13. Volume and Surface Area

Exercise 13A

1. Question

Find the volume, the lateral surface area and the total surface area of the cuboid whose dimensions are:

(i) length = 12 cm, breadth = 8 cm and height = 4.5 cm

(ii) length = 26 m, breadth = 14 m and height = 6.5 m

(iii) length = 15 m, breadth = 6 m and height = 5 dm

(iv) length = 24 m, breadth = 25 cm and height = 6 m

Answer

(i) length = 12 cm, breadth = 8 cm and height = 4.5 cm

Volume of cuboid = (length × breadth × height) = (12 × 8 × 4.5) = 432 cm^3

Lateral surface area of cuboid = 2(length + breadth) × height = 2(12 + 8) × 4.5 = 180 cm^2

Total surface area of cuboid = 2(length × breadth + breadth × height + height × length)

= 2(12 × 8 + 8 × 4.5 + 4.5 × 12) = 2(96 + 36 + 54) = 2 × 186 = 372 cm^2

(ii) length = 26 m, breadth = 14 m and height = 6.5 m

Volume of cuboid = (length × breadth × height) = (26 × 14 × 6.5) = 2366 m^3

Lateral surface area of cuboid = 2(length + breadth) × height = 2(26 + 14) × 6.5 = 520 m^2

Total surface area of cuboid = 2(length × breadth + breadth × height + height × length)

= 2(26 × 14 + 14 × 6.5 + 6.5 × 26) = 2 × 624 = 1248 m^2

(iii) length = 15 m, breadth = 6 m and height = 5 dm = (0.5m)

Volume of cuboid = (length × breadth × height) = (15 × 6 × 0.5) = 45 m^3

Lateral surface area of cuboid = 2(length + breadth) × height = 2(15 + 6) × 0.5 = 21 m^2

Total surface area of cuboid = 2(length × breadth + breadth × height + height × length)

= 2(15 × 6 + 6 × 0.5 + 0.5 × 15) = 2(90 + 3.0 + 7.5) = 2 × 100.5 = 201 m^2

(iv) length = 24 m, breadth = 25 cm and height = 6 m

Volume of cuboid = (length × breadth × height) = (24 × 0.25 × 6) = 36 m^3

Lateral surface area of cuboid = 2(length + breadth) × height = 2(24 + 0.25) × 6 = 291 m^2

Total surface area of cuboid = 2(length × breadth + breadth × height + height × length)

= 2(24 × 0.25 + 0.25 × 6 + 6 × 24) = 303 m^2

2. Question

Find the capacity of a closed rectangular cistern whose length is 8 m, breadth 6 m and depth 2.5 m. Also, find the area of the iron sheet required to make the cistern.

Answer

Given,

Dimensions of closed rectangular cistern = 8m × 6m × 2.5 m

∴ Capacity of tank = volume of tank = (l × b × h) = 8 × 6 × 2.5 = 120 m^3

Area of iron sheet required to make the tank = 2(lb + bh + hl) = 2(8 × 6 + 6 × 2.5 + 2.5 × 8)=2(48 + 15 + 20)= 2 × 83 = 166m^2

3. Question

The dimensions of a room are $(9\ m \times 8\ m \times 6.5\ m)$. It has one door of dimensions $(2\ m \times 1.5\ m)$ and two windows, each of dimensions $(1.5\ m \times 1\ m)$. Find the cost of white washing the walls at Rs. 6.40 per square metre.

Answer

Given,

Dimensions of room = 9m × 8m × 6.5m

Area of 4 walls = 2 (length + breadth) × height = 2 (9 + 8) × 6.5 = 13 × 17 = 221 m^2

Dimensions of one door = 2m × 1.5m

Area of door = length × breadth = 2 × 1.5 = 3.0 m^2

Dimensions of windows = 1.5m × 1m

Area of 2 windows = 2 (l × b) = 2 (1.5 × 1) = 3.0 m^2

Hence,

Area required for white-washing = Area of 4 walls – (area of door + area of 2 windows)

= 221 – (3 + 3) = 221 – 6 = 215 m^2

∵ cost of white-washing 1 m^2 area = Rs. 6.40

∴ cost of white-washing 215 m^2 = 6.40 × 215 = Rs. 1376.

4. Question

How many planks of dimensions (5m × 25cm × 10cm) can be stored in a pit which is 20 m long, 6 m wide and 80 cm deep?

Answer

Given,

Dimensions of plank;

l = 5m

b = 25cm = 0.25 m

h = 10cm = 0.10 m

Dimensions of pit;

l = 20m

b = 6m

h = 80m

$$number\ of\ planks = \frac{volume\ of\ pit}{volume\ of\ plank} = \frac{20 \times 6 \times 0.80}{5 \times 0.25 \times 0.10} = 768$$

5. Question

How many bricks will be required to construct a wall 8 m long, 6 m high and 22.5 cm thick if each brick measures (25 cm × 11.25 cm × 6 cm)?

Answer

Given,

Dimensions of wall = 8m × 6m × 22.5 cm = 800 cm × 600 cm × 22.5 cm

Dimensions of each brick = 25 cm × 11.25cm × 6 cm

Hence,

Number of bricks required = $\frac{volume\ of\ wall}{volume\ of\ one\ brick} = \frac{800 \times 600 \times 22.5}{25 \times 11.25 \times 6} = 6400\ bricks.$

6. Question

A wall 15 m long, 30 cm wide and 4 m high is made of bricks, each measuring $(22\ cm \times 12.5\ cm \times 7.5\ cm)$. If $\frac{1}{12}$ of the total volume of the wall consists of mortar, how many bricks are there in the wall?

Answer

Given,

Dimensions of wall = 15m × 30cm × 4m = 1500 cm × 30 cm × 400 cm

Dimensions of each brick = 22 cm × 12.5 cm × 7.5 cm

Volume of wall = l × b × h = 1500 × 30 × 400 = 180000000 cm^3

Area of mortar = 1/12 × volume of wall = $\frac{1}{12} \times 180000000 = 15000000\ cm^3$

Hence,

Area occupied by bricks only = 180000000 – 15000000 = 165000000 cm^3

Number of bricks required = $\frac{volume\ for\ bricks\ only}{volume\ of\ one\ brick} = \frac{165000000}{22 \times 12.5 \times 7.5} = 8000\ bricks.$

7. Question

An open rectangular cistern when measured from outside is 1.35 m long, 1.08 m broad and 90 cm deep. It is made up of iron, which is 2.5 cm thick. Find the capacity of the cistern and the volume of the iron used.

Answer

Given,

External Dimensions of cistern = 1.35m × 1.08m × 90cm = 135cm × 108cm × 90cm

External volume of cistern = l × b × h = 135 × 108 × 90 = 1312200 cm^3

Internal dimensions of cistern = length = 135 – (2.5 × 2) = 130 cm

Breadth = 108 – (2.5 × 2) = 103 cm

Height = 90 – 2.5 = 87.5 cm

∴ internal volume of cistern = 130 × 103 × 87.5 = 1171625 cm^3

Volumeof iron used = (External volume – Internal volume)

= 1312200 – 1171625 = 140575 cm^3

8. Question

A river 2 m deep and 45 m wide is flowing at the rate of 3 km per hour. Find the volume of water that runs into the sea per minute.

Answer

Given,

Depth of river (h) = 2 m

Breadth of river (b) = 45 m

Rate of flowing = 3 km/h

∴ Length = $\frac{3000}{60}$ meter/min.

Volume of water = l × b × h = $\frac{3000}{60}$ × 2 × 45 = 90 × 50 = 4500 m^3

9. Question

A box made of sheet metal costs Rs 1620 at Rs 30 per square metre. If the box is 5 m long and 3 m wide, find its height.

Answer

Given,

Total cost of box made of sheet metal = Rs. 1620

Cost of per square meter metal = Rs. 30

∴ Area of box = $\frac{1620}{30}$ = 54 m^2

Dimensions of box = 5m × 3m × height

Let height of box = h meter

Total surface area of sheet = 2 (lb + bh + hl)

= 54 = 2 (5 × 3 + 3h + 5h)

= $\frac{54}{2}$ = 15 + 8h

= 8h = 27 – 15 = 12

= h = $\frac{12}{8} = 1.5\ m$

Height of box = 1.5 meter.

10. Question

Find the length of the longest pole that can be put in a room of dimensions $(10\ m \times 10\ m \times 5\ m)$.

Answer

Given,

Dimensions of room = 10m × 10m × 5m

∴ length of longest pole can be put in room = diagonal of room

= $\sqrt{l^2 + b^2 + h^2} = \sqrt{10^2 + 10^2 + 5^2} = \sqrt{225} = 15\ m$

11. Question

How many person can be accommodated in a dining hall of dimensions $(20\ m \times 16\ m \times 4.5\ m)$, assuming that each person requires 5 cubic metres of air?

Answer

Given,

Dimensions of dining hall = 20m × 16m × 4.5m

Volume of hall = 20 × 16 × 4.5 = 1440 m^3

Volume of air required by one person = 5 m^3

∴ Number of persons in hall = $\frac{volume\ of\ hall}{volume\ of\ air\ required\ by\ one\ person} = \frac{1440}{5} = 288\ persons.$

12. Question

A classroom is 10 m long, 6.4 m wide and 5 m high. If each student be given $1.6\ m^2$ of the floor area, how many students can be accommodated in the room? How many cubic metres of air would each student get?

Answer

Given,

Dimensions of classroom = 10m × 6.4m × 5m

Area of room = length × breadth = 10 × 6.4 = 64 m^2

Area of floor required by one student = 1.6 m^2

∴ Number of students can sit in classroom = $\frac{Area\ of\ floor}{area\ required\ by\ one\ student} = \frac{64}{1.6} = 40\ students.$

Volume of classroom = 10 × 6.4 × 5 m^3

Air required by each student = $\frac{volume\ of\ room}{number\ of\ students} = \frac{10 \times 6.4 \times 5}{40} = 8\ m^3$

13. Question

The volume of a cuboid is $1536\ m^3$.Its length is 16 m, and its breadth and height are in the ratio 3:2. Find the breadth and height of the cuboid.

Answer

Given,

Volume of cuboid = 1536 m^3

Length of cuboid = 16 m

Ratio of breadth and height = 3 : 2

Let breadth = 3x

Let breadth = 2x

∴ Volume of cuboid = l × b × h

= 1536 = 16 × 3x × 2x

= $6x^2 = \frac{1536}{16} = 96$

= $x^2 = \frac{96}{6} = 16$

= x = $\sqrt{16}$ = 4

Hence,

Breadth of cuboid = 3x = 3 × 4 = 12m

Height of cuboid = 2x = 2 × 4 = 8m

14. Question

The surface area of a cuboid is $758\ cm^2$. Its length and breadth are 14 cm and 11 cm respectively. Find its height.

Answer

Given,

Surface area of cuboid = 758 cm^2

Length of cuboid = 14 cm

Breadth of cuboid = 11 cm

Let height of cuboid = h cm

Total surface area of cuboid = 2 (lb + bh + hl)

= 758 = 2 (14 × 11 + 11h + 14h)

= 154 + 25h = $\frac{758}{2} = 379$

= 25h = 379 – 154 = 225

= h = $\frac{225}{25} = 9$

Height of cuboid = 9 meter.

15. Question

Find the volume, the lateral surface area, the total surface area and the diagonal of a cube, each of whose edges measures (a) 9m, (b) 6.5 cm. [Take $\sqrt{3} = 1.73$.]

Answer

Given,

a) Edge of cube (a) = 9m

Volume of cube = $a^3 = 9^3 = 729\ m^3$

Lateral surface area of cube = $4a^2 = 4 \times 9^2 = 4 \times 81 = 324\ m^2$

Total surface area of cube = $6a^2 = 6 \times 9^2 = 6 \times 81 = 486\ m^2$

Diagonal of cube = $\sqrt{3}$ a = $\sqrt{3} \times 9 = 1.73 \times 9 = 15.57$ m

b) Edge of cube (a) = 6.5 cm

Volume of cube = $a^3 = 6.5^3 = 274.625\ cm^3$

Lateral surface area of cube = $4a^2 = 4 \times 6.5^2 = 4 \times 42.25 = 169\ cm^2$

Total surface area of cube = $6a^2 = 6 \times 6.5^2 = 6 \times 42.25 = 253.5\ cm^2$

Diagonal of cube = $\sqrt{3}$ a = $\sqrt{3} \times 6.5 = 1.73 \times 6.5 = 11.245$ cm

16. Question

The total surface area of a cube is $1176\ cm^2$. Find its volume.

Answer

Given,

Total surface area of cube = $1176\ cm^2$

Let edge of cube = a cm

$= 6a^2 = 1176$

$= a^2 = \frac{1176}{6} = 196$

$= a = \sqrt{196} = 14$ cm

$\therefore$ Volume of cube = $a^3 = 14^3 = 2744\ cm^3$

17. Question

The lateral surface area of a cube is $900\ cm^2$. Find its volume.

Answer

Given,

Lateral surface area of cube = $900\ cm^2$

Let edge of cube = a cm

$4a^2 = 900$

$= a^2 = \frac{900}{4} = 225$

$= a = \sqrt{225} = 15\ cm$

Volume of cube = $a^3 = 15^3 = 3375\ cm^3$

18. Question

The volume of a cube is $512\ cm^3$. Find its surface area.

Answer

Given

Volume of cube = $512\ cm^3$

Let edge of cube = a cm

So,

$= a^3 = 512$

$= a = \sqrt[3]{512} = 8$ cm

Total surface area of cube $= 6a^2 = 6 \times 8 \times 8 = 384$ cm^2

19. Question

Three cubes of metal with edges 3 cm, 4 cm and 5 cm respectively are melted to form a single cube. Find the lateral surface area of the new cube formed.

Answer

Given,

Edge of three cubes $a_1 = 3$ cm , $a_2 = 4$ cm , $a_3 = 5$ cm

Let edge of single cube formed = A cm

Sum of volume of three cubes = volume of single cube formed

$= a_1^3 + a_2^3 + a_3^3 = A^3$

$= 3^3 + 4^3 + 5^3 = A^3$

$A^3 = 27 + 64 + 125 = 216$

$A = \sqrt[3]{216} = 6\ cm$

Lateral surface area of new cube $= 4a^2 = 4 \times 6 \times 6 = 144$ cm^2

20. Question

In a shower, 5 cm of rain falls. Find the volume of water that falls on 2 hectares of ground.

Answer

Given,

Area of field = 2 hectare = 20000 m^2

Height of rainfall = 5 cm = 0.05 m

Volume of water that falls = Area × height

$= 20000 \times 0.05 = 1000$ m^3

Exercise 13B

1. Question

Find the volume and curved surface area of a right circular cylinder of height 21 cm and base radius 5 cm.

Answer

Given,

Height of cylinder = 21 cm

Radius of base = 5 cm

∴ volume of right circular cylinder = $\pi r^2 h = \frac{22}{7} \times 5 \times 5 \times 21 = 1650\ cm^3$

Curved surface area = $2\pi rh = 2 \times \frac{22}{7} \times 5 \times 21 = 660\ cm^2$

2. Question

The diameter of a cylinder is 28 cm and its height is 40 cm. Find the curved surface area, total surface area and the volume of the cylinder.

