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CHAPTER

Dimensional Analysis

Q.1 Euler number or pressure coefficient may be represented as (notations have their usual meanings)

(a) $\frac{\Delta p}{\gamma H}$ (b) $\frac{\Delta p}{(\rho V^2/2)}$

(c) $\frac{\Delta p}{\mu VL}$ (d) $\frac{\Delta p}{\rho Lg}$

Q.2 A spillway of an irrigation project is to be studied by means of a model constructed to a scale of 1 : 9. The prototype discharge is 1000 m^3/s. Neglecting the viscous and surface tension effects, the required flow rate for the model is

(a) 12.35 m^3/s (b) 111.11 m^3/s

(c) 4.12 m^3/s (d) 1.37 m^3/s

Q.3 A 1: 30 scale model of a submarine is to be tested in a wind tunnel for its drag when it is operating at 15 km/h in ocean. The kinematic viscosity of air is 1.51×10^{-5} m^2/s and that for water is 1.02×10^{-6} m^2/s. What is the velocity of air in wind tunnel that should be maintained for kinematic similarity?

(a) 1500 m/s (b) 2000 m/s

(c) 1850.5 m/s (d) 2500.5 m/s

Q.4 The geometrical similarity between model and prototype is achieved by

(a) using different model scales for horizontal and vertical dimension

(b) using the same model scale throughout

(c) having the same radius of curvature at identical places in model and prototype

(d) None of these

Q.5 Which of the following could be a π-parameter of the function $f(F, V, r, m, L) = 0$ when V, r and L are taken as repeating variables?

(a) $\frac{VL\mu^2}{\rho}$ (b) $\frac{F^2}{V^2\rho L}$

(c) $\frac{F}{\rho L^2 V^2}$ (d) $\frac{VL\rho}{\sqrt{\mu}}$

Q.6 A ship's model of scale 1 : 100 had a wave resistance of 1 kg at its design speed. The corresponding wave resistance in prototype will be

(a) 100 kg (b) 10000 kg

(c) 1000000 kg (d) 1000 kg

Q.7 A harbour model has a horizontal scale of 1/150 and a vertical scale of 1/60. The interval between successive high tides in the model will be nearly

(a) 90 min (b) 40 min

(c) 15 min (d) 5 hours

Q.8 A model of reservoir is emptied in 10 minutes. If the model scale is 1:25, the time taken by the prototype to empty itself, would be

(a) 250 min (b) 50 min

(c) 6250 min (d) 2 min

Q.9 Assuming that the thrust T of a propeller depends on the diameter D, speed of advance V, angular velocity ω, dynamic viscosity μ and mass density ρ, which of the following non-dimensional parameters can be derived by dimensional analysis?

1. $\frac{T}{\rho D^2 V^2}$ 2. $\frac{VD}{\mu}$

3. $\frac{D\omega}{V}$ 4. $\frac{VD\rho}{\mu}$

Select the correct answer using the codes given below:

Codes:

(a) 1, 2 and 3 (b) 2, 3 and 4

(c) 1, 3 and 4 (d) 1, 2 and 4

Q.10 Knudsen number is

(a) the ratio of mean free-path of molecules to the characteristic length of flow

(b) a non-dimensional number characterising viscous flow

(c) the ratio of velocity of sound to the velocity of light in the medium

(d) a non-dimensional number relating the heat transfer in the flow.

Q.11 The Buckingham π theorem is widely used in the dimensional analysis and expresses the resulting equation in terms of

(a) geometric, kinematic and dynamic variables

(b) n - dimensionsless variables

(c) $(n - m)$ dimensionless variables

(d) the independent and dependent variables

Q.12 Gas-flows can be treated as incompressible when the Mach number is less than

(a) 0.5 (b) 1.0

(c) 0.2 (d) 0.1

Q.13 Assertion (A) : When both gravitational and viscous forces are predominant in a flow, scale ratio can be chosen at will.

Reason (R) : With both gravitational and viscous forces being predominant, scale ratio depends upon kinematic viscosity of the fluids.

