CHAPTER 6

Mensuration-I Area and Perimeter (2 - Dimensional Plane Figures)

- **Area :** The area of any plane figure is the amount of surface enclosed within its bounding lines. It is always expressed in square units e.g. square metres, square inches etc.
- **Perimeter :** The perimeter of a geometrical figure is the total length of the sides enclosing the figure.

Note : 1 hectare = 10,000 m^2

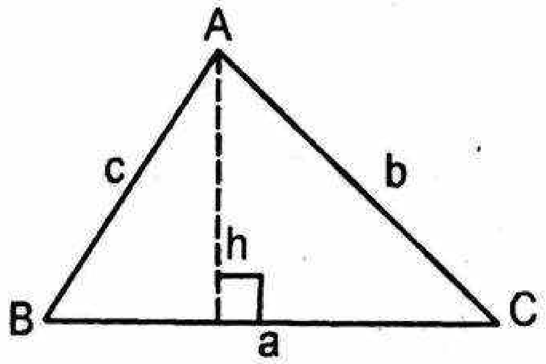
100 hectare = 1,000,000 m^2

1 acre = 100m^2

Weight (mass) = volume × density.

- **Triangle :** A closed figure bounded by three sides.

1. Scalane Triangle :



- Area of a Triangle (A)

(i) $A = \frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2}ah$

(ii) $A = \sqrt{s(s-a)(s-b)(s-c)}$

(Hero's formula)

where $s = \frac{1}{2}(a + b + c)$ = semi-perimeter of the D.

(iii) If lengths of three medians of D ABC are x, y and z units, then :

$$A = \frac{4}{3}\sqrt{Sm(Sm - x)(Sm - y)(Sm - z)}$$

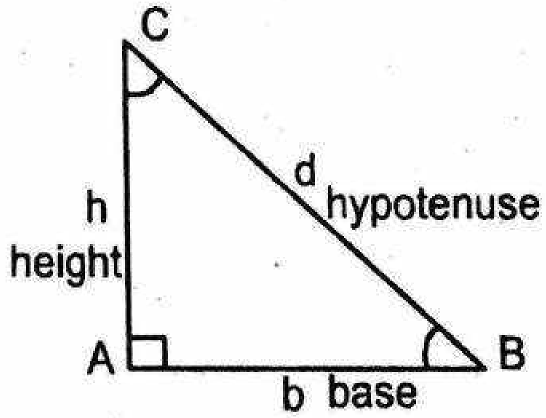
where, $Sm = \frac{x+y+z}{2}$

Perimeter (P) = 2s = a + b + c

Inradius (r) = $\frac{A}{S}$

Circum-radius (R) = $\frac{abc}{4A}$

2. Right Angled Triangle : one angle = 90°

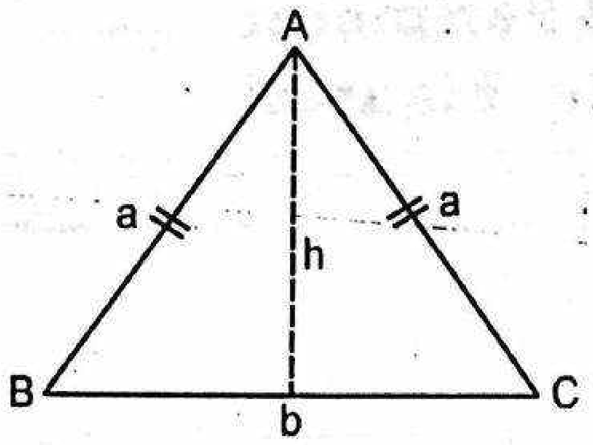


(i) $d^2 = b^2 + h^2$

(ii) $A = \frac{1}{2}bh$

(iii) P = 2s = b + h + d

3. Isosceles Triangle : Two sides are equal.



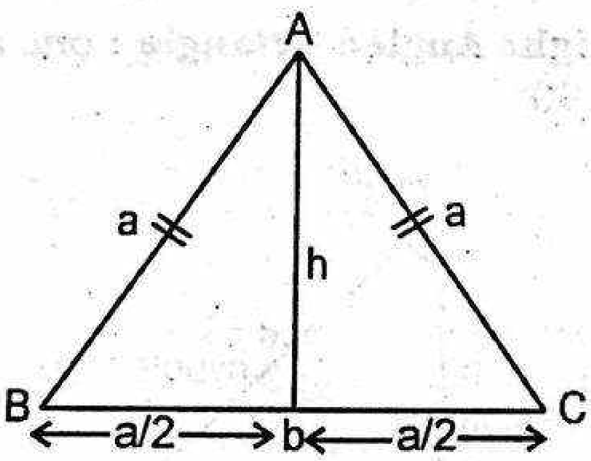
(i) $\angle B = \angle C$

(ii) $A = \frac{1}{4} b\sqrt{4a^2 - b^2}$

(iii) P = 2S = 2a + b

(iv) Altitude (height) = $h = \frac{1}{2}\sqrt{4a^2 - b^2}$

4. **Equilateral Triangle :** All its three sides are equal.



