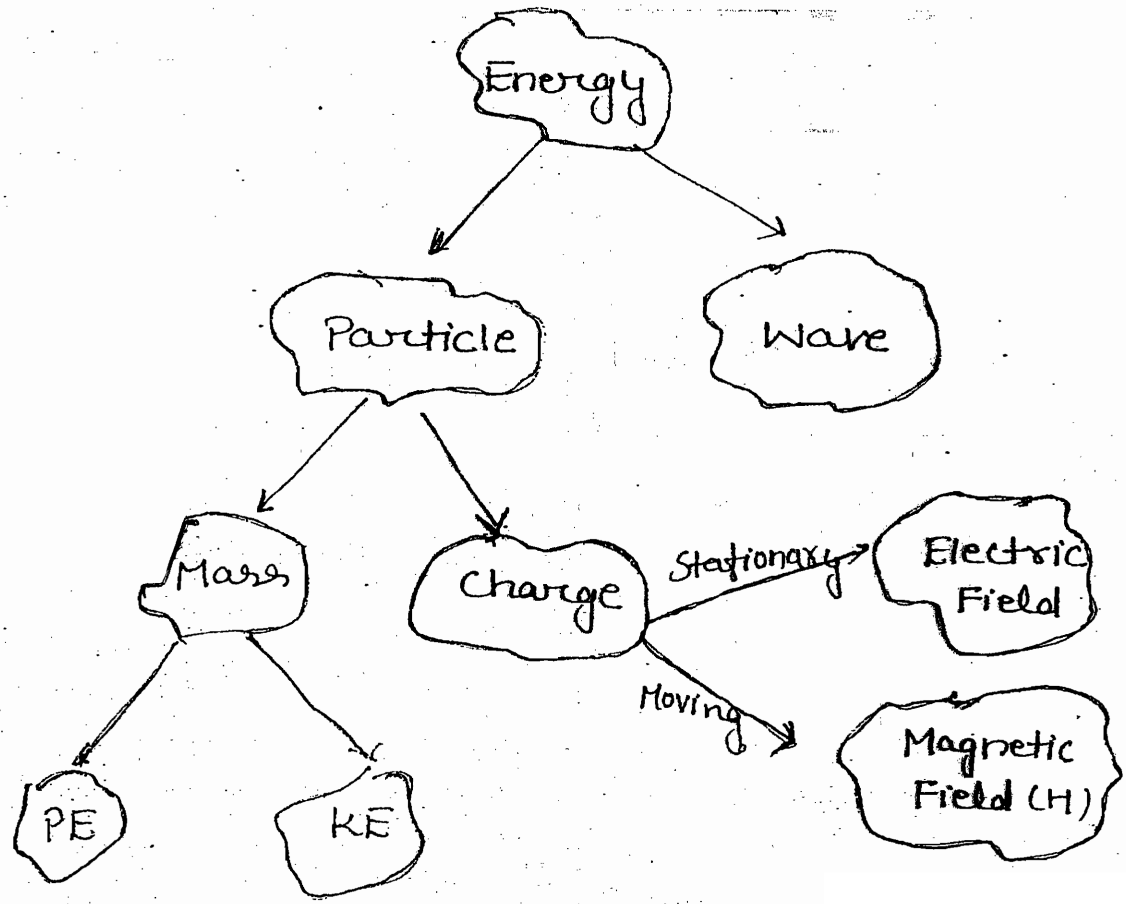
Lecture - I

Static Electromagnetic Fields



Electric Field :-

It is a format of energy that is all around a charge & influences other charges near by

Magnetic Field (H) :-

It is a format of energy that is all around a moving charge & influences other moving charges nearby.

NOTE :-

The word influence in electric fields is a linear accelerating attractive or repulsive force on a charge resulting in straight line path This is Coulomb's law and hence electric

... charge particles ...

$$\boxed{\vec{F} = QE = ma} \longrightarrow \text{Linear path}$$

energised

The word influence in magnetic field is only on moving charges such that forces perpendicular to velocity and the field.

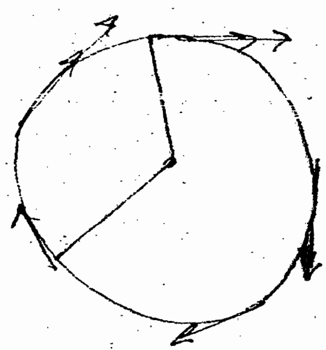
F ⊥ displacement

So that workdone = 0

This is Lorentz law and particle acquire circular path

$$\boxed{\vec{F} = Q(v \times B) = \frac{mv^2}{r}} \longrightarrow \text{Circular path}$$

deflection



Summary :-

1) Charge is stationary then → Electric Field (Static)

→ Time invariant

→ Space Varying

eg :- (I) accumulated charge

(II) D.C voltage

Electric field is energy in the above discussion

2. Charge is moving / flowing without accelerati or with constant velocity or linearly with time

$$Q = Kt$$

$$\frac{dQ}{dt} = K = I \quad (\text{DC current})$$

and DC current cause magnetic field whic is static in nature

→ Magnetic field is the energy in above discussion.

3. Charge is moving with acceleration which creates ~~energy~~ Electric field E(t) ~~an~~ as well as magnetic field H(t) which are power.

Basic Terms & Definitions :-

There is a measure of the field strength at any point in the field

(I) $\vec{E}$ → Electric field Intensity (V/m or N/c)
↓ per unit length

(II) $\vec{H}$ → Magnetic ~~flux density~~ field intensity (Amp/m)

(III) $\vec{B}$ → Magnetic flux density [Weber/m² or Tesla]

(IV) $\vec{D}$ → Electric flux density (coulombs/m²)
↓ per unit area

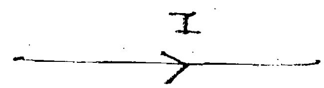
(V) ϵ → Permittivity of the medium

The ability to permit /allow/ hold electric field in that medium

(II) **Current Carrying Wire** :-

→ Scalar

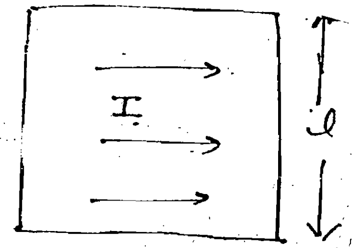
→ Amp



(III) **Surface current** :-

→ $\vec{K} = \frac{dI}{dl} = Amp/m$

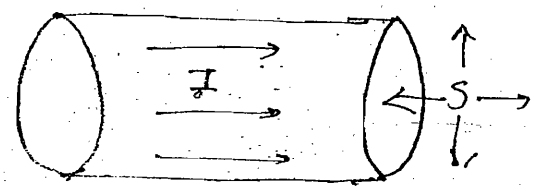
→ Vector



(IV) **Solid conductor (J)** :-

→ vector

→ $J = \frac{dI}{ds} = Amp/m^2$



Vector Calculus :-

It is a study of directional integrations & directional derivatives.

Directional Integration :-

It is the study of the total effects or cummulative effects of a phenomeno in a specific direction in a specific region

Directional derivative :-

It is the study of the instantaneous or rate of change analysis of a phenomenon in a specific direction in a specific region

eg:- Del - operator

$$\nabla \rightarrow \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

→ It is used to study the rate of change of various space varying quantities in 3d-space

→ Del is called as vector spatial derivative operators

∇ (vector operator) → Scalar function f → ∇f → Gradient of Scalar → vector form

∇ (vector operator) → Vector function $\vec{A}$ → $\nabla \cdot \vec{A}$ → Divergence of vector (Scalar function)

∇ (vector operator) → Vector function $\vec{A}$ → $\nabla \times \vec{A}$ → curl of vector (Vector function)

∇ (vector operator) → ∇ → $\nabla \cdot \nabla$ → ∇^2 → Scalar Laplacian operator

NOTE :-

Vector Identities :-

$$\nabla \times \nabla f = 0$$

$$\nabla \times (\nabla f) = 0$$

curl (Grad. of scalar) = 0

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

Div. (curl of vector) = 0

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

Divergence & Out-flow:-

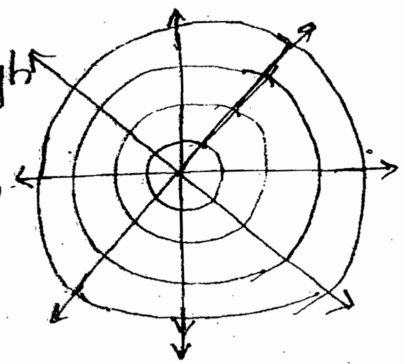
1) Consider a cause or source which has effects spread outward from the cause

eg:-(I) light from a bulb

(II) air velocity from a punctured tyre

For all such phenomenon the strength dec. as the area of expansion inc. such that total outflow is same

The total outflow through any enclosing surface is always the same (constant) and this constant depends on central cause



Total outflow = ↑Strength × Area ↓

= constant. $\propto$ Cause

The strength for all such phenomenon can be expressed as the constant per unit area or cause/area i.e. called as density.

NOTE:-

1) If the cause is Q coulomb charge then the repulsive force or attractive force is called as electric flux and the strength is called as flux density (D) such that

$$\oint D \cdot ds = \Psi_{e\,total} \propto Q$$

If proportionality constant is 1 then this called Gauss Law in integral form

$$\Rightarrow \boxed{\oint D \cdot ds = Q}$$

NOTE 1:-

→ If the surface is not completely enclosing then the effects are the partial flux crossing i.e.

$$\int \vartheta \cdot ds = \psi_e \neq \underset{total}{\psi_e}$$ This is not a Gauss law

→ Every $\underset{\text{vector}}{\underline{\text{Closed surface}}}$ is identified by a $\underset{}{\overset{\text{scalar}}{\underline{\text{finite}}}}$ $\underline{\text{volume}}$

eg:- $4\pi r^2 \xrightarrow{\text{sphere}} \frac{4}{3}\pi r^3$

$2\pi r h \xrightarrow{\text{cylinder}} \pi r^2 h$

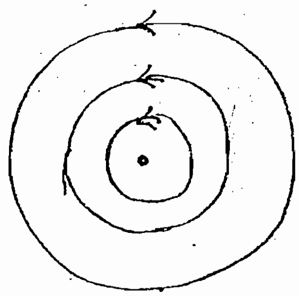
$6a^2 \xrightarrow{\text{cube}} a^3$

→ closed surface having direction while finite volume doesn't having direction

Circulation and curl :-

Cause or Source →

Effects → around the cause



→ The total circulation in any closed length is always a constant and this constant depends on the central cause.

eg:- air velocity under the fan

Total circulation = strength ↓ × length ↑

= constant ∝ Cause

$$\text{Strength} = \frac{\text{constant}}{\text{length}} \text{ or } \frac{\text{cause}}{\text{length}}$$

$\longrightarrow$ Intensity

Cause or Source $\longrightarrow$ I ampere current

Effects $\longrightarrow$ Magnetic line field

Strength $\longrightarrow$ Magnetic field intensity ($\vec{H}$)

$$\boxed{\oint_{closed} H \cdot dl = I}$$

This is ampere's law in integral form

Note -1 :-

Every closed line is identified with a finite area enclosed

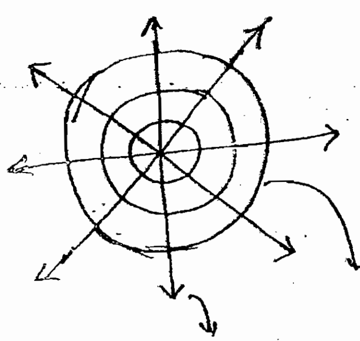
closed line $\rightarrow$ vector

area enclosed $\downarrow$ vector

$$2\pi r \xrightarrow{\text{circle}} \pi r^2$$

$$4a \xrightarrow{\text{square}} a^2$$

Summary 1 :-



Ψ_e total $= Q$ i.e. outflow total $=$ Strength $\times$ area $=$ Cause

$\hookrightarrow$ Electric flux has the units of coulomb's

ϑ is constant

ϑ's direction & direction of divergence

$\hookrightarrow$ closed surface element

Strength around the cause $\vec{D} = \frac{dQ}{ds} = \frac{C}{m^2}$

where ds = any closed surface element

Strength at the cause $= \frac{dQ}{dv} = \frac{d}{dl}\left(\frac{dQ}{ds}\right)$

$= \nabla \cdot D$ = Divergence of D

$= \frac{\text{outflow}}{\text{volume}} = \rho_v$

$$\boxed{\nabla \cdot D = \rho_v}$$ → Gauss law in point form

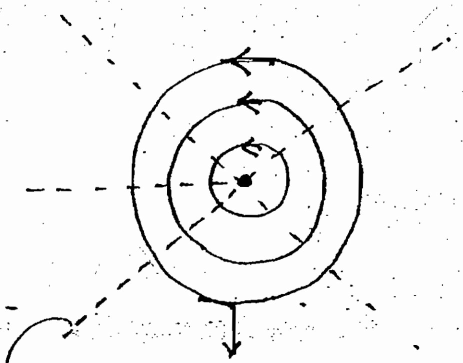
→ The (·) dot product signifies that D's change or derivative exists only when we move in direction of D. This is called as directional derivative

→ The (·) dot product indicates a surface (vector) derivative resulting in volume (scalar) derivative

Summary 2 :-

Total circulation = Strength x length

= current



perpendicular to H & direction of decrease

H's direction & direction in which H is constant

Total Circulation = strength x length

= Current

$$H = \text{Strength around the current} = \frac{dI}{dl} = \frac{\text{Amp}}{\text{m}}$$

where dl = any close line element

$$\text{Strength at the current} = \frac{d}{dl}\left(\frac{dI}{dl}\right) = \nabla \times H$$

$$= \text{curl of } H$$

$$= \frac{\text{Circulation}}{\text{Area}} = J$$

$\boxed{\nabla \times H = J}$ → This is Ampere's law in point form

- The (X) cross product signifies that H changes only when we move perpendicular to H
- The (X) cross product implies a vector (line) derivative resulting a vector (surface) derivative

Summary-3 :-

Col – Q

I – Amps

Cause

Source

Gauss Law ← Mathematical → Ampere's Law

Effects

Density – $Col/m^2 \to D$ — Strength — $\vec{H}$ – amp/m → Intensity

Colomb's Law — ϵ ← Physical → μ — Lorentz's Law

Force b/w Causes

Intensity (V/m) — $\vec{E}$

$\vec{B}$ → $Weber/m^2$ → Density

Summary-4 :-

Divergence & stoke's theorem :-

$$\rightarrow \boxed{\oint D\cdot ds = Q = \int \rho_v\, dv = \int (\nabla\cdot D)\, dv}$$

This is gauss divergence theorem

$$\rightarrow \boxed{\oint H\cdot dl = I = \int J\cdot ds = \int (\nabla \times H)\cdot ds}$$

Stoke's theorem

Note :-

Wrong Notations :-

→ $\int H\cdot dl = \int (\nabla \times H)\cdot ds$ → $\int D\cdot ds = \int (\nabla\cdot D)\, dv$

→ $\oint H\cdot dl = \int (\nabla\cdot H)\, ds$ → $\oint D\, ds = \int (\nabla \times D)\, dv$

→ $\oint H\cdot dl = \oint (\nabla \times H)\cdot ds$

Note :-

Maxwell equations having two formats, 2 field, 2 (·) dot, 2 (X) cross, 2 line, 2 surface, 4 eqn

Summary 5 :-

Maxwell's Equation

Integral form		Point form
1. $\oint D \cdot ds = Q$	[Gauss Law]	1. $\nabla \cdot D = \rho_v$
2. $\oint E \cdot dl = 0$	[Irrotational Vector]	2. $\nabla \times E = 0$
3. $\oint B \cdot ds = 0$	[Solenoidal]	3. $\nabla \cdot B = 0$
4. $\oint H \cdot dl = I$	[Ampere's Law]	4. $\nabla \times H = J$

Note :-

→ $\nabla \times E = 0$. (Always)

curl is generally applied with intensity vector

$$\nabla \times \left(\frac{D}{\epsilon}\right) = 0$$

If ϵ = constant i.e. medium is homogeneous & isotropic

$$\Rightarrow \frac{1}{\epsilon} \nabla \times D = 0 \quad \Rightarrow \boxed{\nabla \times D = 0}$$

→ Acc. to stoke's theorem (For integral form)

$$\oint E \cdot dl = \int (\nabla \times E) \cdot ds = 0$$

Note :-

i) $\nabla \cdot B = 0$ (Always)

Divergence is generally applied with density vector

$$\nabla \cdot (\mu H) = 0$$

If μ is constant i.e. medium is homogeneous and isotropic

$$\mu (\nabla \cdot H) = 0$$

$$\nabla \cdot H = 0 \longrightarrow \text{Also correct}$$

For integral form, apply divergence theorem

$$\oint B \cdot ds = \int (\nabla \cdot B)\, dv = 0$$

(ii) $\oint E \cdot dl = 0 \longrightarrow$ KVL

(iii) $\oint B \cdot ds = 0 \longrightarrow$ KCL

Coordinate Systems

It is a way of adressing/locating points in a 3d-space from a known reference

Reference :-

3 infinite mutually orthogonal planes

$\longrightarrow$ XY, YZ & ZX planes

Cartesian Coordinate System :-

eg:- 1. Uniform plane waves

2. Rectangular wave guides

3. Capacitor plates

$\longrightarrow$ Parameters (x, y, z)

$\longrightarrow$ Unit vector (a_x, a_y, a_z)

Reference :-

1 infinite line

axial   z-axis

Cylinderical coordinate system

eg:- (I) Line charges

(II) I carrying waves wires

(III) cylindrical waveguide

Parameter → ρ , ϕ , z

Unit vectors → a_ρ , a_ϕ , a_z

Reference :-

1 single point

point   origin

Spherical coordinate systems

eg:-(I) point charges

(II) antennas

Parameter → r , θ , ϕ

Unit vectors → a_r , a_θ , a_ϕ

Relation b/w coordinate systems :-

$$x \to y \to z$$

$$\rho \to \phi \to z$$

$$r \to \theta \to \phi$$

Note:-

All the three co-ordinates systems are unit

- orthogonal
- orthonormal
- right handed system

Orthogonal :-

The (·) dot product of any two unit vectors of the same coordinate system is always zero

$\rightarrow a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$

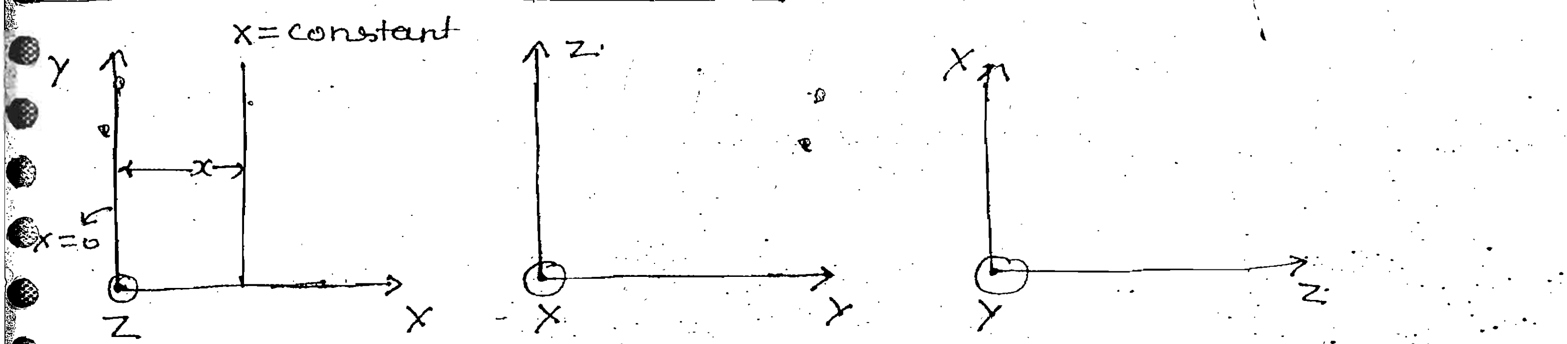
$\rightarrow a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$

Orthonormal :-

The (x) cross product of any two unit vectors of the same co-ordinate system is always the third unit vector

$$a_x \times a_y = a_z$$
$$a_y \times a_z = a_x$$
$$a_z \times a_x = a_y$$

Cartesian Coordinate Systems :-



X $\rightarrow$ It is the shortest distance or perpendicular distance of the point from YZ plane

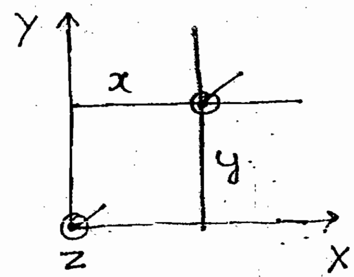
Note:-

The locus of all the points with X = constant is an infinite plane parallel to YZ plane

Range of X → $(-\infty, \infty)$

Note :-

The locus of all the points with $X, Y =$ constant is an infinite line parallel to z-axis

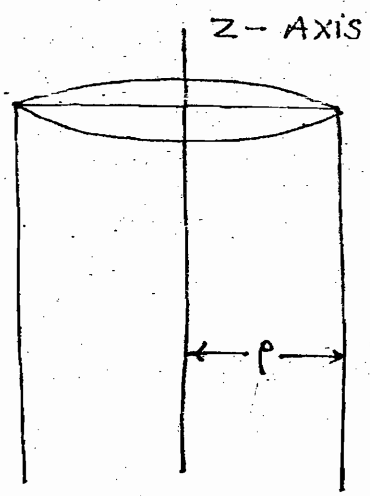


Cylindrical Coordinate System :-

ρ :→ It is the shortest distance / perpendicular distance / radial distance of the point from reference line

Note :-

The locus of all the points with $\rho =$ constant. is a concentric cylinder around a reference line