(a) both A and R are true and R is the correct explanation of A

(b) both A and R are true but R is not a correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

Q.14 The reasons for adopting distorted models are

1. to maintain accuracy in vertical measurements
2. to maintain turbulent flow
3. to obtain suitable roughness condition
4. to obtain suitable bed material and its adequate movement

Which of these statements are correct?

(a) 1, 2 and 3 (b) 1, 3 and 4

(c) 2, 3 and 4 (d) 1, 2, 3 and 4

Q.15 In a model built of Froude law of similarity, a phenomenon lasts for 20 min. If the model scale is 1/25, the duration of the phenomenon in the prototype, in minutes, is

(a) 50 (b) 100

(c) 2500 (d) 4

Q.16 In an automobile traveling at 60 km/h, the power in overcoming wind resistance is 1.5 kW. If the automobile travels at 90 km/h the power in overcoming wind resistance, in kW, is

(a) 2.25 (b) 3.38

(c) 5.06 (d) 7.59

Q.17 In the model of a highway bridge constructed to a scale of 1:25, the force of water on the pipe was measured to be 0.5 kg. The force on the prototype pier will be

(a) 7501.5 kg

(b) 7622.5 kg

(c) 7812.5 kg

(d) 7916.5 kg

Q.18 The height of a hydraulic jump in a stilling pool was found to be 10cm in a model with $l_p/l_m = 36$. The prototype jump height would be

(a) 0.6 m

(b) 3.6 m

(c) 21.6 m

(d) indeterminable for want of adequate data

Q.19 Match List-I (Flow problem under study) with List-II (Model law) and select the correct answer using the codes given below the lists:

List-I

A. Wave resistance of a boat

B. Valve in a pipe

C. Ripples in water tank

D. Singing of transmission line

List-II

1. Mach number
2. Froude number
3. Strouhal number
4. Reynolds number
5. Weber number

Codes:

	A	B	C	D
(a)	5	4	2	3
(b)	4	1	3	5
(c)	2	4	5	3
(d)	2	3	5	2

Q.20 Match List-I (Flow problem under study) with List-II (Model law) and select the correct answer using the codes given below the lists:

List-I

A. Rise of gas bubbles in liquid
B. Flow of gas in a pipe
C. Flow over a spillway dam
D. Flight of supersonic jet

List-II

1. Euler number
2. Froude number
3. Mach number
4. Reynolds number
5. Weber number

Codes:

	A	B	C	D
(a)	3	5	4	1
(b)	3	4	2	1
(c)	5	2	4	3
(d)	5	4	2	3

Q.21 Select the non-dimensional parameter from the following:

(a) Specific weight
(b) Manning's coefficient n
(c) Angular velocity
(d) Specific gravity

Q.22 Which of the following is a dimensionally homogeneous equation?

(a) $V = \frac{1}{n} R^{2/3} S^{1/2}$ (Manning's equation)

(b) $V = C\sqrt{RS}$ (Chezy's equtlon)

(c) $p = \frac{32\mu VL}{D^2}$ (Laminar flow in pipes)

(d) $N_s = \frac{N\sqrt{P}}{H^{5/4}}$ (Specific speed of a turbine)

Q.23 In which of the following situation, the viscous force is unimportant?

(a) flow of incompressible fluids in closed pipes
(b) motion of aeroplanes
(c) capillary waves in channels
(d) resistance to motion of a ship

Q.24 In which of the following situations, the inertia force would be unimportant?

(a) flow over a spillway crest
(b) flow through an open channel transition
(c) flow through a long capillary tube
(d) All of the above

Q.25 In steady, isothermal flow in long pipe lines, the significant value of Mach number for determining trends in flow properties is

(a) $\sqrt{k}$ (b) $\frac{1}{\sqrt{k}}$

(c) k (d) $\frac{1}{k}$

Q.26 Which of the following rules are used in choosing the repeating variables in dimensional analysis?

1. Repeating variables should include the dependent variables.
2. Repeating variables should contain all primary units used in describing the variable in the problem.
3. Repeating variables should combine among themselves.
4. Repeating variables should not contain the dependent variables.

