CBSE Board
Class XI Mathematics

Time: 3 hrs **Total Marks: 100**

General Instructions:
1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

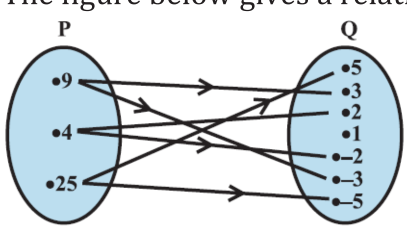
SECTION – A

1. In ΔABC, if a = 2, b = 3 and sinA = $\frac{2}{3}$, find m∠B.

2. Find the value of $\left(\frac{1}{i}\right)^{25}$

3. Write the given statement in the form 'If- then', and state what are the component statements p and q.
If I have the money, i will buy an i-phone.

4. The figure below gives a relation. Write it in the roster form.



SECTION – B

5. Find the equation of the ellipse whose vertices are (±13, 0) and foci are (±5, 0).

6. Find the equation of the circle, the co-ordinates of the end points of whose diameter are (-1, 2) and (4, -3).

7. What is the number of ways in which a set of 5 cards can be chosen out of a deck of 52 cards, if each set of 5 cards has exactly one ace?

8. Prove that $\cos 5x = 16 \cos^5 x - 20 \cos^3 x + 5 \cos x$

9. In how many ways 6 different beads can be arranged on a thread to form a necklace?

10. Find an infinite GP whose first term is 1 and each term is the sum of all the terms which follow it.

11. Find the values of' k' for which $-\frac{2}{7}, k, -\frac{7}{2}$ are in G.P. Find the common ratio/s of the G.P.

12. If $1 + \frac{(1+2)}{2} + \frac{(1+2+3)}{3} + \ldots$ to n terms is S, then find S.

SECTION – C

13. Evaluate: $\left|\frac{1+i}{1-i} - \frac{1-i}{1+i}\right|$

OR

If (a + ib) (c + id) (e + if) (g + ih) = A + iB,

then find the value of $A^2 + B^2$

14. Solve the given quadratic equation: $9x^2 - 12x + 20 = 0$

15. Find the co-efficient of x^5 in the expansion of the product $(1 + 2x)^6(1 - x)^7$

16. Show that: $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

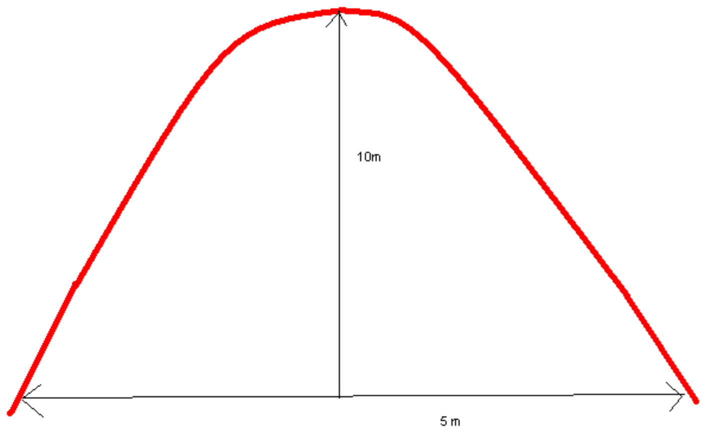
17. Prove that: $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$

OR

Prove that: a(sin B – sin C)+ b(sin C – sin A)+ c(sin A – sin B) = 0.

18. Let A = {a, e, i, o, u} and B = {a, i, k, u}. Find A – B and B – A. Are the two sets A – B and B – A (i) equal (ii) mutually disjoint. Justify your answer.

19. The entrance of a monument is in the form of a parabolic arch with a vertical axis. The arch is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of the parabola?



20. Draw the graph of $f(x)\begin{cases} 3-x, x>1 \\ 1, \quad x=1 \\ 2x, \quad x<1 \end{cases}$ and find the range of f.

OR

Draw the graph of $f(x)\begin{cases} 1, \quad x\geq 1 \\ x \quad -1<x<1 \\ -1, \quad x\leq -1 \end{cases}$ and find the range of f.

21. Let A = {a, b, c, d} and B = {p, q, r}. Write an example of onto and into function from A to B. Does there exist a one-one function from A to B. Justify your answer.

22. Prove that $\cos^2 x+\cos^2(x+\frac{\pi}{3})+\cos^2(x-\frac{\pi}{3})=\frac{3}{2}$

23. The mean of 8, 6, 7, 5, x and 4 is 7. Find (i) the value of x (ii) the mean if each observation was multiplied by 3 (iii) the mean deviation about the median for the original data

SECTION – D

24. When two dice are thrown simultaneously, find the probability that neither a doublet nor a total of 10 will appear.

25. Using mathematical induction prove the following:

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+........+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

26. (i)A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that

(a) all will be blue? (b) at least one will be green?

