CBSE SAMPLE PAPER - 10

Class 11 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = { 1, 2, 3}, B = {x, y} Then the number of functions that can be defined from A into B is **[1]**

a) 12

b) 6

c) 8

d) 3

2. The value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}-1$ is **[1]**

a) -1

b) -4

c) -3

d) -2

3. If |x + 3| $\geq$ 10 , then **[1]**

a) x $\in$ (– 13, 7]

b) x $\in$ (– 10, 7]

c) x $\in$ (– ∞, – 13] $\cup$ [8, ∞)

d) x $\in$ (–∞, – 13] $\cup$ [7, ∞)

4. Two events A and B have probabilities 0.25 and 0.50, respectively. The probability that both A and B occur simultaneously is 0.14. Then, the probability that neither A nor B occurs, is **[1]**

a) None of these

b) 0.39

c) 0.25

d) 0.11

5. If Q = {x : x = $\frac{1}{y}$, where y $\in$ N}, then **[1]**

a) 1 $\in$ Q

b) $\frac{1}{2} \notin$ Q

c) 2 $\in$ Q

d) 0 $\in$ Q

6. The value of $\lim\limits_{x\to 0}\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ is: **[1]**

a) a

b) $-\sqrt{a}$

c) -a

d) $\sqrt{a}$

7. The value of $\left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + \left({}^{21}C_3 - {}^{10}C_3\right) + \left({}^{21}C_4 - {}^{10}C_4\right) + \ldots + \left({}^{21}C_{10} - {}^{10}C_{10}\right)$ is **[1]**

a) 2^{21}- 2^{10}

b) 2^{20}- 2^{10}

c) 2^{20}- 2^{9}

d) 2^{21}- 2^{11}

8. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is: **[1]**

a) 6:7

b) 4:9

c) 10:3

d) 5:8

9. The product (32)$(32)^{\frac{1}{6}}(32)^{\frac{1}{36}}$ ... to ∞ is: **[1]**

a) 16

b) 64

c) 0

d) 32

10. If tan $\theta = \frac{-4}{3}$, then sin θ is **[1]**

a) $\frac{4}{5}$ but not -$\frac{4}{5}$

b) None of these

c) $\frac{-4}{5}$ but not $\frac{4}{5}$

d) $\frac{-4}{5}$ or $\frac{4}{5}$

11. The line which is parallel to X-axis and crosses the curve y = $\sqrt{x}$ at an angle of 45° is: **[1]**

a) y = $\frac{1}{2}$

b) y = 1

c) none of these

d) y = $\frac{1}{4}$

12. If $y = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$ then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{-4x}{x^2-1}$

b) $\frac{1-x^2}{4x}$

c) $\frac{-4x}{\left(x^2-1\right)^2}$

d) $\frac{4x}{x^2-1}$

13. A real value of x satisfies the equation $\left(\frac{3-4ix}{3+4ix}\right)$ = α - iβ (α,$\beta \in$ R) if α^2 + β^2 = **[1]**

a) 2

b) 1

c) -1

d) -2

14. Number of relations that can be defined on the set A = {a, b, c, d} is **[1]**

a) 24

b) 4^4

c) 16

d) 2^{16}

15. The equations of the lines through (-1, -1) and making angles of 45° with the line x + y = 0 are **[1]**

a) none of these

b) x + 1 = 0, y + 1 = 0

c) x – 1 = 0, y – x = 0

d) x + y = 0, y + 1 = 0

16. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, then the value of $C_0 + 2C_1 + 3C_2 + \ldots + (n + 1)C_n$ is **[1]**

a)

b)

(n + 1) 2^n

(n + 2) 2^n

c) (n + 1) 2^{n-1}

d) (n + 2) 2^{n-1}

17. $\lim\limits_{x \to 0} \frac{\sin x^n}{(\sin x)^m}$, n > m > 0 is equal to **[1]**

a) $\frac{m}{n}$

b) 0

c) 1

d) $\frac{n}{m}$

18. If x is a real number and |x| < 5, then **[1]**

a) - 5 < x < 5

b) -5 $\leq$ x $\leq$ 5

c) x $\geq$ 5

d) x $\leq$ -5

19. If A + B + C = 180^o, then **[1]**

Assertion (A): $cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} - \cos^2 \frac{C}{2} = 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$.

Reason (R): cos C + cos D = 2 cos $\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The no. of ways in which 81 different beads can be arranged to form a necklace is 80!. **[1]**

Reason (R): No. of circular arrangement of n identical object is $(n-1)!$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Prove that: $\sin \frac{10\pi}{3} \cos \frac{13\pi}{6} + \cos \frac{8\pi}{3} \sin \frac{5\pi}{6} = -1$. **[2]**

22. Write the set in roster form: J = {x : x $\in$ R and x^2 + x - 12 = 0} **[2]**

23. If odds against an event be 7 : 9, find the probability of non-occurrence of this event. **[2]**

OR

One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be 'a diamond'

24. Show that the line through the points (-2, 6) and (4, 8) is perpendicular to the line through the points (3, -3) and (5, -9). **[2]**

25. Let A and B be two non-empty sets such that n(A) = 5, n(B) = 6 and $n(A \cap B)$ = 3. Find **[2]**

i. $n(A \times B)$
ii. $n(B \times A)$
iii. $n\{(A \times B) \cap (B \times A)\}$

Section C

26. Find the square root of -8 - 6i **[3]**

27. Solve the system of inequations: |X - 1| < 5 , $|x| \geq 2$ **[3]**

OR

solve: $\frac{x^2-2x+5}{3x^2-2x-5} > \frac{1}{2}$

28. Write the number of terms in the expansion of $(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5$. **[3]**

OR

Expand the given expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

29. Show that the points (-2, 3, 5), (1, 2, 3) and (7, 0, -1) are collinear. **[3]**

OR

Four students in traditional dresses represent four states of India, standing at the points represented by O(0,0,0), A(a,0,0), B(0,b,0) and C(0,0,c). Find the place, in terms of coordinates, where a girl representing 'BHARATMATA' be replaced so that 'BHARATMATA is equidistant from the four students. What message does it convey?