(i) $A = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}a^2$

(ii) P = 2s = 3a

(iii) Altitude (h) = $\frac{\sqrt{3}}{2}a$

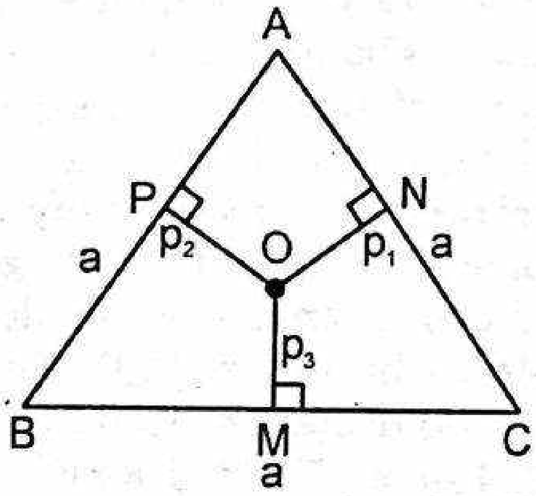
(iv) In-radius = $\frac{a}{2\sqrt{3}}$

(v) circum-radius (R) = $\frac{a}{\sqrt{3}}$

(vi) $\angle A = \angle B = \angle C = 60°$

(vii) $A = \frac{\sqrt{3}}{4}a^2 = \frac{h^2}{\sqrt{3}}$

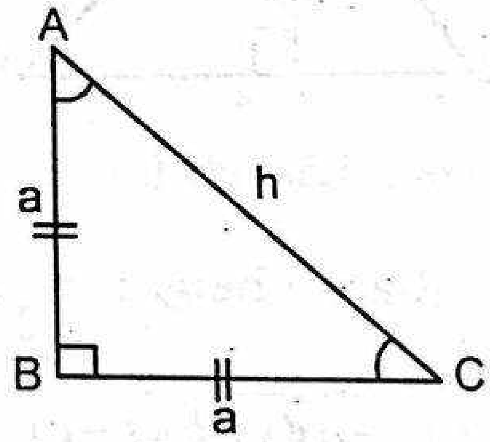
Note : If P_1, P_2 and P_3 are perpendicular lengths from any interior point (O) of an equilateral D ABC to all its three sides respectively, then:-



$$P_1 + P_2 + P_3 = \frac{\sqrt{3}}{2}a = h$$

$$\Rightarrow a = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$$

5. **Isosceles Right-angled Triangle :**



(i) AB = BC = a and $\angle B = 90°$

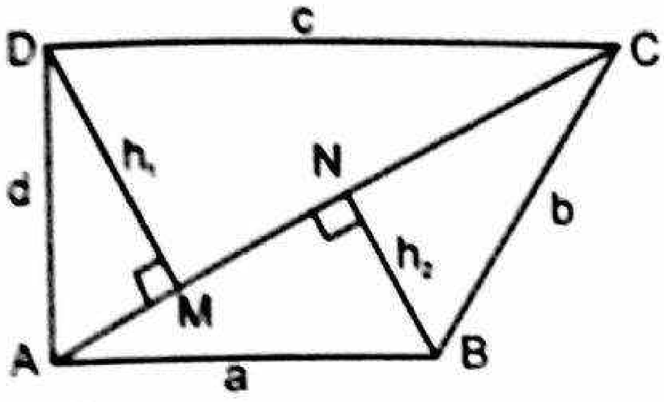
(ii) h = $a\sqrt{2}$

(iii) A = $\frac{1}{2}a^2$

(iv) P = 2s = 2a + h

Quadrilateral : A closed figure bounded by four sides.

(i) $\angle A + \angle B + \angle C + \angle D = 360°$



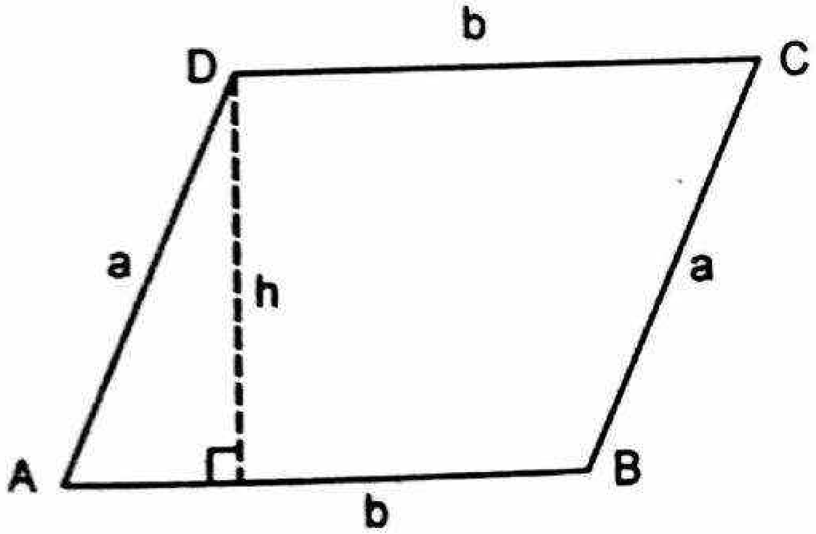
(ii) $\frac{1}{2}$ × one diagonal × (sum of per pendicular to it from opposite vertices)

$= \frac{1}{2}(AC)(h_1 + h_2)$

(iii) P = a + b + c + d

Special cases of Quadrilateral :

(1) Parallelogram : Opposite sides are equal and parallel.



(i) A = base × height = bh

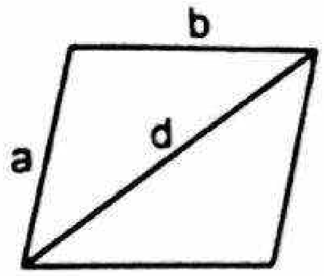
(ii) P = 2(a + b)

(iii) $d_1^2 + d_2^2 = 2(a^2 + b^2)$ (d_1, d_2 = length of diagonals)

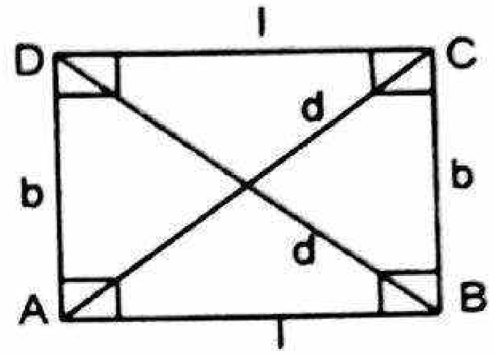
(iv) Area = A = $2\sqrt{s(s-a)(s-b)(s-d)}$

where a & b are adjacent sides, d is the length of diagonal connecting the ends of the two sides and,

$s = \frac{a+b+d}{2}$



2. Rectangle : Its opposite sides equal and all the four angles equal to 90°.



(i) Its diagonals are equal & bisect each other.