(ii) A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

(a)P(2) (b) P(1 or 3) (c) P(not 3)

27. Find the derivative of

(i)sin(x + 1) by the abinitio method

(ii)$\frac{x}{1 + \tan x}$

OR

Evaluate the limits of the following two functions of x:

(i) $\lim_{x\to 0}\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]$

(ii) $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}$

28. Find the length of the perpendicular drawn from the points,

$(\sqrt{a^2-b^2},0)$ and $(-\sqrt{a^2-b^2},0)$ to the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

Show that their product is b^2.

29. Solve the inequalities and represent the solution graphically

$5(2x-7)-3(2x + 3)\leq 0$; $2x + 19\leq 6x + 47$ and $7\leq\frac{(3x+11)}{2}\leq 11$

OR

How many litres of water will have to be added to 1125 litres of a 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

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Solution

SECTION – A

1. We have,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{2}{\frac{2}{3}} = \frac{3}{\sin B}$$

$$\Rightarrow 3 = 3\sin B$$

$$\Rightarrow \sin B = 1$$

$$\Rightarrow m\angle B = 90^\circ$$

2.

$$\left(\frac{1}{i}\right)^{25} = \left(\frac{1}{i}\times\frac{i}{i}\right)^{25} = \left(\frac{1}{i^2}\right)^{25}(i)^{25} = (-1)(i)^{25} = (-1)(i)^{24}i = (-1)\left(i^2\right)^{12}i$$

$$= (-1)(-1)^{12}i = (-1)(1)i = -i$$

3. p: I have the money;
q: I will buy an i-phone,
$p \rightarrow q$: If I have the money $(\rightarrow)$ then I will buy an i-phone

4. Relation R from P to Q is R = {(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}

SECTION – B

5. The vertices are on the x-axis, so the equation will be of the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \ldots(1)$$

Given that a = 13, ae = 5 $\Rightarrow e=\frac{5}{13}$

Now, we have, $b^2=a^2\left[1-e^2\right]$

Therefore, we get

$$b^2=13^2\left[1-\left(\frac{5}{13}\right)^2\right]$$

$$=169\left[1-\frac{25}{169}\right]$$

$$=169\left[\frac{169-25}{169}\right]$$

$$=169-25$$

$$=144$$

Substituting the values of a^2 and b^2 in equation (1), we have

$$\frac{x^2}{13^2}+\frac{y^2}{12^2}=1$$

$$\Rightarrow \frac{x^2}{169}+\frac{y^2}{144}=1$$

6. We know that the equation of the circle described on the line segment joining (x_1, y_1) and (x_2, y_2) as a diameter is,

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Here, $x_1 = -1$, $x_2 = 4$, $y_1 = 2$ and $y_2 = -3$

Thus, the equation of the required circle is

$(x + 1)(x - 4) + (y - 2)(y + 3) = 0$

$\Rightarrow x^2 + y^2 - 3x + y - 10 = 0$

7. One ace can be selected from 4 aces in $^4C_1$.

Other 4 cards which are non-aces can be selected out of 48 cards in $^{48}C_4$ ways.

The total number of ways = $^4C_1 \times {}^{48}C_4$

= 4 x 2 x 47 x 46 x 45 = 778320

8. Let ,

$\cos 5x = (\cos 5x + \cos x) - \cos x$

$= 2\cos\frac{5x+x}{2}\cos\frac{5x-x}{2} - \cos x$

$= 2\cos 3x \cos 2x - \cos x$

$= 2(4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - \cos x$

$= 16\cos^5 x - 20\cos^3 x + 5\cos x$

9. 6 beads have to be arranged in a circular fashion which can be done in (6-1)!
But anticlockwise and clockwise arrangement of beads in a necklace are same so
(6-1)!×(1/2) = 60

10.

Given that the first term is 1.

Also given that each term is the sum of all the terms which follow it.

Let 1, r, r^2,... be an infinite G.P., where r is the common ratio.

Sum of terms of an infinite G.P., $S = \frac{a}{1-r}$

Here, a = r

Thus, $S = \frac{r}{1-r}$

From the given statement of the problem, we have,

$1 = \frac{r}{1-r}$

$\Rightarrow 1 - r = r$

$\Rightarrow r + r = 1$

$\Rightarrow 2r = 1$

$\Rightarrow r = \frac{1}{2}$

Thus the required G.P. is:

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, ...$

11. $-\frac{2}{7}, k, -\frac{7}{2}$ are in G.P.