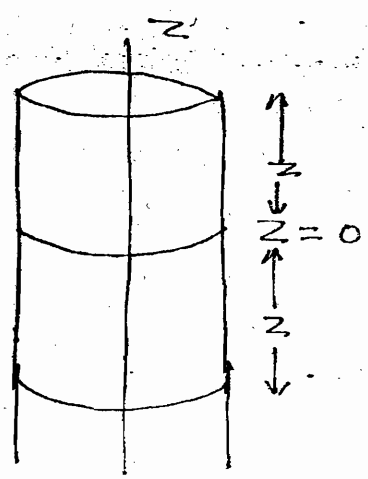


Range → $[0, \infty)$

Z :→

It is the height of the point along the reference line

Range :- $(-\infty, \infty)$

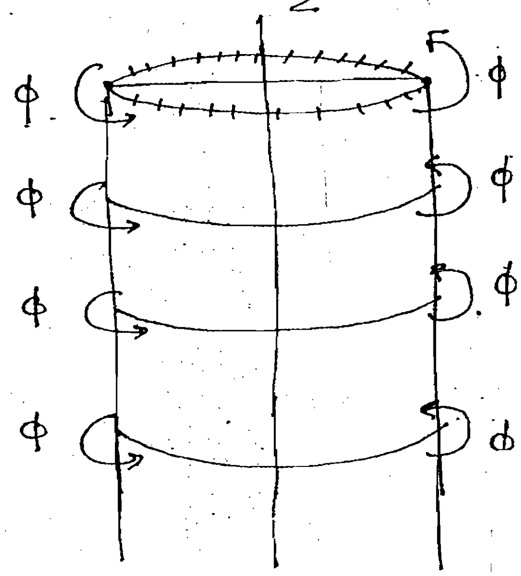


Note:-

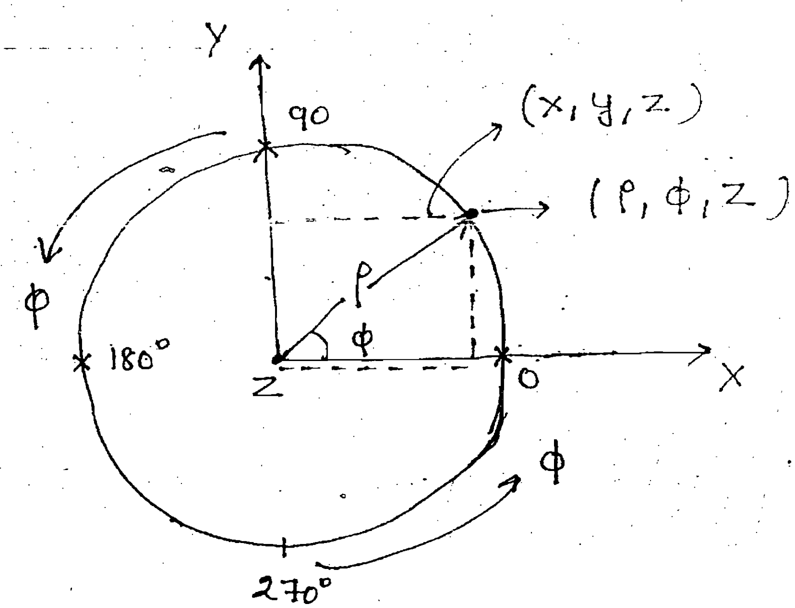
The locus of all the points with ρ, Z = constant is a cocentric circle around the line

ϕ :-

It is the orientation of the point around the reference line



Range :- $[0, 2\pi)$



Point Transformation :-

Cartesian $\longleftrightarrow$ cylindrical

$x = \rho \cos \phi$

$y = \rho \sin \phi$ $\qquad \rho = \sqrt{x^2 + y^2}$

$z = z$ $\qquad \phi = \tan^{-1}(y/x)$

Spherical Coordinate Systems!—

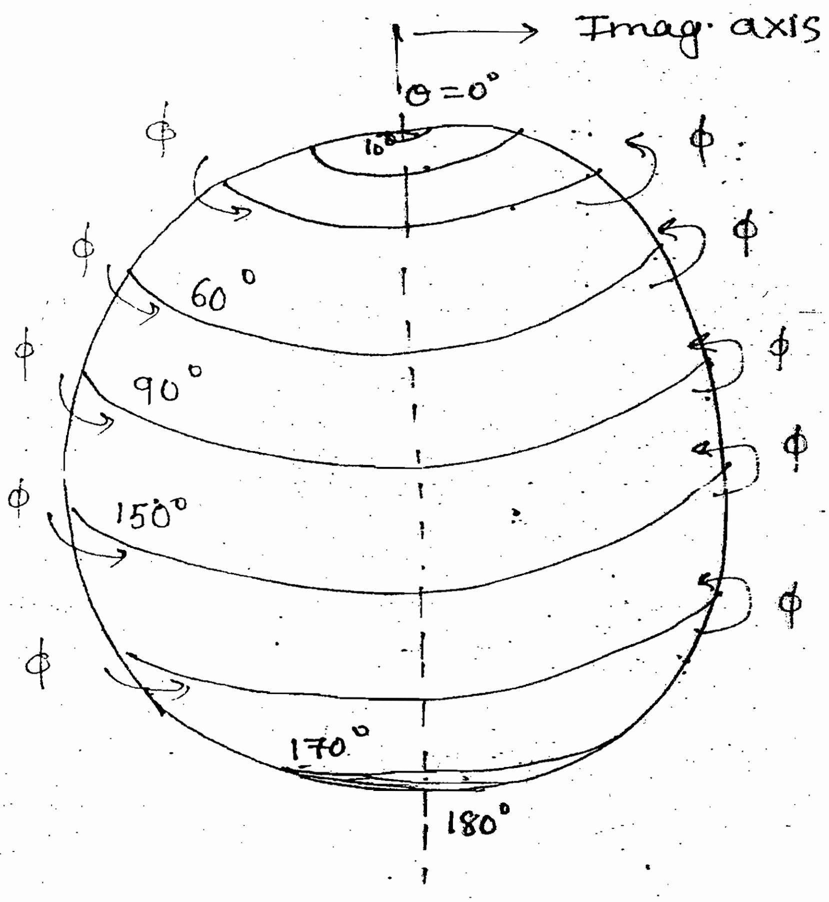
r :→

It is the radial distance or shortest distance of the point from reference point.

Note:-

The locus of the all the points with r is constant is concentric sphere around the origin

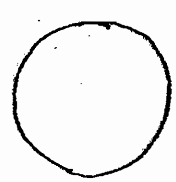
Range → $[0, \infty)$



θ :—

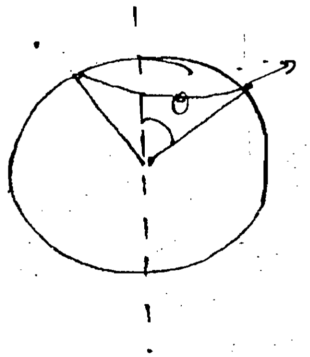
It is the angle of cone whose base is a circle on the r = constant sphere

Range → $[0, \pi]$



Identification of θ constant Circle :-

Consider an imag. axis through the centre of the sphere and take a radial segment inclined by θ with imaginary axis. Rotate the radial segment which results in a cone whose base is a circle on the r = constant sphere



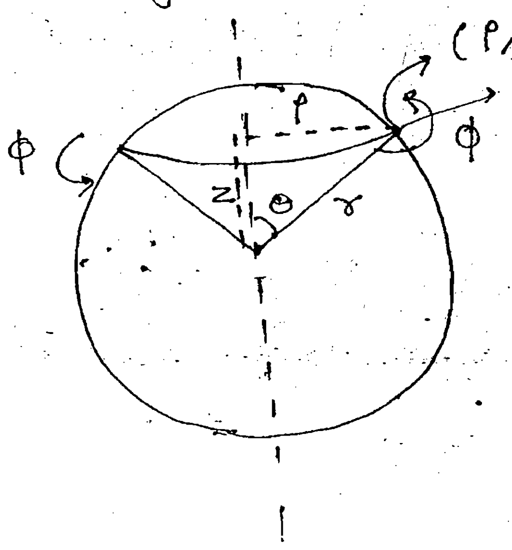
ϕ :-

It is the orientation angle around the imaginary axis

Range of $\phi \rightarrow [0, 2\pi]$

Point transformation :-

If z-axis of cylindrical coordinates coincide with imaginary axis of spherical coordinates then ϕ is same in both the coordinate system



Cylindrical ⟷ Spherical

$$\rho = r\sin\theta$$

$$\phi = \phi$$

$$z = r\cos\theta$$

Point Transformation :-

Cartesian $\longleftrightarrow$ Spherical

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \cdot \sin\phi$$

$$z = r \cos\theta \qquad r = \sqrt{x^2+y^2+z^2}$$

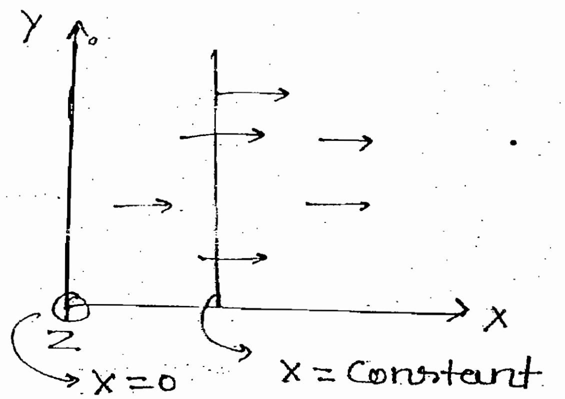
Unit Vectors & Orthogonality in coordinate system :-

Unit vector of Parameter :-

It has unit magnitude & a direction in which the parameter increases

I) Cartesian coordinate system :-

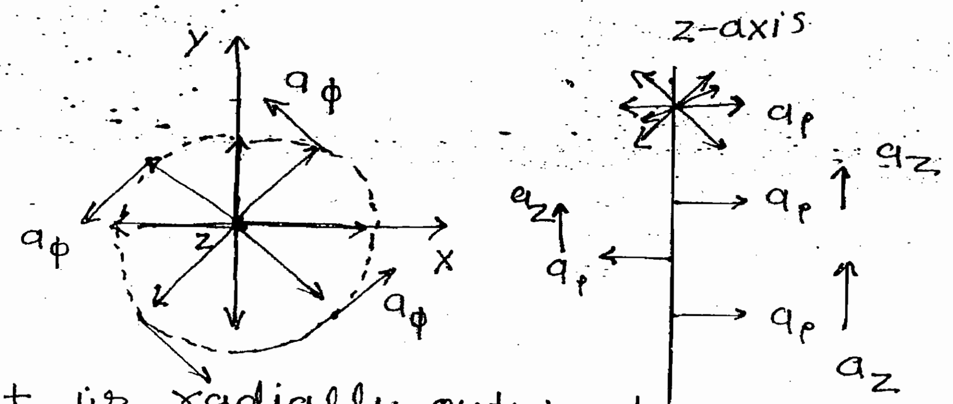
a_x :-



It is always normal to YZ plane or any plane which has X = constant

II) Cylindrical coordinate system :-

a_ρ :-



It is radially outward from the reference line

a_ϕ :-

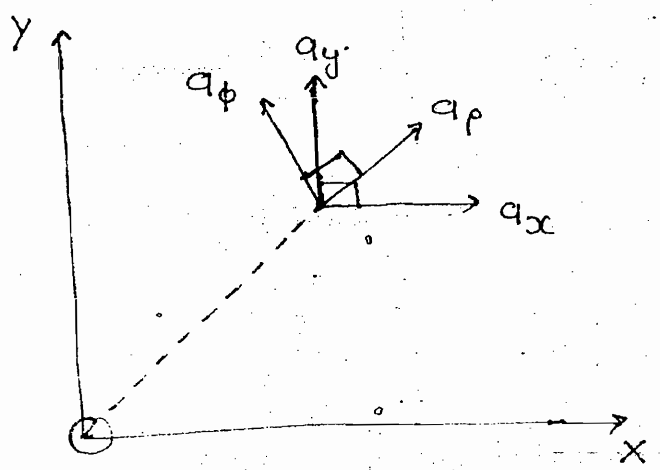
It is tangentially around the reference line

** $\boxed{a_\rho \perp a_\phi \perp a_z}$

a_z :-

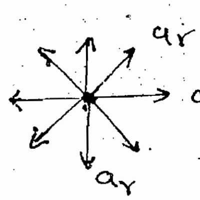
It is linearly along the reference line

Summary :-



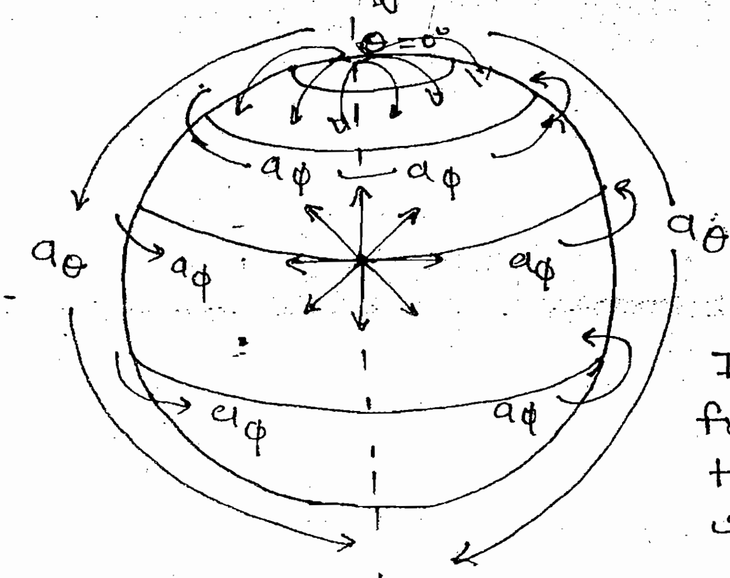
Spherical Coordinate System :-

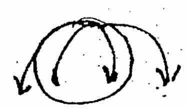
a_r :-



It is radially outward from the origin

a_θ :-



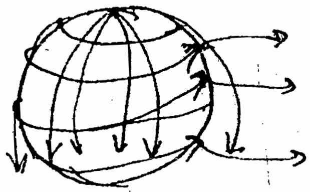


It is vertically from the top to the bottom of the sphere

It is horizontally around the imaginary axis.

ϕ = constant , $\theta \uparrow$

ϕ = constant ; $\theta \uparrow$



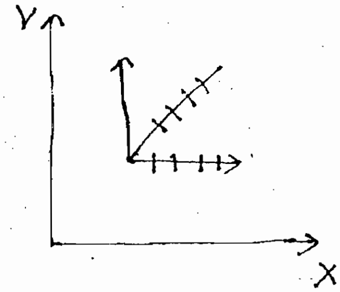
$$\boxed{a_r \perp a_\theta + a_\phi}$$

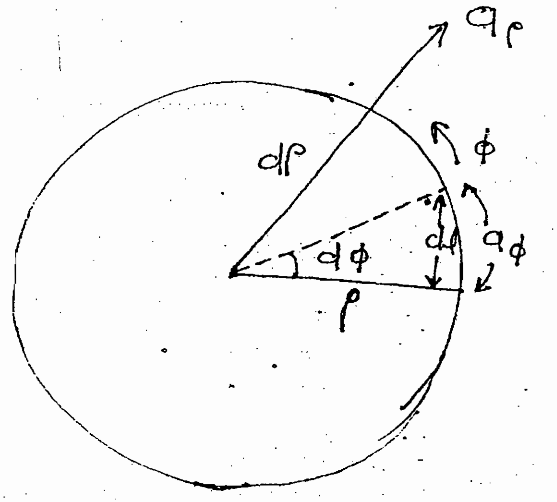
Lecture - 3

Line as a Vector:-

It has a magnitude equal to its length and a direction in which the length parameter increase

Cartesian :-

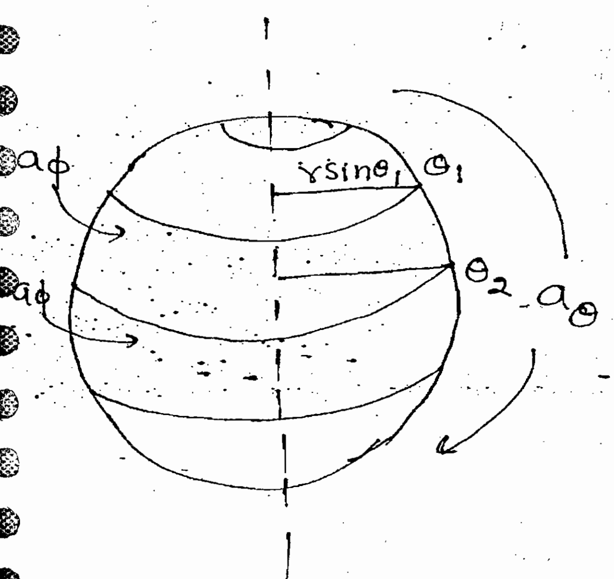


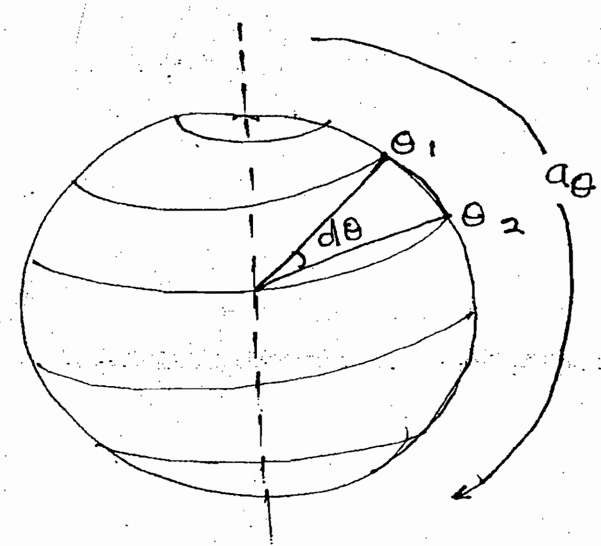


$$d\vec{l} = dx\, a_x + dy\, a_y + dz\, a_z$$

Spherical :-

$$d\vec{l} = dr\, a_r + r\, d\theta \cdot a_\theta + r\sin\theta \cdot d\phi\, a_\phi$$





In angular direction length is curvature

Note :-

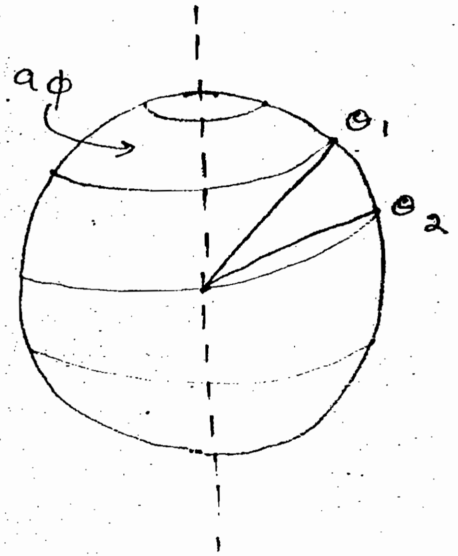
In angular directions length is a curvature not straight line or linear

Curvature length = Radius x angle

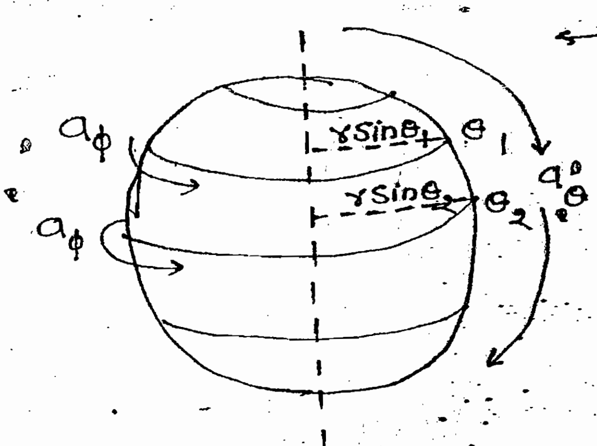
Cartesian → $\vec{dl} = dx\,a_x + dy\,a_y + dz\,a_z$

Cylindrical → $\vec{dl} = d\rho\,a_\rho + \rho\,d\phi\,a_\phi + dz\,a_z$

Spherical → $\vec{dl} = dr\,a_r + r\,d\theta\,a_\theta + r\sin\theta\,d\phi\,a_\phi$



← Top View



Note:-

In ϕ direction in spherical coordinating system curvature length is height on the sphere dependent i.e. it depends on the θ value of the circle.

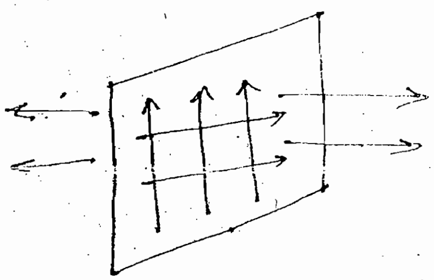
Summary :-

Parameters :-			Scaling Factors		
x	y	z	1	1	1
ρ	ϕ	z	1	ρ	1
r	θ	ϕ	1	r	$r\sin\theta$
u	v	w	h_1	h_2	h_3

$$\vec{dl} = h_1 du a_x + h_2 dv a_v + h_3 dw a_w$$

Surface as a vector :-

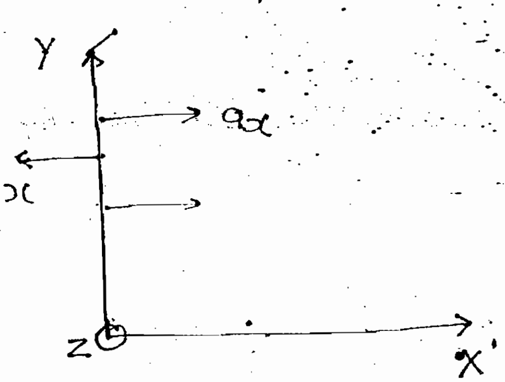
It has a magnitude equal to its area and a direction normal to the plane of surface

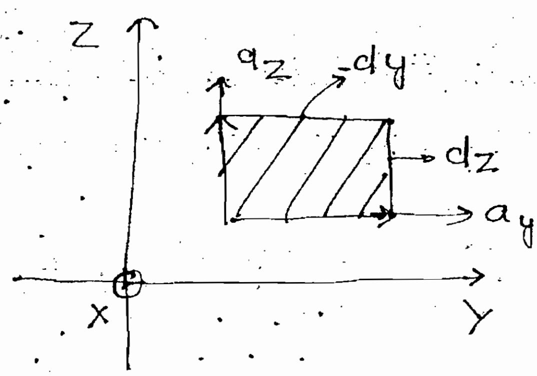


Direction is unique only when we take normal to the surface as there can be two tangential directions.