Answer

Given,

Diameter of cylinder = 28 cm

Height of cylinder = 40 cm

Radius of cylinder = $\frac{diameter}{2} = \frac{28}{2} = 14\ cm$

∴ Curved surface area of cylinder = $2\pi rh = 2 \times \frac{22}{7} \times 14 \times 40 = 44 \times 40 \times 2 = 3520\ cm^2$

∴ total surface area of cylinder = $2\pi rh + 2\pi r^2 = 2\pi r(h + r) = 2 \times \frac{22}{7} \times 14 \times 54 = 88 \times 54 = 4752\ cm^2$

∴ Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 40 = 24640\ cm^3$

3. Question

Find the weight of a solid cylinder of radius 10.5 cm and height 60 cm if the material of the cylinder weigh 5 g per cm^3.

Answer

Given,

Radius of cylinder = 10.5 cm

Height of cylinder = 60 cm

∴ Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times 10.5 \times 10.5 \times 60 = 20790\ cm^3$

∴ Weight of cylinder = volume of cylinder × wt. of cylinder per gram

= 20790 × 5 g = 103950 g = 103.95 kg

4. Question

The curved surface area of a cylinder is $1210\ cm^2$ and its diameter is 20 cm. Find its height and volume.

Answer

Given,

Curved surface area of cylinder = 1210 cm^2

Diameter of cylinder = 20 cm

Radius of cylinder = $\frac{20}{2}$ = 10 cm

Let height of cylinder = h cm

Curved surface area = 2πrh

= 2πrh = 1210

= 2 × $\frac{22}{7}$ × 10 × h = 1210

= h = $\frac{1210 \times 7}{44 \times 10} = 19.25$ cm

∴ Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times 10 \times 10 \times 19.25 = 6050\ cm^3$

5. Question

The curved surface area of a cylinder is $4400\ cm^2$ and the circumference of its base is 110 cm. Find the height and the volume of the cylinder.

Answer

Given,

Curved surface area of cylinder = 4400 cm^2

Circumference of its base = 110 cm

2πr = 110

= r = $\frac{110}{2\pi} = \frac{110 \times 7}{44} = \frac{35}{2} cm$

Let height of cylinder = h cm

C.S.A = 4400

2πrh = 4400

= 2 × $\frac{22}{7}$ × $\frac{35}{2}$ × h = 4400

= h = $\frac{4400 \times 7 \times 2}{44 \times 35} = 40\ cm$

∴ Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 40 = 110 \times 350 = 38500\ cm^3$

6. Question

The radius of the base and the height of a cylinder are in the ratio 2:3. If its volume is $1617\ cm^3$, find the total surface area of the cylinder.

Answer

Given,

Volume of cylinder = 1617 cm^3

Ratio of radius of base and height = 2 : 3

Let base radius = 2x cm

Let height = 3x cm

Volume = $\pi r^2 h$

$= \frac{22}{7} \times 4x^2 \times 3x = 1617$

$= x^3 = \frac{1617 \times 7}{22 \times 12}$

$= x^3 = 42.875$

$= x = \sqrt[3]{42.875} = 3.5\ cm$

Hence,

Radius of cylinder = 2 × 3.5 = 7 cm

Height of cylinder = 3 × 3.5 = 10.5 cm

Total surface area of cylinder = $2\pi rh + 2\pi r^2 = 2\pi r\ (h + r) = 2 \times \frac{22}{7} \times 7 \times 17.5 = 770\ cm^2$

7. Question

The total surface area of a cylinder is $462\ cm^2$. Its curved surface area is one-third of its total surface area. Find the volume of the cylinder.

Answer

Given,

Total surface area of cylinder = 462 cm^2

$2\pi r\ (h + r) = 462$

$\Rightarrow r\ (h + r) = \frac{462}{2\pi}$

$\Rightarrow r^2 + rh = \frac{(462 \times 7)}{44} = \frac{(21 \times 7)}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

$CSA = \frac{1}{3}\ TSA\ (given)$

$2\pi rh = \frac{1}{3} \times 462 = 154$

$\Rightarrow rh = \frac{154}{2\pi} = \frac{(154 \times 7)}{44} = \frac{49}{2} \ldots\ldots\ldots\ldots\ldots (ii)$

Putting value of rh in equation (i)

$\Rightarrow r^2 + \frac{49}{2} = \frac{147}{2}$

$\Rightarrow r^2 = \frac{147}{2} - \frac{49}{2} = \frac{98}{2} = 49$

$\Rightarrow r = \sqrt{49} = 7\ cm$

From (ii)

$\Rightarrow rh = \frac{49}{2}$

$\Rightarrow h = \frac{49}{2 \times 7} = \frac{7}{2} cm$

$\therefore$ Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times \frac{7}{2} = 532\ cm^3$

8. Question

The total surface area of a solid is $231\ cm^2$ and its curved surface area is $\frac{2}{3}$ of the total surface area. Find the volume of the cylinder.

Answer

Given,

Total surface area of solid = $231\ cm^2$

$2\pi r(h + r) = 231$

$\Rightarrow r\ (r + h) = \frac{(231 \times 7)}{44}$

$\Rightarrow r^2 + rh = \frac{231 \times 7}{44} \ldots\ldots\ldots\ldots\ldots\ldots (i)$

$CSA = \frac{2}{3} TSA\ given$

$2\pi rh = \frac{2}{3} \times 231$

$\Rightarrow rh = \frac{2 \times 231 \times 7}{2 \times 3 \times 22} = \frac{49}{2} \ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Putting value of rh in (i) we get,

$\Rightarrow r^2 + \frac{49}{2} = \frac{231 \times 7}{44}$

$\Rightarrow r^2 = \frac{1617}{44} - \frac{49}{2} = \frac{1617 - 1078}{44} = \frac{539}{44} = 12.25$

⇒ r = $\sqrt{12.25} = 3.5\ cm$

From equation (ii)

⇒ rh = $\frac{49}{2}$

⇒ h = $\left(\frac{49}{3.5 \times 2}\right) = 7\ cm$

∴ Volume of cylinder = πr^{2}h = $\frac{22}{7}$ × 3.5 × 3.5 × 7 = 269.5 cm^3

9. Question

The sum of the height and radius of the base of a solid cylinder is 37 m. If the total surface area of the cylinder be 1628 m^2, find its volume.

Answer

Given,

Total surface area of cylinder = 1628 m^2

Sum of height and radius = (h + r) = 37 m

2πr (h + r) = 1628

2πr × 37 = 1628

⇒ r = $\frac{1628 \times 7}{2 \times 22 \times 37} = 7\ m$

∵ r + h = 37

⇒ h = 37 − 7 = 30 m

∴ Volume of cylinder = πr^{2}h = $\frac{22}{7}$ × 7 × 7 × 30 = 4620 m^3

10. Question

The ratio between the curved surface area and the total surface area of a right circular cylinder is 1:2. Find the volume of the cylinder if its total surface area is 616 cm^2.

Answer

Given,

Total surface area of cylinder = 616 cm^2

⇒ $\frac{C.S.A}{T.S.A} = \frac{1}{2}$

⇒ $\frac{2\pi rh}{\{2\pi r(h+r)\}} = \frac{1}{2}$

⇒ $\frac{h}{h+r} = \frac{1}{2}$

⇒ 2h = r + h

⇒ h = r.....(i)

2πr (h + r) = 616

⇒ r (r + r) = $\frac{616 \times 7}{44} = 198$

⇒ $r^2 = \frac{198}{2} = 49$

⇒ r = $\sqrt{49} = 7\ cm$

H = 7 cm

∴ Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 7 = 22 \times 49 = 1078\ cm^3$

11. Question

$1\ cm^3$ of gold is drawn into a wire 0.1 mm in diameter. Find the length of the wire.

Answer

Given,

Diameter of wire = 0.1 mm = 0.01 cm

Radius of wire = $\frac{0.01}{2}\ cm$

Volume of gold = $1\ cm^3$

⇒ $\pi r^2 h = 1$

⇒ $\frac{22}{7} \times \left(\frac{0.01}{2}\right)^2 \times h = 1$

⇒ h = $\frac{200 \times 200 \times 7}{22 \times 1 \times 1}$ = 12727.27 cm or 127.27 m

Length of wire = 127.27 meter.

12. Question

The radii of two cylinders are in the ratio 2:3 and their heights are in the ratio 5:3. Calculate the ratio of their volumes and the ratio of their curved surfaces.

Answer

Given,

Ratio of radii of two cylinders = $R_1 : R_2 = 2 : 3$

Ratio of their heights = $H_1 : H_2 = 5 : 3$

∴ Ratio of volumes of cylinders = $\frac{V_1}{V_2} = \frac{\pi R_1^2 H_1}{\pi R_2^2 H_2} = \frac{4 \times 5}{9 \times 3} = \frac{20}{27}\ OR\ 20 : 27.$

∴ Ratio of their curved surface area = $\frac{A_1}{A_2} = \frac{2\pi R_1 H_1}{2\pi R_2 H_2} = \frac{2 \times 5}{3 \times 3} = \frac{10}{9}\ or\ 10 : 9.$

13. Question

A powder tin has a square base with side 12 cm and height 17.5 cm. another is cylindrical with diameter of its base 12 cm and height 17.5 cm. Which has more capacity and by how much?

Answer

Given,

Side of square base = 12 cm

Height = 17.5 cm

Volume of tin = lbh = 12 × 12 × 17.5 = 2520 cm^3

Diameter of cylindrical base = 12 cm

Radius = $\frac{12}{2} = 6\ cm$

Height of cylinder = 17.5 cm

Volume of tin in cylinder = $\pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 17.5 = \frac{13860}{7} = 1980\ cm^3$

Hence,

Capacity of square tin is more by = 2520 – 1980 = 540 cm^3

14. Question

A cylindrical bucket, 28 cm in diameter and 72 cm high, is full of water. The water is emptied into a rectangular tank, 66 cm long and 28 cm wide. Find the height of the water level in the tank.

Answer

Given,

Diameter of cylindrical bucket = 28 cm

Radius of bucket = $\frac{28}{2} = 14\ cm$

Height of bucket = 72 cm

Volume of water in bucket = $\pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 72\ cm^3$

Length of rectangular tank = 66 cm

Width of tank = 28 cm

Let rise in water level in rectangular tank = h cm

∵ Volume of cylinder = Volume of rectangular tank

$\Rightarrow \frac{22}{7} \times 14 \times 14 \times 72$ = 66 × 28 × h

$\Rightarrow h = \frac{22 \times 14 \times 14 \times 72}{7 \times 66 \times 28} = 24\ cm.$

15. Question

If $1\ cm^3$ of cast iron weighs 21 g, find the weight of a cast iron pipe of length 1 m with a bore of 3 cm in which the thickness of the metal is 1 cm.

Answer

Given,

Weight of 1 cm^3 cast iron = 21 g

Length of wire = h = 1 m = 100 cm

Internal radius (r_1) = $\frac{3}{2} = 1.5\ cm$

Thickness of metal = 1 cm

So, External radius (r_2) = 1.5 + 1 = 2.5 cm

Volume of metal = (External volume – internal volume)

= $\pi r_2^2 h - \pi r_1^2 h = \pi h (r_2^2 - r_1^2) = \frac{22}{7} \times 100\ (2.5 + 1.5)\ (2.5 - 1.5)$

= $\frac{22}{7} \times 100 \times 4 \times 1\ cm^3$

Weight of metal = $\frac{22}{7} \times 100 \times 4 \times 1 \times 21 = 26400\ gm = 26.4\ kg.$

16. Question

A cylindrical tube, open at both ends, is made of metal. The internal diameter of the tube is 10.4 cm and its length is 25 cm. The thickness of the metal is 8 mm everywhere. Calculate the volume of the metal.

Answer

Given,

Internal diameter of tube = 10.4 cm

Internal radius of tube = $\frac{10.4}{2} = 5.2\ cm$

Thickness of metal = 8 mm = 0.8 cm

External radius of tube = 5.2 + 0.8 = 6 cm

Length of tube = 25 cm

∴ Volume of metal = (external volume – internal volume)

= $\pi h (6^2 - 5.2^2) = \frac{22}{7} \times 25 \times 11.2 \times 0.8 = 22 \times 32 = 704\ cm^3$

17. Question

The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used up on writing 33o words on an average. How many words would use up a

bottle of ink containing one-fifth of a litre?

Answer

Given,

Length of cylindrical barrel (h) = 7 cm

Diameter = 5 mm

Radius = $\frac{5}{2} = 2.5\ mm = 0.25\ cm$

Volume of cylindrical barrel = $\pi r^2 h = \frac{22}{7} \times 0.25 \times 0.25 \times 7 = \frac{5.5}{4} cm^3$

$\because \frac{5.5}{4}$ cm^3 volume of barrel is used for writing = 330 words

$\therefore \frac{5.5}{4} \times 1000\ cm^3$ will be used for writing = $330 \times \frac{4}{5.5} \times \frac{1}{8} \times 1000 = 48000\ words$

18. Question

A lead pencil consists of a cylinder of wood with a solid cylinder of graphite fitted into it. The diameter of the pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 10 cm. calculate the weight of the whole pencil, if the specific gravity of the wood is $0.7\ g/cm^3$ and that of the graphite is $2.1\ g/cm^3$.

Answer

Given,

Diameter of pencil = 7 mm

Radius of pencil = $\frac{7}{2}\ mm = \frac{0.7}{2} cm$

Diameter of graphite = 1 mm

Radius of graphite = $\frac{1}{2} mm = \frac{0.1}{2} cm$

Volume of graphite = $\pi r^2 h = \frac{22}{7} \times \frac{0.1}{2} \times \frac{0.1}{2} \times 10 = \frac{0.55}{7} cm^3$

Weight of graphite = volume × specific gravity of graphite

$= \frac{0.55}{7} \times 2.1 = 0.165\ g$

Volume of wood = volume of pencil – volume of graphite

$= \frac{22}{7} \times ((0.35)^2 - (0.05)^2) \times 10 \times 0.7$

$= \frac{22}{7} \times (0.1225 - 0.0025) \times 7 = 2.64\ g$

∴ Total weight of the pencil = weight of wood + weight of graphite

= 0.165 + 2.64 = 2.805 g.

Exercise 13C

1. Question

Find the volume, curved surface area and the total surface area of a cone having base radius 35 cm and height 84 cm.

Answer

Given,

Radius of the cone = 35cm

Height of the cone = 84cm

Curved surface area = πrl

So, we need to find out the l;

l = slant height

$$l = \sqrt{h^2 + r^2}$$

$$l = \sqrt{84^2 + 35^2} = \sqrt{7056 + 1225} = \sqrt{8281}$$

l = 91cm

Curved surface area = $\frac{22}{7} \times 35 \times 91$

= 110 × 91 = 10010cm^2

Volume of the cone = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 84$

= 88 × 1225

= 107800cm^2

Total surface area = πrl + πr^2

$$= 10010 + \frac{22}{7} \times 35 \times 35$$

= 10010 + 3850 = 13860

Total surface area = 13860cm^2

2. Question

Find the volume, curved surface area and the total surface area of a cone whose height and slant height are 6 cm and10 cm respectively. (Take $\pi = 3.14$)

Answer

Given,

Height (h) = 6cm

Slant height (l) = 10cm

r = $\sqrt{l^2 - h^2}$

$r = \sqrt{(10)^2 - (6)^2}$

r = $\sqrt{100 - 36} = \sqrt{64}$

r = 8cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times 6$

$= \frac{8448}{21}$ = 401.92cm^2

Curved surface area = πrl

Curved surface area = $\frac{22}{7} \times 8 \times 10$

$= \frac{1760}{7}$ = 251.2cm^2

Total surface area = πr(r + l)

$= \frac{22}{7} \times 8 \times (8 + 10)$

$= \frac{22}{7} \times 8 \times 18$

$= \frac{3168}{7}$ = 452.16 cm^2

3. Question

The volume of a right circular cone is (100π) cm^3 and its height is 12 cm. Find its slant height and its curved surface area.

