 90°, AC − AB = 2 cm

Steps of Construction:

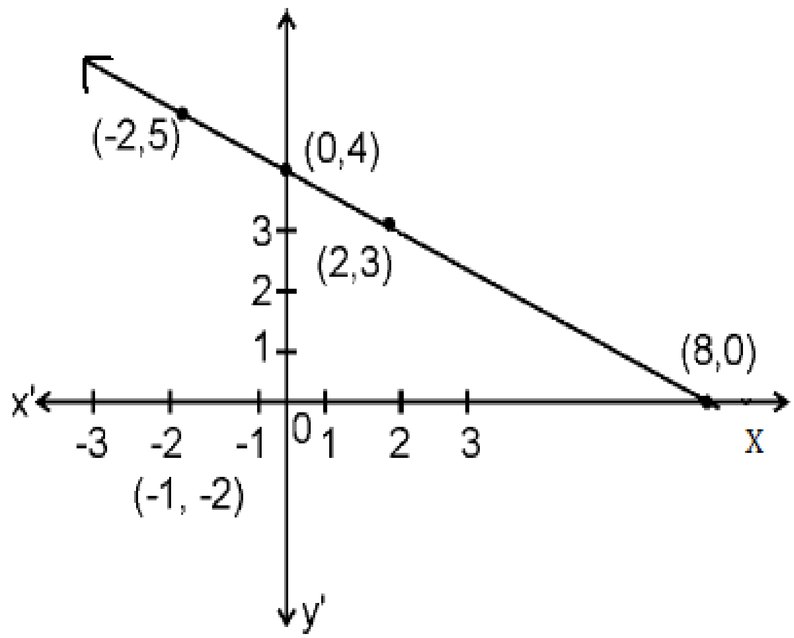
1. Draw BC = 4cm
2. Draw a ray BY such that m∠CBY = 90° and produce YB to form a line YBY'.
3. From ray BY' cut off DB = 2 cm
4. Join CD
5. Construct perpendicular bisector of CD intersecting BY at A
6. Join AC

ΔABC is the required triangle.



$\Rightarrow y = \frac{1}{2}(8 - x)$

x	−2	0	2
y	5	4	3



From the graph it is clear that (−1, −2) does not lie on the line.

Therefore, (−1, −2) is not a solution of line x + 2y = 8.