30. From a well-shuffled deck of 52 cards, 4 cards are drawn at random. What is the probability that all the drawn cards are of the same colour? **[3]**

31. Let f :R → R: f(x) = x^3 for all $x \in R$. Find its domain and range. Also, draw its graph. **[3]**

Section D

32. Calculate the mean, median and standard deviation of the following distribution: **[5]**

Class-interval:	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70
Frequency:	2	3	8	12	16	5	2	3

33. In an university, out of 100 students 15 offered Mathematics only; 12 offered Statistics only; 8 offered Physics only; 40 offered Physics and Mathematics ; 20 offered Physics and Statistics; 10 offered Mathematics and Statistics; 65 offered Physics. Find the number of students who (i) offered Mathematics (ii) offered statistics (iii) did not offer any of the above three subjects. **[5]**

OR

i. In a group of 70 people, 37 like coffee, 52 like tea and each person like at least one of the two drinks. How many people like both coffee and tea?
ii. In a group of 65 people, 40 like Cricket, 10 like both Cricket and Tennis. How many like tennis only and not Cricket? How many like Tennis?

34. Find the axes, eccentricity, latus-rectum and the coordinates of the foci of the hyperbola **[5]**

$25x^2 - 36y^2 = 225$.

OR

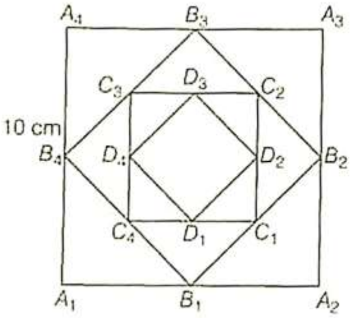
Find the equation of the hyperbola whose foci are $(\pm\sqrt{5}, 0)$ and the eccentricity is $\sqrt{\frac{5}{3}}$.

35. Evaluate: $\lim\limits_{x\to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6}$. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A student of class XI draw a square of side 10 cm. Another student join the mid-point of this square to form new square. Again, the mid-points of the sides of this new square are joined to form another square by another student. This process is continued indefinitely.



(i) The sum of the perimeter of all the square formed is (in cm)

a) 40 b) $40 + 40\sqrt{2}$

c) $80 + 40\sqrt{2}$ d) None of these

(ii) The sum of areas of all the square formed is (in sq cm)

a) 250 b) None of these

c) 200 d) 150

(iii) The perimeter of the 7th square is (in cm)

a) $\frac{5}{2}$ b) 10

c) 5 d) 20

OR

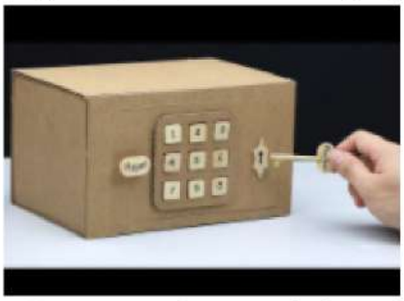
The area of the fifth square is (in sq cm)

a) $\frac{25}{4}$ b) 25

c) 50 d) $\frac{25}{2}$

37. **Read the text carefully and answer the questions:** **[4]**

A locker in a bank has 3 digit lock. Each place's digits may vary from 0 to 9. Mahesh has a locker in the bank where he can put all his property papers and important documents. Once he needs one of the document but he forgot his password and was trying all possible combinations.



(i) He took 6 seconds for each try. How much time will be needed by Mahesh to try all the combinations:

a) 120 minutes b) 90 minutes

c) 100 minutes d) 60 minutes

(ii) How many permutations of 3 different digits are there, chosen from the ten digits 0 to 9 inclusive:

a) 720 b) 504

c) 84 d) 120

(iii) The total number of 9-digit numbers that have all different digits is:

a) 9! b) $10 \times 10!$

c) 10! d) $9 \times 9!$

OR

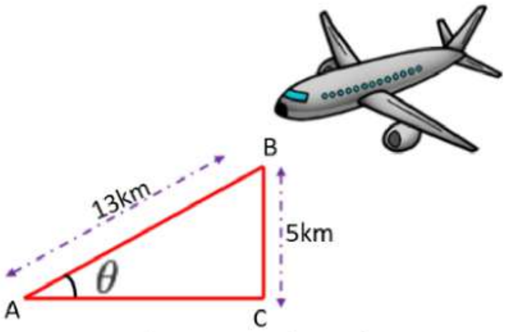
A bank has 6 digit account number with no repetition of digits within an account number. The first and last digit of the account numbers is fixed to be 4 and 7. How many such account numbers are possible:

a) 5040 b) 1680

c) 890 d) 10080

38. **Read the text carefully and answer the questions:** **[4]**

An airplane is observed to be approaching a point that is at a distance of 13 km from the point of observation and makes an angle of elevation of θ and the height of the airplane above the ground is 5km. Based on the above information answer the following questions.



(i) Find the value of $\sin 2\theta$.

(ii) Find the value of $\cos 2\theta$.

Solution

CBSE SAMPLE PAPER - 10

Class 11 - Mathematics

Section A

1. **(c)** 8

   Explanation: Since A has 3 elements and B has 2 elemets, then number of functions from A to B is $2^3 = 8$

2. **(d)** -2

   Explanation: -2

   $\frac{i^{502}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}} - 1$

   $= \frac{i^{4\times148}+i^{4\times147+2}+i^{4\times147}+i^{4\times146+2}+i^{4\times146}}{i^{4\times145+2}+i^{4\times145}+i^{4\times144+2}+i^{4\times144}+i^{4\times143+2}} - 1$ [∵ i^4 = 1 and i^2 =-1]

   $= \frac{1+i^2+1+i^2+1}{i^2+1+i^2+1+i^2} - 1$

   $= \frac{1}{-1} - 1$

   = -2

3. **(d)** x ∈ (–∞, – 13] ∪ [7, ∞)

   Explanation: since |x + 3| ≥ 10 , ⇒ x + 3 ≤ – 10 or x + 3 ≥ 10

   ⇒ x ≤ – 13 or x ≥ 7

   ⇒ x ∈ (– ∞, – 13] ∪ [7, ∞)

   solution set = (– ∞, – 13] ∪ [7, ∞)