$\Rightarrow k^2 = \left(-\frac{2}{7}\right) \times \left(-\frac{7}{2}\right) = 1$

$\Rightarrow k^2 = 1$

$\Rightarrow k = \pm 1$

When $k = 1$; G.P.: $-\frac{2}{7}, 1, -\frac{7}{2}$

$r = \frac{1}{-\frac{2}{7}} = -\frac{7}{2}$

When $k = -1$; GP: $-\frac{2}{7}, -1, -\frac{7}{2}$

$r = \frac{-1}{-\frac{2}{7}} = \frac{7}{2}$

12. $a_n = \frac{(1+2+3+....n)}{n} = \frac{n(n+1)}{2n}$

$S_n = \sum_n a_n$

$= \frac{1}{2}\sum_{i=1}^{n}(n+1)$

$= \frac{1}{2}\frac{n(n+1)}{2} + \frac{n}{2}$

$= \frac{(n^2+n)}{4} + \frac{n}{2}$

$= \frac{n(n+3)}{4}$

SECTION – C

13.

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$

$$=\frac{\left(1+i^2+2i\right)-\left(1+i^2-2i\right)}{(1-i)(1+i)}$$

$$=\frac{(1-1+2i)-(1-1-2i)}{\left(1-i^2\right)}=\frac{(2i)-(-2i)}{\left(1-i^2\right)}$$

$$=\frac{2i+2i}{(1-(-1))}=\frac{4i}{(2)}=2i$$

$$\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{0^2+2^2}=\sqrt{4}=2$$

OR

Consider the given equation:

(a + ib) (c + id) (e + if) (g + ih) = A + iB

Let us take modulus on both sides ,

$|(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|$

We know, $|z_1z_2|=|z_1||z_2|$

$\therefore |(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|$

$\Rightarrow |(a + ib)||(c + id)||(e + if)||(g + ih)| = |A + iB|$

$\Rightarrow \sqrt{a^2+b^2}.\sqrt{c^2+d^2}.\sqrt{e^2+f^2}.\sqrt{g^2+h^2}=\sqrt{A^2+B^2}$

$\Rightarrow \left[\sqrt{a^2+b^2}.\sqrt{c^2+d^2}.\sqrt{e^2+f^2}.\sqrt{g^2+h^2}\right]^2=\left[\sqrt{A^2+B^2}\right]^2$

$\Rightarrow \left(a^2+b^2\right).\left(c^2+d^2\right).\left(e^2+f^2\right).\left(g^2+h^2\right)=A^2+B^2$

Thus, the value of $A^2+B^2=\left(a^2+b^2\right).\left(c^2+d^2\right).\left(e^2+f^2\right).\left(g^2+h^2\right)$

14. Consider the given quadratic equation:

$9x^2 - 12x + 20 = 0$

$\Rightarrow 3x^2 - 4x + \frac{20}{3} = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times \frac{20}{3}}}{2.3} = \frac{4 \pm \sqrt{16 - 4 \times 20}}{6}$$

$$= \frac{4 \pm \sqrt{16 - 80}}{6} = \frac{4 \pm \sqrt{-64}}{6} = \frac{4 \pm 8\sqrt{-1}}{6} = \frac{2 \pm 4i}{3}$$

$$\Rightarrow x = \frac{2}{3} \pm \frac{4}{3} i$$

15. To find the coefficient of x^5 in the expansion of the product,

$(1 + 2x)^6(1 - x)^7$

Let us find the expansions of the 2 binomials.

$$(1+2x)^6 = {}^6C_0(2x)^0 + {}^6C_1(2x)^1 + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6$$

$$= 1\times1 + 6\times(2x) + 15\times(2x)^2 + 20\times(2x)^3 + 15\times(2x)^4 + 6\times(2x)^5 + 1\times(2x)^6$$

$$= 1 + 12x + 60x^2 + 20\times(2x)^3 + 15\times(2x)^4 + 6\times(2x)^5 + 1\times(2x)^6$$

$$(1-x)^7 = {}^7C_0(-x)^0 + {}^7C_1(-x)^1 + {}^7C_2(-x)^2 + {}^7C_3(-x)^3 + {}^7C_4(-x)^4 + {}^7C_5(-x)^5 + {}^7C_6(-x)^6 + {}^7C_7(-x)^7$$

$$= 1\times1 - 7\times(x) + 21\times(x)^2 - 35\times(x)^3 + 35\times(x)^4 - 21\times(x)^5 + 7\times(x)^6 - 1\times(x)^7$$

$$= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$$

Product of $(1 + 2x)^6(1 - x)^7$

$$= \left[\boxed{1} + \boxed{12}x + \boxed{60}x^2 + \boxed{160}x^3 + \boxed{240}x^4 + \boxed{192}x^5 + 64x^6\right]$$

$$\times\left[\boxed{1}\boxed{-7}x + \boxed{21}x^2\boxed{-35}x^3 + \boxed{35}x^4\boxed{-21}x^5 + 7x^6 - x^7\right]$$

$\therefore$ Coefficient of x^5 in the product

$= 1\times(-21) + 12\times35 + 60\times(-35) + 160\times21 + 240\times(-7) + 192\times1$