X = constant surface

X = 0 plane, YZ plane

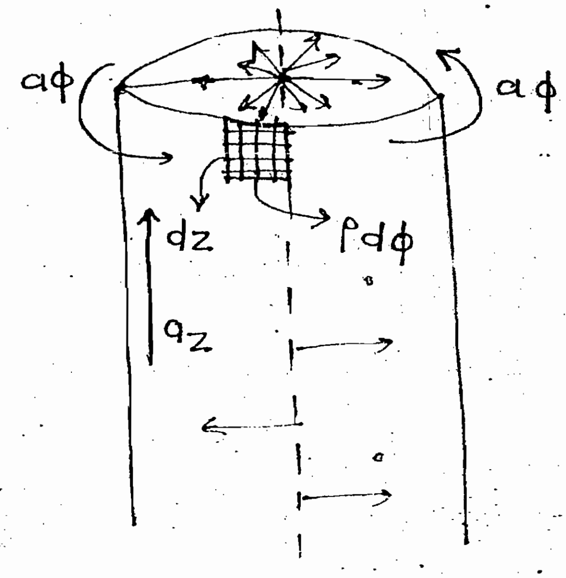


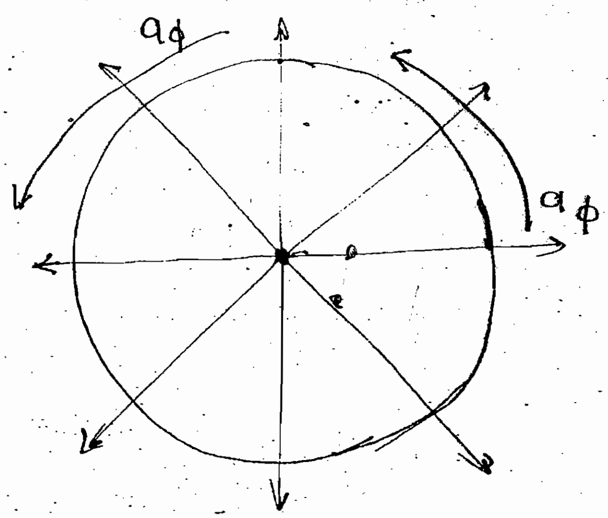


Cartesian :-

$$\boxed{\vec{ds} = \underbrace{dy\,dz}_{\text{mag.}}\ \underbrace{a_x}_{\text{direction}} + dz\,dx\,a_y + dx\,dy\,a_z}$$

Cylindrical Coordinate system :-





Top view

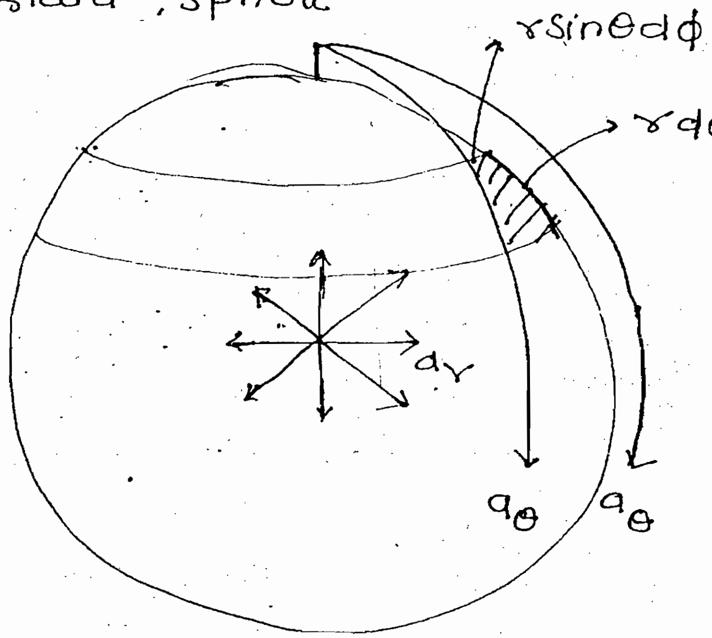
a_ρ = normal i.e. direction

a_ϕ, a_z = tangential → ∥ to surface

$$\vec{ds} = \rho\,d\phi\,dz\,a_\rho + dz\,d\rho\,a_\phi + \rho\,d\rho\,d\phi\,a_z$$

Spherical coordinate System :-

$r =$ constant, sphere



a_r = normal
↓
direction

a_θ, a_ϕ = tangential

$$\boxed{d\vec{s} = r^2 \sin\theta \, d\theta \, d\phi \, \underline{a_r} + r\sin\theta \, d\phi \, dr \, \underline{a_\theta} + r \, dr \, d\theta \, \underline{a_\phi}}$$

$$\boxed{ds = h_2 h_3 \, dv \, dw \, a_u + h_3 h_1 \, dw \, du \, a_v + h_1 h_2 \, du \, dv \, a_w}$$

Summary :-

1 parameter = constant
2 parameter = variable
→ surface

Surface direction = constant direction
= UNIQUE

2 Parameter = Constant
1 Parameter = Variable
→ line

Lines direction = variable's direction
= UNIQUE

3 Parameters = variable] → Volume

3 Parameters = constant] → Point

Volume as a scalar :-

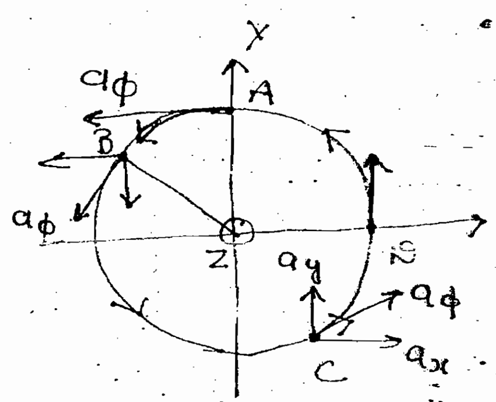
It has no unique direction & it is the scalar triple product of lengths in all three dimensions

$$dv = dx\,dy\,dz$$
$$= \rho\,d\rho\,d\phi\,dz$$
$$= r^2 \sin\theta\,dr\,d\theta\,d\phi$$
$$= h_1 h_2 h_3\,du\,dv\,dw$$

Workbook -1

2. At Point A

$a_\phi = -a_x$



divided by $\sqrt{2}$ to make magnitude = 1

At Point B

$$a_\phi = \frac{-a_x - a_y}{\sqrt{2}}$$

At Point C

$$a_\phi = \frac{a_x + a_y}{\sqrt{2}}$$

At point D

$$a_\phi = a_y$$

3. $\vec{D} = xy\,a_x + yz\,a_y + zx\,a_z \quad C/m^2$

$y = 2 \qquad 0 < x < 4$

$\qquad\qquad 0 < z < 2$

Since only 2 variable are present. Hence it is not a closed surface

$$ds = dx\,dz\,a_y$$

$$\int D \cdot ds = \Psi_e$$

$$\Rightarrow \Psi_e = \int_{x=0}^{4} \int_{z=0}^{2} yz \, dx \, dz \Bigg|_{y=2}$$

$$= 2 \frac{z^2}{2}\Bigg|_0^2 \cdot x \Bigg|_0^4 = 16 \, C \longrightarrow \text{flux}$$

4. $\vec{D} = 5(\rho - 3)^2 a_\rho$

$\rho = 4$, $0 < \phi < \pi$

$-5 < z < 5$

$$ds = \rho \, d\phi \, dz \, a_\rho$$

$$\Psi_e = \int_{\phi=0}^{\pi} \int_{z=-5}^{5} 5(\rho-3)^2 a_\rho \cdot \rho \, d\phi \, dz \, a_\rho \Bigg|_{\rho=4}$$

$$= 20 \, \phi \Big|_0^{\pi} z \Big|_{-5}^{5}$$

$$= 200\pi$$

Divergence and curl of vector :-

$$\vec{A} = A_u a_u + A_v a_v + A_w a_w$$

$$\nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}(h_2 h_3 A_u) + \frac{\partial}{\partial v}(h_3 h_1 A_v) + \frac{\partial}{\partial w}(h_1 h_2 A_w)\right]$$

Cartesian :-

$$\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

cylindrical :-

$$\nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cdot A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Spherical :-

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

Workbook - 1

$$\vec{D} = \rho\cdot z\cos^2\phi\, a_z$$

$$\rho_v = ? \quad \text{at } (1, \pi/4, 3)$$

$$\nabla\cdot D = \rho_v$$

$$\nabla\cdot D = \frac{\partial(\rho z\cos^2\phi)}{\partial z}$$

$$= \rho\cos^2\phi = 1\cdot\frac{1}{2} = 0.5\ C/m^3$$

5. $$\vec{D} = \rho\cdot z\cdot\cos^2\phi\, a_\rho$$

$$\nabla\cdot D = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\cdot\rho z\cos^2\phi)$$

$$= \frac{1}{\rho} z\cos^2\phi\cdot 2\rho = 2\cdot 3\cdot\frac{1}{2} = 3\ C/m^3$$

7. $$\vec{F} = \rho a_\rho + \rho\sin^2\phi\, a_\phi - z a_z$$

$$\nabla\cdot\vec{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\cdot\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho\cdot\sin^2\phi) + \frac{\partial}{\partial z}(-z)$$

$$= 2 + 2\sin\phi\cos\phi - 1$$

$$= 1 + \sin 2\phi$$

Ans - D

Ques - GATE 2012

$$\vec{A} = k r^n a_r \qquad n = ?$$

If $\nabla\cdot A = 0$

Soln:- $\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 k r^n)$

$= \frac{k}{r^2}(n+2) r^{n+1} = 0$

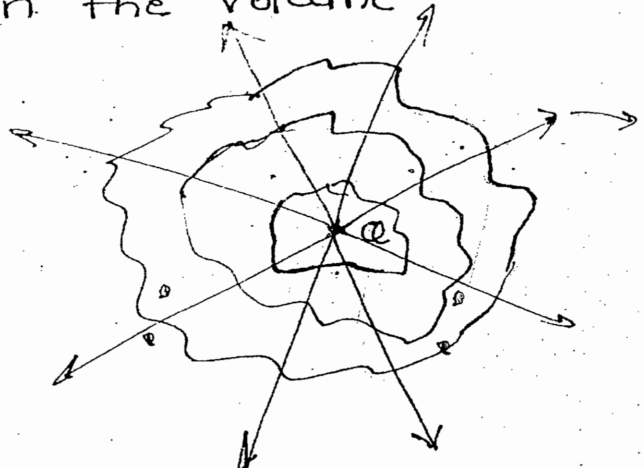
$\Rightarrow n = -2$, Ans

Curl of a vector $\vec{A}$:-

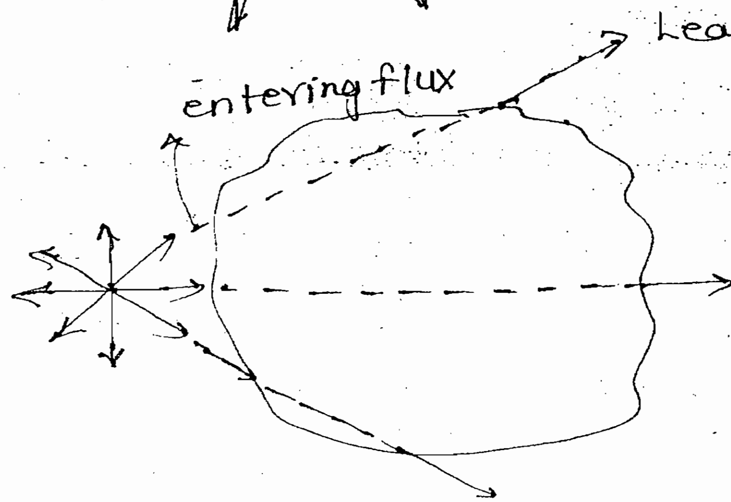
$$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1 a_u & h_2 a_v & h_3 a_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_v & h_2 A_v & h_3 A_w \end{vmatrix}$$

Static Electric Fields :-

Gauss Law :- The net (total) electric flux leaving any closed surface is always equal to the charge (effects) enclosed in the volume (cause)



$\oint D \cdot ds = Q$



$\oint D \cdot ds = 0$

12

The word electric flux means the attractive or repulsive force on any +ve test charge placed in the electric field. Hence electric field or electric flux or lines of force physically represent the direction in which a test charge moves away when placed in the field

→ When the surface is closed the total outflow or flux leaving is independent of density and area. i.e.

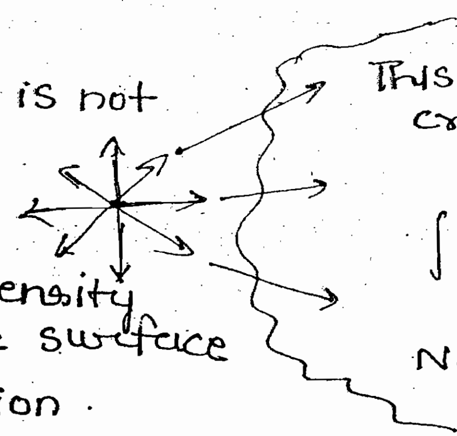
$$\oint D \cdot ds = Q$$

→ If the charge is outside the effects still exists but net effects is zero

$$\text{entering flux} = \text{leaving flux}$$

$$\oint D \cdot ds = 0$$

→ If the surface is not completely enclosing the flux crossing depends on the density of the flux and the surface area of consideration.



This is crossing flux

$$\int D \cdot ds = \psi_c$$

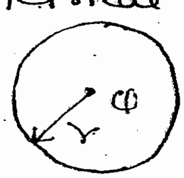
↓

Not Gauss Law

Application of Gauss Law! -

1) Electric field strength of a point charge! -

Consider a concentric symmetrical surface such that D is same everywhere



$$\oint D \cdot ds = Q$$ r = constant, a_r const directed

$$\Rightarrow \quad D \cdot S = Q$$

$$\Rightarrow \quad D = \frac{Q}{S}$$

$$\Rightarrow \quad \boxed{D = \frac{Q}{4\pi r^2} a_r} \quad C/m^2$$

→ Coulomb had a different measure of field strength i.e. the force measured & he related this with charge measure using ϵ. This is called as intensity

$$\vec{E} = \frac{\vec{F}}{Q_t} = \frac{\vec{D}}{\epsilon} = \frac{Q}{4\pi\epsilon r^2} a_r$$

↓

Intensity (N/C)

$$\Rightarrow \boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon r^2}}$$

This is coulomb's law

Static Magnetic Fields :-

Biot - Savart's Law (Ampere's law for current elements) :-

It is derived from Ampere's law & consider a $d\vec{l}$ length I (DC current) carrying as a basic cause of H field

$I\,d\vec{l}$ — (Amp-m) is basic cause of H field

$$\vec{H} = \frac{I\,dl \times a_r}{4\pi r^2}$$

↓

Intensity (Amp/m).

→ $\vec{H}$ direction = I flow direction [cross] × Radial vector from current

→ Lorentz had a different measure of field strength i.e. density measure which is physically force per basic cause

$$\boxed{\vec{B} = \mu\vec{H} = \frac{\mu I dl \times a_r}{4\pi r^2} = \frac{\vec{F}}{\overrightarrow{Idl}} = \frac{\text{Newton}}{\text{Amp-m}}}$$

$$\vec{F} = q\,(V \times B)$$

$$\Rightarrow dF = dQ\left(\frac{dl}{dt}\cdot B\cos\theta\right) = IdlB$$

$$\Rightarrow B = \frac{F}{Idl}$$

Note:-

→ $$D\cdot E = \frac{\text{col}}{m^2} \times \frac{\text{Newton}}{\text{col}} = \frac{\text{Newton}}{m^2}$$

= Electric pressure at that point

→ $$B\cdot H = \frac{\text{Newton}}{\text{Amp-m}} \times \frac{\text{Amp}}{m} = \frac{\text{Newton}}{m^2}$$

= Magnetic Pressure.

→ $$\frac{\text{Newton} \times m}{m^2 \times m} = \frac{\text{Joules}}{m^3} = \text{Energy density}$$

→ $$\frac{1}{2}D\cdot E = \frac{1}{2}\epsilon E^2 = \text{Energy density in electric field}$$

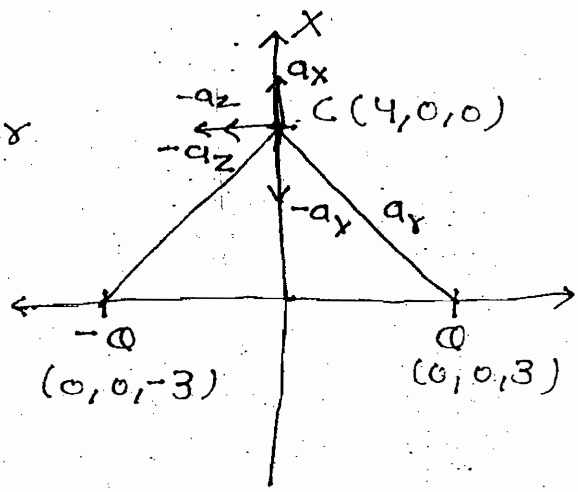
→ $$\frac{1}{2}B\cdot H = \frac{1}{2}\mu H^2 = \text{Energy density in H field}$$

Workbook

$Q \rightarrow (0,0,3)$

$-Q \rightarrow (0,0,-3)$

E at $C\ (4,0,0)$



$$E = \frac{Q}{4\pi\epsilon r^2} a_r$$

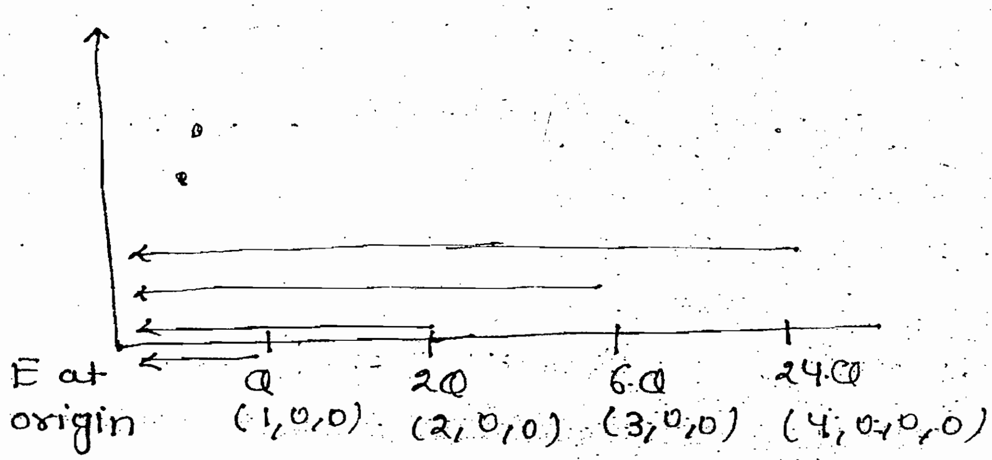
$E_1 = (a_x, -a_z)$
(Q)

$$= \frac{4a_x - 3a_z}{5}$$

$E_2 = (-a_x, -a_z)$
$(-Q)$

$$= \left(\frac{-4a_x - 3a_z}{5}\right)$$

$E_{total} = -a_z$
(direction)



$$E_T = E_1 + E_2 + E_3 + E_4$$

$$E = \frac{Q}{4\pi\epsilon r^2} a_r$$

$$a_r = -a_x$$

$$E_T = \left(\frac{Q}{4\pi\epsilon(1)^2} + \frac{2Q}{4\pi\epsilon(2)^2} + \frac{6Q}{4\pi\epsilon(3)^2} + \frac{24Q}{4\pi\epsilon(4)^2} \right) a_x$$

$$= \frac{Q}{4\pi\epsilon}\left(1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{2}\right)(-a_x)$$

:- Net flux leaving $= \oint D \cdot ds = Q =$ charge enclosed

Sphere $\longrightarrow$ centre = origin
radius = 6m

$2C \longrightarrow (4,8,3) \longrightarrow d_1 = \sqrt{4^2+8^2+3^2} > 6$

$8C \longrightarrow (2,-1,-3) \longrightarrow d_2 = \sqrt{2^2+1^2+3^2} < 6$

$-12C \longrightarrow (-4,0,1) \longrightarrow d_3 = \sqrt{4^2+1^2} < 6$

Net flux leaving = $-4C$

:- Centre $- (2,-3,2)$

$d_1 = \sqrt{2^2 + 11^2 + 1^2} > 6 \longrightarrow$ out

$d_2 = \sqrt{0^2+2^2+5^2} < 6 \longrightarrow$ in.