4. **(b)** 0.39

   Explanation: Given, P(A) = 0.25, P(B) = 0.50, P(A ∩ B) = 0.14

   ∴ P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

   = 0.25 + 0.50 - 0.14 = 0.61

   Now, $P(\overline{A \cup B})$ = 1 - P(A ∪ B) = 1 - 0.61 = 0.39

5. **(a)** 1 ∈ Q

   Explanation: N is set of natural number, so

   x = $\frac{1}{y}$

   When y = 1 then x = 1

   So, 1 ∈Q

6. **(b)** $-\sqrt{a}$

   Explanation: $\lim_{x\to0} \frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$

   Multiplying $\left(\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2}\right)$ and $\left(\sqrt{a+x}+\sqrt{a-x}\right)$ in N^r and D^r

   = $\lim_{x\to0} \frac{(a^2+x^2-ax-a^2-ax-x^2)(\sqrt{a+x}+\sqrt{a-x})}{(a+x-a+x)(\sqrt{x^2-ax+x^2}+\sqrt{a^2+ax+x^2})}$

   = $\lim_{x\to0} \frac{-2ax(\sqrt{a+x}+\sqrt{a-x})}{2x(\sqrt{a^2+ax+x^2}+\sqrt{a^2+ax+x^2})}$

   = $\frac{-a(2\sqrt{a})}{2a}$ = -$\sqrt{a}$

7. **(b)** 2^{20}- 2^{10}

   Explanation: $\left({}^{21}C_1 -{}^{10} C_1\right) + \left({}^{21}C_2 -{}^{10} C_2\right) + \left({}^{21}C_3 -{}^{10} C_3\right) + \ldots + \left({}^{21}C_{10} -{}^{10} C_{10}\right)$

   $= \left({}^{21}C_1 +{}^{21} C_2 + \ldots +{}^{21} C_{10}\right) - \left({}^{10}C_1 +{}^{10} C_2 + \ldots +{}^{10} C_{10}\right)$

   $= \frac{1}{2}\left({}^{21}C_1 +{}^{21} C_2 + \ldots +{}^{21} C_{20}\right) - \left(2^{10} - 1\right)$

   $= \frac{1}{2}\left({}^{21}C_1 +{}^{21} C_2 + \ldots +{}^{21} C_{21} - 1\right) - \left(2^{10} - 1\right)$

   $= \frac{1}{2}\left(2^{21} - 2\right) - \left(2^{10} - 1\right) = 2^{20} - 1 - 2^{10} + 1 = 2^{20} - 2^{10}$

8. **(b)** 4:9

   Explanation: Since mean of x_1, x_2, x_3, x_4 and x_5 is 5

   ⇒ $x_1 + x_2 + x_3 + x_4 + x_5 = 25$

   ⇒ $1 + 3 + 8 + x_4 + x_5 = 25$

$\Rightarrow x_4 + x_5 = 13$ ...(i)

$\because \frac{\sum_{i=1}^{5} x_i^2}{5} - (5)^2 = 9.2 \Rightarrow \sum_{i=1}^{5} x_i^2 = 5(25 + 9.2)$

$= 125 + 46 = 171$

$\Rightarrow (1)^2 + (3)^2 + (8)^2 + x_4^2 + x_5^2 = 171$

$\Rightarrow x_4^2 + x_5^2 = 97$ ...(ii)

$\Rightarrow (x_4 + x_5)^2 - 2x_4x_5 = 97$

$\Rightarrow 2x_4x_5 = 13^2 - 97 = 72 \Rightarrow x_4x_5 = 36$ ...(iii)

(i) and (iii)

$\Rightarrow x_4 : x_5 = \frac{4}{9}$ or $\frac{9}{4}$

9. **(b)** 64

Explanation: $(32)(32)^{\frac{1}{6}}(32)^{\frac{1}{36}} \ldots = (32)^{1+\frac{1}{6}+\frac{1}{36}+-\,-}$

$= (32)^{\frac{1}{1-\left(\frac{1}{6}\right)}}$

$= (32)^{\frac{1}{\frac{5}{6}}}$

$= (32)^{\frac{6}{5}} = 2^6 = 64$

10. **(d)** $\frac{-4}{5}$ or $\frac{4}{5}$

Explanation: Since $\tan\theta = -\frac{4}{3}$ is negative and $\tan\theta$ is negative in either second quadrant or in fourth quadrant. Thus $\sin\theta = \frac{4}{5}$ if θ lies in the second quadrant or $\sin\theta = -\frac{4}{5}$, if θ lies in the fourth quadrant.

11. **(a)** $y = \frac{1}{2}$

Explanation: The equation of the line which is a tangent to the curve $y = \sqrt{x}$ is

$y = mx + a/m$

Since it makes and angle of 45^o, $m = 1$

$y^2 = x$ implies $a = \frac{1}{4}$

Hence the equation of the tangent is $y = x + \frac{1}{4}$

That is the y-intercept is $\sqrt{\frac{1}{4}} = \frac{1}{2}$

Hence the equation of the line is $y = \frac{1}{2}$

12. **(c)** $\frac{-4x}{\left(x^2-1\right)^2}$

Explanation: Given $y = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}} \Rightarrow y = \frac{x^2+1}{x^2-1}$

$\therefore \quad \frac{dy}{dx} = \frac{(x^2-1)\cdot 2x - (x^2+1)\cdot 2x}{\left(x^2-1\right)^2}$

$= \frac{2x\left(x^2-1-x^2-1\right)}{\left(x^2-1\right)^2} = \frac{2x(-2)}{\left(x^2-1\right)^2} = \frac{-4x}{\left(x^2-1\right)^2}$

13. **(b)** 1

Explanation: Given that $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta$

$8 \Rightarrow \left(\frac{3-4ix}{3+4ix} \times \frac{3-4ix}{3-4ix}\right) = \alpha - i\beta$

$\Rightarrow \frac{9-12ix-12ix+16i^2x^2}{9-16i^2x^2} = \alpha - i\beta$

$\Rightarrow \frac{9-24ix-16x^2}{9+16x^2} = \alpha - i\beta$

$\Rightarrow \frac{9-16x^2}{9+16x^2} - \frac{24x}{9+16x^2}i = \alpha - i\beta$ ....(i)

$\Rightarrow \frac{9-16x^2}{9+16x^2} + \frac{24x}{9+16x^2}i = \alpha + i\beta$ .....(ii)

multiplying eqn. (i) and (ii) we g et

$\left(\frac{9-16x^2}{9+16x^2}\right)^2 + \left(\frac{24x}{9+16x^2}\right)^2 = \alpha^2 + \beta^2$