Select the correct answer using the codes given below:

Codes:

(a) 1 and 2 (b) 2 and 3
(c) 2 and 4 (d) 3 and 4

Q.27 With n variables and m fundamental dimensions in a system, which one of the following statements relating to the application of the Buckingham's pi theorem is incorrect?

(a) With experience, pi terms can be written simply by inspection of variables in a flow system.
(b) Buckingham's pi theorem is not directly applicable in compressive flow problem
(c) Buckingham's pi theorem yields dimensionless pi terms given by the difference between the number of variables and the number of fundamental dimensions
(d) Buckingham's pi theorem reduces the number of variables by the number of fundamental dimensions involved.

Q.28 Flow of air can be considered to be incompressible within 1% error if the Mach number of flow is less than

(a) 0.1 (b) 0.2
(c) 0.4 (d) 0.6

Q.29 In order that results obtained in model studies correctly represents the behaviour of the prototype, which of the following similarities must be ensured between the model and the prototype?

(a) Geometric similarity
(b) Kinematic similarity
(c) Dynamic similarity
(d) All of the above

Q.30 The scale effect in models can be

(a) positive only
(b) negative only
(c) both positive and negative
(d) none of the above

Q.31 Which of the following statements are correct?

1. Kinematic similarity is necessary and sufficient condition for dynamic similarity.
2. Geometrical similarity is necessary condition for dynamic similarity.
3. Geometrical similarity is necessary condition for kinematic similarity.
4. Dynamic similarity is necessary condition for kinematic similarity.

Codes:

(a) 1 and 2 (b) 2 and 3
(c) 1, 2 and 3 (d) 1, 2 and 4

Q.32 A laboratory model of a river built to a geometrical scale of 1 : 100, the fluid used in the model is oil of density 900 kg/m^3. The highest flood in the river is 10000 m^3/s, the corresponding discharge in the model shall be

(a) 0.093 m^3/s (b) 0.1 m^3/s
(c) 0.105 m^3/s (d) 10.5 m^3/s

Q.33 Two geometrical similar pumps are running at the same speed of 100 rpm and lifting water against head of 25 m and 16 m respectively. The first pump is having impeller diameter of 300 mm. What is the impeller diameter of second pump

(a) 192 mm (b) 240 mm
(c) 300 mm (d) 469 mm

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Answers **Dimensional Analysis**

1. (b)	2. (c)	3. (c)	4. (b)	5. (c)	6. (c)	7. (b)	8. (b)	9. (c)	10. (d)
11. (c)	12. (c)	13. (d)	14. (d)	15. (b)	16. (c)	17. (c)	18. (b)	19. (c)	20. (d)
21. (d)	22. (c)	23. (c)	24. (c)	25. (b)	26. (c)	27. (c)	28. (b)	29. (d)	30. (c)
31. (b)	32. (b)	33. (b)							

2. (c)

Discharge ratio, $Q_r = L_r^{2.5}$

Prototype discharge = 1000 m³/s

Model discharge = $\frac{1000}{(9)^{2.5}}$ = 4.12 m³/s

3. (c)

The Reynold's model law shall be applied in the case of fully submerged objects

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$\therefore \quad V_m = V_p\left(\frac{\nu_m}{\nu_p}\right)\left(\frac{L_p}{L_m}\right)$$

$$= 15 \times \frac{1.51\times10^{-5}}{1.02\times10^{-6}} \times 30$$

= 6661.8 km/h = 1850.5 m/s

The Reynold's model law ensures dynamic, kinematic and geometric similarities.

4. (b)

Geometric similarity exists between the model and the prototype if the ratios of corresponding length dimensions in the model and the prototype are equal. Different model scales for horizontal and vertical dimension are used in a distorted model. Geometric similarity is achieved by using same model scale throughout.

5. (c)

A π parameter is constant and has zero dimensional unit i.e. $M^0L^0T^0$.