$= -21 + 420 - 2100 + 3360 - 1680 + 192$

$= 3972 - 3801$

$= 171$

16. Consider, $3x = 2x + x$

Rewriting $\tan 3x = \tan(2x + x)$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$

or $\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

or $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

17. We need to prove that,

$$\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$$

Consider L.H.S.

$$\frac{\frac{1\text{-}\cos 8A}{\cos 8A}}{\frac{1-\cos 4a}{\cos 4a}} = \frac{2\sin^2 4A}{\cos 8A} \times \frac{\cos 4A}{2\sin^2 2A}$$

$$= \frac{2\sin 4A \times \cos 4A \times \sin 4A}{\cos 8A \times 2\sin 2A \times \sin 2A}$$

$$= \frac{\sin 8A}{\cos 8A} \times \frac{2\sin 2A \times \cos 2A}{2\sin 2A \times \sin 2A} = \frac{\tan 8A}{\tan 2A}$$

OR

Consider the L.H.S of the given equation:

L.H.S=$a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B)$

Let us use the identity,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Thus, $\text{L.H.S} = a\sin B - a\sin C + b\sin C - b\sin A + c\sin A - c\sin B$

$\Rightarrow \text{L.H.S} = b\sin A - c\sin A + c\sin B - b\sin A + c\sin A - c\sin B$

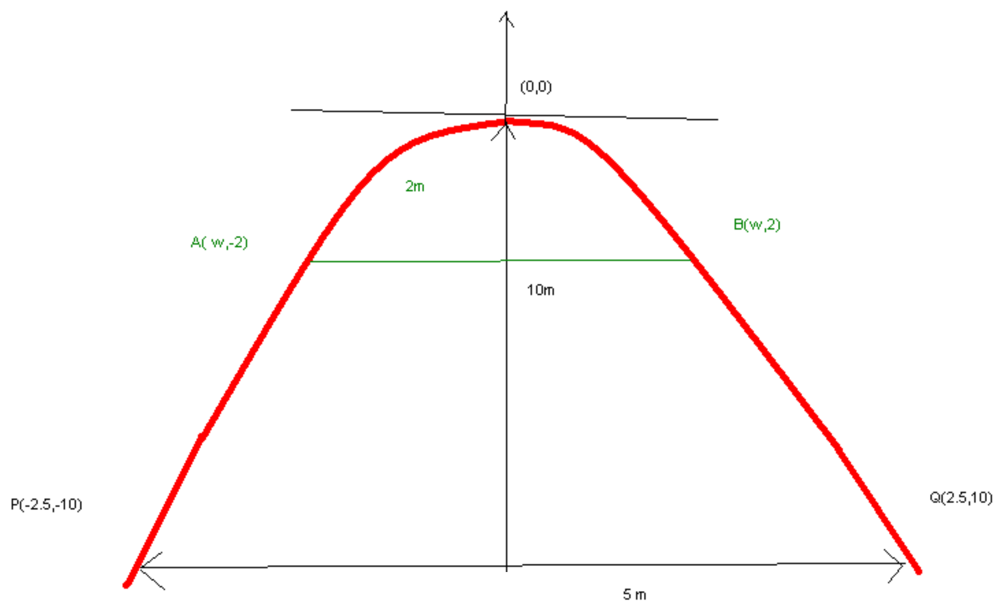
$\Rightarrow \text{L.H.S} = 0$

18. A – B = {*e, o*}, since the elements *e, o* belong to A but not to B and B – A = {*k*}, since the element *k* belongs to B but not to A.

(i)We note that A – B ≠B – A.

(ii)The sets (A – B) and (B – A) are mutually disjoint sets, i.e. the intersection of these two sets is a null set.

19.



This parabola has its axis on the y-axis and opens downwards. Hence, its equation is of the type,

$x^2 = -4ay$

The top of the parabola is its vertex passing through the origin. The width of the base is 5 m, therefore the co-ordinates of the points P and Q are (-2.5, -10) and (2.5, 10) respectively. P and Q lie on the parabola.

Substituting the co-ordinates of the point P in the equation of the parabola, we have

$(-2.5)^2 = -4a(-10)$

$6.25 = 40\ a$

$a = 6.25 / 40 = 5/32$

Let 2w be the width of the arch at 2 m below the vertex. Therefore the co-ordinates of the points are A(-w, -2) and B(w, -2)

A and B lie on the parabola.