$\Rightarrow \frac{(9-16x^2)^2+(24x)^2}{(9+16x^2)^2} = \alpha^2 + \beta^2$

$\Rightarrow \frac{81+256x^4-288x^2+576x^2}{(9+16x^2)^2} = \alpha^2 + \beta^2$

$\Rightarrow \frac{81+256x^4+288x^2}{(9+16x^2)^2} = \alpha^2 + \beta^2$

$\Rightarrow \frac{(9+16x^2)^2}{(9+16x^2)^2} = \alpha^2 + \beta^2$

So, $\alpha^2 + \beta^2 = 1$

Hence, the correct option is (a)

14. **(d)** 2^{16}

Explanation: No. of elements in the set A = 4 . Therefore , the no. of elements in $A \times A = 4 \times 4 = 16.$ As, the no. of relations in $A \times A$ = no. of subsets of $A \times A = 2^{16}$.

15. **(b)** x + 1 = 0, y + 1 = 0

Explanation: The lines x + 1 = 0 and y + 1 = 0 are perpendicular to each other.

The slope of the line x + y = 0 is -1

Hence the angle made by this line with respect to X-axis is 45^o

In other words, the angle made by this line with x + 1 = 0 is 45^o

Clearly the other line with which it can make 45^o is y + 1 = 0

16. **(d)** $(n + 2) 2^{n-1}$

Explanation: We have,

$C_0 + 2C_1 + 3C_2 + ... + (n + 1) C_n$

$= \sum_{r=0}^{n} (r + 1)C_r$

$= \sum_{r=0}^{n} (r + 1)\ {}^nC_r$

$= \sum_{r=0}^{n} (r\cdot {}^nC_r + {}^nC_r)$

$= \sum_{r=0}^{n} r\cdot {}^nC_r + \sum_{r=0}^{n} {}^nC_r$

$= \sum_{r=1}^{n} r \cdot \frac{n}{r}\ {}^{n-1}C_{r-1} + \sum_{r=0}^{n} {}^nC_r ...\ [\because {}^nC_r = \frac{n}{r}\cdot {}^{n-1}C_{r-1}]$

$= n(\sum_{r=1}^{n} {}^{n-1}C_{r-1}) + (\sum_{r=0}^{n} {}^nC_r)$

$= n({}^{n-1}C_0 + {}^{n-1}C_1 + ... + {}^{n-1}C_{n-1}) + ({}^nC_0 + {}^nC_1 + ... + {}^nC_n)$

$= n\cdot 2^{n-1} + 2^n$

$= n\cdot 2^{n-1} + 2\cdot(2^{n-1})$

$= (n + 2)\cdot 2^{n-1}$

17. **(b)** 0

Explanation: $\lim_{x\to 0} \frac{\sin x^n}{(\sin x)^m} \cdot \frac{x^{m+n}}{x^{m+n}}$

$\Rightarrow \lim_{x\to 0} \frac{\sin x^n}{x^n} \cdot \frac{x^m}{(\sin x)^m} \cdot \frac{x^n}{x^m}$

$\Rightarrow 1.1^m . x^{n-m}$

$\Rightarrow 1(0) = 0$

18. **(a)** - 5 < x < 5

Explanation: |x| < 5

$\Rightarrow -5 < x < 5$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: A + B + C = 180^o

$\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} - \cos^2 \frac{C}{2}$

$= \cos^2 \frac{A}{2} + \sin\left(\frac{c}{2} + \frac{\pi}{2}\right) \cdot \sin\left(\frac{c}{2} - \frac{\pi}{2}\right)$ $\{\because \cos^2 B - \cos^2 A = \sin(A + B)\cdot\sin(A - B)\}$

$= \cos^2 \frac{A}{2} + \sin\left(90^\circ - \frac{A}{2}\right) \cdot \sin\left(\frac{C}{2} - \frac{B}{2}\right)$
$= \cos^2 \frac{A}{2} + \cos \frac{A}{2} \cdot \sin\left(\frac{C}{2} - \frac{B}{2}\right)$
$= \cos \frac{A}{2}\left[\cos \frac{A}{2} + \sin\left(\frac{C}{2} - \frac{B}{2}\right)\right]$
$= \cos \frac{A}{2}\left[\sin\left(\frac{C}{2} + \frac{B}{2}\right) + \sin\left(\frac{C}{2} - \frac{B}{2}\right)\right]$
$= \cos \frac{A}{2} \cdot 2 \sin \frac{C}{2} \cos \frac{B}{2}$
$= 2 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \sin \frac{C}{2}$

Proved

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Since clockwise and anticlockwise arrangements are not different so, required no of arrangement is $\frac{80!}{2}$.

Section B

21. We have to prove that: $\sin \frac{10\pi}{3} \cos \frac{13\pi}{6} + \cos \frac{8\pi}{3} \sin \frac{5\pi}{6} = -1$.

$\frac{10\pi}{3} = 600^\circ, \frac{13\pi}{6} = 390^\circ$, $\frac{8\pi}{3} = 480^\circ, \frac{5\pi}{6} = 150^\circ$

Consider LHS = sin 600° cos 390° + cos 480° sin 150°

= sin (90° × 6 + 60°) cos (90° × 4 + 30°) + cos (90° × 5 + 30°) sin (90° × 3 + 60°)

= [-sin 60°] cos 30° + [-sin 30°] cos 60°

= - sin 60° cos (30°) - sin 30° cos 60°

$= -\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= -\frac{3}{4} - \frac{1}{4}$

= -1

= RHS

Hence proved.

22. The given equation is:

$x^2 + x - 12 = 0$

$\Rightarrow x^2 + 4x - 3x - 12 = 0$

$\Rightarrow x(x + 4) - 3(x + 4) = 0$

$\Rightarrow (x - 3)(x + 4) = 0$

$\Rightarrow x - 3 = 0$ or $x + 4 = 0$

$\Rightarrow x = 3$ or $x = -4$

thus, the solution set of the given equation can be written in roaster form as {3, -4}

Therefore, J = {3, -4}

23. We have to find the probability of non-occurrence of this event.

Given: odds against of event is 7:9

Formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

we have to find the probability of non-occurrence of this event

Total possible outcomes are 7k + 9k = 16k

Therefore n(S) = 16k

Let E be the event that it occurs

n(E) = 9k

Probability of occurrence is

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{9k}{16k} = \frac{9}{16}$

Therefore, the probability of non-occurrence of the event is

p(E') = 1 - P(E)

$\text{p}\left(E'\right) = 1 - \frac{9}{16} = \frac{7}{16}$

OR

When a card is drawn from a well shuffled deck of 52 cards,

Total number of possible outcomes = 52.