Answer

Given,

h = 12 cm

Volume of the cone = 100π cm^3

$= \frac{1}{3}\pi r^2 h = 100\pi$

r^{2}h = 100 × 3

$r^2 \times 12 = 100 \times 3$

$r^2 = \frac{100 \times 3}{12} = 25$

r = 5cm

$$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{(12)^2 + (5)^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169} = 13$$

Curved surface area = πrl

= π × 5 × 13

= (65π) cm^2

4. Question

The circumference of the base of a cone is 44 cm and its slant height is 25 cm. Find the volume and curved surface area of the cone.

Answer

Given,

Circumference of the base of the cone = 44cm

2πr = 44cm

$$r = \frac{44 \times 7}{2 \times 22} = 7$$

r = 7cm

$$h = \sqrt{l^2 - r^2}$$

$$= \sqrt{(25)^2 - (7)^2}$$

$$= \sqrt{625 - 49}$$

$$= \sqrt{576} = 24$$

$$volume = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$$

= 22 × 56 = 1232

Volume = 1232 cm^3

Curved surface area = πrl

$$= \frac{22}{7} \times 7 \times 25$$

= 550 cm^2

5. Question

A cone of slant height 25 cm has a curved surface area $550\ cm^2$. Find the height and volume of the cone.

Answer

Given,

Curved surface area = 550cm^2

πrl = 550

$$\frac{22}{7} \times r \times 25 = 550$$

$$r = \frac{550 \times 7}{22 \times 25}$$

r = 7cm

$$h = \sqrt{l^2 - h^2} = \sqrt{25^2 - 7^2}$$

$$= \sqrt{625 - 49} = \sqrt{576} = 24$$

h = 24cm

Volume = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$$

= 24 × 56

Volume = 1232cm^3

6. Question

Find the volume of a cone having radius of the base 35 cm and slant height 37 cm.

Answer

Given,

r = 35cm

l = 37cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 38 \times 35 \times h$$

$$= \frac{3850}{3} h$$

$$h = \sqrt{l^2 - h^2} = \sqrt{37^2 - 35^2}$$

$$= \sqrt{1369 - 1225}$$

$$= \sqrt{144} = 12$$

$$volume = \frac{3850}{3} \times 12$$

Volume = 1540 cm^3

7. Question

The curved surface area of a cone is $4070\ cm^2$ and its diameter is 70 cm. Find its slant height.

Answer

Given,

$$r = \frac{70}{2} = 35cm$$

Curved surface area = 4070

πrl = 4070

$$l = \frac{4070}{\pi r}$$

$$= \frac{4070 \times 7}{22 \times 35}$$

$= \frac{814}{22}$ = 37cm

8. Question

How many metres of cloth, 2.5 m wide, will be required to make a conical tent whose base radius is 7 m and height 24 metres?

Answer

Given,

Radius = 7cm

h = 24cm

Curved surface area of the conical tent = πrl

$$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{(24)^2 + (7)^2}$$

$$= \sqrt{576 + 49}$$

$$= \sqrt{625}$$

= 25m

Curved surface area of the tent = πrl

$$= \frac{22}{7} \times 7 \times 25$$

= 550 m^2

Length of cloth = $\frac{area}{width}$

$$= \frac{550}{2.5} = 220m$$

9. Question

A right circular cone is 3.6 cm high and the radius of its base is 1.6 cm. It is melted and recast into a right circular cone having base radius 1.2 cm. Find its height.

Answer

When we melt any shape, and recast into another shape than volume of both shapes remain same.

Radius of the circular cone (r_1) = 1.6cm

Height of the circular cone (h_1) = 3.6cm

Radius of the new circular cone (r_2) = 1.2 cm

Let height of the new circular cone be h_2

Volume of the circular cone = volume of the new circular cone

$$\frac{1}{3}\pi r_1^2 h = \frac{1}{3}\pi r_2^2 h$$

$(1.6)^2 \times (3.6) = (1.2)^2 \times h_2$

$$h_2 = \frac{(1.6)^2 \times 3.6}{(1.2)^2} = \frac{1.6 \times 1.6 \times 3.6}{1.2 \times 1.2} = 64$$

h_2 = 64 cm

So, the height of the new circular cone will be 64cm

10. Question

Two cones have their heights in the ratio 1:3 and the radii of their bases in the ratio 3:1. Show that their volumes are in the ratio 3:1.

Answer

Given,

Ratio of the heights = $h_1 : h_2 = 1:3$

Let the heights of the cones be x and 3x,

Ratio of radius of base of the two cones = $r_1:r_2 = 3:1$

So,

Let the radius be 3x and x for the cones and volume will be v_1 and v_2

$$\frac{v_1}{v_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$= \frac{(3x)^2 \times x}{x^2 \times 3x}$$

$$= \frac{9x^2}{3x^2}$$

$$\frac{v_1}{v_2} = \frac{3}{1}$$

So, ratio of the volume of the two cones will be 3:1

11. Question

A circus tent is cylindrical to a height of 3 meters and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent.

Answer

Given,

Cylindrical height of the tent = 3m

Diameter of the base of the cone = 105

Radius = $\frac{105}{2}$

Height of the conical portion = 53m

Area of canvas = curved surface area of conical part + curved surface area of cylindrical part

= πrl + 2πrh

$= \frac{22}{7} \times \frac{105}{2} \times 53 + 2 \times \frac{22}{7} \times \frac{105}{2} \times 3$

= 8745 + 990

= 9735m^2

Length of canvas = area/width

$= \frac{9735}{5} = 1947m$

Hence the length of the canvas will be 1947m

12. Question

A conical tent is to accommodate 11 persons. Each person must have $4\ m^2$ of the space on the ground and $20\ m^3$ of air to breath. Find the height of the cone.

Answer

Given,

Number of person in the room = 11

Each person covers area = $4m^2$

Total area covered by all = $44m^2$

$\pi r^2 = 44$

$r^2 = 44 \times \frac{7}{22} = 14$

Volume of the cone = $220m^3$

We know that,

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi r^2 h = 220$

$\frac{1}{3} \times \frac{22}{7} \times 14 \times h = 220$

$h = \frac{220 \times 3 \times 7}{22 \times 14} = 15m$

Hence, the height of the cone will be 15m

13. Question

A cylindrical bucket, 32 cm high and 18 cm of radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and the slant height of the heap.

Answer

Let the radius of the heap be r and the slant height h,

So, we have

Height of the cylindrical bucket = 32cm

Radius of the base of cylindrical bucket = 18cm

Height of the conical heap = 24cm

Volume of cylinder = volume of cone

$\pi r^2 h = \frac{1}{3}\pi r^2 h$

$r^2 = \frac{18 \times 18 \times 32 \times 3}{24}$

$r^2 = 18 \times 8 \times 4$

$r = 18 \times 2$

$r = 36cm$

slant height $l = \sqrt{h^2 + r^2}$

$= \sqrt{(24)^2 + (36)^2}$

$= \sqrt{576 + 1296}$

$= \sqrt{1872}$

$l = 43.27cm$

14. Question

A cylinder and a cone have equal radii of their bases and equal heights. If their curved surface areas are in the ratio 8:5, show that the radius and height of each has the ratio 3:4.

Answer

Given,

Curved surface area of cylinder = curved surface area of cone = 8:5

Let C.S.A of cylinder = 8x

C.S.A of cone = 5x

As mention above cone and cylinder have equal radius and equal height

$\frac{CSA\ of\ cylinder}{CSA\ of\ cone} = \frac{8x}{5x}$

$\frac{2\pi rh}{\pi rl} = \frac{8}{5}$

$\frac{2h}{\sqrt{h^2 + r^2}} = \frac{8}{5}$

$$\left(\frac{2h}{\sqrt{h^2+r^2}}\right)^2 = \left(\frac{8}{5}\right)^2$$

$$\frac{4h^2}{h^2+r^2} = \frac{64}{25}$$

$100h^2 = 64h^2 + 64r^2$

$100h^2 - 64h^2 = 64r^2$

$36h^2 = 64r^2$

$$\frac{r^2}{h^2} = \frac{36}{64}$$

$$\frac{r}{h} = \frac{6}{8}$$

$$\frac{r}{h} = \frac{3}{4}$$

15. Question

An iron pillar consists of a cylindrical portion 2.8 m high and 20 cm in diameter and a cone 42 cm high is surmounting it. Find the weight of the pillar, given that $1 cm^3$ of iron weighs 7.5 g

Answer

Given,

Height of cylinder R = 2.8m = 280cm

Radius = $\frac{20}{2} = 10cm$

Height of cone = 42cm

Volume of pillar = volume of cone + volume of cylinder

$$= \frac{1}{3}\pi r^2 h + \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 42 + \frac{22}{7} \times 10 \times 10 \times 280$$

$$= \frac{22}{7} \times 100[14 + 280]$$

$$= \frac{22}{7} \times 100 \times 294$$

$$= \frac{646800}{7} = 92400$$

Given that,

Weight of 1 cm^3 iron = 7.5gm

Weight of the pillar = 92400 × 75

Weight of the pillar = 693000g

= 693kg

16. Question

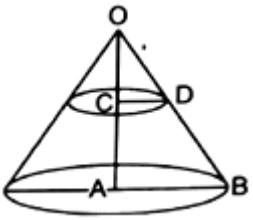
The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base, the section has been made?

Answer

Let's suppose the smaller cone have the radius r and height h cm

And radius of the given cone be R cm

Height of the original cone = 30cm



In triangle ΔOAB and ΔOCD

∠COD = ∠AOB (common angle)

∠OCD = ∠OAB (90°)

∴ Δ OAB ~ ΔOCD [by A-A criteria]

Then,

$\frac{r}{R} = \frac{h}{30}$

$r = \frac{Rh}{30}$.....(i)

Volume of small cone = $\frac{1}{27}$ volume of original cone

$\frac{1}{3}\pi r^2 h = \frac{1}{27} \times \frac{1}{3}\pi R^2 \times 30$

From equation (i)

$\left(\frac{Rh}{30}\right)^2 h = \frac{1}{27} \times R^2 \times 30$

$\frac{R^2h^2}{30 \times 30} = \frac{1}{27} \times R^2 \times 30$

$h^3 = \frac{1}{27} \times 30 \times 30 \times 30$

$h^3 = 1000$

h = 10cm

Height of the small cone = 10cm

AC = OA-OC

AC = 30-10 = 20

Hence selection has been made at height of 20cm above the base.

17. Question

From a solid right circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and base is removed. Find the volume of the remaining solid. (Take $\pi = 3.14$.)

Answer

Height of the cylinder = 10cm

Radius of the cylinder = 6cm

The height and base of the cone is equals to the height and base of the cylinder.

Volume of the remaining solid = volume of cylinder – volume of cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times 3.14 \times 6 \times 6 \times 10$$

Volume of remaining solid = $753.6cm^3$

18. Question

Water flows at the rate of 10 meters per minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the surface 40 cm and depth 24 cm?

Answer

Given,

Radius of the cylindrical pipe = $\frac{5}{2}$ = 2.5mm

= 0.25cm [as we know 10mm = 1cm]

Water flowing per minute through cylindrical pipe = $\pi(0.25)^2 \times 1000$

Radius of the conical vessel = $\frac{40}{2} = 20cm$

Depth of the vessel = 24cm

Volume of the vessel = $\frac{1}{3}\pi(20)^2 \times 24$

Let the time to fill the conical vessel be x minute,

Water flowing per minute through cylindrical pipe x x = volume of conical vessel

$x = \frac{\frac{1}{3}\pi(20)^2 \times 24}{\pi(0.25)^2 \times 1000}$

$x = \frac{20 \times 20 \times 8}{0.25 \times 0.25 \times 1000}$

x = 51min 12 sec.

Hence the required time to fill a conical vessel is 51min 12 sec

Exercise 13D

1. Question

Find the volume and surface area of a sphere whose radius is:

(i) 3.5 cm

(ii) 4.2 cm

(iii) 5 m

Answer

(i) Radius of sphere = 3.5cm

Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$

= 179.67cm^3

Surface area = 4πr^2

$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$

= 2 × 22 × 3.5 = 154 cm^2

(ii) R = 4.2cm

Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2$

= 310.464cm^2

Surface area = 4πr^2

$= 4 \times \frac{22}{7} \times 4.2 \times 4.2$

= 4 × 22 × .6 × 4.2

= 221.76cm^2

(iii) R = 5cm

Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 5 \times 5 \times 5$

$= \frac{11000}{21}$ = 523.80cm^3

Surface area = 4πr^2

$= 4 \times \frac{22}{7} \times 5 \times 5$

$= \frac{20 \times 110}{7}$ = 314.28cm^2 +

2. Question

The volume of a sphere is $38808\ cm^3$. Find its radius and hence its surface area.

Answer

Volume of sphere = 38808cm^3

$\frac{4}{3}\pi r^3 = 38808$

$r^3 = \frac{38808 \times 3}{4\pi}$

$r^3 = \frac{38808 \times 3 \times 7}{4 \times 22}$

r^3 = 441 × 21

r^3 = 21 × 21 × 21

r = 21cm

surface area = 4πr^2

$= 4 \times \frac{22}{7} \times 21 \times 21$

= 5544cm^3

3. Question

Find the surface area of a sphere whose volume is $606.375\ m^3$.

Answer

Given,

Volume = 606.375cm^3

$\frac{4}{3}\pi r^3 = 606.375$

$r^3 = \frac{606.375 \times 3}{4\pi}$

$r^3 = \frac{606.375 \times 3 \times 7}{4 \times 21}$

$r^3 = \frac{12733.875}{88}$

$r^3 = 144.703$

r = 5.25m

Surface area = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 5.25 \times 5.25$

$= 346.5m^2$

4. Question

The surface area of a sphere is $394.24\ m^2$. Find its radius and volume.

Answer

Given,

Surface area = $394.24m^2$

$4\pi r^2 = 394.24$

$4 \times \frac{22}{7} \times r^2 = 394.24$

$r^2 = \frac{394.24 \times 7}{22 \times 4}$

$r^2 = \frac{2759.68}{88}$

$r^2 = 31.36$

r = 5.67cm

Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 5.6 \times 5.6 \times 5.6$

$= \frac{4}{3} \times 22 \times 0.8 \times 5.6 \times 5.6$

$= 735.91cm^3$

5. Question

The surface area of a sphere is $(576\pi)\ cm^2$. Find its volume.

Answer

Given,

Surface area = 576π

$4\pi r^2 = 576\pi$

$r^2 = \frac{576\pi}{4\pi} = 144$

r = 12cm

Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 12 \times 12 \times 12$

= $2304cm^3$

6. Question

The outer diameter of a spherical shell is 12 cm and its inner diameter is 8 cm. Find the volume of metal contained in the shell. Also, find its outer surface area.