Substituting the co-ordinates of the point A in the equation of the parabola, we have

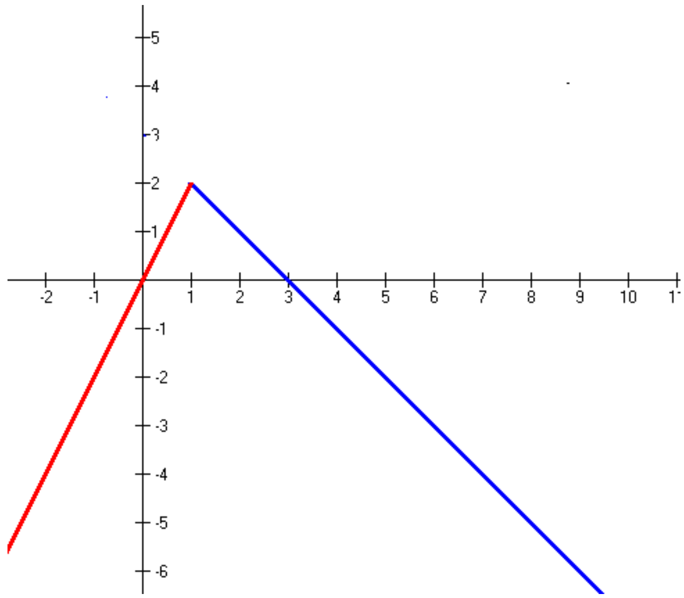
$w^2 = -\ 4\ (5/32)\ (-2)$

$w^2 = 5/4$

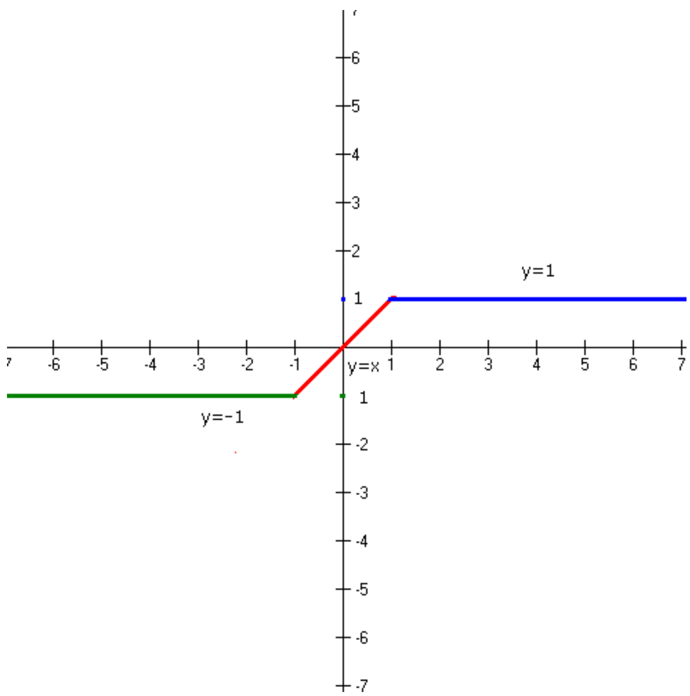
$w = \sqrt{5}/2$

$2w = \sqrt{5} = 2.23$ m

20. Range of f = (-∞, 2)



OR



Range f = [-1, 1]

21. Into Function: {a, p), (b, q), (c,p),(d,p)} range must be the proper subset of set B

Onto: {a, p), (b, q) (c, r), (d, r)} range must be same as set B

No one-one function can be defined from A to B because n(B) < n(A)

22. $\cos^2 x + \cos^2(x + \frac{\pi}{3}) + \cos^2(x - \frac{\pi}{3})$

$$= \cos^2 x + \cos^2(x + \frac{\pi}{3}) + 1 - \sin^2(x - \frac{\pi}{3})$$

$$= 1 + \cos^2 x + \left[\cos^2(x + \frac{\pi}{3}) - \sin^2(x - \frac{\pi}{3})\right]$$

$$= 1 + \cos^2 x + \left[\cos(x + \frac{\pi}{3} + x - \frac{\pi}{3})\cos(x + \frac{\pi}{3} - x + \frac{\pi}{3})\right] \quad \left[\because \cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)\right]$$

$$= 1 + \cos^2 x + \cos(2x)\cos\left(\frac{2\pi}{3}\right)$$

$$= 1 + \cos^2 x + \cos(2x)\left(-\frac{1}{2}\right)$$

$$= 1 + \cos^2 x + \left(2\cos^2 x - 1\right)\left(-\frac{1}{2}\right)$$

$$= 1 + \cos^2 x + \left(-\cos^2 x + \frac{1}{2}\right)$$

$$= 1 + \frac{1}{2} = \frac{3}{2}$$

23. (i)

$$7=\frac{8+6+7+5+x+4}{6}$$

$$\Rightarrow 42=8+6+7+5+x+4$$

$$\Rightarrow 42=30+x$$

$$\Rightarrow x=12$$

(ii)If each observation is multiplied by 3, the mean will also be multiplied by 3, hence the mean is 21.

(iii) Arranging in the ascending order 4, 5, 6, 7, 8, 12

Median = $\frac{6+7}{2}=6.5$ ....