Let A be the event 'the card drawn is a diamond'

Clearly the number of elements in set A is 13.

Total number of favorable outcomes = 13

Therefore, P(A) $= \frac{13}{52} = \frac{1}{4}$

i.e. probability of being a diamond card = $\frac{1}{4}$

24. For two lines to be perpendicular, their product of slope must be equal to -1.

Given points are A(-2, 6), B(4, 8) and C(3, -3), D(5, -9)

Therefore,slope = $\left(\frac{y_2-y_1}{x_2-x_1}\right)$

Slope of line AB × slope of line CD = -1

$\Rightarrow \left(\frac{8-6}{4+2}\right) \times \left(\frac{-9+3}{5-3}\right) = -1$

$\Rightarrow \left(\frac{2}{6}\right) \times \left(\frac{-6}{2}\right) = -1$

⇒ -1 = -1

⇒ L.H.S = R.H.S

25. Here we are given that , A and B are two non-empty sets such that n(A) = 5, n(B) = 6 and = 3

   i. $n(A \times B) = n(A) \times n(B) = (5 \times 6) = 30$
   ii. $n(B \times A) = n(B) \times n(A) = (6 \times 5) = 30$
   iii. Given: $n(A \cap B) = 3$

      ∴ A and B have 3 elements in common

      So, $(A \times B)$ and $(B \times A)$ have 3^2 = 9 elements in common.

      Hence, $n\{(A \times B) \cap (B \times A)\} = 9$

Section C

26. Let x + yi $= \sqrt{-8-6i}$

Squaring both sides, we get

$x^2 - y^2 + 2x yi = -8 - 6i$

Equating the real and imaginary parts

$x^2 - y^2 = -8$.....(i)

and 2xy = -6

$\therefore xy = -3$

Now using the identity

$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$= (-8^2 + 4(-3)^2$

= 64 + 36

= 100

$\therefore x^2 + y^2 = 10$ ..... (ii) [Neglecting (-) sign as $x^2 + y^2 > 0$]

Solving (i) and (ii) we get

$x^2 = 1$ and $y^2 = 9$

$\therefore x = \pm 1$ and $y = \pm 3$

Since the sign of xy is negative

∴ if x = 1, y = -3

and if x = -1, y = 3

$\therefore \sqrt{-8-6i} = \pm(1-3i)$

27. The first inequation is $|x-1| \leq 5$

Using $|x| \leq a \Leftrightarrow -a \leq x \leq a$ we get

$|x-1| \leq 5 \Rightarrow -5 \leq x-1 \leq 5$

$= -4 \leq x \leq 6 \Rightarrow x \in [-4, 6]$ ....(1)

The second inequation is Ixl > 2.

$|x| \geq 2 \Rightarrow x \leq -2$ or $x \geq 2$

$\Rightarrow x \in (-\infty, -2] \cup [2, \infty)$ .....(2)

From (1) and (2) the solution set for the given system is

$(-\infty, -2] \cup [2, \infty)\} \cap -4, 6] = [-4, -2] \cup [2, 6]$

OR

Solve $\frac{x^2-2x+5}{3x^2-2x-5} > \frac{1}{2}$

$\frac{x^2-2x+5}{3x^2-2x-5} > \frac{1}{2}$

$\Rightarrow \quad \frac{x^2-2x+5}{3x^2-2x-5} - \frac{1}{2} > 0$

$\Rightarrow \quad \frac{2(x^2-2x+5)-(3x^2-2x-5)}{2(3x^2-2x-5)} > 0$

$\Rightarrow \quad \frac{-x^2-2x+15}{2(3x^2-2x-5)} > 0$

$\Rightarrow \quad \frac{-(x^2+2x-15)}{2(3x^2-2x-5)} > 0$

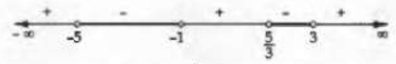
Multiplying inequality by -ve sign

$\Rightarrow \quad \frac{x^2+2x-15}{2(3x^2-2x-5)} < 0$

$\Rightarrow \quad \frac{x^2+2x-15}{3x^2-2x-5} < 0$

$\Rightarrow \quad \frac{(x+5)(x-3)}{(x+1)(3x-5)} < 0$

On equating all factors to zero, we get x = - 5, -1 ,5 / 3, 3. Plotting these points on number line



$x \in (-5,-1) \cup (\frac{5}{3}, 3)$

28. To find:number of terms in the expansion of $(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5$

Binomial expansion of $(x + y)^n$ is given by,

$(x + y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} \times y^r$

Thus,

$(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5$

$= ((\sqrt{2})^5 + (\sqrt{2})^4 \binom{5}{1} + \ldots + \binom{5}{5}) = ((\sqrt{2})^5 - (\sqrt{2})^4 \binom{5}{1} + \ldots - \binom{5}{5})$

So, the no. of terms left would be 6

Thus, the number of terms in the expansion of $(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5$ is 6.

OR

Using binomial theorem for the expansion of $\left(\frac{2}{x}-\frac{x}{2}\right)^5$ we have

$\left(\frac{2}{x}-\frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^5 + {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{-x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{-x}{2}\right)^2 + {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{-x}{2}\right)^3$

$+ {}^5C_4\left(\frac{2}{x}\right)\left(\frac{-x}{2}\right)^4 + {}^5C_5\left(\frac{-x}{2}\right)^5$

$= \frac{32}{x^5} + 5 \cdot \frac{16}{x^4} \cdot \frac{-x}{2} + 10 \cdot \frac{8}{x^3} \cdot \frac{x^2}{4} + 10 \cdot \frac{4}{x^2} \cdot \frac{-x^3}{8} + 5 \cdot \frac{2}{x} \cdot \frac{x^4}{16} + \frac{-x^5}{32}$

$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$

29. Let A(-2, 3, 5), B (1, 2, 3) and C(7, 0, -1) be three given points.

Then $AB = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{9+1+4} = \sqrt{14}$

$BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14}$

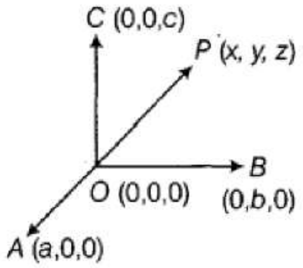
$AC = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} = \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$

Now AC = AB + BC

Therefore,A,B,C are collinear.