Answer

Given,

Outer Diameter of spherical shell = 12cm

Radius of the outer sphere r_1 = 6cm

Inner diameter of spherical shell = 8cm

Radius of the inner sphere r_2 = 4cm

Volume of metal = outer volume – inner volume

$= \frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_2^3$

$= \frac{4}{3}\pi[r_1^3 - r_2^3]$

$= \frac{4}{3} \times \frac{22}{7} \times [6^2 - 4^3]$

$= \frac{4}{3} \times \frac{22}{7}[216 - 64]$

$= \frac{4}{3} \times \frac{22}{7} \times 152$

= $636.95cm^3$

Surface area of outer surface = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 6 \times 6$

= $452.571cm^2$

7. Question

How many lead shots, each 3 mm in diameter, can be made from a cuboid with dimensions $(12\ cm \times 11\ cm \times 9\ cm)$?

Answer

Given,

Dimensions of cuboid l = 12cm

b = 11cm

h = 9cm

Diameter of sphere (d) = 3mm

r = $\frac{3}{2}mm$ = 1.5mm

r = 0.15 cm

When we melt any object, and convert it into another then the volume of both the object will be same.

So,

Volume of cuboid = n × volume of sphere

n = no. of sphere

l × b × h = $n \times \frac{4}{3}\pi r^3$

12 × 11 × 9 = $n \times \frac{4}{3} \times \frac{22}{7} \times 0.15 \times 0.15 \times 0.15$

n = $\frac{12 \times 11 \times 9 \times 3 \times 7 \times 100 \times 100 \times 100}{4 \times 22 \times 0.15 \times 0.15 \times 0.15} = \frac{3 \times 7 \times 20 \times 20 \times 20}{2}$

n = 84000

8. Question

How many lead balls, each of radius 1 cm, can be made from a sphere of radius 8 cm?

Answer

Given,

Radius of big sphere (R) = 8cm

Radius of small sphere (r) = 1cm

Volume of big sphere = 2 × volume of small sphere

n = no. of sphere

$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$

$\frac{4}{3}\pi(8)^3 = n \times \frac{4}{3}\pi(1)^3$

512 = n

n = 512 ball

9. Question

A solid sphere of radius 3 cm is melted and then cast into smaller spherical balls, each of diameter 0.6 cm. Find the number of small balls thus obtained.

Answer

Given,

Radius of big ball = 3cm

Diameter of small ball = 0.16cm

$r = \frac{0.16}{2} = 0.3cm$

Volume of big ball = n × volume of small ball

$\frac{4}{3}\pi(3)^3 = n \times \frac{4}{3}\pi(0.3)^3$

$n = \frac{(3)^3}{(0.3)^3} = \frac{3 \times 3 \times 3}{0.3 \times 0.3 \times 0.3} = \frac{3 \times 3 \times 3}{3 \times 3 \times 3} \times 1000$

n = 1000

10. Question

A metallic sphere of radius 10.5 cm is melted and then recast into smaller cones, each of radius 3.5 cm and height 3 cm. How many cones are obtained?

Answer

Given,

Sphere radius = 10.5cm

Cone radius = 3.5cm

h = 3cm

When any object is melt and recast into another so the volume of both the object will be same

Volume of sphere = n × volume of cone

$\frac{4}{3} \times \pi r^3 = n \times \frac{1}{3}\pi r^3 h$

$\frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5 = n \times \frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 3$

$n = \frac{4\pi \times 10.5 \times 10.5 \times 10.5 \times 3}{3\pi \times 3.5 \times 3.5 \times 3}$

n = 126

11. Question

How many spheres 12 cm in diameter can be made from a metallic cylinder of diameter 8 cm and height 90 cm?

Answer

Given,

Diameter of cylinder = 8cm

Radius = 4cm

Height = 90cm

Diameter of sphere = 12cm

Radius = 6cm

When we convert any object into another shape the volume will remain same.

Volume of cylinder = n × volume of sphere

$$\pi r^2 h = n \times \frac{4}{3}\pi r^3$$

$$\pi \times 4 \times 4 \times 90 = n \times \frac{4}{3}\pi (6)^3$$

$$n = \frac{\pi \times 4 \times 4 \times 90 \times 3}{4\pi \times (6)^3}$$

$$n = \frac{4 \times 90 \times 3}{6 \times 6 \times 6} = \frac{90}{18}$$

n = 5

12. Question

The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. Find the length of the wire.

Answer

Given,

Diameter sphere = 6cm

r = 3cm

radius of wire = $\frac{2}{2} = 1mm$

r = 0.1cm

let us consider length of wire = h cm

When we convert any object into another shape the volume will remain same.

Volume of sphere = volume of cylinder

$\frac{4}{3}\pi r^3 = \pi r^2 h$

$\frac{4}{3}\pi(3)^3 = \pi(1)^2 h$

$h = \frac{4\pi \times 3 \times 3 \times 3}{3 \times 0.1 \times 0.1 \times \pi} = \frac{4 \times 3 \times 3}{.001}$

h = 36 × 100 =3600

h = 36m

13. Question

The diameter of a copper sphere is 18 cm. It is melted and drawn into a long wire of uniform cross section. If the length of the wire is 108 m, find its diameter.

Answer

Given,

Radius of sphere = 9cm

Let us consider diameter at cylinder = d cm

Radius = r cm

Height = 108 m = 10800 cm

When we convert any object into another shape the volume will remain same.

Volume of sphere = volume of cylinder

$\frac{4}{3}\pi r^3 = \pi r^2 h$

$\frac{4}{3}\pi(9)^3 = \pi r^2 \times 10800$

$r^2 = \frac{4 \times 9 \times 9 \times 9}{3 \times 10800}$

$r^2 = \frac{4}{3} \times \frac{729}{10800}$

r^2 = 0.09

r = 0.03 cm

Diameter = 2 × 0.03 = 0.06 cm

14. Question

A sphere of diameter 15.6 cm is melted and cast into a right circular cone of height 31.2 cm. Find the diameter of the base of the cone.

Answer

Given,

When we convert any object into another shape the volume will remain same.

Radius of sphere = $\frac{15.6}{2} = 7.8cm$

Radius of cone = r cm

Volume of sphere = volume of cone

$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$

$\frac{4}{3}\pi(7.8)^3 = \frac{1}{3}\pi r^2 \times 31.2$

$r^2 = \frac{4\pi \times 3 \times 7.8 \times 7.8 \times 7.8}{3\pi \times 31.2} = \frac{4 \times 474.552}{31.2}$

$r^2 = \frac{1898.208}{31.2}$

r^2 = 60.84

r = 7.8cm

d = 2 × r = 2 × 7.8 =15.6 cm

15. Question

A spherical cannonball 28 cm in diameter is melted and cast into a right circular cone mould, whose base is 35 cm in diameter. Find the height of the cone.

Answer

Given,

Radius of sphere (r_3) = 14cm

Diameter of cone = 35cm

$r_c = \frac{35}{2}$

When we convert any object into another shape the volume will remain same.

Volume of sphere = volume of cone

$\frac{4}{3}\pi r_3^3 = \frac{1}{3}\pi r_c^2 h$

$4r_3^3 = r_c^2$

$4 \times (14)^3 = \left(\frac{35}{2}\right)^2 \times h$

$h = \frac{4 \times 14 \times 14 \times 14}{\left(\frac{35}{2}\right)^2}$

$h = \frac{4 \times 14 \times 14 \times 14 \times 2 \times 2}{35 \times 35} = 35.84cm$

16. Question

A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of these balls are 1.5 cm and 2 cm. Find the radius of the third ball.

Answer

Given,

Radius of big ball (R) = 3cm

Radius of smaller ball (r_1) = 1.5 cm

Radius of second smaller ball (r_2) = 2 cm

Let r_3 be the radius of 3rd smaller ball

$V = v_1 + v_2 + v_3$

$$\frac{4}{3}\pi(R)^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$(R)^3 = r_1^3 + r_2^3 + r_3^3$$

$(3)^3 = (1.5)^3 + (2)^3 + (r_3)^3$

$27 = 2.817 + 8 + (r_3)^3$

$(r_3)^3 = 27- (2.817 + 8) = 16.875$

$r_3 = 2.5$ cm

17. Question

The radii of two spheres are in the ratio 1:2. Find the ratio of their surface areas.

Answer

Given,

Ratio of radii of spheres = $R_1 : R_2 = 1 : 2$

Ratio of their surface areas = $\frac{A_1}{A_2} = \frac{4\pi R_1^2}{4\pi R_2^2} = \frac{R_1^2}{R_2^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$

18. Question

The surface areas of two spheres are in the ratio 1:4. Find the ratio of their volumes.

Answer

Given,

Ratio of Surface area of two spheres = $A_1 : A_2 = 1 : 4$

Let radius of these sphere are resp. = R_1 and R_2

$= \frac{A_1}{A_2} = \frac{1}{4}$

$= \frac{4\pi R_1^2}{4\pi R_2^2} = \frac{1}{4}$

$= \frac{R_1^2}{R_2^2} = \frac{1}{4}$

$= \frac{R_1}{R_2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

Ratio of their volumes $= \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \frac{R_1^3}{R_2^3} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

19. Question

A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball?

Answer

Given,

Radius of cylinder = 12 cm

Height = 20 cm

Before drop a ball volume of water = $v_1 = \pi r^2 h = \pi r^2 \times 20\ cm^3$

After droping rise in water level = 6.75 cm

New height = 20 + 6.75 = 26.75 cm

New volume = $\pi r^2 \times 26.75\ cm^3$

Volume of spherical ball = πr^2 (26.75 – 20)

$= \pi r^2 \times 6.75 = \frac{22}{7} \times 12 \times 12 \times 6.75 = 3054.85\ cm^3$

$= \frac{4}{3}\pi R^3 = 3054.85$

$= R^3 = \frac{3054.85 \times 3 \times 7}{4 \times 22} = 729$

$= R = \sqrt[3]{729} = 9\ cm$

20. Question

A cylindrical bucket with base radius 15 cm is filled with water up to a height of 20 cm. A heavy iron spherical ball of radius 9 cm is dropped into the bucket to submerge completely in the water. Find the increase in the level of water.

Answer

Given,

Radius of spherical ball = 9 cm

Volume of spherical ball = $\frac{4}{3}$πr^3 = $\frac{4}{3}\pi(9)^3\ cm^3$

Radius of cylinder = 15 cm

Let the increase in level = h cm

= $\frac{4}{3}$ π × 729 = π × 15 × 15 × h

= h = $\frac{4}{3} \times \frac{729}{225} = 4.326\ cm$

21. Question

A hemisphere of lead of radius 9 cm is cast into a right circular cone of height 72 cm. Find the radius of the base of the cone.

Answer

Given,

Radius of hemisphere = (R) = 9 cm

Height of cone = 72 cm

Let radius of cone = r cm

We know that,

Volume of hemisphere = volume of cone

= $\frac{2}{3}$πR^3 = $\frac{1}{3}$πr^{2}h

= $\frac{2}{3} \times 9^3$ = r^2 × 72

= r^2 = $\frac{2 \times 729}{3 \times 72} = \frac{81}{4}$

= r = $\sqrt{\frac{81}{4}} = \frac{9}{2} = 4.5\ cm$

Radius of base of cone = 4.5 cm.

22. Question

A hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles are required to empty the bowl?

Answer

Given,

Radius of hemisphere (R) = 9 cm

Radius of cylinder (r) = $\frac{3}{2} = 1.5\ cm$

Height of cylinder = 4 cm

Volume of hemisphere = n × volume of cylinder

$= \frac{2}{3}\pi R^3 = n \times \pi r^2 h$

$= \frac{2}{3} \times 9^3 = n \times 1.5^2 \times 4$

$= n = \frac{2 \times 9 \times 9 \times 9}{3 \times 1.5 \times 1.5 \times 4} = 54.$

23. Question

A hollow spherical shell is made of a metal of density 4.5 g per cm^3. If its internal and external radii are 8 cm and 9 cm respectively, find the weight of the shell.

Answer

Given,

Internal radius of sphere (r_i) = 8 cm

External radius of sphere (r_e) = 9 cm

Volume of shell = (external volume – internal volume)

$= \frac{4}{3}\pi r_e^3 - \frac{4}{3}\pi r_i^3 = \frac{4}{3}\pi(9^3 - 8^3) = \frac{4}{3} \times \frac{22}{7} \times 217 = 909.33\ cm^3$

Weight of sphere = 909.33 × 4.5 = 4092 gm = 4.092 kg

24. Question

A hemispherical bowl is made of steel 0.5 cm thick. The inside radius of the bowl is 4 cm. Find the volume of steel used in making the bowl.

Answer

Given,

In-radius of bowl (r_i) = 4 cm

Thickness of steel = 0.5 cm

External radius of bowl (r_e)= 4 + 0.5 = 4.5 cm

Volume of metal = $\frac{2}{3}\pi r_e^3 - \frac{2}{3}\pi r_i^3$

$= \frac{2}{3}\pi (r_e^3 - r_i^3) = \frac{2}{3}\pi(4.5^3 - 4^3) = \frac{2}{3} \times \frac{22}{7} \times (91.25 - 64)$

$= \frac{2}{3} \times \frac{22}{7} \times 27.125 = 56.83\ cm^3$

CCE Questions

1. Question

The length, breadth and height of a cuboid are 15 cm, 12 cm, and 4.5 cm respectively. Its volume is

A. $243\ cm^3$

B. $405\ cm^3$

C. $810\ cm^3$

D. $603\ cm^3$

Answer

Given: Length = 15 cm

Breadth = 12 cm

Height = 4.5 cm

Volume of a cuboid = Length × Breadth × Height

Volume = 15 cm ×12 cm ×4.5 cm = 810 cm^3

2. Question

A cuboid is 12 cm long, 9 cm broad and 8 cm high. Its total surface area is

A. $864\ cm^2$

B. $552\ cm^2$

C. $432\ cm^2$

D. $276\ cm^2$

Answer

Given: Length = 12 cm

Breadth = 9 cm

Height = 8 cm

Total surface area of a cuboid = 2[(Length ×Breadth) + (Breadth ×Height) + (Height ×Length)]

Total surface area = 2[(12×9) + (9×8) + (8×12)] cm^2 = 2(108+72+96) cm^2

= 2(276) cm^2 = 552 cm^2

3. Question

The length, breadth and height of a cuboid are 15 m, 6 m and 5 dm respectively. The lateral surface area of the cuboid is

A. $45\ m^2$

B. $21\ m^2$

C. $201\ m^2$

D. $90\ m^2$

Answer

Given: Length = 15 m

Breadth = 6 m

Height = 5 m

Lateral surface area of a cuboid = 2(Length +Breadth) × Height

1 m = 10dm

⇒ 5dm =0.5m

Lateral surface area = $2(15+6) \times 0.5\ m^2 = 1 \times 21\ m^2$

$= 21\ m^2$

4. Question

A beam 9 m long, 40 cm wide and 20 cm high is made up of iron which weighs 50 kg per cubic metre. The weight of the beam is

A. 27 kg

B. 48 kg

C. 36 kg

D. 56 kg

Answer

Given: Length = 9 cm

Breadth = 40 cm

Height = 20 cm

Volume of a cuboid = Length × Breadth × Height

1 m =100 cm

⇒ 40 cm= 0.4 m and 20 cm =0.2 m

Volume = $9\ m \times 0.4m \times 0.2m = 0.72\ m^3$

Given that $1\ cm^3$ weighs 50 kg

⇒ $0.72\ m^3$ weighs 50×0.72 kg = 36 kg

5. Question

The length of the longest rod that can be placed in a room of dimensions (10m × 10m × 5m) is

A. 15 m

B. 16 m

C. $10\sqrt{5}$ m

D. 12 m

Answer

Longest rod = diagonal of the cuboid = $\sqrt{(l^2 + b^2 + h^2)}$

Length of longest rod = $\sqrt{(10^2 + 10^2 + 5^2)} = \sqrt{(100 + 100 + 25)}$

= $\sqrt{225}$ = 15m

6. Question

What is the maximum length of a pencil that can be placed in a rectangular box of dimensions (8cm × 6cm × 5cm)?(Given $\sqrt{5} = 2.24$)

A. 8 cm

B. 9.5 cm

C. 19 cm

D. 11.2 cm

Answer

Maximum length of a pencil = diagonal of the cuboid

Now, the diagonal of cuboid is = $\sqrt{l^2 + b^2 + h^2}$

Thus,

Length of longest rod

= $\sqrt{(8^2 + 6^2 + 5^2)}$

= $\sqrt{(64 + 36 + 25)}$

= $\sqrt{125}$

= 5√5 cm

= 5(2.24) cm

= 11.2 cm

7. Question

The number of planks of dimensions (4m × 5m × 2m) that can be stored in a pit which is 40, long 12 m wide and 16 m deep, is

A. 190

B. 192

C. 184

D. 180

Answer

Volume of a cuboid = Length × Breadth × Height

Volume of pit = 40 m × 12 m × 16 m = 7680 m^3

Volume of plank = 4 m × 5 m × 2 m = 40 m^3

No. of planks = $\frac{\text{Volume of Pit}}{\text{volume of Plank}} = \frac{7680}{40}$

= 192

8. Question

How many planks of dimensions (5m × 25cm × 10cm) can be stored in a pit which is 20 m long, 6 m wide and 50 cm deep?