$d_3 = \sqrt{6^2+3^2+1^2} > 6 \longrightarrow$ out

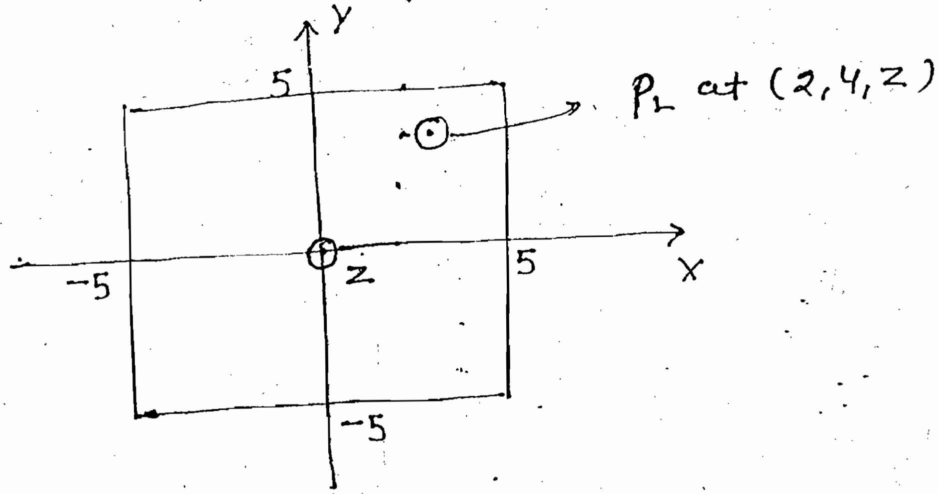
Net flux leaving = $8C$

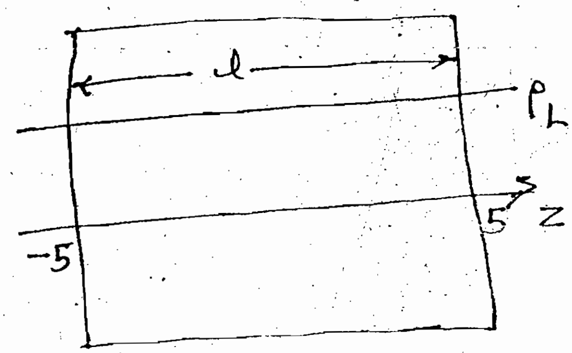
:- Net flux leaving $= Q = \rho_L l$

$\rho_L = 15\, nC/m$

at $x=2, y=4$ for all z

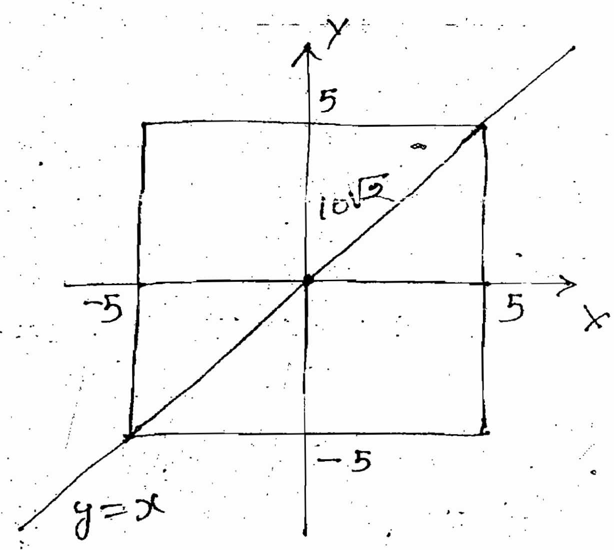
Closed surface $= -5 < x < 5$
$-5 < y < 5$
$-5 < z < 5$





$$Q = 15\,\frac{nC}{m} \times 10$$
$$= 150\ nC \quad \text{, Ans}$$

13 :-

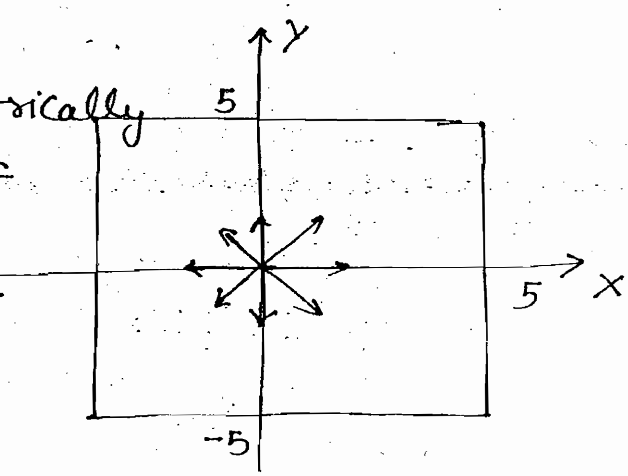


$$Q = 15 \times 10\sqrt{2}$$
$$= 150\sqrt{2}$$

14 :-

$Q = 6\ nC$

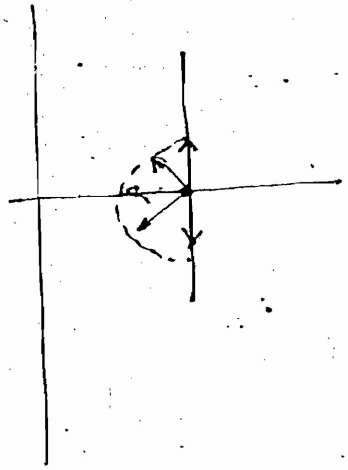
6nC → flux symmetrically crosses 6 sides of cube Inc-flux cross per 1 side



5.

$z = 5m$-plane

$3nC$, $4\pi\gamma$



Note:-

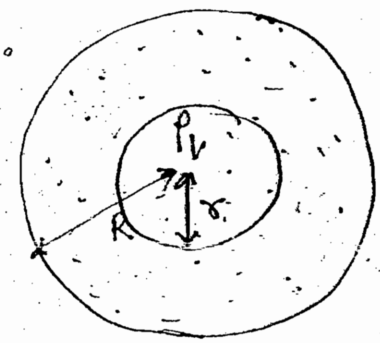
A point charge and a infinite sheet near by has half the flux crossing through

$$\oint_s D \cdot ds = Q$$

$\downarrow$ constant

$$D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$



$$= \frac{\rho_v \frac{4}{3}\pi r^3}{4\pi r^2} = \frac{\rho_v r}{3}$$

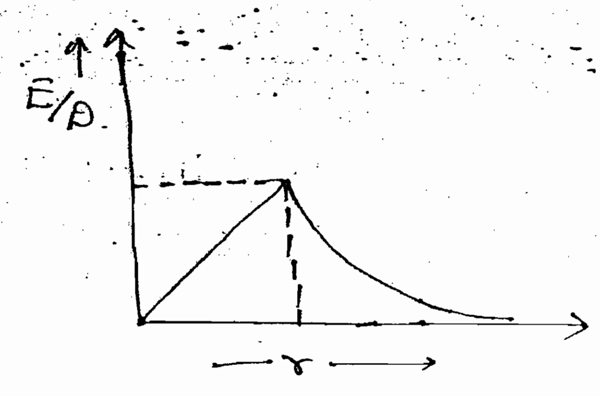
$$E = \frac{\rho_v r}{3\epsilon}$$

$$\boxed{E\left(\frac{R}{2}\right) = \frac{\rho_v R}{6\epsilon}}$$

$$D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

$$= \frac{\rho_v \frac{4}{3}\pi R^3}{4\pi r^2}$$



$$D = \frac{\rho_V R^3}{3r^2}$$

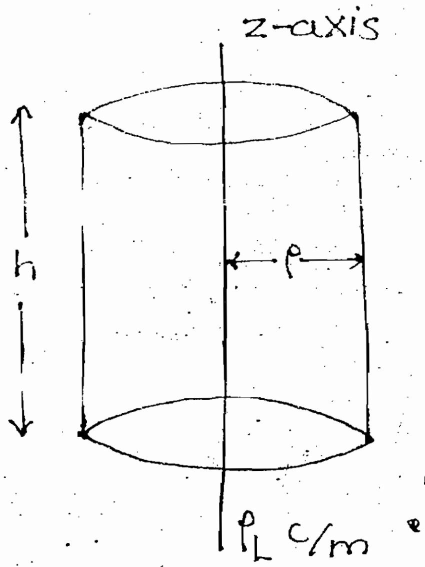
$$E = \frac{\rho_V R^3}{3\epsilon r^2}$$

$$\boxed{E(2R) = \frac{\rho_V R}{12\epsilon}}$$

Gauss Law Application-2 :-

Electric field strength of an infinite length ρ_L c/m line charge :-

Consider a cocentric symmetric Gaussian surface which is a cylinder around the charge



ρ = Constant

$$\oint D \cdot dS = Q$$

a_ρ directed

Constant

$$D \cdot S = Q$$

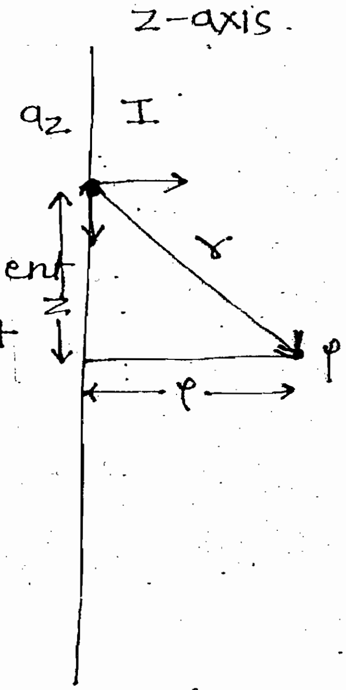
$$\Rightarrow D = \frac{Q}{S}$$

$$\Rightarrow D = \frac{\rho_L \cdot \cancel{h}}{2\pi\rho \cancel{h}}$$

$$\Rightarrow D = \frac{\rho_L}{2\pi\rho} a_\rho$$

$$\Rightarrow \boxed{E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho}$$

Magnetic field strength of an infinite length I carrying wire :-



$$H = \frac{I\,dl}{4\pi r^2} \times a_r$$

Consider a small dl length I current element at the z point height above the point

$$dl = dz\, a_z$$

$$r = \sqrt{\rho^2+z^2}$$

$$a_r = \frac{\vec{r}}{|r|}$$

$$= \frac{\rho a_\rho - z a_z}{\sqrt{\rho^2+z^2}}$$

$$dH = \frac{I\,dz\,a_z}{4\pi(\rho^2+z^2)} \times \left(\frac{\rho a_\rho - z a_z}{\sqrt{\rho^2+z^2}}\right)$$

$$H = \int_{z=-\infty}^{\infty} dH$$

$$\Rightarrow H = \frac{I\rho}{4\pi}\int_{z=-\infty}^{\infty} \frac{dz\,(a_z \times a_\rho)}{(\rho^2+z^2)^{3/2}}$$

Put $z = \rho\tan\theta$

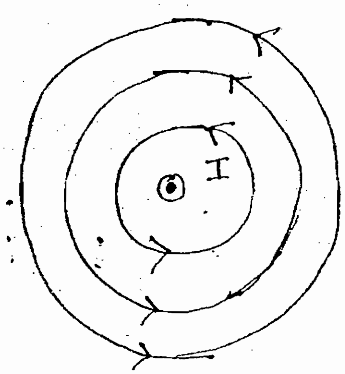
$$dz = \rho\sec^2\theta\, d\theta$$

$$(\rho^2+z^2)^{3/2} = \rho^3\sec^3\theta$$

$$H = \frac{I\rho}{4\pi}\int_{\theta=-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta\, d\theta}{\rho^3 \sec^3\theta} = \frac{I}{4\pi\rho}\int_{-\pi/2}^{\pi/2} \cos\theta \cdot d\theta$$

$$\Rightarrow \quad H = \frac{I}{2\pi\rho}(a_z \times a_\rho) = \frac{I}{2\pi\rho}\, a_\phi$$

Note :-



$$\oint H \cdot dl = I$$

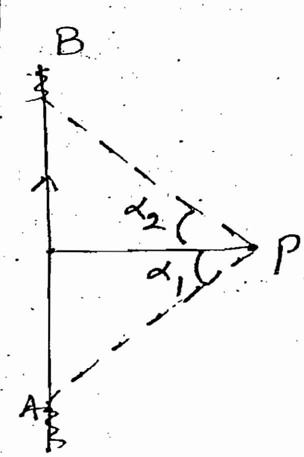
$$H \cdot \text{Length} = \text{current}$$

$$H = \frac{I}{2\pi\rho}\, a_\phi$$

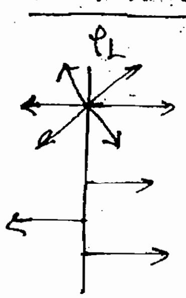
Extension :-

H-field of finite length I wire :-

$$\boxed{H = \frac{I}{4\pi\rho}(\sin\alpha_1 + \sin\alpha_2)\, a_\phi}$$



Summary :-



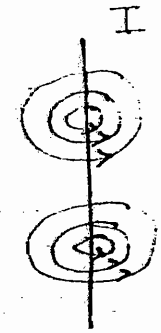
$\rho_L \longrightarrow$ Line charge

$E \longrightarrow$ Field

$$D = \frac{\rho_L}{2\pi\rho}\, a_\rho \qquad E = \frac{\rho_L}{2\pi\epsilon\rho}\, a_\rho$$

I carrying H-field

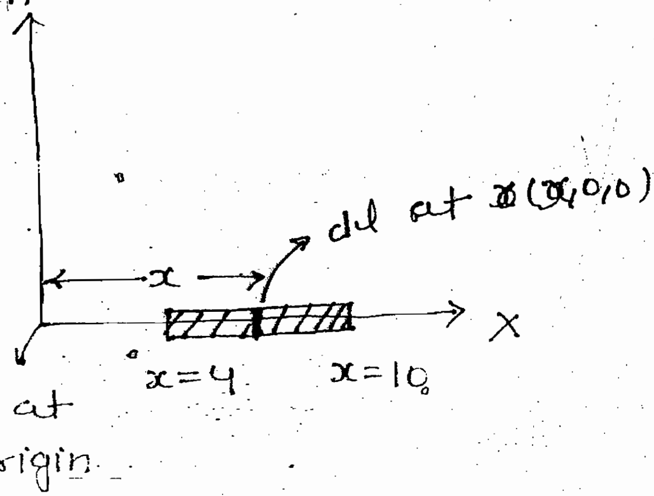
$$H = \frac{I}{2\pi\rho} a_\phi$$



$$B = \frac{\mu I}{2\pi\rho} a_\phi$$

7 :-

consider a dl length on the charge and apply the $\frac{Q}{4\pi\epsilon r^2} a_r$



Field calculation

$$Q = \rho_L dx$$

$$r = x$$

$$a_r = -a_x$$

$$dE = \frac{\rho_L dx}{4\pi\epsilon x^2} (-a_x)$$

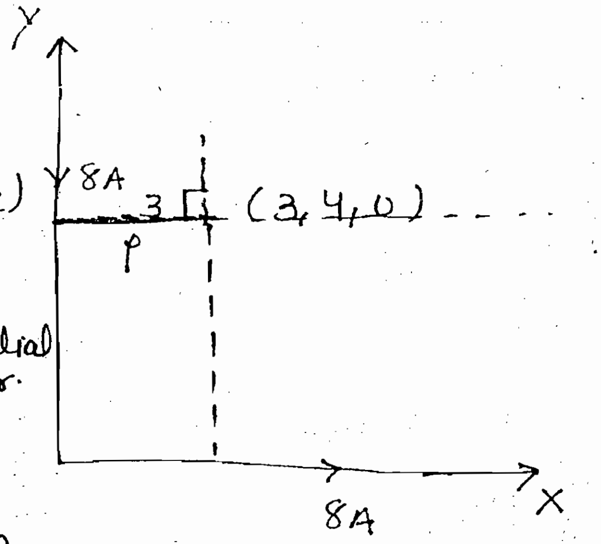
$$E = \frac{\rho_L}{4\pi\epsilon} \int_{x=4}^{10} \frac{dx}{x^2} (-a_x)$$

$$= \frac{\rho_L}{4\pi\epsilon} \left(-\frac{1}{x}\right)_4^{10} (-a_x)$$

8

$$\underset{(Y\text{-axis})}{H} = \frac{8}{4\pi 3}\left(1+\frac{4}{5}\right)$$

$(-a_y \times a_x)$



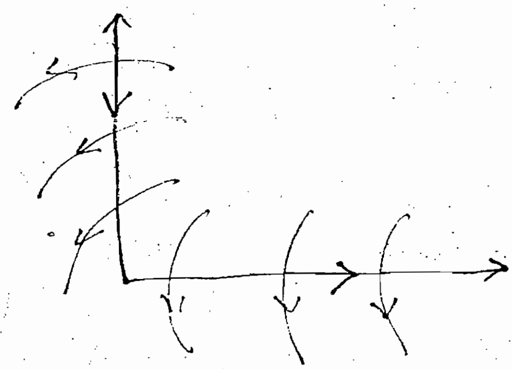
$$a_\phi = a_z \times a_\rho$$

$$H\text{-direction} = \underset{\text{direction}}{I} \times \underset{\text{dir.}}{\text{Radial}}$$

$$= \frac{8}{4\pi 3} \times \frac{9}{5}\,(a_z)$$

$$\underset{(x\text{-axis})}{H} = \frac{8}{4\pi \cdot 4}\left(\frac{3}{5}+1\right)(a_x \times a_y)$$

$$= \frac{8}{16\pi} \times \frac{8}{5}\,(a_z)$$



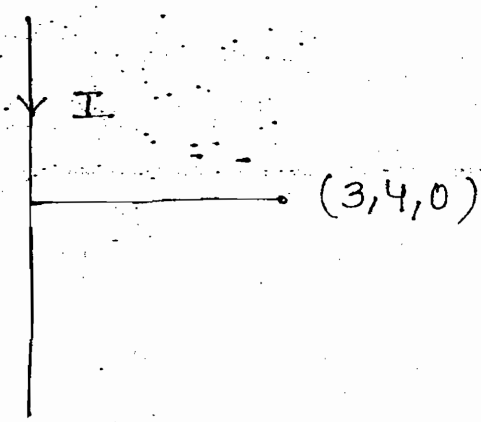
$$\underset{(\text{total})}{H} = \frac{2}{\pi}\,a_z$$

Ques:- Repeat the above problem if current is only entire y-axis

Soln:-

$$H = \frac{8}{2\pi \cdot 3}\,a_z$$

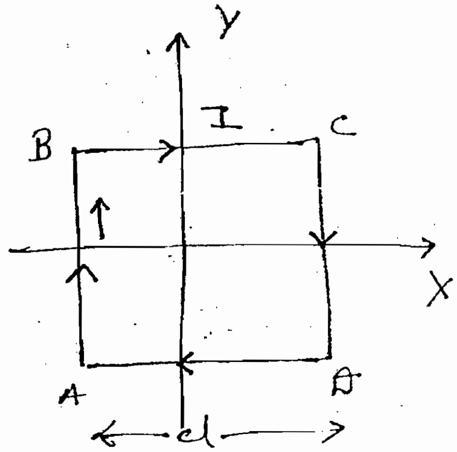
$$= \frac{1.33}{\pi}\,a_z$$



19

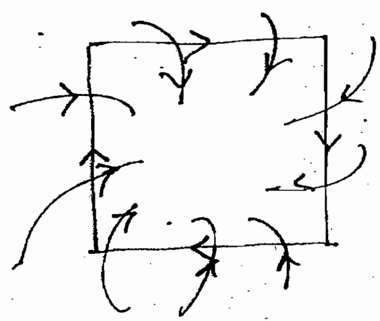
$$H_{AB} = a_y \times a_x = -a_z$$

$$H_{BC} = a_x \times -a_y = -a_z$$



Note:-

→ For any symmetric current distribution H field at geometric centre is always max



→ For any symmetric charge distribution E field at the geometric centre is always zero

$$H = \frac{I}{4\pi \cdot \frac{d}{2}} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \times 4 \, (-a_z)$$

$$= \frac{2\sqrt{2}\, I}{\pi d} (-a_z) = 0.9 \frac{I}{d} (-a_z)$$

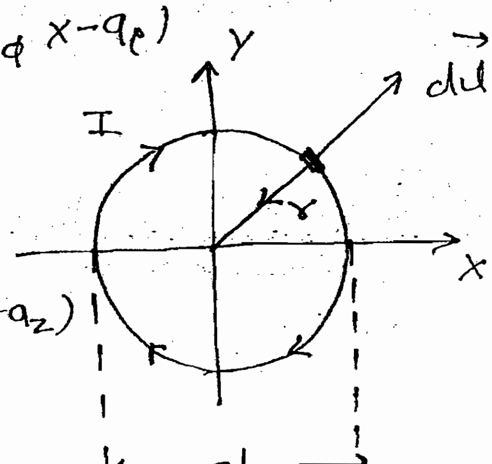
20

$$dH = \frac{I \, \vec{dl}}{4\pi r^2} (-a_\phi \times -a_\rho)$$

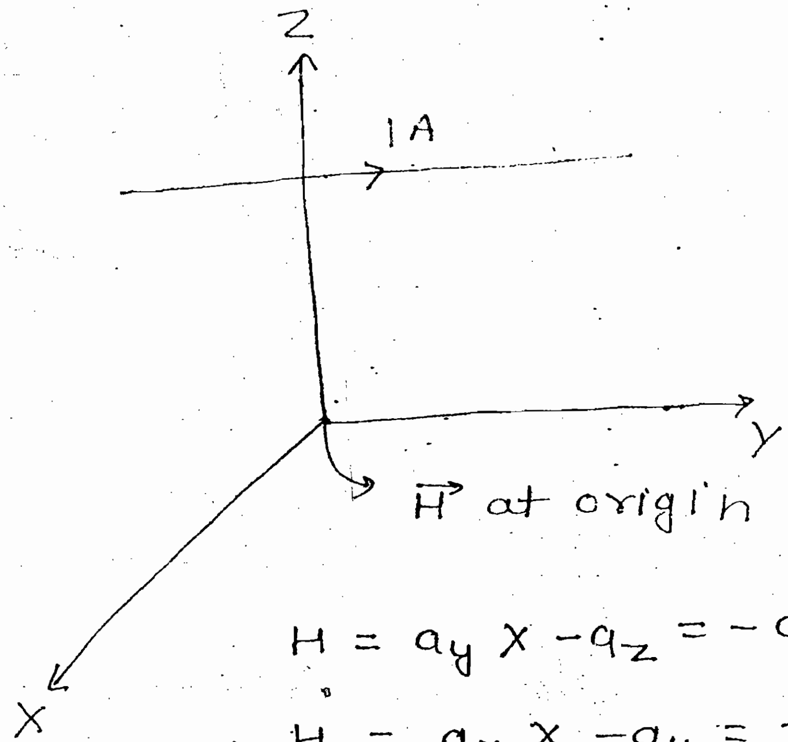
$$H = \int dH$$

$$= \frac{I \, 2\pi r}{4\pi r^2} (-a_z)$$

$$= \frac{I}{2r} = \frac{I}{d} (-a_z)$$



22 :-



$H = a_y \times -a_z = -a_x$

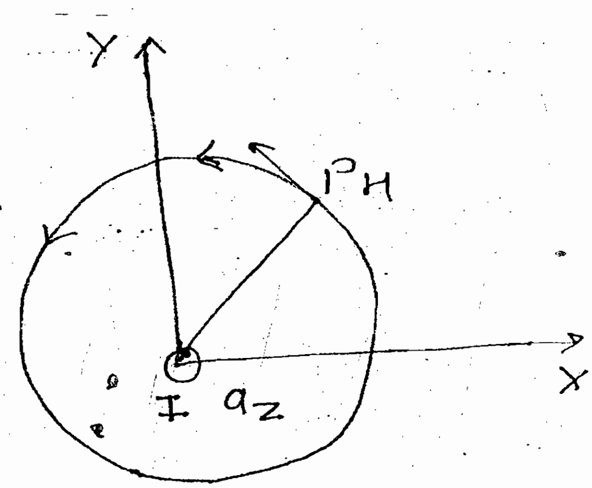
$H = a_x \times -a_y = -a_z$

Ans - d

23.