OR

Let A (a, 0, 0), B(0, b, 0), C(0, 0, c) and O (0, 0, 0) be four points equidistant from the point P(x, y, z).



Then, PA = PB = PC = OP

Now, OP = PA ⇒ $OP^2 = PA^2$

⇒ $x^2 + y^2 + z^2 = (x - a)^2 + (y - 0)^2 + (z - 0)^2$ [∵ distance = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$]

⇒ $x^2 + y^2 + z^2 = x^2 + a^2 - 2ax + y^2 + z^2$

⇒ $0 = -2ax + a^2$

⇒ $x = \frac{a}{2}$

Similarly, OP = PB ⇒ $y = \frac{b}{2}$ and OP = PC

⇒ $z = \frac{c}{2}$

Hence, the coordinates of the required points are $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$.

All the people of different states are equal and part of India.

30. We have to find the probability that all the drawn cards are of the same colour

Out of 52 cards, four cards can be randomly chosen in $^{52}C_4$ ways.

∴ $n(S) = {}^{52}C_4$

Let A = event where the four cards drawn are red

and B = event where the four cards drawn are black

Then, n(A) = ^{26}C4 and n(B) = $^{26}C_4$

⇒ P(A) = $\frac{^{25}C_4}{^{52}C_4}$ and PB = $\frac{^{25}C_4}{^{52}C_4}$

A and B are mutually exclusive events.

i.e. $P(A \cap B) = 0$

By addition theorem, we have:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{26}{\sqrt[3]{2}C_4} + \frac{25C_4}{652C_4} - 0$

$= \frac{46}{17\times49} + \frac{46}{17\times49}$

$= 2 \times \frac{46}{17\times49} = \frac{92}{838}$

Hence, the probability that all the drawn cards are of the same colour is $\frac{92}{833}$.

31. We have, f :R → R: f(x) = x^3 for all x ∈ R

dom (f) = R and range (f) = R.

We have,

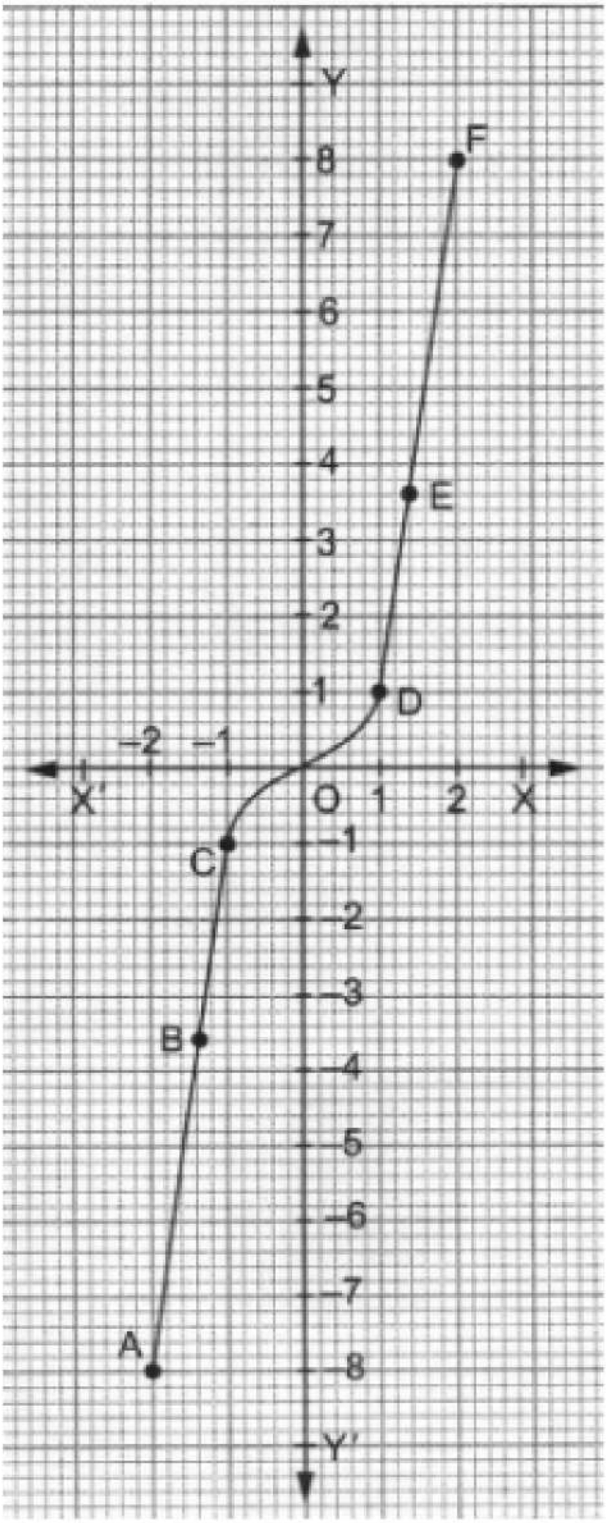
x	-2	-1.5	- 1	0	1	1.5	2
$f(x) = x^3$	-8	-3.375	- 1	0	1	3.375	8

On a graph paper, we draw X' OX and YOY' as the x-axis and the y-axis respectively.

We take the scale as 5 small divisions = 1 unit.

Now, we plot the points A(-2, - 8), B(-1.5, -3.375), C(-1, -1), 0(0, 0), D(l, 1), E(1.5, 3.375) and F(2, 8).

We join these points freehand successively to obtain the required curve shown in the figure below.



Section D

32. 1st of all we will prepare the below table with the help of given information.