Forming the table, we get

x_i	4	5	6	7	8	12
$x_{i-6.5}$	-2.5	-1.5	-0.5	0.5	1.5	5.5
$\lvert x_i-6.5\rvert$	2.5	1.5	0.5	0.5	1.5	5.5

$$\sum_{i=1}^{n}\lvert x_i-M\rvert=\sum_{i=1}^{n}\lvert x_i-6.5\rvert=12$$

$$M.D(M)=\frac{\sum_{i=1}^{n}\lvert x_i-M\rvert}{n}=\frac{\sum_{i=1}^{n}\lvert x_i-6.5\rvert}{6}$$

$$=\frac{12}{6}=2.0$$

SECTION – D

24. Let S be the sample space. Then n(S) = 36

Let $E_1 =$ event that a doublet appears

Let $E_2 =$ event of getting a total of 10.

Then, $E_1 = [(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)]$,

and $E_2 = [(4,6),(5,5),(6,4)]$

$\therefore E_1 \cap E_2 = [(5,5)]$

So, $n(E_1)=6, n(E_2)=3$ and $n(E_1 \cap E_2)=1$.

Thus,

$$P(E_1)=\frac{n(E_1)}{n(S)}=\frac{6}{36}=\frac{1}{6},$$

$$P(E_2)=\frac{n(E_2)}{n(S)}=\frac{3}{36}=\frac{1}{12}$$

and

$$P(E_1 \cap E_2)=\frac{n(E_1 \cap E_2)}{n(S)}=\frac{1}{36}$$

Therefore, the probability of getting a doublet or a total of 10

$=P(E_1 \text{ or } E_2)$

$=P(E_1 \cup E_2)$

$=P(E_1)+P(E_2)-P(E_1 \cap E_2)$

$=\frac{1}{6}+\frac{1}{12}-\frac{1}{36}$

$=\frac{8}{36}$

$=\frac{2}{9}$

Thus P(getting neither a doublet nor a total of 10)

$=P(\overline{E_1} \text{ and } \overline{E_2})$

$=P(\overline{E_1} \cap \overline{E_2})$

$=P(\overline{E_1 \cup E_2})$

$=1-P(E_1 \cup E_2)$

$=1-\frac{2}{9}$

$=\frac{7}{9}$

25. Let the statement P(n) be: $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+......+\frac{1}{(n)(n+1)(n+2)}=\frac{(n)(n+3)}{4(n+1)(n+2)}$

Consider P(1): $\frac{1}{1.2.3}=\frac{(1)(1+3)}{4(1+1)(1+2)}$

$\Rightarrow \frac{1}{1.2.3}=\frac{(1)(1+3)}{4(1+1)(1+2)}=\frac{1.4}{4.2.3}=\frac{1}{1.2.3}$ [P(1) is true]

Let us assume that P(k) is true

$P(k): \frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{(k)(k+1)(k+2)}=\frac{(k)(k+3)}{4(k+1)(k+2)}$

To prove:

$P(k+1): \frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{(k)(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$

$=\frac{(k+1)(k+4)}{4(k+2)(k+3)}$

$LHS=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{(k)(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$

$=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{k(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$

$=\frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$ (using P(k)

$=\frac{1}{(k+1)(k+2)}\left[\frac{k(k+3)}{4}+\frac{1}{(k+3)}\right]$

$=\frac{1}{(k+1)(k+2)}\left[\frac{k(k+3)^2+4}{4(k+3)}\right]$

$=\frac{1}{(k+1)(k+2)}\left[\frac{k(k^2+9+6k)+4}{4(k+3)}\right]$

$=\frac{1}{(k+1)(k+2)}\left[\frac{k^3+9k+6k^2+4}{4(k+3)}\right]$

$=\frac{1}{(k+1)(k+2)}\left[\frac{(k+1)^2(k+4)}{4(k+3)}\right]$

$=\frac{(k+1)(k+4)}{4(k+2)(k+3)}=$ R.H.S.

26.

(i) There are a total of 60 marbles out of which 5 marbles are to be selected

Number of ways in which 5 marbles are to be selected out of 60 = $^{60}C_5$

(a)

Out of 20 blue marbles , 5 can be selected in= $^{20}C_5$

$$P(\text{all 5 blue marbles}) = \frac{^{20}C_5}{^{60}C_5} = \frac{\frac{20!}{5!15!}}{\frac{60!}{5!55!}} = \frac{20!5!55!}{5!15!60!}$$

$$= \frac{20.19.18.17.16}{60.59.58.57.56} = \frac{34}{11977}$$

(b) Number of ways in which 5 marbles are to be selected out of 60= $^{60}C_5$.