$H_\phi = a_z \times a_\rho$

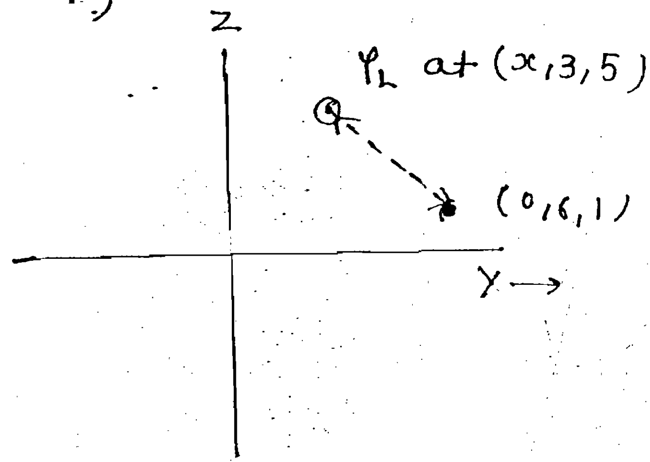
Ans - (c)



2) ρ_L at $y = 3$, $z = 5$ for all x

Given $\vec{E}$ at $(0, 6, 1)$

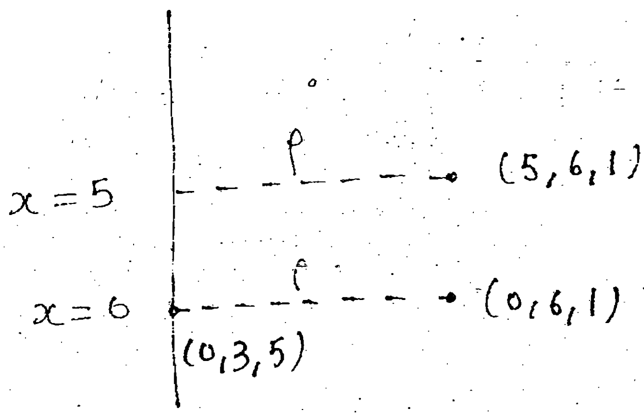
$\vec{E} = ?$ at $(5, 6, 1)$



$$E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho$$

$$E \propto \frac{1}{\rho}$$

ρ_L at $(x, 3, 5)$



For both point ρ is same

$$\rho = \sqrt{3^2 + 4^2} = 5$$

Note :-

In general z-axis $\longrightarrow$ $(0, 0, z)$

Any point (x, y, z)

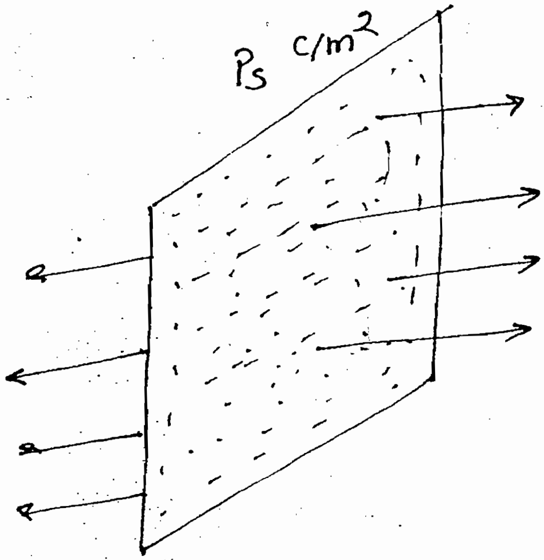
$$\rho = \sqrt{x^2 + y^2}$$

Any line parallel to x-axis $\longrightarrow$ (x, a, b)

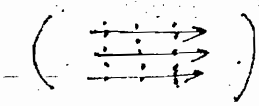
Any point $\longrightarrow$ (x, y, z)

$$\rho = \sqrt{(y-a)^2 + (z-b)^2}$$

Electric field strength of an infinite sheet of ρ_s C/m^2 charge density :-



→ The electric field is completely normal to the sheet and the field lines are parallel to themselves

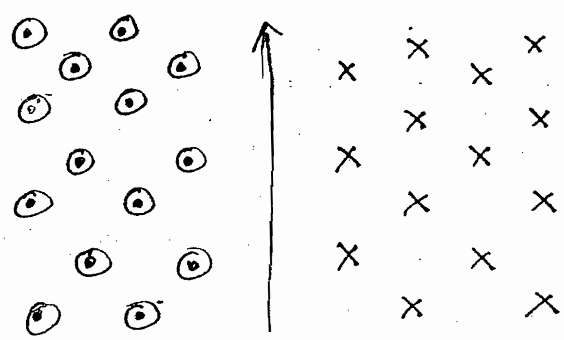


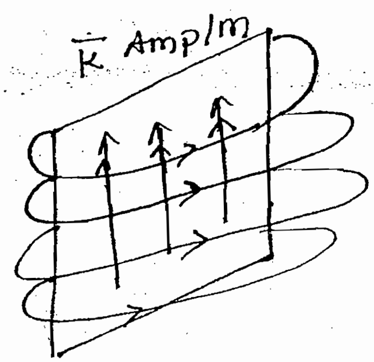
→ The flux density is same everywhere. The field is uniform.

→ $$D = \frac{\rho_s}{2} a_N$$

→ $$E = \frac{D}{\epsilon} = \frac{\rho_s a_N}{2\epsilon} \Rightarrow \boxed{E = \frac{\rho_s}{2\epsilon} a_N}$$ **

Magnetic field strength of an infinite sheet of ρ_s C/m^2 charge density :-





→ The magnetic field is completely normal to the current and the field lines are parallel to themselves and to the sheet

→ The flux density is same everywhere i.e. field is uniform

→ $$H = \frac{\vec{K}}{2} \times a_N \quad \text{Amp/m}$$

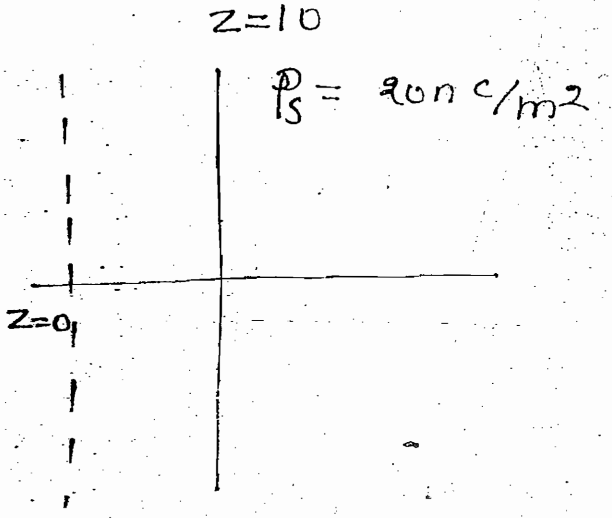
and $$\boxed{B = \frac{\mu \vec{K}}{2} \times a_N.}$$

24. $a_N = \pm a_z$

At $z = 0$, $a_N = -a_z$

$$E = \frac{\rho_S}{2\epsilon} a_N$$

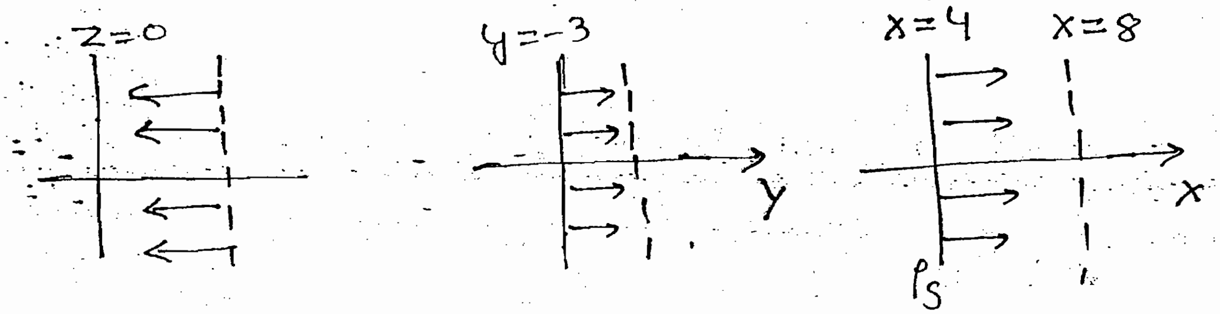
$$= \frac{20 \times 10^{-9}}{2 \times \frac{1}{36\pi \times 10^{9}}} (-a_z)$$



25. $x = 4$ $\rho_{S_1} = 18\, nC/m^2 \longrightarrow E_1 = \pm a_x$

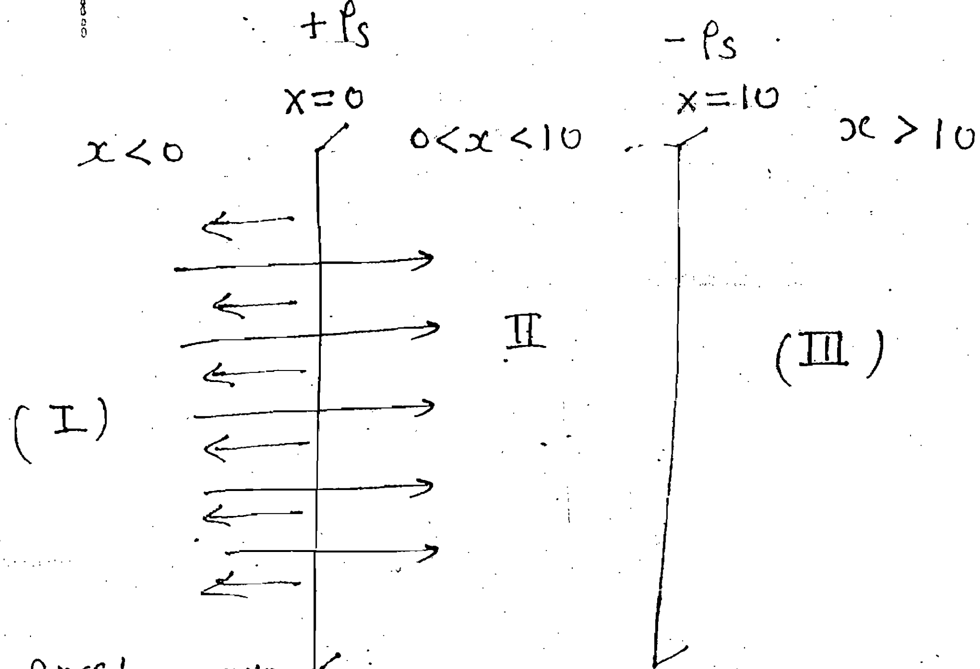
$y = -3$ $\rho_{S_2} = 9\, nC/m^2 \longrightarrow E_2 = \pm a_y$

$z = 0$ $\rho_{S_3} = -24\, nC/m^2 \longrightarrow E_3 = \pm a_z$



$$E_3 = \frac{24}{2\epsilon}(-a_z) \qquad E_2 = \frac{9}{2\epsilon} a_y \frac{nN}{C} \qquad E_1 = \frac{18}{2\epsilon} a_x \frac{nN}{C}$$

$$E_T = \frac{3}{2\epsilon}\left(6a_x + 3a_y - 8a_z\right) \frac{nN}{C}$$



In first region

$$E_1 = \frac{\rho_s}{2\epsilon}(-a_x)$$

$$E_2 = \frac{\rho_s}{2\epsilon}(a_x)$$

$$\vec{E}_{Net} = 0$$

In third region

$$\vec{E}_{Net} = 0$$

In second region :-

$$E_1 = \frac{\rho_s}{2\epsilon} a_x \qquad E_2 = \frac{\rho_s}{2\epsilon} a_x$$

$$E = \frac{\rho_s}{\epsilon} a_x \quad 0 < x < 10$$

$$= 0 \quad \text{elsewhere}$$

$\vec{K} = 30\, a_z$ mA/m $\rightarrow$ $Y = 0$ plane $\Rightarrow$ zx plane

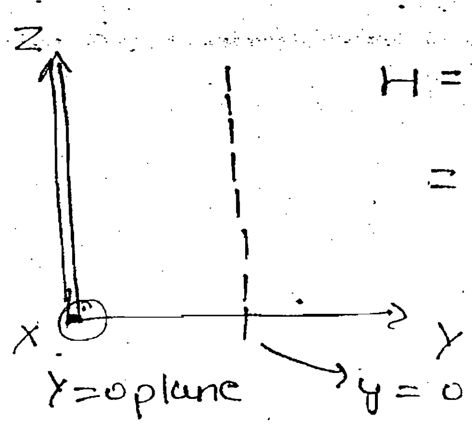
$H = \frac{\vec{K}}{2} \times a_N$

at $y = 2$

$a_N = \pm a_y$

for $y = 20$

$a_N = a_y$



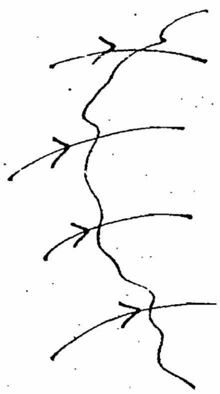
$H = \frac{30}{2} a_z \times a_y$

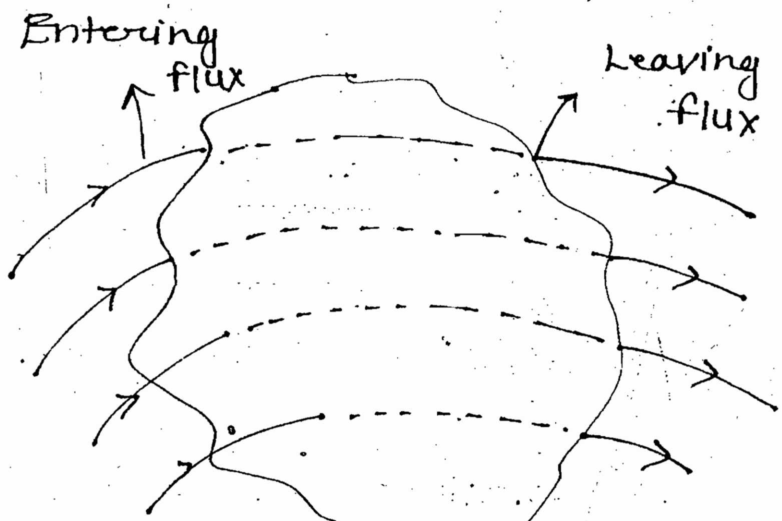
$= -15 a_x$ mA/m

Ans-(A)

Closed surface Integral of B — Maxwell's III Equation :-

→ Magnetic field always forms closed lines around the current and has no starting / ending point for flux lines i.e. No sources/sinks exists for B flux lines





$$\int B \cdot ds = \psi_m$$

This is not Maxwell's equation

$$\oint B \cdot ds = 0$$

This is Maxwell III equation

→ For any closed surface

Entering flux = Leaving flux

$$\oint B \cdot ds = 0$$

B field is solenoidal (No divergence)

$$\nabla \cdot B = 0$$

Note :-

$$\oint D \cdot ds = 0 \; \begin{cases} \rightarrow Q = 0 \\ \rightarrow +Q, -Q \end{cases} \Rightarrow \boxed{\oint B \cdot ds = 0}$$

Always

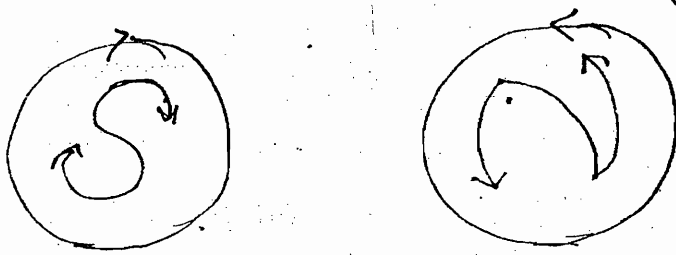
→ By comparison, equals of charge in E fields donot exist in B fields i.e. every cause of B field is a dipole, i.e. Magnetic monopoles don't exist.

eg:- Bar Magnet , N/S - Dipole

best example:- I current carrying wire

→ I always flows only when both the polarities exist and are connected hence it is always treated as magnetic dipoles.

→ I always flows in closed circuits and every closed I wire is a magnetic dipole



Clockwise → South pole

Anticlockwise → North pole

→ Cause - current - closed

Entering current = Leaving current (Junction)

Effect - B field - closed around the cause

Entering flux = Leaving flux (closed surface)

$$\boxed{\oint B \cdot ds = 0} \longrightarrow \text{KCL in Magnetic Field}$$

Potential, Gradient, closed line Integral in E field:-

Potential is a ~~square~~ scalar measure of E field strength in terms of energy at any point in the field

$$\text{V (Volts) Potential at any point} = \frac{\text{Workdone by the charge to reach the point}}{\text{charge}}$$

$$V = \frac{W}{Q} = \frac{\text{Joules}}{\text{Coulomb}}$$

$\Rightarrow$ Work = Force . displacement

$\Rightarrow$ $dW = \vec{F} \cdot \vec{dl} = Q\vec{E} \cdot \vec{dl}$

= Workdone in field / force direction

= Workdone on the charge

→ Work is done on the charge when it moves in the field direction and hence the charge acquires energy

$$\boxed{V = \frac{W}{Q} = -\int \vec{E} \cdot \vec{dl}} \rightarrow \text{Scalar potential function of space}$$

$$V_{AB} = -\int_{\text{Ref } B}^{\text{Ref } A} \vec{E} \cdot \vec{dl} = \text{Potential diff b/w } A \,\&\, B$$

If ref. B has $V_B = 0$, then V_A is called as absolute potential at A.

Potential function of a point charge :-

$$V = -\int \vec{E} \cdot \vec{dl}$$

Spherical coord.

$$= -\int \frac{Q}{4\pi\epsilon r^2} a_r \, dr \, a_r = \frac{Q}{4\pi\epsilon_0 r}$$

$$\Rightarrow \boxed{V = \frac{Q}{4\pi\epsilon r}}$$

$\vec{E} \rightarrow$ Intensity $- \frac{1}{r^2}$ decrease

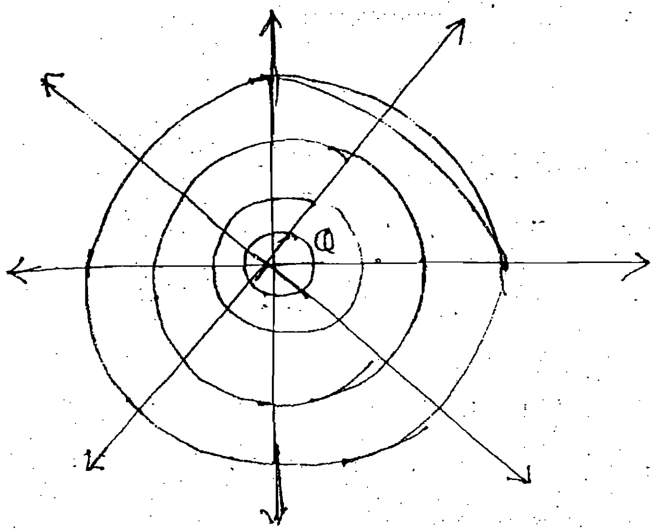
a_r directed vector

$V \longrightarrow$ Potential $- \frac{1}{r}$ decrease

– scalar –

If $r =$ constant then $V =$ constant

$\longrightarrow$ cocentric sphere $\rightarrow$ surface



→ The family of ~~cocentri~~ equipotential surfaces represent the potential distribution and variation of potential in the region and similar to vector intensity line

Potential function of a line Charge:–

$$V = -\int \vec{E} \cdot \vec{dl}$$

$$= -\int \frac{\rho_L}{2\pi\epsilon\rho} a_\rho \, d\rho \, a_\rho$$

$$\Rightarrow V = \frac{\rho_L}{2\pi\epsilon} \ln\left(1/\rho\right)$$

If $\rho =$ constant $\Rightarrow V =$ constant

Equipotential surfaces are cocentric cylinders

$\vec{E}$ — Intensity — $\frac{1}{\rho}$ decrease

a_ρ directed vector

$V \rightarrow$ Potential — logarithmic decrease

— scalar

Potential function of a sheet charge (Uniform):-

$$V = -\int \vec{E} \cdot \vec{dl}$$

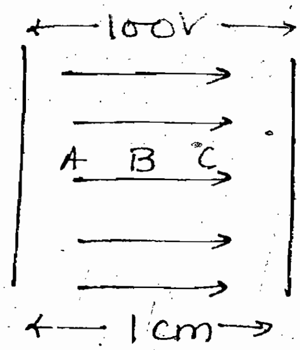
$$\Rightarrow V = -El$$

$$\Rightarrow E = -\frac{V}{l}$$

$\vec{E} \rightarrow$ Intensity — Uniform

V — Potential — linear decrease

eg:- Capacitor Plates

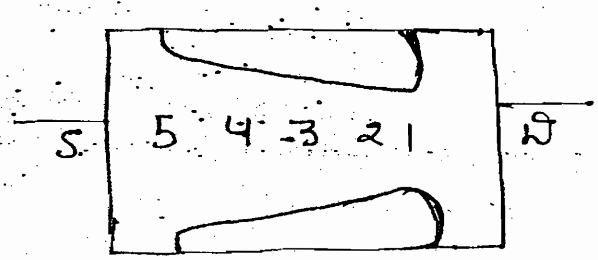


E_A or E_B or E_C

$= \frac{100}{10} = 10\ kV/m$

$V_A > V_B > V_C$

FET Channel



28.

$$V = -\int \vec{E}\cdot\vec{dl} = -\int_{(0,0,0)} (y a_x + x a_y)\cdot(dx\, a_x + dy\, a_y).$$

$$V = -\int_{x=0}^{4} y\, dx - \int_{y=0}^{1} x\, dy$$

Using the straight line path or equation

$$\frac{y-1}{1-0} = \frac{x-4}{4-0}$$

$$x = 4y$$

$$= -\int_0^4 \frac{x}{4}\, dx - \int_0^1 4y\, dy$$

$$= -\frac{1}{\cancel{8}}\left(16\right)^2 - \frac{\cancel{4}}{\cancel{2}} \times 1$$

$$= -4V \text{ , Ans.}$$

Note :-

In line integral,

$\int f(x,y,z)\,dx$ We have to transform y & z in terms of x using the known relationship

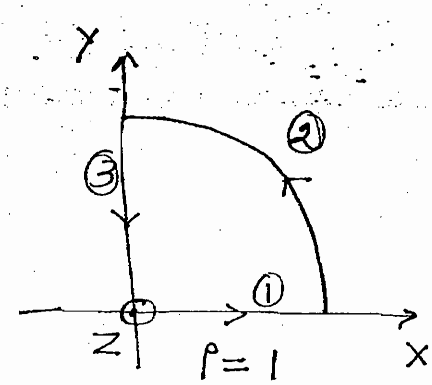
i.e. path of integration

$$\int f(x,y,z)\,dx = \int g(x)\,dx$$

29.

$$\vec{A} = 2\rho\cos\phi\, a_\rho$$

$$\oint A\cdot dl = 3 \rightarrow \int A\cdot dl$$



(1) $dl = d\rho\, a_\rho$

$$\phi = 0$$

$$A = 2\rho\, a_\rho$$

$$\int A\cdot dl = \int_{\rho=0}^{1} 2\rho\, a_\rho\, d\rho \cdot a_\rho$$

$$= \rho^2\Big|_0^1 = 1$$

2 → $dl = \rho\, d\phi\, a_\phi$ $\quad (\because a_\phi \cdot a_\rho = 0)$

$$\int A\cdot dl = 0$$

3 → $dl = d\rho\, a_\rho$

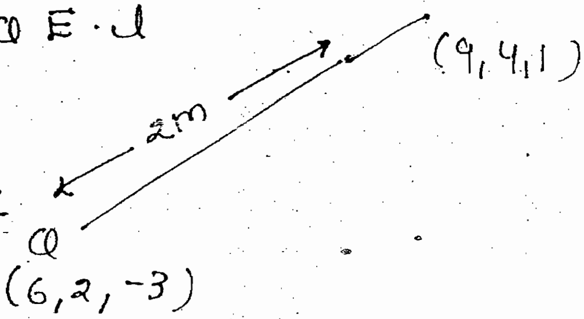
ρ from 1 to 0

$\phi = 90 \Rightarrow A = 0$

30 → $\vec{E} = 4a_x - 3a_y + 5a_z \longrightarrow$ Uniform

$$W = Q\int \vec{E}\cdot d\vec{l} = Q\, E\cdot l$$

Unit length vector $= \dfrac{3a_x + 2a_y + 4a_z}{\sqrt{3^2+2^2+4^2}} \times 2$ 5C



$(\because$ 2m length vector$)$

$$W = 5\,(4a_x - 3a_y + 5a_z)\cdot \frac{(3a_x + 2a_y + 4a_z)\times 2}{\sqrt{29}}$$

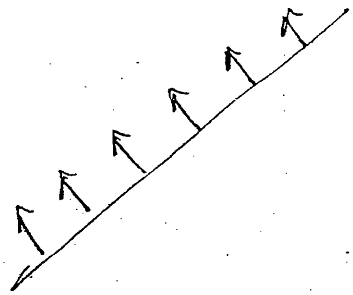
$$= \frac{260}{\sqrt{29}}\ \text{J}$$

Potential Gradient :-

Scalar surface equation (f) $\xrightarrow{\nabla f}$ Vector (Normal) to the surface (direction)

eg:- Linear surface

$$f = 4x + 7y - 15z = 18$$



$$\nabla f = 4a_x + 7a_y - 15a_z$$

eg:- Non-linear surface

$$f = 4xy - 15x^2yz$$

$$\nabla f = (4y - 30xyz)a_x + (4x - 15xz^2)a_y + 15x^2y\,a_z$$

$$V = -\int \vec{E}\cdot\vec{dl}$$

$$dV = -\vec{E}\,dl\cos\theta$$

$$\Rightarrow \boxed{\frac{dV}{dl} = -E\cos\theta}$$

Case-(1) :-

If $\theta = 90^\circ \Rightarrow V = \text{constant}$

→ The direction of E field is always normal to equi-potential surfaces.

