# Chapter - 2
# Polynomial

## Exercise – 2. 2

**Q. 1** Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) 6x2 - 3 - 7x

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x – 4$

**Solution:**

Zeroes of the polynomial are the values of the variable of the polynomial when the polynomial is put equal to zero.

Let p(x) be a polynomial with any number of terms any number of degree. Now, zeroes of the polynomial will be the values of x at which p(x) = 0. If $p(x) = ax^2 + bx + c$ is a quadratic polynomial (highest power is equal to 2) and its roots are α and β, then

Sum of the roots = $\alpha + \beta = - b/a$

Product of roots = $\alpha\beta = c/a$

(i) $p(x) = x^2 - 2x - 8$

So, the zeroes will be the values of x at which p(x) = 0.

Therefore,

$\Rightarrow x^2 - 4x + 2x - 8 = 0$

(We will factorize 2 such that the product of the factors is equal to 8 and difference is equal to 2)

$\Rightarrow$ x(x - 4) + 2(x - 4) = 0= (x - 4)(x + 2)

The value of $x^2$ - 2x - 8 is zero when x − 4 = 0 or x + 2 = 0,

i.e, x = 4 or x = −2

Therefore, The zeroes of $x^2$ - 2x - 8 are 4 and −2.

Sum of zeroes = 4 + (-2) = 2

$$= \frac{-(-2)}{1} = \frac{-(-coefficient\ of\ x)}{(coefficient\ of\ x^2)}$$

Hence, it is verified that, sum of Zeros $= \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$

Product of zeroes = $4\ +\ (-2) =\ -8 = \frac{(-8)}{1} = \frac{constant\ term}{coefficient\ of\ x^2}$

Hence, it is verified, Product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

(ii) $4s^2$ - 4s +1

= $(2s)^2$ - 2(2s)1 + $1^2$

As, we know $(a - b)^2 = a^2 - 2ab + b^2$, the above equation can be written as = $(2s - 1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, when, s = 1/2 , 1/2.

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(coefficient\ of\ s)}{coefficient\ of\ s^2}$

Product of zeroes =

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{constant\ term}{coefficient\ of\ s^2}$$

Hence Verified.

**(iii)** $6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

(We will factorize 7 such that the product of the factors is equal to 18 and the difference is equal to - 7)= $6x^2 + 2x - 9x - 3$ $= 2x(3x + 1) - 3(3x + 1) = (3x + 1)(2x - 3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$,

i.e. $x = \frac{-1}{3}\ or\ \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}\ or\ \frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$

Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{constant\ term}{coefficient\ of\ x^2}$

Hence, verified.

**(iv)** $4u^2 + 8u$

$= 4u^2 + 8u + 0$

$= 4u\ (u + 2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$,

i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and $-2$.

Sum of zeroes = $0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(coefficient\ of\ u)}{coefficient\ of\ u^2}$

Product of zeroes = $0 + (-2) = 0 = \frac{0}{4} = \frac{-constant\ term}{coefficient\ of\ u^2}$

**(v)** $t^2 – 15$

$= t^2 – (\sqrt{15})^2$

$= (t - \sqrt{15})(t + \sqrt{15})$ [As, $x^2 - y^2 = (x - y)(x + y)$]

The value of $t^2 - 15$ is zero when $(t - \sqrt{15}) = 0$ or $(t + \sqrt{15}) = 0$,

i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(coefficient\ of\ t)}{coefficient\ of\ t^2}$


$$\sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(coefficient\ of\ t)}{coefficient\ of\ t^2}$$


Product of zeroes =

$$(\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{-constant\ term}{coefficient\ of\ x^2}$$

Hence verified.

**(vi)** $3x^2 – x – 4$

(We will factorize 1 in such a way that the product of factors is equal to 12 and the difference is equal to 1) $= 3x^2 - 4x + 3x - 4$ $= x(3x - 4) + 1(3x - 4) = (3x – 4)(x + 1)$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$,

when $x = \frac{4}{3}$ $or$ $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1

Sum of zeroes = $\frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeroes = $\frac{4}{3}(-1) = \frac{-4}{3} = \frac{constant\ term}{coefficient\ of\ x^2}$

Hence, verified.

**Q. 2** Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$
(ii) $\sqrt{2}, \frac{1}{3}$
(iii) $0, \sqrt{5}$
(iv) $1, 1$
(v) $-\frac{1}{4}, \frac{1}{4}$
(vi) $4, 1$

**Solution:** If $\alpha, \beta$ are roots of an equation, then the quadratic form of this equation can be given by $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

(i) $\frac{1}{4}$ -1

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zeros are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be, $ax^2 + bx + c$, then

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$

$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

Let a = 4, then b = -1, c = -4

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be, $ax^2 + bx + c$, then

$\alpha + \beta = \sqrt{2} = \frac{-b}{a}$

and

If a = 3, then b = $-3\sqrt{2}$, and c = 1

Therefore, the quadratic polynomial is $3x2 - 3\sqrt{2}x + 1$

(iii) 0, $\sqrt{5}$

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial b $ax^2 + bx + c$, then

$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$

$\alpha\beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$

If a = 1, then b = 0, c = $\sqrt{5}$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) 1, 1

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be $ax^2 + bx + c$ , then

$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$

$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$

If a = 1, then b = -1, c = 1

Therefore, the quadratic polynomial is $x^2 - x + 1$

(v) $-\frac{1}{4}, \frac{1}{4}$

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be $ax^2 + bx + c$, then

$\alpha + \beta = 1 = \frac{-1}{4} = \frac{-b}{a}$

$\alpha\beta = \frac{1}{4} = \frac{c}{a}$

If a = 4, then b = 1, c = 1

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

(vi) 4, 1

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be $ax^2 + bx + c$, then

$$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$$

$$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If a = 1, then b = -4, c = 1

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.