Class Interval	f_i	Mid point x_i	$u_i = \frac{x_i - 53}{4}$	u_i^2	$f_i u_i$	$f_i u_i^2$
31-35	2	33	-5	25	-10	50
36-40	3	38	-3.75	14.06	-11.25	42.18
41-45	8	43	-2.5	6.25	-20	50
46-50	12	48	-1.25	1.56	-15	18.72
51-55	16	53	0	0	0	0
56-60	5	58	1.25	1.56	6.25	7.8

61-65	2	63	2.5	6.25	5	12.5
66-70	3	68	3.75	14.06	11.25	42.18
	N = 51					$\sum_{i=1}^{n} f_i u_i^2$ = 223.38

X = a + h $\left(\frac{\sum_{i=1}^{n} f_i u_i}{N}\right)$

= 53 + 4 $\left(\frac{-33.75}{51}\right)$

= 50.36

$\sigma^2 = h^2\left(\frac{\sum_{i=1}^{n} f_i u_i^2}{N} - \left(\frac{\sum_{i=1}^{n} f_i u_i}{N}\right)^2\right)$

= 16 $\left(\frac{223.38}{51} - \frac{1139.06}{2601}\right)$

= 63.07

$\sigma = \sqrt{63.07}$

= 7.94

f_i	Cumulative frequency
2	2
3	5
8	13
12	25
16	41
5	46
2	48
3	51

$\sum f_i$ = 51 = N

$\frac{N}{2}$ = 25.5

Median class interval is 51 - 55

L = 51

F = 25

f = 16

h = 4

Median = L + $\frac{\frac{N}{2} - F}{f} \times$ h

= 51 + $\frac{25.5 - 25}{16} \times 4$

= 51 + $\frac{0.5}{4}$

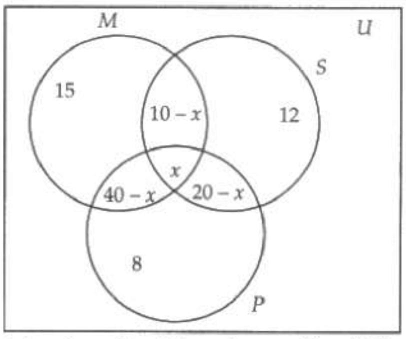
= 51.125

33. Let M, S and P be the sets of students who offered Mathematics, Statistics and Physics respectively. Let x be the number of students who offered all the three subjects.

Given: n(U) = 100, No. of students offered only Mathematics = 15, No. of students who offered only Statistics = 12, No. of students who offered only Physics = 8,

n (P ∩ M) = 40, n (P ∩ S) = 20, n (M ∩ S) = 10, n(P) = 65

It is given that 10 students offered Mathematics and Statistics. Therefore, number of students who offered Mathematics and Statistics but not Physics is 10 - x. Similarly, number of students who offered Statistics and Physics but not Mathematics is 20 - x and number of students who offered Mathematics and Physics but not Statistics is 40 - x. The Venn diagram is shown in figure.



It is given that 65 students offered Physics.

So, (40 - x) + x + (20 - x) + 8 = 65

⇒68 - x = 65

⇒ x = 3

i. The number of students who offered Mathematics, n(M) = 15 + (10 - x) + x + (40 - x) = 65 - x = 65 - 3 = 62
ii. The number of students who offered Statistics, n(S) = 12 + (10 - x) + x + (20 - x) = 42 - x = 42 - 3 = 39
iii. The number of students who offered any of three subjects, n(M ∪ S ∪ P)

= 15 + 12 + 8 + (10 - x) + (40 - x) + (20 - x) + x

= 105 - 2x

= 105 - 2 × 3 = 99

∴ Number of students who did not offer any of the three subjects = n(U) - n(M ∪ S ∪ P) = 100 - 99 = 1

OR

i. Let A = Set of people like coffee and

B = Set of people who like tea

Then, A ∪ B = Set of people who like at least one of the two drinks and

A ∩ B = Set of people who like both the drinks.

Given, $n(A) = 37$

$n(B) = 52$

$n(A \cup B) = 70$

We know that,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore 70 = 37 + 52 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 89 - 70 = 19$

Hence, 19 people like both coffee and tea.

ii. Let C be the set of people who like Cricket and T be the set of people who like Tennis.

Then, $n(C \cup T) = 65$

$n(C) = 40$ and

$n(C \cap T) = 10$

We know that,

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\therefore 65 = 40 + n(T) - 10$

$n(T) = 65 - 40 + 10 = 35$

Number of people who like only Tennis

$= n(T) - n(C \cap T)$

$= 35 - 10$

$= 25$

Hence, the number of people who like Tennis only and not cricket is 25 and the number of people who like tennis is 35.

34. We have,

$25x^2 - 36y^2 = 225$

$\Rightarrow \frac{25x^2}{225} - \frac{36y^2}{225} = 1$

$\Rightarrow \frac{x^2}{9} - \frac{4y^2}{25} = 1$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{\frac{25}{4}} = 1$

$\Rightarrow \frac{x^2}{(3)^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where a = 3 and b = $\frac{5}{2}$

The length of the transverse axis = 2a

= 2 × 3 = 6

The length of the conjugate axis is 2b = 2 × $\frac{5}{2}$ = 5

The eccentricity e is given by,

e = $\sqrt{1+\frac{b^2}{a^2}}$

= $\sqrt{1+\frac{\frac{25}{4}}{9}}$

= $\sqrt{1+\frac{25}{36}}$

= $\sqrt{\frac{61}{36}}$

= $\frac{\sqrt{61}}{6}$

Length of Latusrectum = $\frac{2b^2}{a} = \frac{25}{6}$

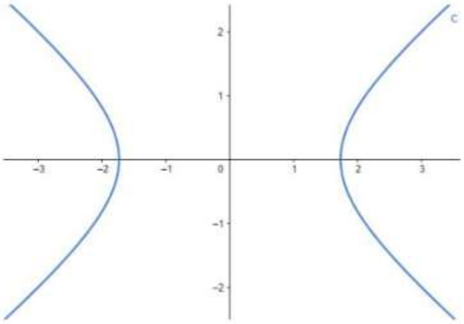
and Foci ($\pm\frac{\sqrt{61}}{2}$,0)