Out of 30 non-green marbles , 5 can be selected in $^{30}C_5$

$$P(\text{all 5 non-green marbles}) = \frac{^{30}C_5}{^{60}C_5} = \frac{\frac{30!}{5!25!}}{\frac{60!}{5!55!}} = \frac{30!5!55!}{5!25!60!}$$

$$= \frac{30.29.28.27.26.}{60.59.58.57.56} = \frac{117}{4484}$$

$$P(\text{atleast one green marble}) = 1 - P(\text{all 5 non} - \text{green marbles})$$

$$= 1 - \frac{117}{4484}$$

$$= \frac{4484 - 117}{4484} = \frac{4367}{4484}$$

(ii)On the dice two faces are with number '1', three faces are with number '2' and one face is with number '3'

$$\therefore P(1) = \frac{2}{6} = \frac{1}{3}; P(2) = \frac{3}{6} = \frac{1}{2}; P(3) = \frac{1}{6}$$

$$\therefore \text{(i) } P(2) = \frac{1}{2}$$

(ii) $P(1 \text{ or } 3) = P(1) + P(3)$ [The events are mutually exclusive]

$$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

(iii) $P(\text{not}3) = 1 - P(3) = 1 - \frac{1}{6} = \frac{5}{6}$

27. (i) $\sin(x+1)$ by the definition method

Let $y=\sin(x+1)$

$$\Rightarrow y+\delta y=\sin(x+\delta x+1)$$

$$\Rightarrow \delta y=(y+\delta y)-y=\sin(x+\delta x+1)\ -\ \sin(x+1)$$

$$=2\cos\left(\frac{x+\delta x+1+x+1}{2}\right)\sin\left(\frac{x+\delta x+1-(x+1)}{2}\right)$$

$$=2\cos\left(\frac{2x+\delta x+2}{2}\right)\sin\left(\frac{\delta x}{2}\right)$$

$$\frac{\delta y}{\delta x}=\frac{1}{\delta x}2\cos\left(\frac{2x+\delta x+2}{2}\right)\sin\left(\frac{\delta x}{2}\right)$$

$$\Rightarrow \frac{\delta y}{\delta x}=\cos\left(\frac{2x+\delta x+2}{2}\right)\frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$$

$$\Rightarrow \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\cos\left(\frac{2x+\delta x+2}{2}\right)\frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$$

$$\Rightarrow \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\cos\left(\frac{2x+\delta x+2}{2}\right)\lim_{\delta x\to 0}\frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$$

$$\Rightarrow \frac{dy}{dx}=\cos\left(\frac{2x+2}{2}\right).1$$

$$\Rightarrow \frac{dy}{dx}=\cos(x+1)$$

(ii) Let $y=\dfrac{x}{1+\tan x}$

$$\Rightarrow \frac{dy}{dx}=\frac{(1+\tan x)\frac{d}{dx}(x)-(x)\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{(1+\tan x).1-(x)(\sec^2 x)}{(1+\tan x)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1+\tan x-x\sec^2 x}{(1+\tan x)^2}$$

OR

$$\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]=\left[\frac{x-2}{x(x-1)}-\frac{1}{x(x^2-3x+2)}\right]$$

$$=\left[\frac{x-2}{x(x-1)}-\frac{1}{x(x-1)(x-2)}\right]$$

$$=\left[\frac{x^2-4x+4-1}{x(x-1)(x-2)}\right]$$

$$=\frac{x^2-4x+3}{x(x-1)(x-2)}$$

$$\lim_{x\to 1}\left[\frac{x^2-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]=\lim_{x\to 1}\frac{x^2-4x+3}{x(x-1)(X-2)}$$

$$=\lim_{x\to 1}\frac{(x-3)(x-1)}{x(x-1)(x-2)}$$

$$=\lim_{x\to 1}\frac{x-3}{x(x-2)}=\frac{1-3}{1(1-2)}=2$$

(ii) $$\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}=\lim_{x\to 0}\left[\frac{\sin 4x}{4x}\times\frac{2x}{\sin 2x}\times 2\right]$$

$$=2\times\lim_{x\to 0}\left[\frac{\sin 4x}{4x}\right]\div\left[\frac{\sin 2x}{2x}\right]$$

$$=2\times\lim_{4x\to 0}\left[\frac{\sin 4x}{4x}\right]\div\lim_{2x\to 0}\left[\frac{\sin 2x}{2x}\right]$$

$$=2\times 1\times 1=2 \qquad (\text{as } x\to 0, 4x\to 0 \text{ and } 2x\to 0)$$

28. Perpendicular distance of the point (m,n) from the line ax+by+c=0, is given by

$$\left|\frac{a(m)+b(n)+c}{\sqrt{a^2+b^2}}\right|$$

$\therefore$ Perpendicular distance of the point $(\sqrt{a^2-b^2},0)$ from the line

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta-1=0$, is given by

$$\left|\frac{\frac{\cos\theta}{a}(\sqrt{a^2-b^2})+\frac{\sin\theta}{b}(0)\;-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|=\left|\frac{\frac{\cos\theta}{a}(\sqrt{a^2-b^2})\;-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