Case-(II):-

If $\theta = 0^\circ/180^\circ \Rightarrow \left.\frac{dV}{dl}\right|_{max} = |E|$

→ The magnitude of E intensity is always the maximum rate of change of potential per unit length ~~If~~

Case-(III):-

If $\theta = 0^\circ \Rightarrow |E| = -\left.\frac{dV}{dl}\right|_{max}$

→ The direction of E intensity is always the direction in which potential decreases by maximum

Note:-

The operation is physically called as gradient

Hence $E = -\nabla V$ = potential gradient is field intensity.

→ Gradient signifies the maximum rate of change of scalar along with the direction of change.

eg:- Given $V = 4(x^2-y^2)$ for all z

Find the equation of equipotential surface passing through (3,1,9)

Soln:- V at $(3,1,9) = 4(9-1) = 32V$

All x & y when V = 32V is the equipotential surface

$\Rightarrow x^2 - y^2 = 8$

Summary:-

Equation of equipotential surface OR Voltage function (V) $\longrightarrow \nabla V \rightarrow$ Vector Intensity function (E) which is normal to equi-potential surfaces

Mathematically, ∇V.

$$\boxed{\nabla V = \frac{1}{h_1}\frac{\partial V}{\partial u} a_u + \frac{1}{h_2}\frac{\partial V}{\partial v} a_v + \frac{1}{h_3}\frac{\partial V}{\partial w} a_w}$$

eg:- If $V = \frac{4\cos\theta}{r^2}$

Find $\vec{E}$ at $(2, \pi/4, \pi)$

Soln:- $E = -\nabla V$

$$\nabla V = \frac{1}{1}\frac{\partial}{\partial r}\left(\frac{4\cos\theta}{r^2}\right) a_r + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{4\cos\theta}{r^2}\right) a_\theta$$

$$= 4\cos\theta\left(\frac{-2}{r^3}\right) a_r + \frac{4}{r^3}(-\sin\theta) a_\theta$$

$$E = \frac{4}{r^3}\left(2\cos\theta\, a_r + \sin\theta\, a_\theta\right)\Big|_{(2, \pi/4, \pi)}$$

$$= \frac{1}{2}\left(\sqrt{2}\, a_r + \frac{1}{\sqrt{2}} a_\theta\right)$$

32.

(a) $V = 0$ $\quad \nabla V = 6xy\, a_x + (3x^2 - z) a_y - y a_z$

$\neq 0$ at $(1, 0, -1)$

(b) $x^2 y = 1$ in XY plane

$V = 3(1) - y(0)$

$V = 3 \longrightarrow$ equipotential surface

(c) At $(2, -1, 4) \Rightarrow V = -8$

$V = 3(4)(-1) - (-1)4 = -8$

(d) Normal vector at P $= \frac{\nabla V}{|\nabla V|}\Big|_{(2,-1,4)} = \frac{-12a_x + 8a_y - a_z}{\sqrt{12^2 + 8^2 + 1}}$

$= (-0.83\hat{x} + 0.55\hat{y} + 0.07\hat{z})$

Lecture - 6

Closed line integral of electric field - Maxwell's II Eqn :-

$$\oint E \cdot dl = 0$$

→ Potential difference exists b/w 2 different points but b/w the same point potential difference is zero

→ Potential is unique at a point at a time

→ Workdone in moving a charge in any closed path is always zero

→ In any closed path energy acquired = energy lost

i.e Energy acquired in field direction = energy lost opposite to field direction.

→ Electric field is a conservative field & never forms closed loops.

i.e. Irrotational vector $(\nabla \times E = 0)$

→ Workdone in moving a charge b/w two points is independent of path of consideration.

→ This is KVL in electric fields.

Potential, Vector Potential and Ampere's Law in H field :-

→ Potential is called as MMF or Magneto Motive force and is a scalar measure of magnetic field strength at any point in the field.

$$V_m = MMF = \int \vec{H} \cdot \vec{dl} = Amp$$

$$H = \nabla V_m$$

$$\boxed{\oint H \cdot dl = I}$$

This is Ampere's law in Integral form

$$\boxed{\nabla \times H = J}$$

This is Ampere's law in point form

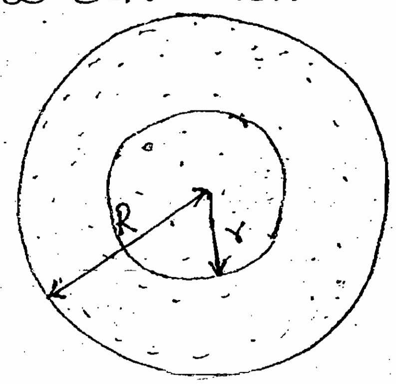
Statement :-

effects

The net circulation of magnetic field in any closed line is always equal to the current crossing the surface enclosed.

Circulation = Strength × length
= current

Consider a closed ~~ring~~ line symmetrical & cocentric with the I flow direction such that H = constant everywhere



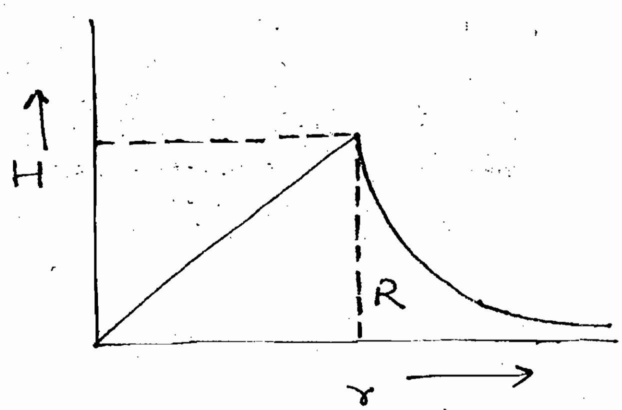
$$\oint \underbrace{H}_{\text{constant}} \cdot dl = I$$

H · Length = current crossing $r < R$

$$H = \frac{\frac{I}{\pi R^2} \cdot \pi r^2}{2\pi r} = \frac{Ir}{2\pi R^2}$$

$r > R$

$$H = \frac{I}{2\pi r}$$



Note :-

Point → $1/r^2$

Line → $1/\rho$

Sheet → Uniform

Solid → r

Vector Potential :-

Limitation of Scalar Potential (MMF) :-

→ The definition of potential in electric field is consistent with its nature i.e. irrotational nature

$$E = -\nabla V$$

$$\nabla \times E = 0$$

$$\nabla \times (-\nabla V) = 0$$

$$\text{curl (Grad of scalar)} = 0$$

→ A similar definition in magnetic field for MMF is not consistence with Ampere's law

$$H = \nabla V_m$$

$$\nabla \times H = J$$

$$\nabla \times (\nabla V_m) = 0 = J$$

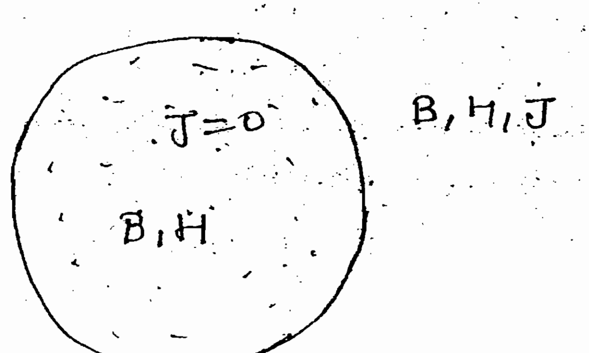
Hence MMF is defined and exists in only those regions where $J = 0$

i.e. outside current flowing regions

i.e. free space conditions and outside conductor

eg:-(i) MMF b/w windings of solenoids

(ii) MMF in air gaps in machines



Vector Potential A :-

→ The basic definition of vector potential is in accordance to nature of magnetic field i.e. solenoidal nature. Hence if curl of A = B then A is called as vector potential

$$B = \nabla \times \vec{A}$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

Div. (curl of vector) = 0

→ The term vector potential has a units weber/m which physically signifies the work for current element

$$\vec{A} = \text{Weber/m} = \frac{\text{Joules}}{\text{Amp-m}}$$

$$= \frac{W}{I\,\vec{dl}}$$

and as $I\vec{dl}$ is a vector, $\vec{A}$ is a vector quantity.

Note:-

$$\frac{\text{Weber}}{\text{Second}} = \text{Volts}$$

$$\frac{\text{Weber}}{\text{second}} = \frac{\text{Joules}}{\text{Coulomb}}$$

$$\frac{\text{Weber}}{\text{m}} = \frac{\text{Joules}}{\text{Amp-m}}$$

$$V = \frac{W}{Q} = \frac{Q}{4\pi\epsilon r}$$

$$\boxed{\vec{A} = \frac{W}{I\vec{dl}} = \frac{\mu I \vec{dl}}{4\pi r}}$$

A's direction is along I flow direction & is a solenoidal vector.

Summary 1 :-

→ Q – coulomb – point charge – Basic cause – E field
(scalar)

→ $I\vec{dl}$ – Amp-m – current element – H field
(vector)

→ Q – $\rho_L\,dl$ – $\rho_s\,ds$ – $\rho_v\,dv$

→ $I\vec{dl}$ – $\vec{K}\,ds$ – $\vec{J}\,dv$

→ $\vec{E}$ – Intensity – Strength – Force – $\frac{\vec{F}}{Q}$ – $\frac{1}{\epsilon}$ dependent

→ $\vec{B}$ – Density – Strength – Force – $\frac{\vec{F}}{I\vec{dl}}$ – μ dependent

→ $\vec{D}$ – Density – strength – $\frac{dQ}{ds} = C/m^2$

→ $\vec{H}$ – Intensity – strength – $\frac{dI}{dl}$ = Amp/m

→ V – Potential – strength – Work = W/Q → Scalar
(Volts)

⇒ $\vec{A}$ – Potential – strength – Work = $\frac{W}{Idl}$ → Vector.
(Weber/m)

Summary :-

Points — Scalar function $\xrightarrow[\text{Gradient}]{\nabla}$ Vector f^n Intensity (per m) — Line

eg:- V volts $\longrightarrow$ ∇V $\longrightarrow$ E volts/m

Line — Vector function Intensity (per m) $\xrightarrow[\text{curl}]{\nabla \times}$ Vector f^n Density (per m^2) — Surface

eg:- Amp/m $\vec{H}$ $\longrightarrow$ $\nabla \times H$ $\longrightarrow$ J Amp/m^2

Web/m $\vec{A}$ $\longrightarrow$ $\nabla \times \vec{A}$ $\longrightarrow$ $\vec{B}$ Web/m^2

Surface — Vector function Density (per m^2) $\xrightarrow{\nabla \cdot}$ Scalar f^n Volume (per m^3) — Volume

eg:- $\vec{D}$ (C/m^2) $\longrightarrow$ $\nabla \cdot \vec{D}$ $\longrightarrow$ ρ_v C/m^3

33 $\oint A \cdot dl = \frac{\text{Weber}}{m} \times m = \text{Webers}$ ↓ flux

OR

$$\oint \vec{A} \cdot \vec{dl} = \int (\nabla \times A) \cdot ds = \int B \cdot ds = \psi_m$$

34.- Stoke's theorem

$\vec{V} = \nabla \times \vec{A}$

B

C $\rightarrow$ closed line

S_c $\rightarrow$ surface

$$\oint A \cdot dl = \int (\nabla \times A) \cdot ds = \int \vec{V} \cdot \vec{ds}$$

Laplace / Poisson's Equations :-

→ They are second order differential equations relating volume charge and potential developed inside / outside the charge

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot [\epsilon(-\nabla V)] = \rho_v$$

If ϵ = constant then

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \longrightarrow \text{Poisson's Equation}$$

For $\rho_v = 0$, i.e. outside the charge, charge free region

$$\boxed{\nabla^2 V = 0} \longrightarrow \text{Laplace Equation.}$$

Note :-

Inspite of being II order differential equation they always have a unique solution i.e. voltage is a single valued function of space

This is called as ~~UNQ~~ Uniqueness theorem

Mathematically

$$\nabla^2 V = \nabla \cdot (\nabla V)$$

$$\nabla \cdot (\nabla V) = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial V}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3 h_1}{h_2} \cdot \frac{\partial V}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial V}{\partial w}\right)\right]$$

Extension :—

$\vec{A}$ Vs $\vec{J}$ in Magnetic Fields

$$\nabla \times H = \vec{J}$$

$$\Rightarrow \nabla \times \left(\frac{B}{\mu}\right) = J$$

$$\Rightarrow \nabla \times (\nabla \times A) = \mu J$$

$$\Rightarrow \nabla(\nabla \cdot A) - \nabla^2 A = \mu J$$

$$\Rightarrow \boxed{\nabla^2 A = -\mu J}$$

For current free region

$$\boxed{\nabla^2 A = 0}$$

35

$$V = -\frac{6r^5}{\epsilon_0}$$

1. $\nabla^2 V = -\rho_v/\epsilon$ 2. $Q = \int \rho_v \, dv$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(-\frac{6r^5}{\epsilon}\right) = -\frac{\rho_v}{\epsilon}\right.$$

$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot 6 \cdot 5 \cdot r^4\right) = \rho_v$$

$$\frac{30}{r^2} 6 \cdot r^5 = \rho_v$$

$$\Rightarrow \rho_v = 180 r^3$$

$$Q = \int_{r=0}^{1}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 180 r^3 \cdot r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= 180 \frac{r^6}{6}\Big|_0^1 (-\cos\theta)_0^{\pi} (\phi)_0^{2\pi} = 120\pi C, \text{ Ans.}$$

36. $\rho_v = -10^{-8}(1+10\rho)$

V on $\rho = 5cm$

Given $\rho = 2cm$ V & $E = 0$

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \cdot \frac{\partial V}{\partial \rho}\right) = \frac{10^{-8}(1+10\rho)}{\frac{1}{36\pi \times 10^9}}$$

$$= 360\pi(1+10\rho)$$

$$\rho \frac{\partial V}{\partial \rho} = 360\pi \int (\rho + 10\rho^2)\, d\rho$$

$$\rho \frac{\partial V}{\partial \rho} = 360\pi\left(\frac{\rho^2}{2} + \frac{10\rho^3}{3}\right)$$

$$\Rightarrow V = 360\pi \int \left(\frac{\rho}{2} + \frac{10\rho^2}{3}\right) d\rho$$

$$= 360\pi \left(\frac{\rho^2}{4} + \frac{10\rho^3}{9}\right)_{2\times10^{-3}}^{5\times10^{-2}}$$

$$\Rightarrow \boxed{V = 0.5V}$$

37. $$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{\rho_v}{\epsilon}$$

$$= 20\cdot 3\cdot 2\cdot x + 10\cdot 4\cdot 3\, y^2\Big|_{(2,0)} = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow \boxed{\rho_v = -240\,\epsilon_0}$$

38. ϕ — 1 dimension function — satisfying - laplace

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} = 0$$

$$\frac{\partial\phi}{\partial x} = c_1 = \text{Rate of change is constant}$$

$$\frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2d}$$

$$\phi = c_1 x + c_2 \longrightarrow \text{Linear equation}$$

Ans - B

39. $V = \sinh x \cdot \cos ky \, e^{pz}$

$$\nabla^2 V = 0 \qquad \frac{e^x \mp e^{-x}}{2} = \sinh x$$

$$\nabla^2 V = 1V - k^2 V + p^2 V = 0$$

$$\Rightarrow \boxed{k = \sqrt{1+p^2}}$$

Boundary Conditions for Electric fields

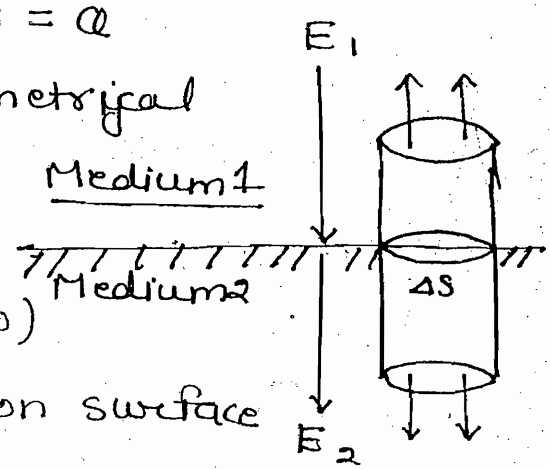
→ If a field is spread out into two different medium and known the field in one region. The field in the adjacent region can be calculated under two conditions

Case-(I) :-

Electric field is normal to the boundary.

Using Gauss law $\oint D \cdot ds = Q$

consider a surface symmetrical in both the medium



$$\boxed{D_2 \cdot \Delta S - D_1 \Delta S = 0}$$

$(\because Q = 0)$

If charge is present on surface then

$$\boxed{D_2 \Delta S - D_1 \Delta S = \rho_s \Delta S}$$

$D_{n_1} = D_{n_2}$

$D_{n_2} - D_{n_1} = \rho_s$

Statement :-

The normal components of electric flux density is same on either side but otherwise discontinous when a surface charge density exists on the boundary

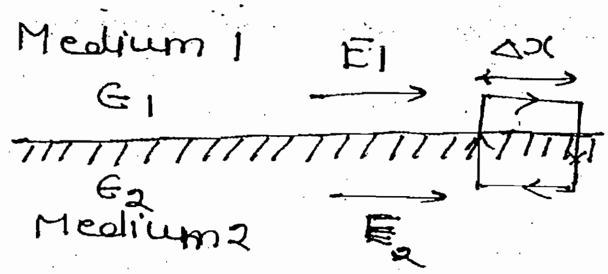
Case-(II) :-

Electric field is ~~parallel~~ tangential to boundary :-

Using $\oint E \cdot dl = 0$.

$E_1 \Delta x - E_2 \Delta x = 0$

$\Rightarrow \boxed{E_1 = E_2}$



$E_{t_1} = E_{t_2}$

The tangential components of electric field intensity is always continous

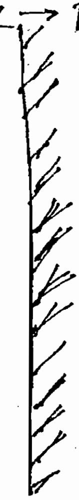
40

$x = 0$ Plane

$YZ \rightarrow$ Plane

$x < 0$ $\qquad$ $x > 0$

$\epsilon_{R_1} = 1.5$ $\qquad$ $\epsilon_{R_2} = 2.5$



$E_{n_1} = 2a_x$

$E_{t_1} = -3a_y + a_z$

$E_1 = 2a_x - 3a_y + a_z$

$E_{t_1} = E_{t_2} = -3a_y + a_z$

$D_{n_1} = 1.5 \times 2a_x \times \epsilon_0 \quad (D_{n_1} = \epsilon_0 \epsilon_R \times E_n)$

$\cancel{= 3a_x} \quad \cancel{= D_{n_2}}$

$= 3\epsilon_0 a_x \longrightarrow D_{n_2} = 3\epsilon_0 a_x$

$E_{n_2} = \dfrac{3\epsilon_0 a_x}{2.5\epsilon_0} = 1.2 a_x$

$D_1 = \epsilon_0 (3a_x - 4.5a_y + 1.5a_z)$

$D_2 = \epsilon_0 (3a_x - 7.5a_y + 2.5a_z)$

$\boxed{E_2 = 1.2a_x - 3a_y + a_z}$ Ans

Note :-

→ The value of vector dec. then its projection dec. i.e moves away from normal and moves towards the surface

→ The projection of normal component is dec. in E_2 when compare to E_1. Hence the field is shifting away from the normal. It can be verified with inc. tangential component when comparing D_1 & D_2

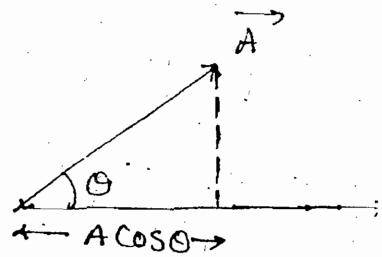
Sur

A's scalar projection on B

$$= A\cos\theta$$

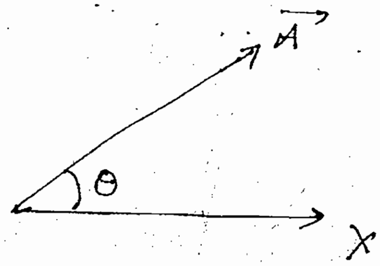
$$= \cancel{A}\,\frac{A\cdot B}{\cancel{A}\,B}$$

$$= \frac{A\cdot B}{|B|}$$



A's projection in x

$$= \frac{A\cdot a_x}{|a_x|}$$



A's vector projection on B

$$= (A\cos\theta)\cdot\hat{B}$$

of

$$= \frac{(A\cdot B)}{|B|}\hat{B}$$

13.