(ii) A = length × breadth = $l \times b$

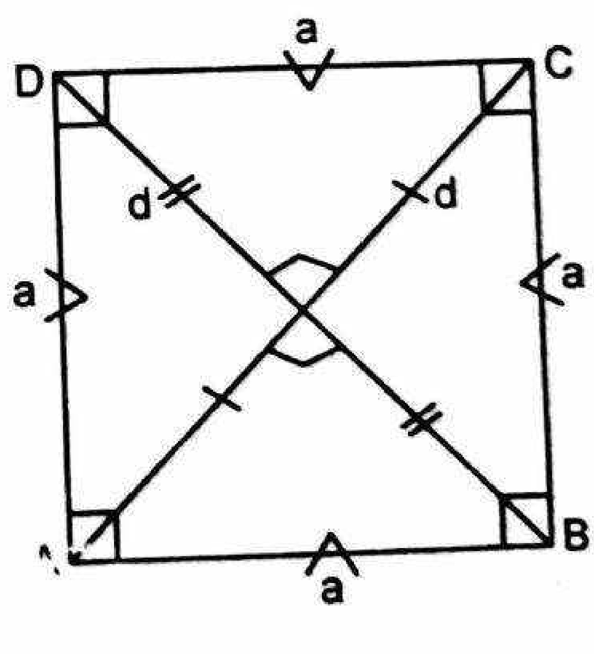
(iii) d = AC = BD = $\sqrt{l^2 + b^2}$

(iv) P = 2 (l + b)

3. Square : Its all four sides are equal and all the four angles equal to 90°

(i) Its diagonals are equal and bisect each other at 90°

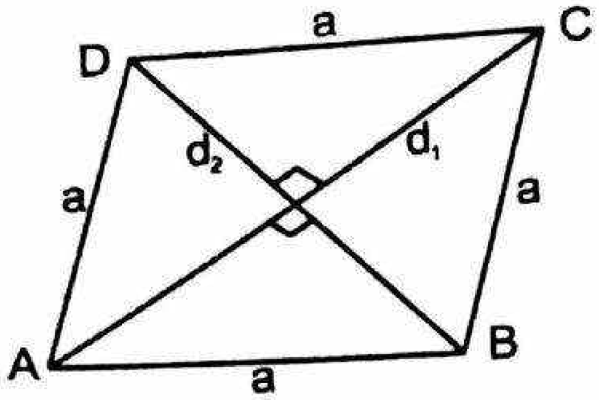
(ii) $A = (\text{side})^2 = a^2 = \frac{d^2}{2} = \frac{P^2}{16}$



(iii) diagonal (d) = $AC = BD = a\sqrt{2} = \frac{P}{2\sqrt{2}}$

(iv) P = 4a.

4. **Rhombus :** It is a ||gm whose all four sides are equal.



(i) Diagonals bisect each other at 90°

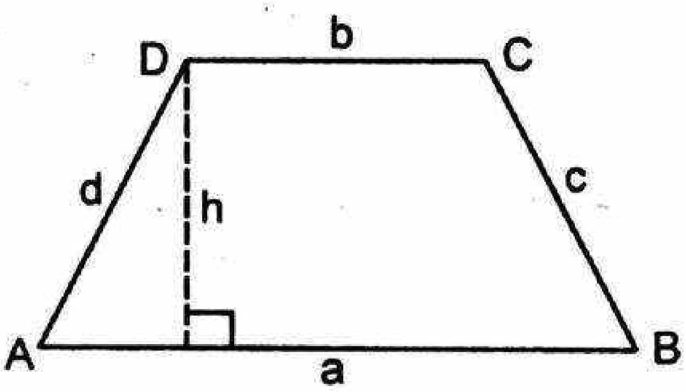
(ii) $A = \frac{1}{2}(d_1 \times d_2)$

(iii) $a = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

(iv) P = 4a

(v) $d_1^2 + d_2^2 = 4a^2$

5. **Trapezium :** It is a quadrilateral whose any two opposite sides are aparallel.



here, AB || CD

(i) A

$= \frac{1}{2}$(sum of || sides) × (distance b/w them)