A. 480

B. 450

C. 320

D. 360

Answer

Volume of a cuboid = Length × Breadth × Height

1 m =100 cm

Volume of pit = 20 m × 6 m × 0.5 m = 60 m^3

Volume of plank = 5 m × 0.25 m × 0.1 m = 0.125 m^3

No. of planks = $\frac{\text{Volume of Pit}}{\text{volume of Plank}} = \frac{60}{0.125}$

= 480

9. Question

How many bricks will be required to construct a wall 8 m long, 6 m high and 22.5 cm thick if each brick measures (25cm × 11.25 cm × 6cm)?

A. 4800

B. 5600

C. 6400

D. 5200

Answer

Volume of a cuboid = Length × Breadth × Height

1 m =100 cm

Volume of wall = 8 m × 6 m × 0.225 m = 10.8 m^3

Volume of a brick = 0.25 m × 0.1125 m × 0.06 m = 0.0016875 m^3

No. of bricks = $\frac{\text{Volume of wall}}{\text{volume of brick}} = \frac{10.8}{0.0016875}$

= 6400

10. Question

How many persons can be accommodated in a dining hall of dimensions (20m × 15m × 4.5m), assuming that each person requires 5m^3 of air?

A. 250

B. 270

C. 320

D. 300

Answer

Volume of a cuboid = Length × Breadth × Height

Volume of hall = 20 m × 15 m × 4.5 m = 1350 m^3

Volume of air required by 1 person = 5 m^3

No. of persons = $\frac{\text{Volume of hall}}{\text{Volume of air required by 1 person}} = \frac{1350}{5}$

=270

11. Question

A river 1.5 m deep and 30 m wide is flowing at the rate of 3 km per hour. The volume of water that runs into the sea minute is

A. 2000 m^3

B. 2250 m^3

C. 2500 m^3

D. 2750 m^3

Answer

Volume of a cuboid = Length × Breadth × Height

Length of the river = Speed of river = 3km (in an hr)

1km =1000 m and 1 hour =60 min

Speed in m per minute = $3 \times \frac{1000}{60} = 50\text{m per min}$

Volume of water that runs in a minute = 1.5 m × 30 m × 50 m = 2250 m^3

12. Question

The lateral surface area of a cube is 256m^2. The volume of the cube is

A. 64 m^3

B. 216 m^3

C. 256 m^3

D. 512 m^3

Answer

Lateral surface area of a cube = 4(side)^2

Given Lateral surface area = 256 m^2

⇒ 4(side)^2 = 256m^2

⇒ (side)^2 = $\frac{256}{4}m^2$

⇒ (side) = √64m= 8m

Volume of a cube = (side)^3

⇒ Volume = $(8)^3$ m^3 = 512 m^3

13. Question

The total surface area of a cube is 96 cm^2. The volume of the cube is

A. 8 cm^3

B. 27 cm^3

C. 64 cm^3

D. 512 cm^3

Answer

Total surface area of a cube = $6(\text{side})^2$

Given Total surface area = 96 cm^2

$\Rightarrow 6(\text{side})^2 = 96m^2$

$\Rightarrow (\text{side})^2 = \frac{96}{6}cm^2$

$\Rightarrow$ (side) = $\sqrt{}16cm = 4cm$

Volume of a cube = $(\text{side})^3$

$\Rightarrow$ Volume = $(4)^3\ cm^3 = 64\ cm^3$

14. Question

The volume of a cube is 512 cm^3. Its total surface area is

A. $256\ cm^2$

B. $384\ cm^2$

C. $512\ cm^2$

D. $64\ cm^2$

Answer

Volume of a cube = $(\text{side})^3$

Given volume = 512 cm^3

$\Rightarrow (\text{side})^3 = 512\ cm^3$

$\Rightarrow$ side = $\sqrt[3]{512} = 8$ cm

Total surface area of a cube = $6(\text{side})^2$

$\Rightarrow$ Total surface area = $6(8)^2cm^2 = 384\ cm^2$

15. Question

The length of the longest rod that can fit in a cubical vessel of side 10 cm, is

A. 10 cm

B. 20 cm

C. $10\sqrt{2}$ cm

D. $10\sqrt{3}$ cm

Answer

Length of the longest rod = diagonal of the cube = side $\sqrt{3}$

Length of longest rod =10 $\sqrt{3}$ cm

16. Question

If the length of diagonal of a cube is $8\sqrt{3}$ cm, then its surface area is

A. $192\ cm^2$

B. $384\ cm^2$

C. $512\ cm^2$

D. $768\ cm^2$

Answer

Diagonal of the cube = side $\sqrt{3}$

Given diagonal =8 $\sqrt{3}$ cm = side $\sqrt{3}$

⇒ side = 8 cm

Total surface area of a cube = $6(side)^2$

⇒ Surface area = $6(8)^2 = 6\times64 = 384\ cm^2$

17. Question

If each edge of a cube is increased by 50%, then the percentage increase in its surface area is

A. 50%

B. 75%

C. 100%

D. 125%

Answer

Let original side be x, on increasing it by 50% i.e. $\frac{50}{100} = \frac{1}{2}$

New side will be $x + \frac{1}{2}x = \frac{3}{2}x$

Total surface area of a cube = $6(side)^2$

Original surface area = $6\ (x)^2$

New surface area = $6\ (\frac{3}{2}x)^2 = 6\times\frac{9}{4}x^2 = \frac{27}{2}x^2$

Change in surface area = $\frac{27}{2}x^2 - 6\ (x)^2$

Taking LCM of 2 and 1 = 2

$\Rightarrow \frac{27x^2 - 12x^2}{2} = \frac{15}{2}x^2$

The percentage increase in its surface area is $\frac{\frac{15}{2}x^2}{6x^2} \times 100\% = 125\%$

18. Question

Three cubes of metal with edges 3 cm, 4 cm and 5 cm respectively are melted to form a single cube. The lateral surface area of the new cube formed is

A. $72\ cm^2$

B. $144\ cm^2$

C. $128\ cm^2$

D. $256\ cm^2$

Answer

Here, the volume of three cubes = volume of the new cube

Volume of a cube = $(side)^3$

Volume of three cubes = $(3)^3 + (4)^3 + (5)^3 = (27 + 64 + 125)\ cm^3 = 216\ cm^3$

$\Rightarrow$ Volume of new cube = $216\ cm^3 = (side)^3$

$\Rightarrow (side)^3 = (6\ cm)^3$

$\Rightarrow$ side = 6cm

Lateral surface area = $4(side)^2 = 4(6)^2 = 144cm^2$

19. Question

In a shower, 5 cm of rain falls. What is the volume of water that falls on 2 hectares of ground?

A. $500\ m^3$

B. $750\ m^3$

C. $800\ m^3$

D. $1000\ m^3$

Answer

1 hectare = $10000\ m^2$

2 hectares = $20000\ m^2$

1 cm = 0.01 m ⇒ 5cm= 0.05 m

Volume of water that falls on 2 hectares of ground = 20000× 0.05 m^3 = 1000 m^3

20. Question

Two cubes have their volumes in the ratio 1 : 27. The ratio of their surface areas is

A. 1 : 3

B. 1 : 8

C. 1 : 9

D. 1 : 18

Answer

Volume of a cube = $(side)^3$

Let the sides be x and y

Ratio of volumes = $\frac{x^3}{y^3} = \frac{1}{27}$

$\Rightarrow \frac{x}{y} = \frac{1}{3}$

Surface area of a cube = $6(side)^2$

Ratio of surface areas = $\frac{x^2}{y^2} = \frac{1}{3^2} = \frac{1}{9}$ = 1: 9

21. Question

If each side of a cube is doubled, then its volume

A. is doubled

B. becomes 4 times

C. becomes 6 times

D. becomes 8 times

Answer

Let original side be x, New side will be 2x

Volume of a cube = $(side)^3$

Original volume = $(x)^3$

New volume = $(2x)^3 = 8x^3$

So, the volume is 8 times of the original volume

22. Question

The diameter of the base of a cylinder is 6 cm and its height is 14 cm. The volume of the cylinder is

A. $198\ cm^3$

B. $396\ cm^3$

C. $495\ cm^3$

D. $297\ cm^3$

Answer

Volume of a cylinder = $\pi r^2 h$

Diameter = 6cm ⇒ radius = 3cm

⇒ Volume = $\frac{22}{7} \times 3^2 \times 14$

= $22 \times 9 \times 2 = 396 cm^3$

23. Question

If the diameter of a cylinder is 28 cm and its height is 20 cm, then its curved surface area is

A. $880\ cm^2$

B. $1760\ cm^2$

C. $3520\ cm^2$

D. $2640\ cm^2$

Answer

Curved surface area of a cylinder = $2\pi rh$

Diameter = 28 cm ⇒ radius = 14 cm

⇒ Curved surface area = $2 \times \frac{22}{7} \times 14 \times 20$

= $44 \times 40 = 1760\ cm^2$

24. Question

If the curved surface area of a cylinder is $1760\ cm^2$ and its base radius is 14 cm, then its height is

A. 10 cm

B. 15 cm

C. 20 cm

D. 40 cm

Answer

Curved surface area of a cylinder = $2\pi rh$

⇒ Curved surface area = $2 \times \frac{22}{7} \times 14 \times h = 1760\ cm^2$

⇒ $h = \frac{1760}{44 \times 2} = 20\ cm$

25. Question

The height of a cylinder is 14 cm and its curved surface area is 264 cm^2. The volume of the cylinder is

A. 308 cm^3

B. 396 cm^3

C. 1232 cm^3

D. 1848 cm^3

Answer

Curved surface area of a cylinder = $2\pi rh$

⇒ Curved surface area = 1760 cm^2

⇒ $2 \times \frac{22}{7}$ × r × 14 = 1760 cm^2

⇒ $r = \frac{1760}{44 \times 2} = 20\ cm$

Volume of a cylinder = $\pi r^2 h$

Volume = $\frac{22}{7} \times 20^2 \times 14$

= 17,600 cm^3

26. Question

The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. The height of the pillar is

A. 4 m

B. 5 m

C. 6 m

D. 7 m

Answer

Curved surface area of a cylinder = $2\pi rh$

⇒ Curved surface area = $2 \times \frac{22}{7} \times r \times h = 264\ m^2$

$\Rightarrow r = \frac{264 \times 7}{44 \times h} = \frac{42}{h}$

Volume of a cylinder = $\pi r^2 h$

Volume = $\frac{22}{7} \times \left(\frac{42}{h}\right)^2 \times h = 924 m^3$

$\Rightarrow h = \frac{22 \times 42 \times 6}{924}$

$\Rightarrow h = \frac{42 \times 6}{42} = 6\ m$

27. Question

The radii of two cylinders are in the ratio 2 :3 and their heights are in the ratio 5 :3. The ratio of their curved surface area is

A. 2 : 5

B. 8 : 7

C. 10 : 9

D. 16 : 9

Answer

Let the radii be 2x and 3x respectively and heights be 5y and 3y respectively.

Curved surface area of a cylinder = $2\pi rh$

⇒ Ratio of their Curved surface area = $\frac{2\pi rh}{2\pi RH} = \frac{2x \times 5y}{3x \times 3y} = \frac{10}{9}$

28. Question

The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5:3. The ratio of their volumes is

A. 27 : 20

B. 20 : 27

C. 4 : 9

D. 9 : 4

Answer

Let the radii be 2x and 3x respectively and heights be 5y and 3y respectively.

Volume of a cylinder = $\pi r^2 h$

⇒ Ratio of their Volumes = $\frac{\pi r^2 h}{\pi R^2 H} = \frac{2x^2 \times 5y}{3x^2 \times 3y} = \frac{20}{27}$

29. Question

The ratio between the radius of the base and the height of a cylinder is 2 : 3. If its volume is $1617\ cm^3$, then its total surface area is

A. $308\ cm^2$

B. $462\ cm^2$

C. $540\ cm^2$

D. $770\ cm^2$

Answer

Let the radius be 2x and height be 3x respectively.

Volume of a cylinder = $\pi r^2 h$

⇒ Volume = $\frac{22}{7} \times (2x)^2 \times 3x = 1617$

$\Rightarrow \frac{22}{7} \times x^3 \times 12 = 1617$

$\Rightarrow x^3 = \frac{1617 \times 7}{22 \times 12} = \frac{343}{8}$

$\Rightarrow x = \frac{7}{2} = 3.5$

So, radius = 2× 3.5 =7 cm and height = 3× 3.5 = 10.5 cm

Total surface area of a cylinder = $2\pi r(r + h)$

⇒ T.S.A. $= 2 \times \frac{22}{7} \times 7(7 + 10.5) = 44 \times 17.5 = 770\ cm^2$

30. Question

Two circular cylinders of equal volume have their heights in the ratio 1 : 2. The ratio of their radii is

A. $1 : \sqrt{2}$

B. $\sqrt{2} : 1$

C. $1 : 2$

D. $1 : 4$

Answer

Let the heights be h = x and H=2x respectively of the two cylinders.