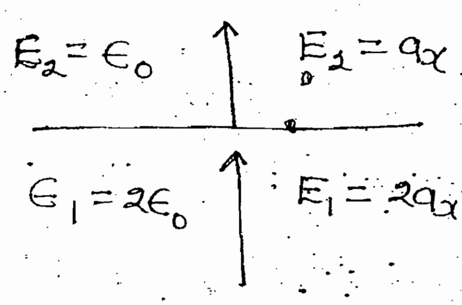
$$= \frac{(A\cdot B)}{|B|^2}\vec{B}$$

41.

$D_2 - D_1 = \rho_s$

$1\cdot\epsilon_0 - 2\cdot 2\epsilon_0 = \rho_s$

$\boxed{\rho_s = -3\epsilon_0}$



42.

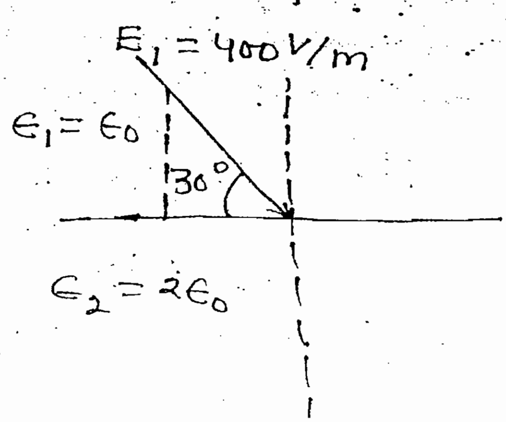
$E_{t_1} = E_{t_2}$

$E_1 \cos 30 = E_{t_2}$

$\Rightarrow \quad E_{t_2} = 200\sqrt{3}$

$\epsilon_0 E_1 \sin 30^\circ = 20\,\epsilon_0 E_{n_2}$

$\Rightarrow \quad E_{n_2} = 10$



Ohm's law and Continuity Equation:

$$\int (\nabla \cdot J)\, dv = \oint J \cdot ds = I = \frac{dQ}{dt} = \frac{d}{dt}\int \rho_v\, dv = \int \frac{\partial \rho_v}{dt}\, dv$$

i.e. $$I = \frac{dQ}{dt} = \frac{d}{dt}\int \rho_v\, dv = \int \frac{\partial \rho_v}{dt}\, dv$$

$$\Rightarrow \quad I = \oint J \cdot ds = \int (\nabla \cdot J)\, dv$$

Divergence theorem

$$\boxed{\nabla \cdot J = \frac{\partial \rho_v}{\partial t}}$$

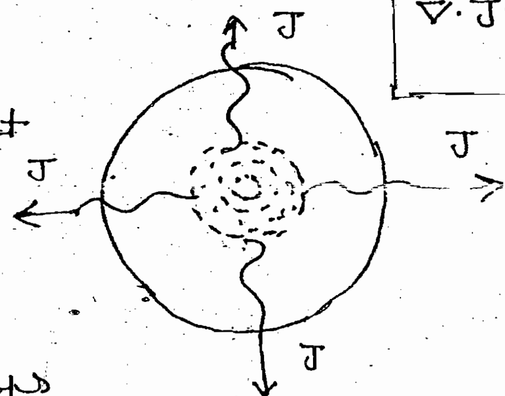
→ current has outflow and divergence at a rate depending on decrease of volume charges with time

$$\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$

→ Practically

Entering current = Leaving current

charge can neither be created nor be destroyed



$$\left.\begin{aligned} \oint J \cdot ds &= 0 \\ \nabla \cdot J &= 0 \end{aligned}\right] \longrightarrow \text{Always}$$

→ For a uni-directional current flow

$$\frac{\partial J}{\partial x} = \frac{\partial \rho_v}{\partial t}$$

$$\partial J = \partial \rho_v \frac{\partial x}{\partial t}$$

Integrating on both sides

$$J = \rho_v V_d$$

where $V_d = \frac{\partial x}{\partial t}$ = drift velocity

V_f = free velocity = $\sqrt{\frac{2qV}{m}}$

= vacuum tubes, CRO

= few $10^6 - 10^7$ m/s

$$\boxed{V_f \propto \sqrt{E}}$$

where $V_d = \frac{\delta x}{\delta t}$ = drift velocity

= charge in a material (bulk)

= few cm/sec

$V_d \propto E \Rightarrow \boxed{V_d = \mu E}$

μ → mobility of the particle

$J = \rho_v \vec{v}_d$

$J = \rho_v \mu E$

$\Rightarrow \boxed{J = \sigma E}$ → Ohm's law in point form.

where σ = conductivity (mho/m) $= \rho_v \mu$

= ability to allow current

= available free carriers × mobility of carriers

Case-(1) :-

$\sigma = \infty$ → very good conductor

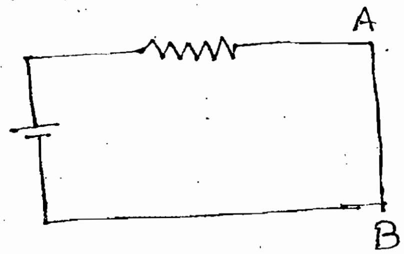
$\frac{J}{\sigma} = E = 0$

→ Electric field cannot exist in a very good conductor

→ $V = \int 0 \cdot dl$ = constant ;

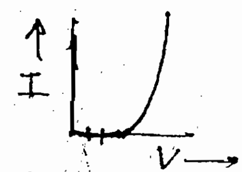
conductor is always a equi-potential region

→ Potential difference or voltage cannot exist in a very good conductor only current can flow but voltage difference ~~good~~ conductor cannot exist



$V_{AB} = 0$ but current flows

$V\uparrow \quad V_{AB} = 0 \quad I_{AB}\uparrow$



eg:-(1) diode current after cuting voltage

(II) Accumulation — does not exist — only flow exists

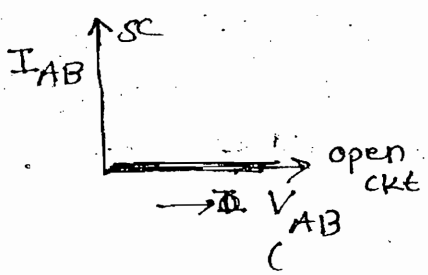
Case-(II) :-

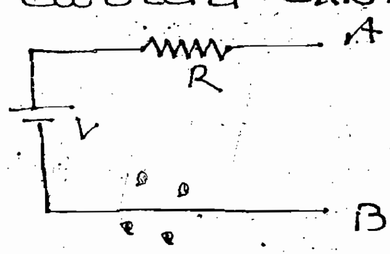
$\sigma = 0$ very good dielectric

$J = 0$

→ current cannot exist in a very good dielect

→ Voltage or potential difference or Electric field can exist but flow current exist





→ Accumulation can exist but flow can't exist.

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

$$J = \sigma E$$

$$\nabla \cdot (\sigma E) = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \sigma (\nabla \cdot E) = -\frac{\partial \rho_v}{\partial t}$$

• The ρ_v is time solution is

$$\boxed{\rho_v(t) = \rho_{v_0} \cdot e^{-\frac{t\sigma}{\epsilon}}}$$

Note:-

Every charge density exponentially spread on the medium at a rate depending on $\frac{\epsilon}{\sigma}$ that called the relaxation time

$$\frac{\epsilon}{\sigma} = \text{Relaxation time} = \frac{\text{Farad}}{m \times \frac{mho}{m}}$$

$$= \text{ohms} \times \text{farad}$$

$$= \text{second}$$

Boundary conditions at conductor surfaces:-

(I) $E_{t_1} = E_{t_2}$ (General)

As $\sigma = \infty$ along the conductor

$\frac{J}{\sigma} = E = 0$ along the surface

$\Rightarrow \boxed{E_{tang.} = 0}$

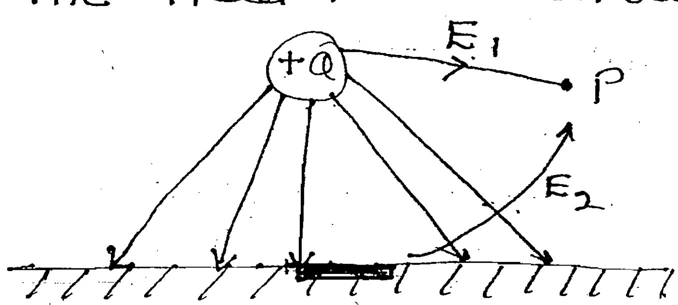
(II) $D_{n_2} - D_{n_1} = \rho_s$ (General)

Electric field can be normal to a conductor which depends on ρ_s on the surface

$$\boxed{D_{normal} = \rho_s}$$

Conductors, Induction, Method of Images:-

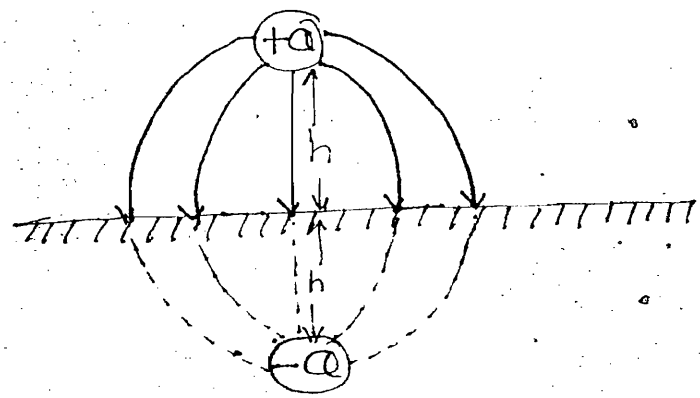
→ When a charge is placed near a conductor surface the field is as shown below



→ The field has tangential component which displaces the free carrier. Hence they are periodically accumulated all ~~the other~~ along the conductor surface. This is called as induced charge

→ The resultant field at any point is the vector sum of field due to actual charge and induced charge

→ This field is such that the tangential component are removed and has only normal components as shown below



→ The field appears to be a dipole field with negative charge called as image charge below the conductor surface

Summary :—

Every charge and its induction effects are represented by a image charge below the conductor obeying all rules of light and optics

44.
$$D_1 = 2(a_x - \sqrt{3}a_z)\ C/m^2 = \rho_s$$

$$\rho_s = |D_n| = 2\sqrt{1+3} = 4$$

45.
$$E_n = 2\ V/m$$

$$D_n = 80\epsilon_0 E_n = \rho_s$$

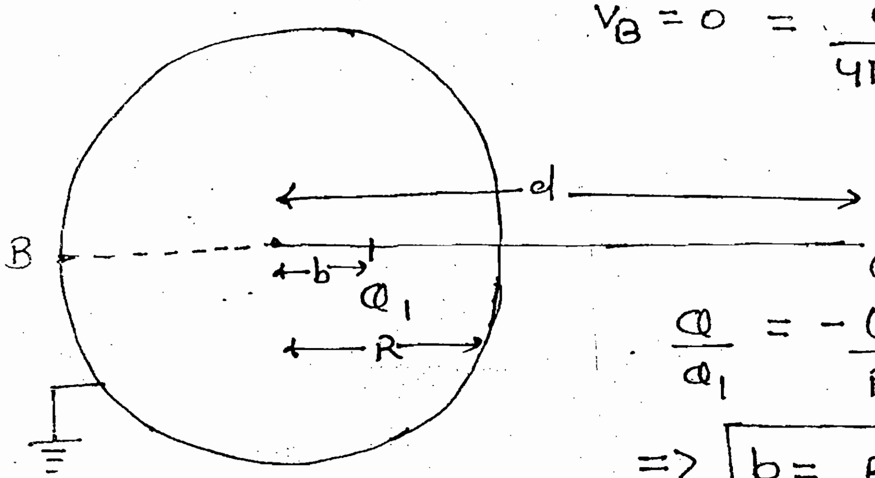
$$\Rightarrow \rho_s = 80 \times 8.8 \times 10^{-12} \times 2$$

$$= 1.41 \times 10^{-9}\ C/m^2$$

6.

$$V_A = 0 = \frac{Q}{4\pi\epsilon(d-R)} + \frac{Q_1}{4\pi\epsilon(R-b)}$$

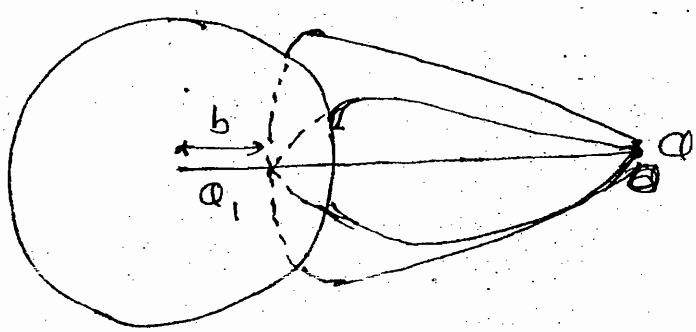
$$V_B = 0 = \frac{Q}{4\pi\epsilon(d+R)} + \frac{Q_1}{4\pi\epsilon(R+b)}$$



$$\frac{Q}{Q_1} = -\frac{(d-R)}{R-b} = -\frac{(d+R)}{(R+b)}$$

$$\Rightarrow \boxed{b = \frac{R^2}{d}} \quad \& \quad \boxed{Q_1 = -\frac{R}{d}}$$

Note:-



7. 4μC, Ans

Energy Density in Electric fields $\left(\frac{dW_E}{dv} = \frac{1}{2}\epsilon E^2\right)$:-

Consider Q_1, Q_2, Q_3 ------ Q_n point charges assemble in a region

Total energy of the Electric field = Total Energy expended in assembling the charges

$W_1 = 0$ $\quad$ V_{21} = Potential at 2nd due to 1st charge

$W_2 = -Q_2 V_{21}$

$W_3 = -Q_3 V_{31} - Q_3 V_{32}$

$W_4 = -Q_4 V_{41} - Q_4 V_{43}$

$\vdots$

$W_n = -Q_n V_{n1} - Q_n V_{n2} - \cdots Q_n V_{n\,n-1}$

$$Q_2 V_{21} = Q_2 \cdot \frac{Q_1}{4\pi\epsilon r_{21}} = Q_1 V_{12}$$

Substitute subscripts can be interchange without change in meaning of value

$$W_1 = 0$$

$$W_2 = -Q_1 V_{12}$$

$$W_3 = -Q_1 V_{13} - Q_2 V_{23}$$

$$W_4 = -Q_1 V_{14} - Q_2 V_{24} - Q_3 V_{34}$$

$$\vdots$$

$$W_n = -Q_1 V_{1n} - Q_2 V_{2n} - \cdots - Q_{n-1} V_{n-1\cdot n}$$

$$W_E = W_1 + W_2 + W_3 + \cdots W_n$$

total energy

$$2W_E = -Q_1 V_1 - Q_2 V_2 - Q_3 V_3 \cdots\cdots Q_n V_n$$

$$\Rightarrow \quad \boxed{W_E = \frac{-1}{2} \sum_{i=1}^{n} Q_i V_i}$$

For a continous charge distribution

$$W_E = \frac{-1}{2} \int \rho_v V dv = \frac{-1}{2} \int (\nabla \cdot D) V dv = \int \frac{1}{2} D(-\nabla V) dv$$

$$\Rightarrow \quad W_E = \int \frac{1}{2} (D \cdot E) dv$$

$$\frac{dW_E}{dv} = \text{Energy density}$$

$$= \text{Strength of energy at any point}$$

$$= \frac{1}{2} (D \cdot E) = \frac{1}{2} \epsilon E^2$$

Extension :-

$$\frac{dW_H}{dv} = \text{Magnetic Energy density}$$

$$= \frac{1}{2} B \cdot H = \frac{1}{2} \mu H^2$$

Jo

Capacitors and Inductors :-

→ Capacitance is the ability to confine Electric field in a finitely small region:

$$C = \text{Farad} = \frac{\oint D \cdot S}{\int E \cdot dl} = \frac{\epsilon \oint E \cdot ds}{\int E \cdot dl} = \frac{Q}{V}$$

→ It is the ratio with charge utilized to the potential developed by the charge.

Specific Geometries :-

→ Parallel Plates
→ cocentric cylinders
→ Cocentric sphere

Inductance :-

Inductance is the ability to confine H field in a finitely small region

$$L = \text{Henry} = \frac{\int B \cdot ds}{\oint H \cdot dl} = \frac{\mu \int H \cdot ds}{\oint H \cdot dl} = \frac{\psi_m}{I}$$

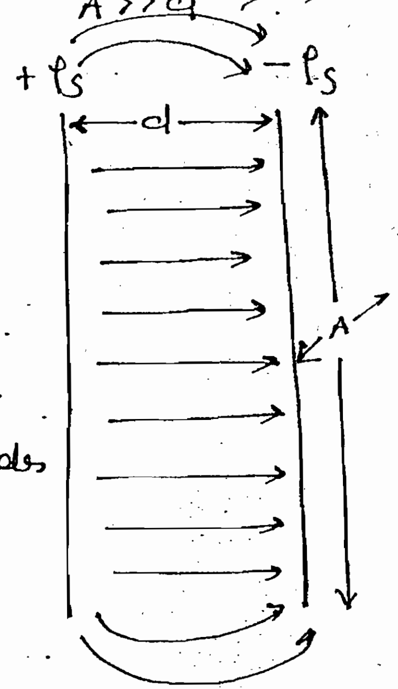
→ It is the ratio of the flux developed to the current utilized by the flux

Specific Geometries :-

→ Solenoids
→ cocentric cylinder
→ Torroids.

Parallel Plate Capacitors :-

Parallel Plate Capacitors :-



$$E = \frac{\rho_s}{\epsilon}$$

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s d}{\epsilon}} = \frac{\cancel{\epsilon A}}{} = \frac{\epsilon A}{d}$$

Capacitance is independent of Q or V and always depends on Area and distance or physical dimensions

$$W_E = \frac{1}{2}\epsilon E^2 (Ad)$$

$$= \frac{1}{2}\frac{\epsilon A}{d}(Ed)^2$$

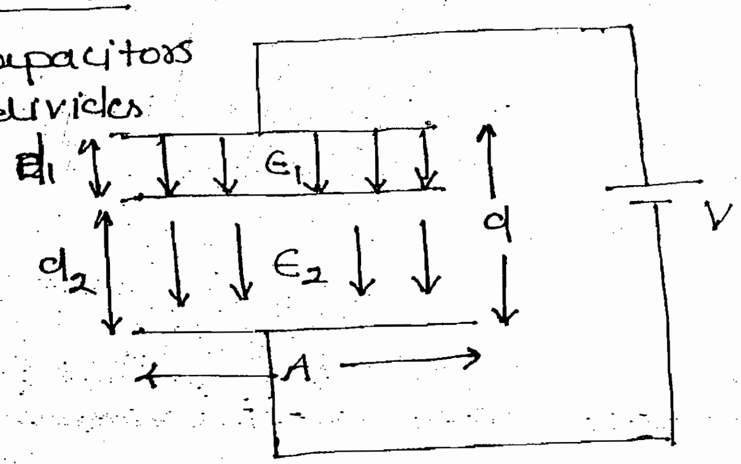
$$= \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

Multiple Dielectrics in Capacitors :-

Case-(1) :-

Equal areas of cross-section unequal width of the dielectrics :-

They are two capacitors in series as voltage divides



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\frac{\epsilon_1 A}{d_1} \cdot \frac{\epsilon_2 A}{d_2}}{\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}}$$

$$D_1 = D_2 \Rightarrow \epsilon_1 E_1 = \epsilon_2 E_2 \Rightarrow \frac{\epsilon_1 V_1}{d_1} = \frac{\epsilon_2 V_2}{d_2}$$

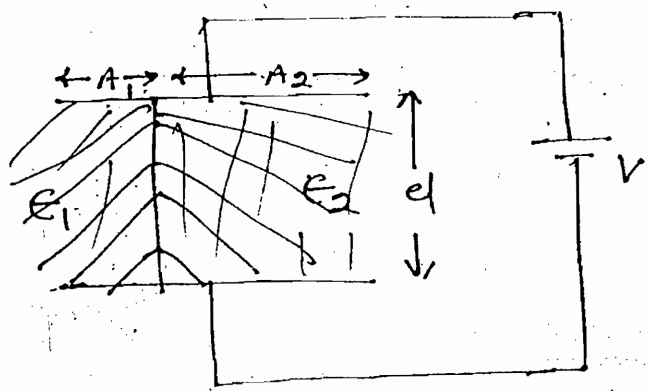
$$\Rightarrow \boxed{\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)\left(\frac{d_1}{d_2}\right)}$$

Case-(II.)