OR

Given: Foci are $(\pm\sqrt{5}, 0)$, and the eccentricity is $\sqrt{\frac{5}{3}}$

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ = 1

The eccentricity, e = $\sqrt{\frac{5}{3}}$



And also given, foci are $(\pm\sqrt{5}, 0) \Rightarrow (\pm\sqrt{5}, 0)$ = $(\pm ae, 0)$

$\Rightarrow$ ae = $\sqrt{5}$

$\Rightarrow a = \frac{\sqrt{5}}{e}$

$\Rightarrow a = \frac{\sqrt{5}}{\sqrt{\frac{5}{3}}}$ [As e = $\sqrt{\frac{5}{3}}$

$\Rightarrow a = \sqrt{3}$ $\Rightarrow$ $a^2 = 3$

We know that, $e = \sqrt{1+\frac{b^2}{a^2}}$

$\Rightarrow \sqrt{1+\frac{b^2}{a^2}} = \sqrt{\frac{5}{3}}$

$\Rightarrow 1+\frac{b^2}{a^2} = \frac{5}{3}$ [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2} = \frac{5}{3} - 1 = \frac{2}{3}$

$\Rightarrow b^2 = \frac{2}{3}a^2 = \frac{2}{3} \times$ (3) = 2 [As a = $\sqrt{3}$]

$\Rightarrow$ $b^2 = 2$

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{3} - \frac{y^2}{2} = 1$

35. We have to find the value of $\lim\limits_{x\to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6}$

We have

$\lim\limits_{x\to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6}$

Divide $x^3 + 3x^2 - 9x - 2$ by $x^3 - x - 6$

$$\begin{array}{r|l} & 1 \\ \hline x^3-x-6 & x^3+3x^2-9x-2 \\ & \underset{-}{+x^3} \quad \underset{+}{-x} \quad \underset{+}{-6} \\ \cline{2-2} & \underline{3x^2-8x+4} \end{array}$$

$\Rightarrow \quad \lim\limits_{x \to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6} = \lim\limits_{x \to 2} 1 + \lim\limits_{x \to 2} \frac{3x^2-8x+4}{x^3-x-6}$

$= 1 + \lim\limits_{x \to 2} \frac{3x^2-2x-6x+4}{x^3-x-6}$

$= 1 + \lim\limits_{x \to 2} \frac{3x^2-2x-6x+4}{x^3-x-6}$

$\Rightarrow \lim\limits_{x \to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6} = 1 + \lim\limits_{x \to 2} \frac{(3x-2)(x-2)}{x^3-x-6}$

Divide $x^3 - x - 6$ by $x - 2$

$$\begin{array}{r|l} & x^2+2x+3 \\ \hline x-2 & x^3-x-6 \\ & \underline{\pm x^3 \underset{+}{-2x^2}} \\ & 2x^2-x-6 \\ & \underline{2x^2 \mp 4x} \\ & \quad 3x-6 \\ & \quad \underline{3x-6} \\ & \quad\ \ \underline{\times} \end{array}$$

$\Rightarrow \quad \lim\limits_{x \to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6} = 1 + \lim\limits_{x \to 2} \frac{(3x-2)(x-2)}{(x-2)\left(x^2+2x+3\right)}$

$= 1 + \lim\limits_{x \to 2} \frac{(3x-2)}{\left(x^2+2x+3\right)}$

$= 1 + \frac{3 \times 2-2}{2^2+2 \times 2+3}$

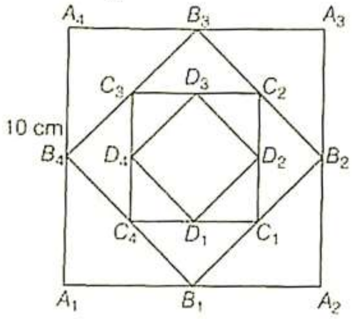
$=1 + \frac{4}{11}$

$= \frac{15}{11}$

Section E

36. **Read the text carefully and answer the questions:**

A student of class XI draw a square of side 10 cm. Another student join the mid-point of this square to form new square. Again, the mid-points of the sides of this new square are joined to form another square by another student. This process is continued indefinitely.



(i) **(c)** $80 + 40\sqrt{2}$

Explanation: $80 + 40\sqrt{2}$

(ii) **(c)** 200

Explanation: 200

(iii) **(c)** 5

Explanation: 5

OR

(a) $\frac{25}{4}$

Explanation: $\frac{25}{4}$

37. **Read the text carefully and answer the questions:**

A locker in a bank has 3 digit lock. Each place's digits may vary from 0 to 9. Mahesh has a locker in the bank where he can put all his property papers and important documents. Once he needs one of the document but he forgot his password and was trying all possible combinations.

(i) **(c)** 100 minutes

Explanation: 100 minutes

(ii) **(a)** 720

Explanation: 720

(iii) **(d)** $9 \times 9!$

Explanation: $9 \times 9!$

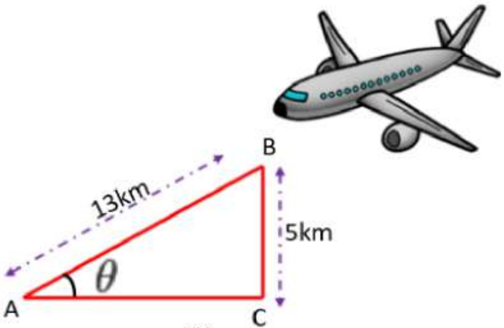
OR

(b) 1680

Explanation: 1680

38. **Read the text carefully and answer the questions:**

An airplane is observed to be approaching a point that is at a distance of 13 km from the point of observation and makes an angle of elevation of θ and the height of the airplane above the ground is 5km. Based on the above information answer the following questions.



(i) $\sin 2\theta = \frac{120}{169}$

From diagram $\tan\theta = \frac{5}{12}$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\left(\frac{5}{12}\right)}{1+\left(\frac{5}{12}\right)^2} = \frac{\frac{10}{12}}{\frac{169}{144}}$$

$\Rightarrow \sin 2\theta = \frac{120}{169}$

(ii) $\cos 2\theta = \frac{119}{169}$

From diagram $\tan\theta = \frac{5}{12}$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\left(\frac{5}{12}\right)^2}{1+\left(\frac{5}{12}\right)^2} = \frac{\frac{119}{144}}{\frac{169}{144}}$$

$\Rightarrow \cos 2\theta = \frac{119}{169}$