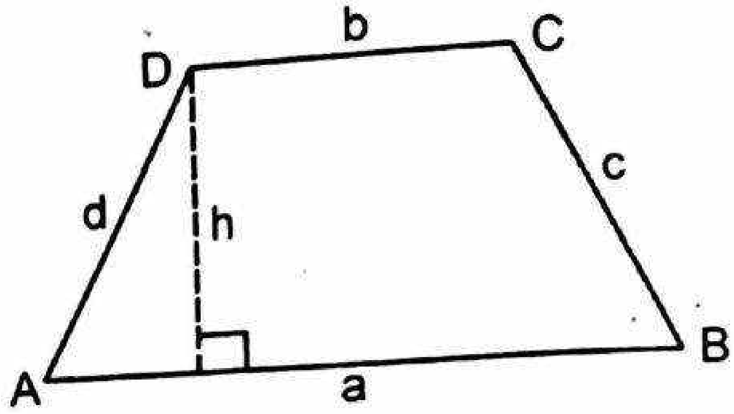
$= \frac{1}{2}(a + b) \times h$

(ii) P = a + b + c + d

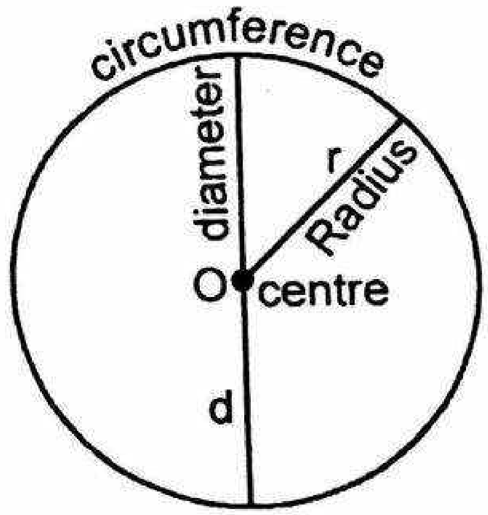
(iii) $\frac{AO}{CO} = \frac{BO}{OD}$

(iv) $d_1^2 + d_2^2 = c^2 + d^2 + 2(ab)$

(sum of squares of non-parallel sides) + (product of parallel sides)



Circle : A circle is a set of points on a plane which lie at a fixed distance from a fixed-point.
The fixed point is known as 'centre' and the fixed disatance is called the 'radius'.



(i) Circumference or Perimeter of circle (P) = $2\pi r = \pi d$ (d = diameter)

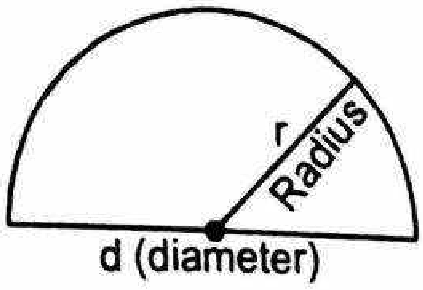
(ii) Area = A = $\pi r^2 = \frac{\pi d^2}{4}$

∴ Diameter of the circle = d = $\sqrt{\frac{4A}{\pi}}$

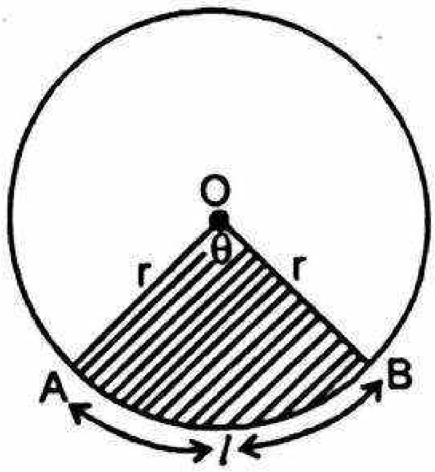
Semi-Circle : It is a figure enclosed by a diameter and the part of the circumference cut off by it.

(i) Circumference (Perimeter) $= \pi r + 2r$
$= \pi r + d$

(ii) Area(A)= $\frac{\pi r^2}{2}$.



Sector : A sector is a figure enclosed by two radii and an arc lying between them.
For sector AOB,



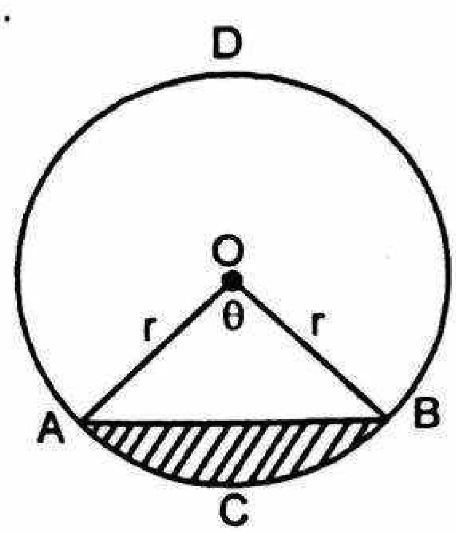
(i) l = Arc AB $= (2\pi r)\frac{\theta}{360°}$

(ii) Area of sector ACBO $= \frac{1}{2}$(arc AB)× radius

$= (\pi r^2)\frac{\theta}{360°}$

(iii) Perimeter (P) = Arc AB + 2r = l + 2r

Segment of a circle : A figure enclosed by a chord and an arc which it cuts off.



(i) Area of segment ACB. (minor segment) = area of sector ACBO - area of DOAB.

(ii) Area of segment ADB (major segment)
= area of circle - area of segment ACB.

(iii) Perimeter (P) = arc AB + $\theta.r$

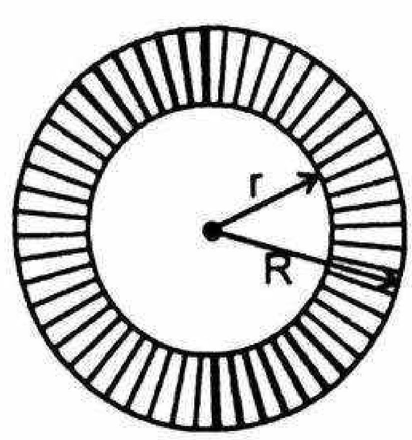
$$= 2r\left[\frac{\pi\theta}{360°} + \sin\left(\frac{\theta}{2}\right)\right]$$

Note : Arc = Angle × Radius

Ring or Circular Path :