Unequal areas of cross-section and equal width of the dielectrics :-

$$C_{eq} = C_1 + C_2$$

$$= \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{d}$$



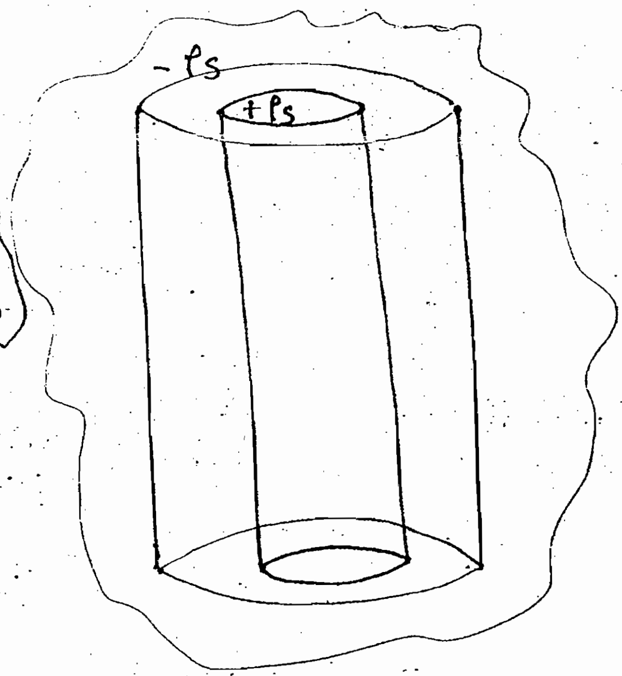
Cocentric Cylinder Capacitance :-

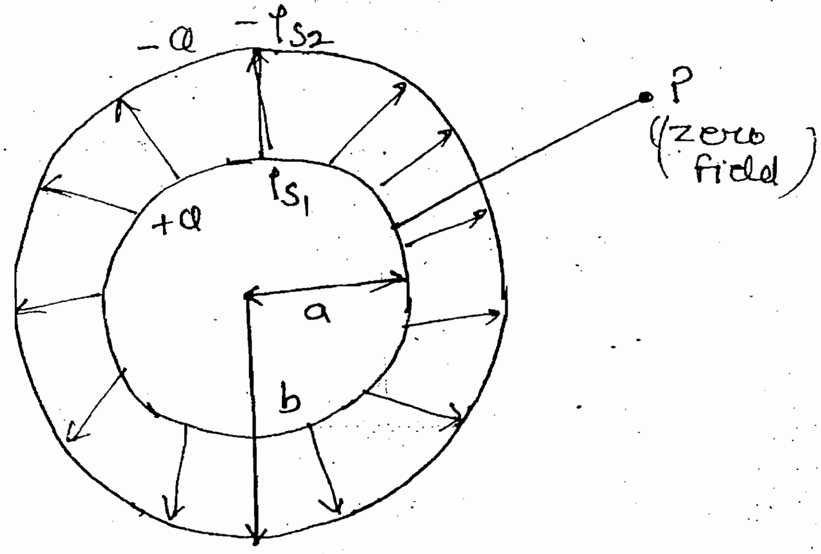
Two cocentric cylinders with equal and opposite charge density may not have confined flux but two equal and opposite charges on concentric cylinders has flux confined only b/w the cylinders

$$Q_{net} = Q_{out} + Q_{in}$$

$$= -\rho_s 2\pi bh + \rho_s 2\pi ah$$

$$\neq 0$$





$D \propto \rho_S \propto \frac{1}{\rho}$

→ The field of a sheet of cylindrical charge obeys same geometry as a line charge

$$C = \frac{Q}{V} = \frac{\rho_L h}{\frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow \boxed{C = \frac{2\pi\epsilon h}{\ln(b/a)}}$$

Extension :-

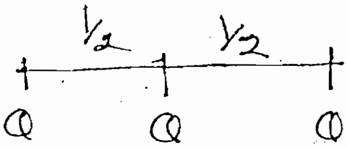
Cocentric sphere capacitance

$$C = \frac{Q}{V}$$

$$\boxed{C = \frac{Q}{\frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\epsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}}$$

20

$$W_{E1} = 0 + Q \cdot \frac{Q}{4\pi\epsilon \, l/2} + Q \cdot \frac{Q}{4\pi\epsilon \, l/2} + Q \cdot \frac{Q}{4\pi\epsilon \cdot l} = \frac{5Q^2}{4\pi\epsilon}$$



$$\boxed{W_{E2} = \frac{5Q^2}{8\pi\epsilon} = \frac{W_{E1}}{2}}$$

Note :-

$$W_E \propto QV \propto \frac{1}{r}$$

21.

$$\frac{1}{2}(J \cdot A) = \frac{\text{Amp}}{m^2} \times \frac{\text{Joules}}{\text{Amp-m}} = \frac{\text{Joules}}{m^3}$$

OR

$$\Rightarrow \frac{1}{2}(J \cdot A) = \frac{1}{2}\left((\nabla \times H) \cdot A\right) = \frac{1}{2}\left(H(\nabla \times A)\right)$$

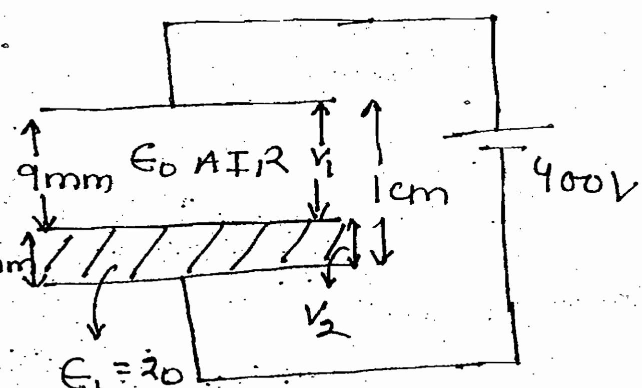
$$= \frac{1}{2} B \cdot H$$

22.

$$E_{AIR} = \frac{V_{AIR}}{9\,mm}$$

$$V_1 + V_2 = 400 \quad \text{—(I)}$$

$$\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)\left(\frac{d_1}{d_2}\right) \quad \text{—(II)}$$



$$\frac{V_1}{V_2} = \frac{20\epsilon_0}{\epsilon_0} \times \frac{9\,mm}{1\,mm} = 180$$

$$V_1 = 180 V_2$$

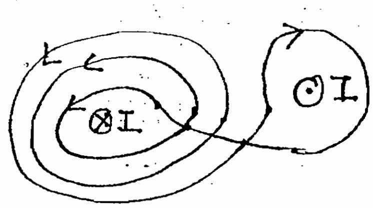
Ans → $V_1 = 44$ kV/m

24.

$$\text{Circulation} = \oint H \cdot dl = I$$

$$= I + I + I - (-I$$

$$= 4I \text{ , Ans.}$$



Lecture - 8

Time Varying Fields and Maxwell's Equation

Static Fields - Maxwell's Equations:-

	Integral Form	Point form
i)	$\oint D \cdot ds = Q$	$\nabla \cdot D = \rho_V$
ii)	$\oint E \cdot dl = 0$	$\nabla \times E = 0$
iii)	$\oint B \cdot ds = 0$	$\nabla \cdot B = 0$
iv)	$\oint H \cdot dl = I$	$\nabla \times H = J$

Note:-

- The surface ~~and~~ Integral and Divergence expressions are consistently the same in static / time varying fields
- The line integral and curl expression are modified in time varying fields to explain AC voltages and currents

Open Integral's (Not Maxwell Equations):-

1. $\int D \cdot ds = \psi_e = $ coulombs
2. $\int E \cdot dl = V = EMF = $ volts
3. $\int B \cdot ds = \psi_m = $ Webers
4. $\int H \cdot dl = V_m = MMF = $ Amps

time

Maxwell 2nd Equation and Faraday's Law:-

Faraday's Law Statement:-

EMF or voltage is induced even in a closed conductor when the magnetic flux crossing the surface changes with time.

i.e. Rate of change of magnetic flux is equal to induced EMF

$$\oint E \cdot dl = V = \overset{\text{due to Lenz's Law}}{-}\frac{d\psi_m}{dt} = -\frac{d}{dt}\int B \cdot ds$$

$$= \int -\frac{\partial B}{\partial t} \cdot ds$$

Lenz's Law :-

The induced EMF ~~allows~~ always opposes the basic changing flux (Cause)

$$\oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot ds$$

$$\oint E \cdot dl = V = -\frac{d\psi_m}{dt} = -\frac{d}{dt}\int B \cdot ds = \int -\frac{\partial B}{\partial t} \cdot ds$$

Apply stoke's theorem $= \int \nabla \times E \cdot ds$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$ Modified Maxwell's Second Equation

Note:-

Potential is unique at a time at a point in a space but changes with time and Hence the modification

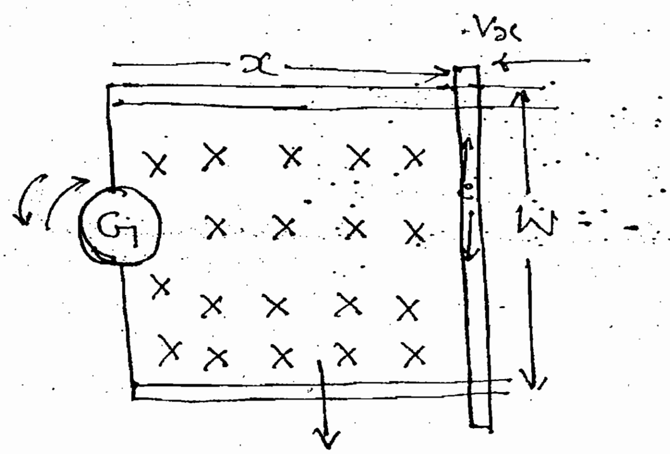
Sliding Rail Experiment :-

$$V = -\frac{d\psi_m}{dt}$$

$$\Rightarrow V = -\frac{d}{dt}(B \cdot A)$$

$$\Rightarrow \boxed{V = -B \cdot W \frac{dx}{dt}}$$

$$\Rightarrow \boxed{V = -B \cdot W \cdot V_x}$$



Static Uniform B field

Note:-

→ When a conducting rod is moving in a magnetic field a force is exist on the electron obeying Lorentz Law. This displaces the electrons towards one side. This accumulation is called as induced voltage

→ As the rod moves to and fro the voltage polarity changes. This is called as AC voltage.

→ The already displaced $\bar{e}$ opposes another further coming $\bar{e}$'s. This is called as Lenz's law

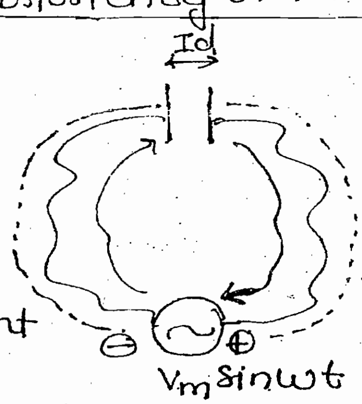
$$F_y = \cancel{q}(V_x \times B_z) = \cancel{q}E$$

$$\Rightarrow \frac{dx}{dt} B \cdot \sin\theta = \frac{V}{W} \quad \cancel{c}$$

$$\Rightarrow V = \frac{d}{dt}(xBW) = \frac{d}{dt}(BA) \quad (\theta = 90^\circ)$$

$$\Rightarrow V = \frac{d\psi_m}{dt}$$

Maxwell's IV Equation and Inconsistency of Ampere's Law:-



→ When a capacitor is connected with AC harmonic voltage there is a current flowing in the wire and plates obeying ohm's law like any linear element

$$I = C\frac{dV}{dt}$$

$$I = C \cdot W \cdot V_m \sin(wt + 90^\circ)$$

$e^{j90} = j$ → Phase ... Phase

$$I = jwC\, V_m \sin wt$$

$$V = \left(\frac{1}{jwC}\right) I$$

→ There is no current flowing b/w plates as their is dielectric Hence the circuit is not closed But Ampere's law state that current flows only

$\because$

closed circuits. Hence Maxwell's modified Ampere's law as

$$\oint H \cdot dl = I_c + I_d$$

$$\nabla \times H = J_c + J_d$$

Using equation of continuity

$$\nabla \cdot J_d = \frac{\partial \rho_v}{\partial t} = \frac{\partial (\nabla \cdot D)}{\partial t} = \nabla \cdot \frac{\partial D}{\partial t} \Rightarrow$$

$$\Rightarrow J_d = \frac{\partial D}{\partial t} \quad \& \quad I_d = \int \frac{\partial D}{\partial t} \cdot ds$$

Maxwell IV Equation is

$$\oint H \cdot dl = I_c + \int \frac{\partial D}{\partial t} \cdot ds$$

$$\Rightarrow \nabla \times H = J_c + \frac{\partial D}{\partial t}$$

where I_c = Conduction current

OR

= Moving $\bar{e}$'s

→ When an AC voltage applied to the capacitor plate the plates are alternatively charge and discharge. This continues takes place as the polarity changes. Hence this self establish continuity

→ As ρ_s on the plates changes with time, D b/w the plates changes with time. Hence $\frac{\partial D}{\partial t}$ is

$$\frac{C/m^2}{\text{second}} = Amp/m^2 = J_d$$

This is also a format of current

This is called as Displacement current density.

Summary 1:—

$$\oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot ds$$

$$\oint H \cdot dl = \int \frac{\partial D}{\partial t} \cdot ds + J_c$$

→ A time varying magnetic flux is a cause of Voltage

→ A time varying electric flux is a format of current

$$\boxed{\frac{\text{Weber}}{\text{second}} = \text{Volts}} \qquad \boxed{\frac{\text{Coulomb}}{\text{Sec}} = \text{Amp}}$$

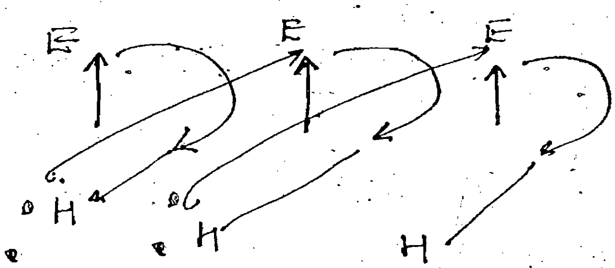
$$\begin{tikzcd}
\overset{\text{Cause}}{\text{Voltage}} \arrow[r] & \overset{\text{Effect}}{\text{Electric field}} \arrow[d, "\text{time}"] \\
\underset{\text{Effect}}{\text{H}\cdot\text{Field}} \arrow[u, "\text{time}"] & \underset{\text{Cause}}{\text{current}} \arrow[l]
\end{tikzcd}$$

Summary 2:—

Space varying Electric field ← $\nabla \times E = -\frac{\partial B}{\partial t}$ → time varying B field

$$\nabla \times H = \frac{\partial D}{\partial t} + J_c$$

→ A time varying E field produces a space varying orthogonal H field and vice-versa



$$\text{H field} \underset{\text{Space}}{\overset{\text{time}}{\rightleftarrows}} \text{E}\cdot\text{Field}$$

→ Accumulation leads to flow and flow leads to accumulation sustaining each other.

Summary 3:—

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} + \sigma E$$

→ μ = Henry/m , ϵ = Farad/m , σ = mho/m →

For E to transform to H and vice-versa the material and its permitting abilities are also important.

i.e. Material consists decides the E/H dynamics in the medium

→ $\vec{E}$ - Volts/m , $\vec{H}$ - amps/m , ∇ - perm

Note:-

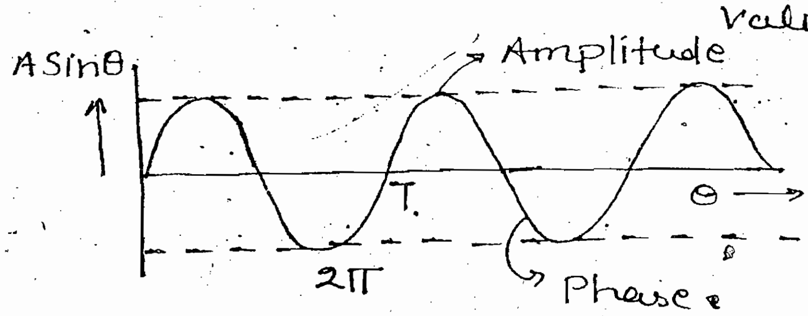
→ For sustain oscillation or ever existing phenomenon are produce when the time derivative of E and H is always back the same function and such a function called as Harmonic function.

Harmonic Functions:-

They are 2 dimension quantities and having 3 formats

→ A Sinθ
→ A Cosθ
→ A $e^{j\theta}$

Amplitude → Domain 1 → Peak value
Phase → Domain 2 → Instanteous value



Property 1:-

The phase should be a linear function of the variable

(I) $\theta \propto t$ t→time Harmonic

$\Rightarrow$ $\theta = \omega t$ ω = Phase shift constant per unit time

$= \frac{2\pi}{T}$ rad/second

(II) $\theta \propto z$ → Space Harmonic

$\Rightarrow$ $\theta = \beta z$ β = Phase ~~ee~~ shift constant per unit length $= \frac{2\pi}{\lambda}$ rad/m

Property 2 :-

The derivative of every harmonic has to be back the same function shifted orthogonally by 90°

$$A \sin \omega t \xrightarrow{\text{I-derivative}} \omega A \sin(\omega t + 90^\circ) \xrightarrow[\text{der.}]{\text{II}} \omega^2 A \sin(\omega t + 180^\circ)$$

$$\xrightarrow{\text{III derivative}} \omega^3 A \sin(\omega t + 270)$$

$$A e^{j\omega t} \xrightarrow[\text{der}]{\text{I}} \omega A e^{j(\omega t + 90)} \xrightarrow[\text{der.}]{\text{II}} \omega^2 A e^{j(\omega t + 180)}$$

$$\xrightarrow{\text{III-der.}} \omega^3 A e^{j(\omega t + 270)}$$

$(\because j^3 = -j = e^{j270})$

$(-1 = e^{j180})$

$(\because j = e^{j90})$

→ All harmonics obey the basic property the second order derivative is back the same function

→ They satisfied the differential equation

$$D^2 - M^2 = 0$$

$$D^2 + M^2 = 0$$

eg:- E/H field equation in free space

Electro magnetic wave equation in free space

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

Taking ∇ on both sides

$$\nabla \times (\nabla \times H) = \epsilon \frac{\partial}{\partial t} (\nabla \times E)$$

$$-\nabla^2 H = \epsilon \frac{\partial}{\partial t} \left(-\mu \frac{\partial H}{\partial t}\right)$$

Space Harmonic

$$\boxed{\begin{aligned} \nabla^2 H &= \epsilon\mu \frac{\partial^2 H}{\partial t^2} \\ \nabla^2 E &= \epsilon\mu \frac{\partial^2 E}{\partial t^2} \end{aligned}}$$

Time Harmonic

These are called as E/H wave equations in free space

eg-(2) :- V/I Equations in LC circuits

$$I = C\frac{dV}{dt}$$

$$V = -L\frac{dI}{dt} = -L\frac{d}{dt}\left(C\frac{dV}{dt}\right)$$

$$\Rightarrow \quad V = -LC\frac{d^2V}{dt^2}$$

$$\Rightarrow \quad \boxed{\frac{d^2V}{dt^2} = -\frac{1}{LC}V}$$

and $\boxed{\frac{d^2I}{dt^2} = -\frac{1}{LC}I}$

By comparison

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

Types of Exponential Functions :-

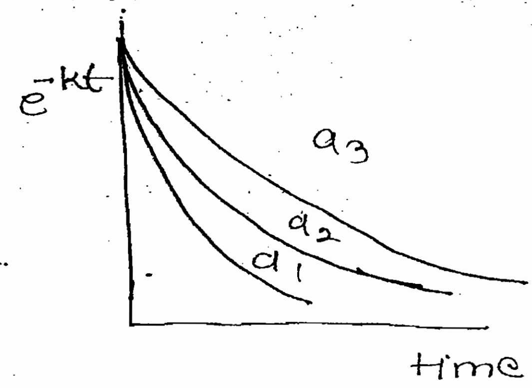
→ e^{-Kt} vs t

(1) Case - (1) :-

$K = a =$ +ve real no.

$a_1 > a_2 > a_3$

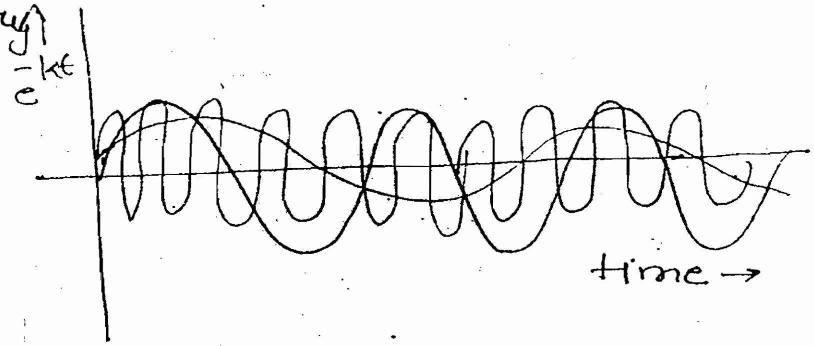
→ Real Exponential.



II) Case-(II) :-

$k = j\omega$ = purely imaginary no.

ω = Phase Shift Constant

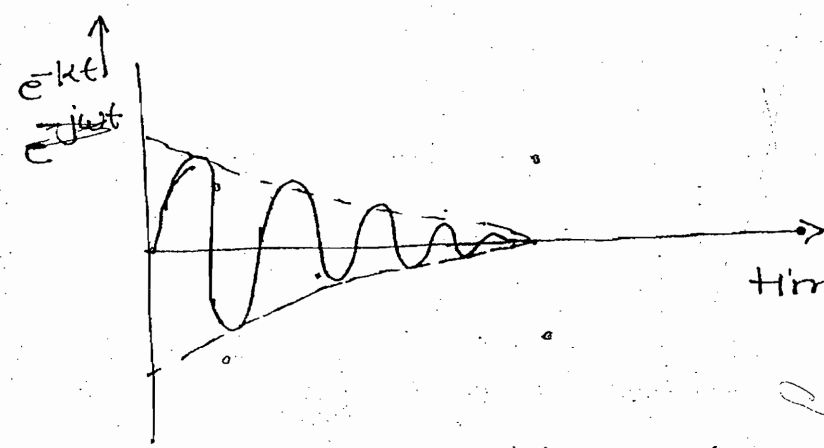


III) Case-(III) :-

$k = a + j\omega$

$(e^{-at})\ (e^{-j\omega t})$

$A \cdot e^{j\theta}$



Note :-

→ j stands for an orthogonal shift from domain 1 to a second independent domain 2

eg :- (I) east and north displacements

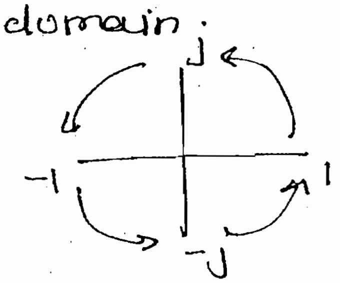
(II) Resistance and reactance of a impedance

(III) Amplitude and phase of a harmonic

→ A scaling of j in domain 1 is a shift of 90° in domain 2

eg :- (I) $j\omega \cancel{t}\, e^{-j\omega t} \longrightarrow \omega e^{j(\omega t + 90^\circ)}$

(II) S-domain :



$1 \times j = j = 1 \angle 90^\circ$

$j \times j = -1 = 1 \angle 180^\circ$

$-1 \times j = -j = 1 \angle 270^\circ$

$-j \times j = 1 \angle 360^\circ$