Volume of a cylinder = $\pi r^2 h$

Given that $\pi r^2 h = \pi R^2 H$

$$\Rightarrow \frac{r^2}{R^2} = \frac{2x}{x}$$

$$\Rightarrow r : R = \sqrt{2} : 1$$

31. Question

The ratio between the curved surface area and the total surface area of a right circular cylinder is 1 : 2. If the total surface area is 616 cm^2, then the volume of the cylinder is

A. 1078 cm^3

B. 1232 cm^3

C. 1848 cm^3

D. 924 cm^3

Answer

Total surface area of a cylinder = $2\pi r(r + h)$

Curved surface area of a cylinder = $2\pi rh$

$$\Rightarrow \frac{2\pi rh}{2\pi r(r+h)} = \frac{1}{2}$$

$\Rightarrow 2h = r + h \Rightarrow h = r$

Given that total surface area = 616 cm^2

$$\Rightarrow 2 \times \frac{22}{7} \times 2r^2 = 616$$

$\Rightarrow r^2 = 7 \times 7$

So, r = h = 7 cm

Volume of a cylinder = $\pi r^2 h$

$$\Rightarrow \frac{22}{7} \times 49 \times 7 = 1078\ cm^3$$

32. Question

In a cylinder, if the radius is halved and the height is doubled, then the volume will be

A. the same

B. doubled

C. halved

D. four times

Answer

Let the radius be r and height be h

Volume of a cylinder = $\pi r^2 h$

When radius = $\frac{1}{2}r$ and height = 2h

Volume = $\pi\left(\frac{r}{2}\right)^2 \times 2h = \pi\frac{r^2}{2}h$

The volume will be halved.

33. Question

The number of coins 1.5 cm in diameter and 0.2 cm thick to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm is

A. 540

B. 450

C. 380

D. 472

Answer

Volume of a cylinder = $\pi r^2 h$

Volume of the coin = $\pi(\frac{1.5}{2})^2 \times 0.2$

Volume of the cylinder = $\pi(\frac{4.5}{2})^2 \times 10$

Number of coins $= \frac{\text{Volume of the cylinder}}{\text{Volume of the coin}}$

$= \frac{\pi\left(\frac{4.5}{2}\right)^2 \times 10}{\pi\left(\frac{1.5}{2}\right)^2 \times 0.2}$

$= 9 \times 50 = 450$

34. Question

The radius of a wire is decreased to one-third. If volume remains the same, the length will become

A. 2 times

B. 3 times

C. 6 times

D. 9 times

Answer

Let radius and length of a wire be r and h respectively

Volume of a wire = $\pi r^2 h$

If radius = $\frac{r}{3}$ and new length = H

Volume of the wire = $\pi(\frac{r}{3})^2 \times H = \pi r^2 h$

⇒ H = 9h i.e. 9 times

35. Question

The diameter of a roller, 1 m long, is 84 cm. If it takes 500 complete revolutions to level a playground, the area of the playground is

A. $1440\ m^2$

B. $1320\ m^2$

C. $1260\ m^2$

D. $1550\ m^2$

Answer

Curved surface area of a cylinder = $2\pi rh$

1m= 100cm, radius = 42 cm = 0.42m

Curved surface area $= 2 \times \frac{22}{7} \times 0.42 \times 1 = 2.64\ m^2$

Area of the playground = 500 × 2.64 m^2 = 1320 m^2

36. Question

2.2 dm^3 of lead is to be drawn into a cylindrical wire 0.50 cm in diameter. The length of the wire is

A. 110 m

B. 112 m

C. 98 m

D. 12 m

Answer

Given volume of the cylindrical wire is 2.2dm^3

Volume of a wire = $\pi r^2 h$

1 dm = 10 cm ⇒ 0.50 cm = 0.05 dm

Volume of the wire = $\pi(0.25)^2 \times \text{length of the wire} = 2.2$

$$\Rightarrow h = \frac{2.2 \times 7}{22 \times 0.0625} = 11.2 \text{ dm}$$

1 m= 10 dm

⇒ 11.2 dm = 112 m

37. Question

The lateral surface area of a cylinder is

A. $\pi r^2 h$

B. πrh

C. $2\pi rh$

D. $2\pi r^2$

Answer

The curved surface area of a cylinder is only the lateral surface area

And, we know that the curved surface area = 2πrh

38. Question

The height of a cone is 24 cm and the diameter of its base is 14 cm. The curved surface area of the cone is

A. 528 cm^2

B. 550 cm^2

C. 616 cm^2

D. 704 cm^2

Answer

Curved surface area of a cone = πrl

where $l = \sqrt{h^2 + r^2}$

Here, r=7cm and h=24cm

$l = \sqrt{24^2 + 7^2}$

$= \sqrt{625}$

$= 25\text{cm}$

$\Rightarrow \frac{22}{7} \times 7 \times 25$

$= 550\text{cm}^2$

39. Question

The volume of a right circular cone of height 12 cm and base radius 6 cm, is

A. $(12\pi)\text{cm}^3$

B. $(36\pi)\text{cm}^3$

C. $(72\pi)\text{cm}^3$

D. $(144\pi)\text{cm}^3$

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3}\pi 6^2 12$

$= \pi \times 36 \times 4$

$= (144\pi)\text{cm}^3$

40. Question

How much cloth 2.5 m wide will be required to make a conical tent having base radius 7 m and height 24 m?

A. 120 m

B. 180 m

C. 220 m

D. 550 m

Answer

Curved surface area of a cone = πrl

where $l = \sqrt{h^2 + r^2}$

Here, r=7m and h=24m

$l = \sqrt{24^2 + 7^2}$

$= \sqrt{625}$

$= 25\text{m}$

$\Rightarrow \frac{22}{7} \times 7 \times 25 = 550\text{m}^2$

The cloth required $= \frac{550}{2.5}$

= 220 m

41. Question

The volume of a cone is 1570 cm^3 and its height is 15 cm. What is the radius of the cone? (Use $\pi = 3.14$)

A. 10 cm

B. 9 cm

C. 12 cm

D. 8.5 cm

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Given volume = 1570 cm^3

$\Rightarrow \frac{1}{3}\pi r^2 \times 15 = 3.14 \times r^2 \times 5 = 15.7r^2 cm^3$

$\Rightarrow 15.7r^2 = 1570$

$\Rightarrow r^2 = 100 \Rightarrow r = 10cm$

42. Question

The height of a cone is 21 cm and its slant height is 28 cm. The volume of the cone is

A. 7356 cm^3

B. 7546 cm^3

C. 7506 cm^3

D. 7564 cm^3

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h$

where $\text{slant height } l = \sqrt{h^2 + r^2}$

Here, l=28 cm and h=21 cm

$r = \sqrt{28^2 - 21^2}$

$= \sqrt{441}$

$= 21\text{cm}$

Volume $= \frac{1}{3} \times \frac{22}{7} \times 21^2 \times 21$

$=$7546 cm^3

43. Question

The volume of a right circular cone of height 24 cm is 1232 cm^3. Its curved surface area is

A. $1254\ cm^2$

B. $704\ cm^2$

C. $550\ cm^2$

D. $462\ cm^2$

Answer

Given:

Volume of cone = 1232 cm^3

As we know, Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232\ cm^3$

$\Rightarrow r^2 = 49$

$\Rightarrow r = 7$ cm

$\text{slant height } l = \sqrt{h^2 + r^2}$

Here, r =7 cm and h = 24 cm

$l = \sqrt{7^2 + 24^2} = \sqrt{625} = 25\text{cm}$

Curved surface area of a cone = πrl

$\Rightarrow \frac{22}{7} \times 7 \times 25$

$= 550\text{cm}^2$

44. Question

If the volumes of two cones be in the ratio 1 : 4. and the radii of their bases be in the ratio 4 : 5, then the ratio of their heights is

A. 1 : 5

B. 5 : 4

C. 25 : 16

D. 25 : 64

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi R^2 H} = \frac{1}{4}$ and $\frac{r^2}{R^2} = \frac{16}{25}$

$\Rightarrow \frac{h}{H} = \frac{1 \times 25}{4 \times 16}$

$\Rightarrow \frac{h}{H} = \frac{25}{64}$

45. Question

If the height of a cone is doubled, then its volume is increased by

A. 100%

B. 200%

C. 300%

D. 400%

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h$

If height is doubled,

volume $= \frac{1}{3}\pi r^2 \times 2h$

$\text{Volume} = \frac{2}{3}\pi r^2 h$

$\text{Increase in volume} = \frac{\frac{2}{3}\pi r^2 h - \frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi r^2 h} \times 100\%$

$= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi r^2 h} \times 100\%$

$= 100\%$

Thus, there will be 100% increase in the volume.

46. Question

The curved surface area of one cone is twice that of the other while the slant height of the latter is twice that of the former. The ratio of their radii is

A. 2 : 1

B. 4 : 1

C. 8 : 1

D. 1 : 1

Answer

Curved surface area of a cone = πrl

Given that curved surface area of 1^{st} = 2× curved surface area of 2^{nd}

And slant height of 2^{nd} = 2× slant height of 1^{st}

⇒ L= 2l

$$\Rightarrow \frac{\pi rl}{\pi RL} = \frac{2}{1}$$

$$\Rightarrow \frac{r}{R} = \frac{2 \times 2l}{1 \times l}$$

⇒ r : R = 4 : 1

47. Question

The ratio of the volumes of a right circular cylinder and a right circular cone of the same base and the same height will be

A. 1 : 3

B. 3 : 1

C. 4 : 3

D. 3 :4

Answer

Given that heights and radii of cone and cylinder are equal

Volume of a cone $= \frac{1}{3}\pi R^2 H$

Volume of a cylinder = $\pi r^2 h$

Ratio of their volumes $= \frac{\pi r^2 h}{\frac{1}{3}\pi R^2 H} = \frac{3}{1}$

{because h=H and r=R}

Ans – 3:1

48. Question

A right circular cylinder and a right circular cone have the same radius and the same volume. The ratio of the height of the cylinder to that of the cone is

A. 3 : 5

B. 2 : 5

C. 3 : 1

D. 1 : 3

Answer

Let height of cylinder and cone be H and h respectively

Given that radii of cone and cylinder are equal

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a cylinder = $\pi r^2 H$

Given $\frac{1}{3}\pi r^2 h = \pi r^2 H$

$\Rightarrow \frac{H}{h} = \frac{1}{3}$

Ans: 1 : 3

49. Question

The radii of the bases of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3. Then, their volumes are in the ratio

A. 9 : 8

B. 8 : 9

C. 3 : 4

D. 4 : 3

Answer

Given that radii of cone and cylinder are 4x and 3x respectively and

height of cylinder and cone are 2y and 3y respectively

Volume of a cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 4x^2 \times 3y$

Volume of a cylinder $= \pi r^2 h = \pi \times 3x^2 \times 2y$

$$\Rightarrow \frac{\text{Volume of a cylinder}}{\text{Volume of a cone}} = \frac{\pi \times 3x^2 \times 2y}{\frac{1}{3}\pi \times 4x^2 \times 3y}$$

$$\Rightarrow \frac{\text{Volume of a cylinder}}{\text{Volume of a cone}} = \frac{9 \times 2}{16}$$

$$\Rightarrow \frac{\text{Volume of a cylinder}}{\text{Volume of a cone}} = \frac{9}{8}$$

50. Question

If the height and the radius of a cone are doubled, the volume of the cone becomes

A. 3 times

B. 4 times

C. 6 times

D. 8 times

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h$

If height and radius are doubled, volume $= \frac{1}{3}\pi(2r)^2 \times 2h = \frac{8}{3}\pi r^2 h$

The volume of the cone becomes 8 times.

51. Question

A solid metallic cylinder of base radius 3 cm and height 5 cm is melted to make n solid cones of height 1 cm and base radius 1 mm. The volume of n is

A. 450

B. 1350

C. 4500

D. 13500

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a cylinder $= \pi r^2 h$

Volume of solid metallic cylinder $= \pi(3)^2 \times 5 = 45\pi \text{ cm}^3$

1cm=10mm

Volume of solid coin $= \frac{1}{3}\pi(0.1)^2 \times 1 = \frac{1}{3} \times 0.01\pi \text{ cm}^3$

No. of coins $= \frac{\text{Volume of solid metallic cylinder}}{\text{Volume of solid coin}}$

$= \frac{45\pi \times 3\ cm^3}{0.01\pi\ cm^3}$

$= 13500$

52. Question

A conical tent is to accommodate 11 persons such that each person occupies $4m^2$ of space on the ground. They have 220 m^3 of air to breathe. The height of the cone is

A. 14 m

B. 15 m

C. 16 m

D. 20 m

Answer

As each person needs 4 m^2 spaces on ground, so 11 persons will need 44 m^2 space on the ground.Therefore, Area of ground = 44 $m^2 \Rightarrow \pi r^2 = 44$

$\Rightarrow \frac{22 \times r^2}{7} = 44$

$\Rightarrow r^2 = 14$Each person needs $= \frac{220}{11} = 20m^3$ of air Therefore volume of tent = 220 m^3

Volume of a cone $= \frac{1}{3}\pi r^2 h \Rightarrow \frac{1}{3}\pi r^2\ h\ =\ 220$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 14 \times\ h\ =\ 220$

$\Rightarrow$ h = 15cm

53. Question

The volume of a sphere of radius 2r is

A. $\frac{32\pi r^3}{3}$

B. $\frac{16\pi r^3}{3}$

C. $\frac{8\pi r^3}{3}$

D. $\frac{64\pi r^3}{3}$

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume $= \frac{4}{3}\pi(2r)^3 = \frac{32}{3}\pi r^3$

54. Question

The volume of a sphere of radius 10.5 cm is

A. $9702\ cm^3$

B. $4851\ cm^3$

C. $19404\ cm^3$

D. $14553\ cm^3$

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume $= \frac{4}{3}\pi(10.5)^3$

$= \frac{4}{3} \times \frac{22}{7} \times (10.5)^3$

$= 4851 cm^3$

55. Question

The surface area of a sphere of radius 21 cm is

A. $2772\ cm^2$

B. $1386\ cm^2$

C. $4158\ cm^2$

D. $5544\ cm^2$

Answer

Surface area of a sphere $= 4\pi r^2$

Surface area $= 4\pi(21)^2$

$= 4 \times \frac{22}{7} \times (21)^2$

$= 5544\text{cm}^2$

56. Question

The surface area of a sphere is 1386 cm^2. Its volume is

A. $1617\,cm^3$

B. $3234\ cm^3$

C. $4851\ cm^3$

D. $9702\ cm^3$

Answer

Surface area of a sphere $= 4\pi r^2$

Given Surface area = 1386 cm^2

$$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 1386$$

$$\Rightarrow r^2 = \frac{1386 \times 7}{88} = \frac{441}{4}$$

$$\Rightarrow r = \frac{21}{2}$$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume $= \frac{4}{3}\pi\left(\frac{21}{2}\right)^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3$

$= 4851\text{cm}^3$

57. Question

If the surface area of a sphere is $(144\pi)m^2$, then its volume is

A. $(288\pi)\ m^3$

B. $(188\pi)\ m^3$

C. $(300\pi)\ m^3$

D. $(316\pi)\ m^3$

Answer

Surface area of a sphere $= 4\pi r^2$

Given Surface area = $(144\pi)m^2$,

$\Rightarrow 4 \times \pi \times r^2 = 144\pi$

$\Rightarrow$ r= 6m

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume $= \frac{4}{3}\pi(6)^3 = 288\pi m^3$

58. Question

The volume of a sphere is 38808 cm^3. Its surface area is

A. $5544\ cm^2$

B. $8316\ cm^2$

C. $4158\ cm^2$

D. $1386\ cm^2$

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Given Volume= $38808\ cm^3$.