R → outer radius

r → inner radius

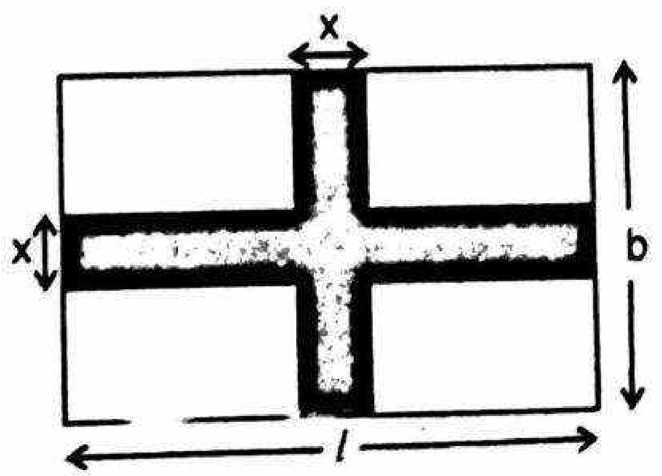


(i) Area (A) = π (R^2 - r^2)

(ii) Perimeter = 2π(R + r)

Pathways running across the middle of a rectangle :

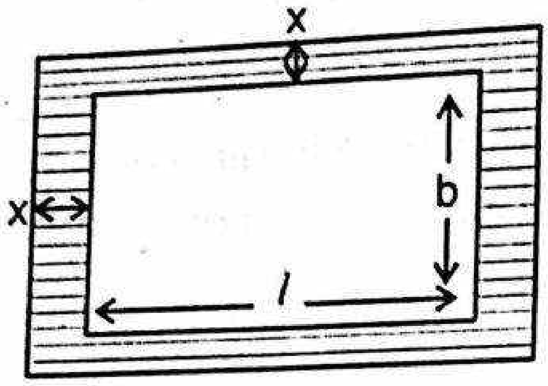
x - width of the path (road)



(i) Area of Path (A) = (l + b - x)x

(ii) Perimeter of Path (P) = 2(l + b) - 4x
= 2(l + b -2x)

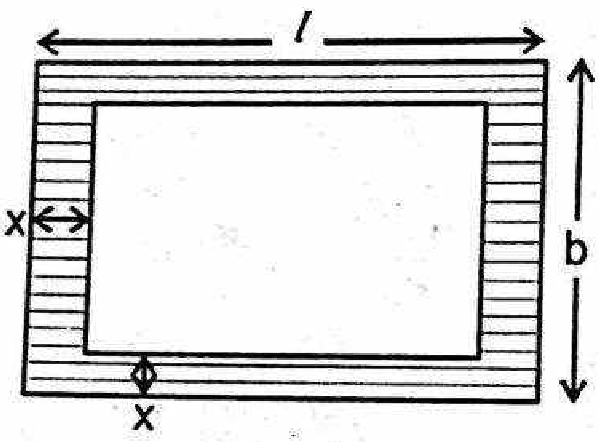
Pathways around a Rectangular Space(Outer Pathways) :



(i) Area (A) = $(l + b + 2x)2x$

(ii) Perimeter (P) = inner Perimeter + outer Perimeter
$= 2(l + b) + 2(l + b + 4x)$
$= 4(l + b + 2x)$

InnerPathways :



(i) Area (A) = $(l + b - 2x)2x$

(ii) Perimeter (P) = inner P + Outer P
$= 2(l + b) + 2(l + b - 4x)$
$= 4(l + b - 2x)$

Room : length(l), breadth(b) & height(h), then :

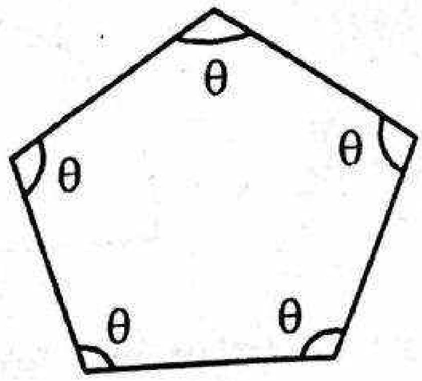
(i) Area of four walls of the room = $2h(l + b)$

(ii) Area of the floor and four walls = $2h(l + b) + lb$

(iii) Area of the floor, roof and four walls = $2[h(l + b) + lb]$

Polygon: A polygon is a plane figure enclosed by four or more straight lines. e.g. Pentagon, Octagon etc.

Regular Polygon: It is a polygon whose all sides are equal.



All the interior angles of a regular polygon are equal.

For a regular Polygon -

(i) Sum of exterior angles = 360° $= 2\pi$

(ii) Sum of interior angles = $(n-2)\pi$ $= (n-2) \times 180°$

(iii) Interior angle + exterior angle = 180°

(iv) No. of diagonals in a polygon

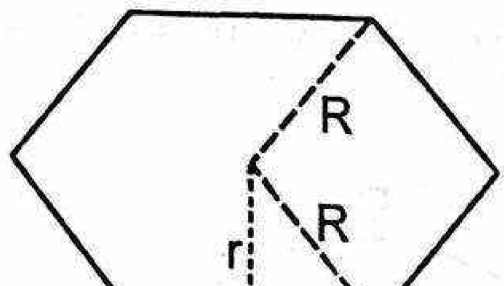
$$= \frac{n(n-3)}{2}$$

(v) Each interior angle $= \frac{(n-2)}{n} \times 180°$

(vi) Each exterior angle $= \frac{360°}{n}$

(vii) Perimeter (P) = $n \times a$
(where n = no. of sides, and, a = length of each side)

(viii) Area (A) $= \frac{1}{2} \times p \times r = \frac{1}{2} \times n \times a \times r$



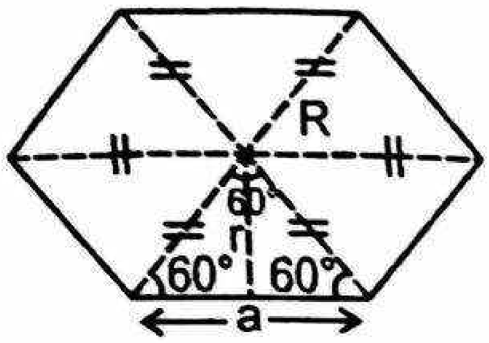
where, r = radius of inscribed circle

$$\Rightarrow A = \frac{1}{2} \times n \times a \times \sqrt{R^2 - \left(\frac{a}{2}\right)^2}$$