6. (c)

$$l_r = \frac{l_m}{l_p} = \frac{1}{100}$$

Froude model law is applicable here because of influence of gravity force. Using Froude model law

$$V_r = \sqrt{l_r}$$

$$\frac{F_m}{F_p} = \frac{1\,\text{kg}}{F_p} = \frac{\rho l_m^2 V_m^2}{\rho l_p^2 V_p^2}$$

$$= (l_r)^2 = l_r^3 = \left(\frac{1}{100}\right)^3$$

or, $F_p = 1000000$ kg $= 10^6$ kg

7. (b)

Time scale

$$t_r = \frac{t_m}{t_p} = \frac{L_r}{V_r} = \frac{X_r}{\sqrt{Y_r}}$$

and Tidal velocity $V = \sqrt{gy}$

$$\therefore \quad t_r = \frac{\frac{1}{150}}{\sqrt{\frac{1}{60}}} = \frac{\sqrt{60}}{150}$$

Taking t_p = 12 hrs.

$$\therefore \quad t_m = 12\times60\times\frac{\sqrt{60}}{150}$$

= 37 min. ≃ 40 min.

8. (b)

$$\frac{t_m}{t_p} = \sqrt{\frac{l_m}{l_p}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$$

$$\therefore \quad t_p = 5\,t_m = 5 \times 10 = 50 \text{ min}$$

9. (c)

Number of variables = 6

Number of fundamental dimensions =3

∴ Number of dimensionless parameters = 6 – 3 = 3.

Take D, V, and ρ as repeating variables.

$\frac{VD}{\mu}$ will not be dimensionless number as its dimension is $[M^{-1}L^3]$

11. (c)

Buckingham's π-theorem states that if there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into $(n-m)$ dimensionless terms. Each term is called π-term.

15. (b)

$$F_p = F_m$$

$$l_r = \sqrt{l_r} = \frac{l_m}{l_p}$$

$$\Rightarrow \quad t_p = \frac{t_m}{t_p} = \frac{20}{\sqrt{1/25}}$$

$$t_p = 5 \times 20$$

= 100 minutes

16. (c)

Power, $P \propto V^3$

∴ When the automobile travels at a speed of 90 km/hr, power

$$P = 1.5\times\frac{90^3}{60^3} = 5.06 \text{ kW}$$

17. (c)

$$F_r = L_r^3$$

$$\Rightarrow \quad \frac{F_m}{F_p} = \left(\frac{L_m}{L_p}\right)^3$$

$$\Rightarrow \quad \frac{0.5}{F_p} = \left(\frac{1}{25}\right)^3$$

Solving, we get

$$F_p = 25^3 \times 0.5$$

= 7812.5 kg

19. (c)

Strouhal number $S = \frac{nd}{V}$

where,

n = frequency of sheading of vortices

d = diameter of cylinder

V = free stream velocity

Alternate shedding of vortices from the two sides of a cylinder gives rise to the phenomenon of "singing" of telephone or power lines in the wind.

31. (b)

It is possible to have kinematic similarity yet no dynamic similarity.

32. (b)

$$L_r = \frac{L_m}{L_p} = 100$$

Q_p = 10000 m³, Q_m = ?

$$Q_r = \frac{Q_m}{Q_p}$$

$$\Rightarrow \quad Q_m = Q_r Q_p$$

Now, $$Q_r = \frac{l^3}{T_r} = \frac{L^3}{\sqrt{L}} = L_r^{2.5}$$

$$\therefore \quad Q_m = L_r^{2.5} \times Q_p$$

$$= \left(\frac{1}{100}\right)^{2.5} \times 10000 = 0.1\,m^3/s$$

33. (b)

$$\frac{\sqrt{H_r}}{N_r D_r} = 1$$

$$\sqrt{H_r} = D_r$$

$$\Rightarrow \quad \sqrt{\frac{H_1}{H_2}} = \frac{D_1}{D_2}$$

$$\Rightarrow \quad \sqrt{\frac{25}{16}} = \frac{300}{D_2}$$

$$\Rightarrow \quad D_2 = 240 \text{ mm}$$

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