$\Rightarrow \frac{4}{3}\pi r^3 = 38808$

$\Rightarrow r^3 = 38808 \times \frac{21}{88} = 9261$

$\Rightarrow$ r=21cm

Surface area of a sphere $= 4\pi r^2$

Surface area $= 4 \times \frac{22}{7}(21)^2 = 5544\ cm^2$

59. Question

If the ratio of the volumes of two spheres is 1 : 8, then the ratio of their surface area is

A. 1 : 2

B. 1 : 4

C. 1 : 8

D. 1 : 16

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Given that $\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{1}{8}$

$\Rightarrow \frac{r}{R} = \frac{1}{2}$

Surface area of a sphere $= 4\pi r^2$

$\Rightarrow \frac{4\pi r^2}{4\pi R^2} = \frac{1}{4}$

60. Question

A solid metal ball of radius 8 cm is melted and cast into smaller balls, each of radius 2cm. The number of such balls is

A. 8

B. 16

C. 32

D. 64

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume of the solid metal ball $= \frac{4}{3}\pi(8)^3$

Volume of smaller ball$= \frac{4}{3}\pi(2)^3$

No. of balls $= \frac{\text{Volume of the solid metal ball}}{\text{Volume of smaller ball}}$

$= \frac{\frac{4}{3}\pi(8)^3}{\frac{4}{3}\pi(2)^3}$

$= 64$

61. Question

A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is

A. 4.2 cm

B. 2.1 cm

C. 2.4 cm

D. 1.6 cm

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2.1)^2 \times 8.4$

$= 12.348\pi \text{ cm}^3$

On recasting a cone into sphere, the volume will remain same

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume of sphere =12.348 π cm^3

$\Rightarrow \frac{4}{3}\pi r^3 = 12.348\pi$

$\Rightarrow r^3 = 12.348 \times \frac{3}{4} = 9.261$

⇒ r = 2.1cm

62. Question

A solid lead ball of radius 6 cm is melted and then drawn into a wire of diameter 0.2 cm. The length of wire is

A. 272 m

B. 288 m

C. 292 m

D. 296 m

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi \text{ cm}^3$

On recasting a sphere into cylinder, the volume will remain same

Volume of a cylinder $= \pi r^2 h$

Radius = 0.1 cm

$\Rightarrow \pi(0.1)^2 h = 288\pi$

$\Rightarrow h = 288 \times \frac{1}{0.01}$

$= 28800\text{cm}$

$= 288 \text{ m}$ (∵ 1m =100 cm)

⇒ h = 288 m

63. Question

A metallic sphere of radius 10.5 cm is melted and then recast into small cones, each of radius 3.5 cm and height 3 cm. The number of such cones will be

A. 21

B. 63

C. 126

D. 130

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10.5)^3\ cm^3$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(3.5)^2 \times 3$

$= \pi(3.5)^2 cm^3$

No. of cones $= \frac{\text{Volume of a sphere}}{\text{Volume of a cone}}$

$= \frac{\frac{4}{3}\pi(10.5)^3}{\pi(3.5)^2}$

$= 126$

64. Question

How many lead shots, each 0.3 cm in diameter, can be made from a cuboid of dimensions 9 cm × 11 cm × 12 cm ?

A. 7200

B. 8400

C. 72000

D. 84000

Answer

Volume of a cuboid = l× b× h = 9× 11× 12 cm^3

Radius of a lead shot = 0.15 cm

Volume of a lead shot $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.15)^3\ cm^3$

No. of lead shot $= \frac{\text{Volume of a cuboid}}{\text{Volume of a lead shot}}$

$= \frac{9\times 11\times 12}{\frac{4}{3}\pi(0.15)^3}$

$$= \frac{9 \times 11 \times 3 \times 3}{\frac{22}{7} \times 0.003375}$$

$$= 84000$$

65. Question

The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. The length of the wire is

A. 12 mB. 18 m

C. 36 m

D. 66 m

Answer

Radius of the sphere = 3 cm

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(3)^3$

$= 36\pi \text{ cm}^3$

On recasting a sphere into cylinder wire, the volume will remain same

Volume of a cylinder $= \pi r^2 h$

1cm=10mm

⇒ 2mm =0.2cm

Radius = 0.1 cm

$\Rightarrow \pi(0.1)^2 h = 36\pi$

$\Rightarrow h = 36 \times \frac{1}{0.01}$

$\Rightarrow h = 3600\text{cm}$

$\Rightarrow h = 36 \text{ m}$ (∵ 1m =100 cm)

66. Question

A sphere of diameter 12.6 cm is melted and cast into a right circular cone of height 25.2 cm. The radius of the base of the cone is

A. 6.3 cm

B. 2.1 cm

C. 6 cm

D. 4 cm

Answer

Radius of the sphere = 6.3 cm

Volume of a sphere $= \frac{4}{3}\pi r^3$

$\Rightarrow \text{Volume of a sphere} = \frac{4}{3}\pi(6.3)^3\ cm^3$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow \text{Volume of a cone} = \frac{1}{3}\pi r^2 \times 25.2$

$= 8.4\pi r^2 cm^3$

On recasting a sphere into a cone, volume will remain same

$\Rightarrow 8.4\pi r^2 = \frac{4}{3}\pi(6.3)^3$

$\Rightarrow r^2 = \frac{4}{3}(6.3)^3 \times \frac{1}{8.4} = 39.69$

⇒ r = 6.3 cm

67. Question

A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of these balls are 1.5 cm and 2 cm. The radius of the third ball is

A. 1 cm

B. 1.5 cm

C. 2.5 cm

D. 0.5 cm

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume of spherical ball $= \frac{4}{3}\pi(3)^3\ cm^3$

Volume of three balls $= \frac{4}{3}\pi(1.5)^3 + \frac{4}{3}\pi(2)^3 + \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(3.375 + 8 + r^3)$On recasting this sphere into three spherical balls, volume will remain same

$\Rightarrow \frac{4}{3}\pi(3.375 + 8 + r^3) = \frac{4}{3}\pi(3)^3$

$\Rightarrow 11.375 + r^3 = 27$

$\Rightarrow r^3 = 15.625$

$\Rightarrow$ r = 2.5 cm

68. Question

The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratio of the surface areas of the balloons in two cases is

A. 1 : 4

B. 1 : 3

C. 2 : 3

D. 1 : 2

Answer

Surface area of a hemisphere = $2\pi r^2$

Radii are 6cm and 12 cm respectively

Ratio of surface areas $= \frac{2\pi r^2}{2\pi R^2} = \frac{6^2}{12^2} = \frac{1}{4}$

Ans 1:4

69. Question

The volumes of the two spheres are in the ratio 64 : 27 and the sum of their radii is 7 cm. The difference of their total surface areas is

A. $38\ cm^2$

B. $58\ cm^2$

C. $78\ cm^2$

D. $88\ cm^2$

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Given Ratio of volumes of two spheres $= \frac{64}{27}$

$\Rightarrow \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{r^3}{R^3} = \frac{64}{27}$

$\Rightarrow \frac{r}{R} = \frac{4}{3}$

So, r = 4x and R = 3x

Also given that the sum of radii = 7

⇒ r +R = 4x +3x =7x =7

⇒ x =1

So r = 4cm and R = 3cm

Surface area of a sphere $= 4\pi r^2$

Difference in total surface area = $4\pi r^2 - 4\pi R^2 = 4\pi(r^2 - R^2)$

$4 \times \frac{22}{7} \times 7 = 88\ cm^2$

70. Question

A hemispherical bowl of radius 9 cm contains a liquid. This liquid is to be filled into cylindrical small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl?

A. 27

B. 35

C. 54

D. 63

Answer

Volume of a hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(9)^3\ cm^3$

Volume of a cylinder $= \pi r^2 h$

Volume of a cylindrical bottle $= \pi(1.5)^2 \times 4$

No. of bottles required $= \frac{\text{Volume of a hemisphere}}{\text{Volume of a cylindrical bottle}}$

$= \frac{\frac{2}{3}\pi(9)^3}{\pi(1.5)^2 \times 4}$

$= \frac{81 \times 3}{2.25 \times 1.5 \times 2} = 54$

Thus, total 54 bottles are required.

71. Question

A cone and a hemisphere have equal bases and equal volumes. The ratio of their heights is

A. 1 : 2

B. 2 : 1

C. 4 : 1

D. $\sqrt{2} : 1$

Answer

Given that Radius of the hemisphere = Radius of cone

And Volume of hemisphere = Volume of cone

Volume of a hemisphere $= \frac{2}{3}\pi r^3$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{h}{r} = \frac{2}{1}$

72. Question

A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is

A. 1 : 2 : 3

B. 2 : 1 : 3

C. 2 : 3 : 1

D. 3 : 2 : 1

Answer

Given that Radius of the hemisphere = Radius of cone = Radius of cylinder

And Height of the hemisphere = Height of cone = Height of cylinder

Volume of a hemisphere $= \frac{2}{3}\pi r^3$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a cylinder $= \pi r^2 h$

$\text{Ratio of their volumes} = \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$

$= h : 2r : 3h = 1 : 2 : 3$

73. Question

If the volume and the surface area of a sphere are numerically the same, then its radius is

A. 1 unit

B. 2 units

C. 3 units

D. 4 units

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Surface area of a sphere $= 4\pi r^2$

Given that volume = surface area

$\Rightarrow \frac{4}{3}\pi r^3 = 4\pi r^2$

$\Rightarrow r = 3 \text{ units}$

74. Question

Which is false in case of a hollow cylinder?

A. Curved surface area of a hollow cylinder $= 2\pi h(R + r)$.

B. Total surface area of a hollow cylinder $= 2\pi(R + r)(h + R - r)$.

C. Inner curved surface area of a hollow cylinder $= 2\pi h(R - r)$.

D. Area of each end of a hollow cylinder $= \pi(R^2 - r^2)$.

Answer

Inner curved surface area of a hollow cylinder = $2\pi rh$

75. Question

Which is false?

A. Volume of a hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$.

B. Volume of a hemisphere $= \frac{2}{3}\pi R^3$.

C. Total surface area of a hemisphere $= 3\pi R^2$.

D. Curved surface area of a hemisphere $= \pi R^2$.

Answer

Curved surface area of a hemisphere = $2\pi r^2$

76. Question

For a right circular cylinder of base radius = 7 cm and height = 14 cm, which is false?

A. Curved surface area $= 616 \text{ cm}^2$.

B. Total surface area $= 924\ cm^2$.

C. Volume $= 2156\ cm^3$.

D. Total area of the end faces $= 154\ cm^2$.

Answer

A) Curved surface area of a cylinder = $2\pi rh$

$$\Rightarrow 2 \times \frac{22}{7}(7) \times 14 = 616\ cm^2$$

B) Total surface area of a cylinder = $2\pi r(r + h)$

$$\Rightarrow 2 \times \frac{22}{7}(7) \times (7 + 14) = 924\ cm^2$$

C) Volume of a cylinder = $\pi r^2 h$

$$\Rightarrow \frac{22}{7}(7^2) \times 14 = 2156\ cm^3$$

D) Total area of the end faces = $2\times \pi r^2$ {Because there are two circular faces}

$$= 2 \times \frac{22}{7} \times 49 = 308\ cm^2$$

77. Question

Which is false?

A metal pipe is 63 cm long. Its inner diameter is 4 cm and the outer diameter is 4.4 cm. Then,

A. its inner curved surface area $= 792\ cm^2$

B. its outer curved surface area $= 871.2\ cm^2$

C. surface area of each end $= 2.64\ cm^2$

D. its total surface area $= 1665.84\ cm^2$

Answer

A) Inner curved surface area = $2\pi rh$

$$\Rightarrow 2 \times \frac{22}{7}(2) \times 63 = 792\ cm^2$$

B) Outer curved surface area = $2\pi Rh$

$$\Rightarrow 2 \times \frac{22}{7}(2.2) \times 63 = 871.2\ cm^2$$

C) Surface area of the end face = π(R ^{2} – r^2){Because there are two circular faces}

$$= 2 \times \frac{22}{7} \times (2.2^2 - 2^2) = 2.64 \text{ cm}^2$$

D) R = 2.2 cm, r = 2 cm and h = 63 cm

Total surface area of a hollow cylinder $= 2\pi(R + r)(h + R - r)$.

$$= 2 \times \frac{22}{7} \times 4.2 \times 63.2 = 1668.48 \text{ cm}^2$$

78. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	**Reason (R)**
The base radius of a cone is 7 cm and its slant height is 25 cm. The volume of the cone is 1232 cm^3.	The volume of a right circular cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Slant height $l = \sqrt{h^2 + r^2}$

Here, r=7cm and l=25cm

$$h = \sqrt{25^2 - 7^2} = \sqrt{576} = 24\text{cm}$$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24 = 1232\text{cm}^2$$

Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

79. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	**Reason (R)**
The surface area of a sphere is $2464\ cm^2$. Its volume is $11498\frac{2}{3} cm^3$. $\left(\pi = \frac{22}{7}\right)$.	The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Surface area of a sphere $= 4\pi r^2$

Given Surface area = 2464 cm^2

$$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 2464$$

$$\Rightarrow r^2 = \frac{2464 \times 7}{88} = 196$$

$$\Rightarrow r = 14cm$$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume $= \frac{4}{3}\pi(14)^3 = \frac{4}{3} \times \frac{22}{7} \times (14)^3 = 11498\frac{2}{3} cm^3$

Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

80. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	**Reason (R)**
The outer and inner radii of a hollow cylinder 2 m 10 cm long are 5 cm and 3 cm respectively. If it is made up of copper, then volume of copper in it is $10560\ cm^3$.	The volume of a hollow cylinder of length h, external radius R and internal radius r is given by $V = \pi h(R^2 - r^2)$.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

The volume of a hollow cylinder with external and internal radii R and r respectively and height h

$= \pi(R^2 - r^2)h$

$= \pi(5^2 - 3^2) \times 210$

$= \frac{22}{7} \times 16 \times 210$

$= 10560 cm^3$

Thus, the volume is $10560 cm^3$

Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

81. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
If the radius of a sphere is doubled then the ratio of the volume of the first sphere to that of the second is 1 : 8.	A cone and a hemisphere have equal bases and equal volumes. The ratio of their heights is 1 : 2.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume $= \frac{4}{3}\pi(2r)^3 = \frac{32}{3}\pi r^3$

Ratio = 1:8

Reason is wrong. Assertion (A) is true and Reason (R) is false.

82. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
The curved surface area of a cone is $550\ cm^2$ and its diameter is 14 cm. Then, its slant height is 25 cm.	The curved surface area of a cone having base radius r and slant height l is πrl.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Curved surface area of a cone = πrl

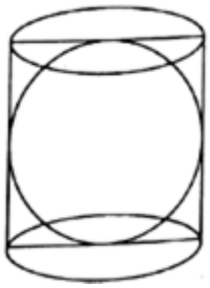
$\Rightarrow \frac{22}{7} \times 7 \times l = 550 cm^2$

$\Rightarrow$ l=25 cm

Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

83. Question

A right circular cylinder just encloses a sphere of radius r (as shown in the figure). Then, the surface area of the sphere is equal to the curved surface area of the cylinder.