R = radius of circumscribed circle.

$$or\ A = \frac{na^2}{4}\cot\frac{\pi}{n}$$

Regular hexagon:



(i) radius of incircle (r) $= \frac{\sqrt{3}}{2}a$

(ii) radius of circum-circle (R) = a

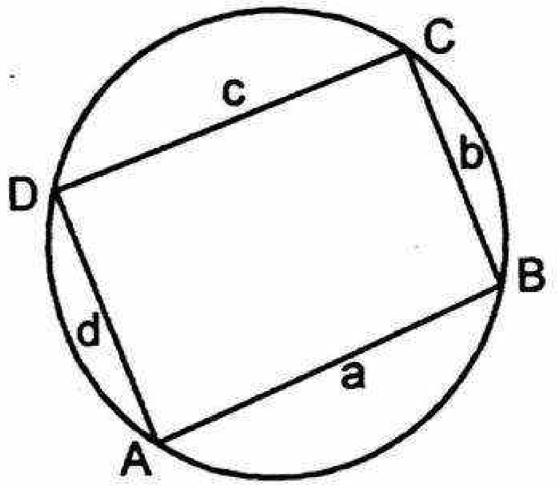
(iii) Area (A) = 6 × (Area of equilateral triangle of side a)

$$= 6\times\left(\frac{\sqrt{3}a^2}{4}\right) = \frac{3\sqrt{3}}{2}a^2$$

⇒ Area of regular Octagon

$$= 2\left(\sqrt{2}+1\right)(side)^2$$

Cyclic Quadrilateral: A quadrilateral whose vertices lie on the circumference of the circle.



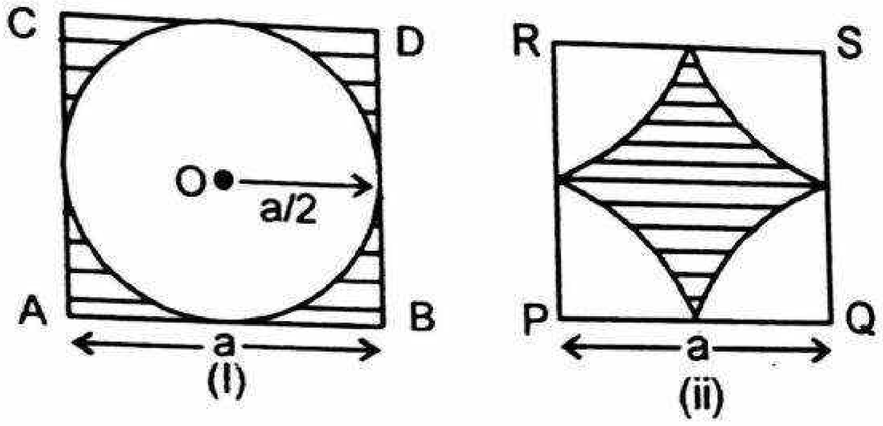
(i) Area (A) = $\sqrt{s(s-a)(s-b)(s-c)(s-d)}$

where, $s = \frac{a+b+c+d}{2}$

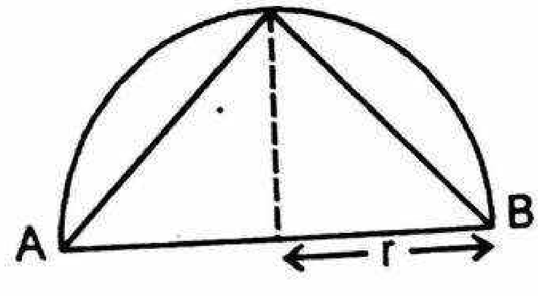
(ii) ∠A + ∠B + ∠C + ∠D = 2π

(iii) ∠A + ∠C = ∠B + ∠D = 180° = π

Some-useful Results :



1. In figure (i) ABCD is a square of side
   (a) O is the centre of the incircle.
   In figure (ii) PQRS is a square of side
   (b) P,Q,R and S are the centres of four quadrant of radius a/2 each.
   In both case- Area of shaded region $= \frac{3}{14}a^2$
2. If the additional of square increases by x times, then the area of the square becomes x^2 times.
3. If each of the defining dimensions or sides of any 2-D figures are increased (or decreased) by x%, its Perimeter also increases (or decreases) by x%.
4. If all the sides of a quadrilateral are increased (or decreased by x%, its diagonals also increases (or decreasses) by x%.
6. If the area of the square is a cm², then the area of the circle formed by the same perimeter is $= \frac{4a}{\pi}cm^2$.
7. The area of largest triangle inscribed in a semi-circle of radius r is r².



Area of the $\Delta ACB = r^2$

7. The number of revolutions made by a circular wheel of radius r in travelling distance 'd' is given by -

(no. of revolutions) $n = \dfrac{d}{2\pi r}$

8. If the length and breadth of a rectangle are increased by x% and y% respectively, then the area of rectangle will increases by :

$\left(x+y+\dfrac{xy}{100}\right)\%$

9. If the length and breadth of a rectangle are dicreased by x% and y% respectively, then the area of rectangle will decreases by :

$\left(x+y-\dfrac{xy}{100}\right)\%$

10. If the length of a rectangle is increased by x%, then its breadth will have to be decreased by $\left(\dfrac{100x}{100+x}\right)\%$ in order to maintain the same area of the rectangle.