Perpendicular distance of the point $(-\sqrt{a^2-b^2},0)$ from the line

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta-1=0$, is given by

$$\left|\frac{-\frac{\cos\theta}{a}(\sqrt{a^2-b^2})+\frac{\sin\theta}{b}(0)\;-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|=\left|\frac{-\frac{\cos\theta}{a}(\sqrt{a^2-b^2})\;-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

Product of the perpendicular distances

$$=\left|\frac{\frac{\cos\theta}{a}(\sqrt{a^2-b^2})\;-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|\times\left|\frac{-\frac{\cos\theta}{a}(\sqrt{a^2-b^2})\;-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

$$=\left|\frac{\left[\frac{\cos\theta}{a}(\sqrt{a^2-b^2})\;-1\right]\left[\frac{\cos\theta}{a}(\sqrt{a^2-b^2})\;+1\right]}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

$$=\left|\frac{\left[\frac{\left(a^2-b^2\right)\cos^2\theta}{a^2}\;-1\right]}{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}\right|$$

$$=\frac{\left|\left[\dfrac{\left(a^2-b^2\right)\cos^2\theta}{a^2}-1\right]\right|}{\dfrac{b^2\cos^2\theta+a^2\sin^2\theta}{a^2b^2}}$$

$$=\frac{a^2b^2\left|\left[\dfrac{a^2\cos^2\theta-b^2\cos^2\theta-a^2}{a^2}\right]\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left|a^2\cos^2\theta-b^2\cos^2\theta-a^2\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left|-a^2(1-\cos^2\theta)-b^2\cos^2\theta\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left|-a^2(\sin^2\theta)-b^2\cos^2\theta\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left|-\left(a^2(\sin^2\theta)+b^2\cos^2\theta\right)\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\frac{b^2\left(a^2(\sin^2\theta)+b^2\cos^2\theta\right)}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$=b^2$$

29. $5(2x-7)-3(2x+3) \leq 0$;

$2x + 19 \leq 6x + 47$

and

$7 \leq \frac{(3x+11)}{2} \leq 11$

Let us solve the inequalities one by one and then work out the common solution.

Inequality 1:

$5(2x-7)-3(2x+3) \leq 0$

$\Rightarrow 10x - 35 - 6x - 9 \leq 0$

$\Rightarrow 4x - 44 \leq 0$

$\Rightarrow 4x \leq 44$

$\Rightarrow x \leq 11$

Inequality 2:

$2x + 19 \leq 6x + 47$

$\Rightarrow 2x - 6x \leq -19 + 47$

$\Rightarrow -4x \leq 28$

$\Rightarrow -x \leq 7$

$\Rightarrow x \geq -7$

Inequality 3:

$7 \leq \frac{(3x+11)}{2} \leq 11$

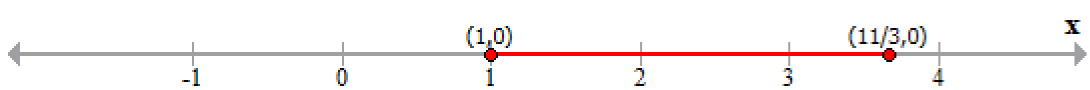
$\Rightarrow 14 \leq 3x+11 \leq 22$

$\Rightarrow 14\text{-}11 \leq 3x \leq 22-11$

$\Rightarrow 3 \leq 3x \leq 11$

$\Rightarrow 1 \leq x \leq \frac{11}{3}$

$x \leq 11, x \geq -7, 1 \leq x \leq \frac{11}{3}$ together $\Rightarrow 1 \leq x \leq \frac{11}{3}$



OR

The amount of acid in 1125lt of the 45% solution=45% of 1125=$\frac{45\times1125}{100}$

Let x lt of the water be added to it to obtain a solution between 25% and 30% solution

$$\Rightarrow 25\% < \frac{1125\times\frac{45}{100}}{1125+x} < 30\%$$

$$\Rightarrow \frac{25}{100} < \frac{1125\times\frac{45}{100}}{1125+x} < \frac{30}{100}$$

$$\Rightarrow \frac{25}{100} < \frac{1125\times45}{(1125+x)\times100} < \frac{30}{100}$$

$$\Rightarrow 25 < \frac{1125\times45}{(1125+x)} < 30$$

$$\Rightarrow \frac{1}{25} > \frac{(1125+x)}{1125\times45} > \frac{1}{30}$$

$$\Rightarrow \frac{1125\times45}{25} > (1125+x) > \frac{1125\times45}{30}$$

$$\Rightarrow \frac{50625}{25} > (1125+x) > \frac{50625}{30}$$

$$\Rightarrow 2025 > (1125+x) > 1687.5$$

$$\Rightarrow 2025 > (1125+x) > 1687.5$$

$$\Rightarrow 2025-1125 > x > 1687.5-1125$$

$$\Rightarrow 900 > x > 562.5$$

$$\Rightarrow 562.5 < x < 900$$

So the amount of water to be added must be between 562.5 to 900 lt