Answer

True

Curved surface area of a sphere $= 4\pi r^2$

Radius of cylinder = r + r = 2r

Curved surface area of a cylinder = $2\pi rh = 2\pi \times r \times 2r = 4\pi r^2$

84. Question

The largest possible right circular cone is cut out of a cube of edge r cm. The volume of the cone is $\frac{1}{12}\pi r^3$.

Answer

True

The dimensions of the cone arediameter = r ; radius = r/2 height = rVolume of a cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(\frac{r}{2})^2 \times r = \frac{1}{12}\pi r^3$

85. Question

If a sphere is inscribed in a cube, then the ratio of the volume of the cube to the volume of the sphere will be $6 : \pi$.

Answer

True

Let the radius of sphere be r so the edge of cube = 2r

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume of a cube $= (2r)^3$

Ratio of their volumes $= 8r^3 : \frac{4}{3}\pi r^3 = 6 : \pi$

86. Question

If the length of diagonal of a cube is $6\sqrt{3}$ cm, then the length of each edge of the cube is 3 cm.

Answer

False

Diagonal of the cube = $\text{side}\sqrt{3}$

Length of longest rod = $6\sqrt{3}$ cm

Side = 6 cm

Formative Assessment (Unit Test)

1. Question

The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of5 : 3. Then, the ratio of their volumes is

A. 10 : 17

B. 20 : 27

C. 17 : 27

D. 20 : 37

Answer

Let the radii be 2x and 3x respectively and heights be 5y and 3y respectively.

Volume of a cylinder = $\pi r^2 h$

⇒ Ratio of their Volumes = $\frac{\pi r^2 h}{\pi R^2 H} = \frac{2x^2 \times 5y}{3x^2 \times 3y}$

Thus, the ratio of two cylinders $= \frac{20}{27}$

2. Question

The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is

A. $2\pi r(l + r)$

B. $\pi r\left(l + \frac{r}{4}\right)$

C. $\pi r(l + r)$

D. $2\pi rl$

Answer

Total surface area of a cone = $\pi r(r + l)$

$= \pi \frac{r}{2}\left(\frac{r}{2} + 2l\right)$

$= \pi r(\frac{r}{4} + l)$

3. Question

A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is

A. 1.6 cm

B. 2.1 cm

C. 2.4 cm

D. 4.2 cm

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2.1)^2 \times 8.4$

$= 12.348\pi \text{ cm}^3$

On recasting a cone into sphere, the volume will remain same

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume of sphere =12.348 π cm^3

$\Rightarrow \frac{4}{3}\pi r^3 = 12.348\pi$

$\Rightarrow r^3 = 12.348 \times \frac{3}{4} = 9.261$

$\Rightarrow$ r = 2.1cm

4. Question

The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratio of the surface areas of the balloon in the two cases is

A. 1 : 4

B. 1 : 3

C. 2 : 3

D. 2 : 1

Answer

Surface area of a hemisphere = $2\pi r^2$

Radii are 6cm and 12 cm respectively

Ratio of surface areas $= \frac{2\pi r^2}{2\pi R^2} = \frac{6^2}{12^2} = \frac{1}{4}$

5. Question

A copper sphere of diameter 6 cm is melted and drawn into 36 cm long wire of uniform circular cross-section. Then, its radius is

A. 2 cm

B. 1.5 cm

C. 1.2 cm

D. 1 cm

Answer

Radius of the sphere = 3 cm

Volume of a sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$

On recasting a sphere into cylinder wire, the volume will remain same

Volume of a cylinder $= \pi r^2 h$

$\Rightarrow \pi(r)^2 36 = 36\pi$

$\Rightarrow r = 1\text{cm}$

6. Question

Find the lateral surface area and the total surface area of a cube of side 8 cm.

Answer

Total surface area of a cube = $6(\text{side})^2$

⇒ Total surface area = $6(8)^2 \text{cm}^2 = 384 \text{ cm}^2$

Lateral surface area of a cube = $4(\text{side})^2$

⇒ Total surface area = $4(8)^2 \text{cm}^2 = 256 \text{ cm}^2$

7. Question

Find the lateral surface area and the total surface area of a cuboid of dimensions
$40 \text{ cm} \times 30 \text{ cm} \times 20 \text{ cm}$.

Answer

Total surface area of a cuboid = 2[(Length ×Breadth) + (Breadth ×Height) + (Height ×Length)]

Total surface area = $2[(40\times30) + (30\times20) + (20\times40)] \text{ cm}^2 = 2(1200+600+800) \text{ cm}^2$

$= 2(2600) \text{ cm}^2 = 5200 \text{ cm}^2$

Lateral surface area of a cuboid = 2(Length +Breadth) ×Height

Lateral surface area = $2(40+30) \times 20 \text{ cm}^2 = 140\times 20 \text{ cm}^2$

$= 2800 \text{ cm}^2$

8. Question

The total surface area of a cylinder is 462 cm^2 and its curved surface area is one-third of its total surface area. Find the volume of the cylinder.

Answer

Total surface area of a cylinder = $2\pi r(r+h)$

Curved surface area of a cylinder = $2\pi rh$

$$\Rightarrow \frac{2\pi rh}{2\pi r(r+h)} = \frac{1}{3}$$

⇒ 3h=r+h ⇒ 2h=r

Given that total surface area = 462 cm^2

$$\Rightarrow 2 \times \frac{22}{7} \times \frac{3}{2} r^2 = 462$$

⇒ $r^2 = 7\times 7$

So, r = 7 cm , h =3.5cm

Volume of a cylinder = $\pi r^2 h$

$\Rightarrow \frac{22}{7} \times 49 \times 3.5 = 539\ \text{cm}^3$

9. Question

The length and breadth of a room are in a ratio 3 : 2. The cost of carpeting the room at Rs 25 per m^2 is Rs 1350 and the cost of papering the four walls at Rs 15 per m^2 is Rs 2580. If one door and two windows occupy $8m^2$, find the dimensions of the room.

Answer

Area of the floor $= \frac{\text{Cost of carpeting}}{\text{rate of carpeting}} = \frac{1350}{25} = 54\ \text{m}^2$

Given that length and breadth are in ratio 3:2, so l = 3x and b = 2x

$\Rightarrow$ l= 9 m and b =6 m

Lateral surface area of a cuboid = 2(Length +Breadth) ×Height

Lateral surface area $= \frac{2580}{15} = 172\ \text{m}^2$

Adding door and window, Lateral surface area = 180 m^2

$\Rightarrow 2(l+b)h = 2(15 \times h) = 180$

$\Rightarrow$ h = 6 m

10. Question

If the radius of a sphere is increased by 10%, prove that its volume will be increased by 33.1%.

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Let the radius be 'r'Increased Radius = 1.1r

Volume $= \frac{4}{3}\pi(1.1r)^3 = \frac{4\times1.331}{3}\pi r^3$

Change in volume $= \frac{\frac{4}{3}\times1.331\pi r^3 - \frac{4}{3}\pi r^3}{\frac{4}{3}\pi r^3} \times 100\% = \frac{0.331}{1} \times 100\% = 33.1\%$

11. Question

The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height and volume of the cone. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Answer

Curved surface area of a sphere $= 4\pi r^2$

Curved Surface area of a cone $= \pi rl$

Given that $\Rightarrow 4\pi r^2 = 5(\pi rl)$

$$\Rightarrow l = \frac{4 \times 25}{5 \times 4} = 5cm$$

l= 5 cm and r= 4 cm

$$h = \sqrt{5^2 - 4^2} = \sqrt{9} = 3cm$$

Volume $= \frac{1}{3} \times \frac{22}{7} \times 4^2 \times 3 = 50.3\ cm^3$

12. Question

A rectangular tank measuring 5m × 4.5m × 2.1 m is dug in the centre of the field measuring 13.5m × 2.5 m. The earth dug out is spread over the remaining portion of the field. How much is the level of the field raised?

Answer

Volume = l× b × h

Volume = 5× 4.5× × 2.1=47.25 m^3

Area over which it is spread = 13.5 x 25 - 5 x 4.5 = 33.75 - 220 = 11.75 mRise in level $= \frac{47.25}{11.75}$= 4.2 m

13. Question

A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Answer

Curved surface area of a cone = πrl

where $l = \sqrt{h^2 + r^2}$

Here, r=7cm and h=24cm

$$l = \sqrt{24^2 + 7^2} = \sqrt{625} = 25cm$$

$$\Rightarrow \frac{22}{7} \times 7 \times 25 = 550cm^2$$

Area of 10 such caps = 5500 cm^2

14. Question

The volume of a right circular cone is 9856 cm^3. If the diameter of its base is28 cm, find the height of the cone.

Answer

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Given volume = 9856 cm^3

Radius of cone = 14 cm

$$\Rightarrow \frac{1}{3}\pi(14)^2 \times h = \frac{22}{21} \times 196 \times h = 9856cm^3$$

$$\Rightarrow h = 48\ cm$$

15. Question

Into a circular drum of radius 4.2 m and height 3.5 m, how many full bags of wheat can be emptied if the space required for wheat in each bag is $2.1\ m^3$?

Answer

Volume of a cylinder = $\pi r^2 h$

Volume of the cylinder = $\pi(4.2)^2 \times 3.5$

Number of bags $= \frac{\text{Volume of the cylinder}}{\text{Volume of the wheat bags}} = \frac{\pi(4.2)^2 \times 3.5}{2.1} = 92$

16. Question

A well with 10 m inside diameter is dug 14 m deep. Earth taken out of it is spread all around to a width of 5 m to form an embankment. Find the height of the embankment.

Answer

Volume of a cylinder = $\pi r^2 h$

Radius of well = 5m , Height of well = 14m

Volume of the well = $\pi(5)^2 \times 14 = \frac{22}{7} \times 25 \times 14 = 1100m^2$

For embankment, radius = 5+5 =10 m and let height be h m

Volume of well = Volume of embankment

$$\Rightarrow \pi[(10)^2 - (5)^2] \times h = \frac{22}{7} \times 75 \times h = 1100$$

⇒ h = 4.67 m

17. Question

How many metres of cloth 5 m wide will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m?

Answer

Curved surface area of a cone = πrl

where $l = \sqrt{h^2 + r^2}$

Here, r=7m and h=24m

$l = \sqrt{24^2 + 7^2} = \sqrt{625} = 25m$

$\Rightarrow \frac{22}{7} \times 7 \times 25 = 550m^2$

The cloth required $= \frac{550}{5} = 110m$

18. Question

The volume of a solid cylinder is 1584 cm^3 and its height is 14 cm. Find its total surface area.

Answer

Volume of a cylinder$= \pi r^2 h$, Given volume = $1584\ cm^3$

$\Rightarrow \frac{22}{7} \times r^2 \times 14 = 1584\ cm^3$

$\Rightarrow r^2 = 36 \Rightarrow r = 6$ cm

Total surface area of a cylinder = $\pi r(r + h)$

$\Rightarrow \frac{22}{7} \times 6 \times (6 + 14) = 754.29\ cm^2$

19. Question

The volume of two spheres are in the ratio 64 : 27. Find the difference of their surface areas if the sum of their radii is 7 cm.

Answer

Volume of a sphere $= \frac{4}{3}\pi r^3$

Given Ratio of volumes of two spheres $= \frac{64}{27}$

$\Rightarrow \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{r^3}{R^3} = \frac{64}{27}$

$\Rightarrow \frac{r}{R} = \frac{4}{3}$

So, r = 4x and R = 3x

Also given that the sum of radii = 7

$\Rightarrow$ r +R = 4x +3x =7x =7

$\Rightarrow$ x =1

So, r = 4cm and R = 3cm

Surface area of a sphere $= 4\pi r^2$

Difference in total surface area = $4\pi r^2 - 4\pi R^2 = 4\pi(r^2 - R^2)$

$4 \times \frac{22}{7} \times 7 = 88\ cm^2$

20. Question

The radius and height of a right circular cone are in the ratio 4 : 3. and its volume is 2156 cm^3. Find the curved surface area of the cone.

Answer

Since the radius and height of a cone is 4:3 so let radius = 4x and height = 3x

Volume of a cone $= \frac{1}{3}\pi r^2 h$, Given volume = $2156\ cm^3$.

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times (4x)^2 \times 3x = 2156\ cm^3$

$\Rightarrow x^3 = \frac{343}{8} \Rightarrow x = \frac{7}{2}$

So , r =14 cm and height = 10.5 cm

$\text{slant height } l = \sqrt{h^2 + r^2}$

Here, r=14 cm and h=10.5 cm

$l = \sqrt{14^2 + 10.5^2} = \sqrt{306.25} = 17.5cm$

Curved surface area of a cone = πrl

$\Rightarrow \frac{22}{7} \times 14 \times 17.5 = 770\ cm^2$

21. Question

The radius of the base of a cone is 14 cm and its height is 24 cm. Find the volume, curved surface area and the total surface area of the cone.

Answer

Curved surface area of a cone = πrl

where $l = \sqrt{h^2 + r^2}$

Here, r=14cm and h=24cm

$l = \sqrt{24^2 + 14^2} = \sqrt{772} = 27.8cm$

$\Rightarrow \frac{22}{7} \times 14 \times 27.8 = 1223\ cm^2$

Total surface area of a cone = $\pi r(r + l)$

$\Rightarrow \frac{22}{7} \times 14 \times (14 + 27.8) = 1839\ cm^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times (14)^2 \times 24 = 4928\ cm^3$

22. Question

Two cylindrical vessels are filled with oil. Their radii are 15 cm and 10 cm respectively and their heights are 25 cm and 18 cm respectively. Find the radius of the cylindrical vessel 33 cm in height which will just contain the oil of the two given vessels.

Answer

Volume of a cylinder$= \pi r^2 h$

Volume of first vessel = $\frac{22}{7} \times 15^2 \times 25 = 17678.57\ cm^3$

Volume of second vessel = $\frac{22}{7} \times 10^2 \times 18 = 5657.14\ cm^3$

The volume of the third vessel = volume of first vessel + volume of second vessel

$\Rightarrow \frac{22}{7} \times r^2 \times 33 = 17678.57 + 5657.14$

$\Rightarrow r^2 = 225 \Rightarrow r = 15\ cm$

23. Question

The ratio of the curved surface area and the total surface area of a circular cylinder is 1:2 and the total surface area is 616 cm^2. Find its volume.

Answer

Total surface area of a cylinder = $2\pi r(r + h)$

Curved surface area of a cylinder = $2\pi rh$

$\Rightarrow \frac{2\pi rh}{2\pi r(r + h)} = \frac{1}{2}$

$\Rightarrow 2h = r + h$

$\Rightarrow h = r$

Given that total surface area = 616 cm^2

$\Rightarrow 2 \times \frac{22}{7} \times 2r^2 = 616$

$\Rightarrow r^2 = 7 \times 7$

So, r = h = 7 cm

Volume of a cylinder = $\pi r^2 h$

$\Rightarrow \frac{22}{7} \times 49 \times 7 = 1078\ \text{cm}^3$