11. If each of the degining dimensions or sides of any 2-0 figure (triangle, rectangle, square, circle, quadrilateral, pentagon, hexagon etc.) is changed by x%, its area changes by

$x\left(2+\dfrac{x}{100}\right)\%$

$x \to +ve$ if increases

$x \to -ve$ if decreases

Exercise

LEVEL - 1

1. In a rhombus, the lengths of two diagonals are 4m and 3m, then side of rhombus is :
(a) 5m (b) 2.5m
(c) 3.0m (d) 4.0m

2. The area of a triangle whose sides are 15m, 16m and 17m is :
(a) $24\sqrt{4}m^2$ (b) $24m^2$
(c) $24\sqrt{21}m^2$
(d) none of these

3. If the sides of a triangle are doubled, its area :
(a) remains same (b) is doubled
(c) becomes half
(d) becomes 4 times

4. The height of an equilateral triangle whose perimeter is 27 cm, is :
(a) $\dfrac{9}{2}\sqrt{3}cm$ b. $\dfrac{9}{2}cm$
(c) 9cm d. none of these

5. Find the area of an isosceles right-angled triangle whose hypotenuse is 16cm :
(a) 32 cm^2 (b) 16 cm^2
(c) 24 cm^2 (d) 64 cm^2

6. If the area of a triangle is 250m^2 and base : height is 4 : 5, find its height :
(a) 25m (b) 20m
(c) 45m (d) 30m

7. A rectangle plot is $116\times 68m^2$. It has a gravel path 2.5m wide all around in it. Find the area of the gravel path?
(a) 945m^2 (b) 900m^2
(c) 895m^2 (d) 885m^2

8. A retangular plot has an area of 490m^2. If the length of the rectangular plot is 24.5m, then find the perimeter of the rectangular plot.
(a) 85m (b) 98m
(c) 89m (d) 87m

9. If the diagonals of two squares are in the ratio 2 : 5, then their areas will be in the ratio :

(a) 4 : 5 (b) 4 : 25

(c) 2 : 5 (d) $\sqrt{2}:\sqrt{5}$

10. The area of a rectangle with length twice of breath is $578m^2$. What is the length of that rectangle ?

(a) 34m (b) 17m

(c) 38m (d) 30m

11. The sides of a rectangular field of $432m^2$ are in the ratio of 4 : 3. Find the sides :

(a) 16m, 12m (b) 12m, 36m

(c) 24m, 18m (d) 32m, 24m

12. Find the area of a rhombus one of whose diagonals measures 8cm and the other 10cm.

(a) $36cm^2$ (b) $80cm^2$

(c) $48cm^2$ (d) $40cm^2$

13. The ratio of the length and breadth of rectangular plot is 5 : 3 respectively. The perimeter of the plot is 80metres. What is the length of the plot in meters ?

(a) 25m (b) 15m

(c) 20m (d) 50m

14. Find the distance between the two parallel sides of a trapezium if the area of the trapezium is 250sq.m and the two parallel sides are equal to 15m and 10m respectively.

(a) 15m (b) 20m

(c) 25m (d) 22m

15. The radius of a circular wheel is 1.75m. The number of revolutions that it will make in covering 11km is:

(a) 500 (b) 10,000

(c) 100 (d) 1000

16. The length of a rectangle is increased by 50%. By what % should the width be decreased to maintain the same area ?

(a) $33\frac{1}{2}\%$ (b) 25%

(c) $33\frac{1}{3}\%$ (d) $37\frac{1}{2}\%$

17. The ratio of the areas of the in-circle and the circum-circle of a square is:

(a) 1 : 2 (b) $\sqrt{2}:1$

(c) $1:\sqrt{2}$ (d) 2 : 1

18. The area of an equilateral triangle inscribed in a circle is $9\sqrt{3}cm^2$. The area of the circle is :

(a) $8\sqrt{3}cm^2$ (b) $13\pi\ cm^2$

(c) $12\pi\ cm^2$ (d) $36\pi\ cm^2$

19. The area of a circle is halved when its radius is decreased by n. Find its radius :

(a) $\frac{2n}{\sqrt{2}-1}$ (b) $\frac{\sqrt{2}n}{\sqrt{2}-1}$

(c) $\frac{\sqrt{3}n}{\sqrt{2}-1}$

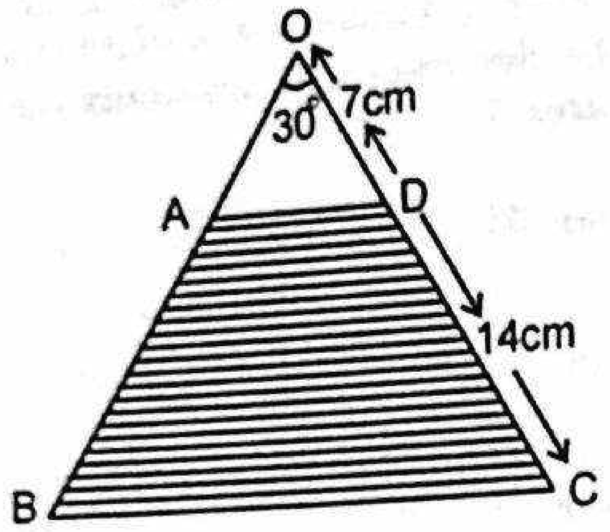
(d) None of these

20. The length of a rectangle is increased by 60%. By what per-cent would the width be decreased so as to maintain the same area ?

(a) $37\frac{1}{2}\%$ (b) 60%

(c) 75% (d) 120%

21. The diagram represents the area swept by the wiper of a car. With the dimensions given in the figure, calculate the shaded area swept by the wiper.



(a) $102.67cm^2$ (b) $205.34cm^2$
(b) $51.33cm^2$ (d) $208.16cm^2$

22. From a point in the interior of an equilateral triangle, the perpendicular distance of the sides are $\sqrt{3}m, 2\sqrt{3}m$ and $5\sqrt{3}$ m respectively. The perimeter of the triangle is :
(a) 16m (b) 32m
(c) 24m (d) 48m

23. The perimeter of an isosceles, right-angled triangle is 2p unit. The area of the same triangle is :
(a) $\left(3-2\sqrt{2}\right)p^2 sq.unit$
(b) $\left(2+\sqrt{2}\right)p^2 sq.unit$
(c) $\left(2-\sqrt{2}\right)p^2 sq.unit$
(d) $\left(3-\sqrt{2}\right)p^2 sq.unit$

24. The area of the largest triangle that can be inscribed in a semi circle of radius x in square unit is :
(a) $4x^2$ (b) x^2
(c) $2x^2$ (d) $3x^2$

25. The length of the side of a square is 14cm. Find out the ratio of the radii of the inscribed and circumscribed circle of the square.
(a) $\sqrt{2}:1$ (b) $1:\sqrt{2}$
(c) $\sqrt{2}:3$ (d) $2:1$

26. The perimeter of a rhombus is 146 cm and one of its diagonals is 55cm. The other diagonal is :
(a) 92 cm (b) 73 cm
(c) 48 cm (d) 72 cm

27. If the length of a chord of a circle at a distance of 12cm from the centre is 10cm, then the diameter of the circle is :
(a) 13 cm (b) 15 cm
(c) 26 cm (d) 30 cm

28. Area of the incircle of an equilateral triangle with side 6cm is :
(a) $\frac{\pi}{2} sq.cm$ (b) $\sqrt{3}\pi$ sq.cm
(c) 6π sq.cm (d) 3π sq.cm

29. If the circumradius of an equilateral triangle be 10cm, then the measure of its in-radius is :
(a) 5 cm (b) 10cm
(c) 20 cm (d) 15cm

30. The adjacent sides of a parallelogram are 36cm and 27 cm in length. If the distance between the shorter sides is 12cm, then the distance between the longer sides is :
(a) 10 cm (b) 12 cm
(c) 16 cm (d) 9 cm

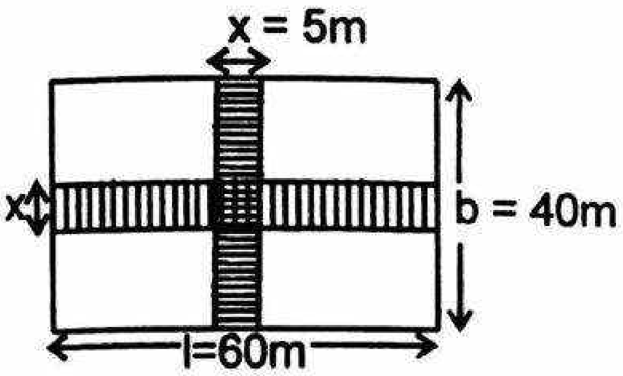
31. What is the area of a triangle having perimeter 32cm, one side 11cm and difference of other two sides 5cm?
(a) $8\sqrt{30}\ cm^2$ (b) $5\sqrt{35}\ cm^2$
(c) $6\sqrt{30}\ cm^2$ (d) $8\sqrt{2}\ cm^2$

32. A circle and a rectangle have the same perimter. The sides of the rectangle area 18cm and 26cm. The area of the circle is :
(a) $125\ cm^2$ (b) $230\ cm^2$
(c) $550\ cm^2$ (d) $616\ cm^2$

33. The perimeter of a semicircular path is 36m. Find the area of this semicircular path.
(a) 42sq.m (b) 54sq.m
(c) 63sq.m (d) 77sq.m

LEVEL - 2

1. A rectangular lawn $60 \times 40m^2$ has two roads each 5m wide running between the park. One is parallel to length and other is parallel to width. Cost of gravelling is 60 paise/m^2. Find the total cost of gravelling ?



(a) ₹285 (b) ₹300
(c) ₹275 (d) ₹270

2. The ratio between the length and width of the rectangular field is 3 : 2. If only length is increased by 5m. The new area of the field is $2600m^2$. What is the width of the rectangular field ?
(a) 60 m (b) 50m
(c) 40 m (d) 65m

3. In a ||gm, the lengths of adjacent sides are 11 and 13cm. If the length of one diagonal is 16cm, find the length of other diagonal.
(a) 17cm b. 18cm
(c) 15cm d. 19cm

4. If the are a of square is $3\sqrt{3}$ times the area of an equilateral triangle, then the ratio of the sides of the square to the side of the equilateral triangle is equal to :
(a) 2 : 3 (b) 4 : 3
(c) 5 : 2 (d) 3 : 2

5. Find the area of a rectangle whose area is equal to the area of a circle with perimter equal to 24π :
(a) 144 (b) 144π
(c) 154π
(d) none of these

6. A wire bent in the form of a circle of radius 2cm is cut and bent in the form of a square, then the ratio of the area of the circle to the area of the square is equal to :
(a) $4 : \pi$ (b) $\pi : 4$
(c) $2 : \pi$ (d) $\pi : 2$

7. A circle is inscribed in an equilateral triangle of side 8cm. The area of the portion between the triangle and the circle is :
(a) $11cm^2$ (b) $10.95cm^2$
(c) $10cm^2$ (d) $10.50cm^2$

8. The base and altitude of a right angled triangle are 12cm and 5cm respectively. the perpendicular distance of its hypotenuse from the opposite vertex is :
(a) $4\frac{4}{13}$cm (b) 7cm
(c) $4\frac{8}{13}$ cm (d) 5cm

9. The length of rectangle, which is 24cm is equal to the length of a square and the area of the rectangle is 176cm less than the area of the square. What is the breadth of the rectangle ?
(a) 15cm (b) 16 cm
(c) $16\frac{1}{3}$ cm (d) $16\frac{2}{3}$ cm

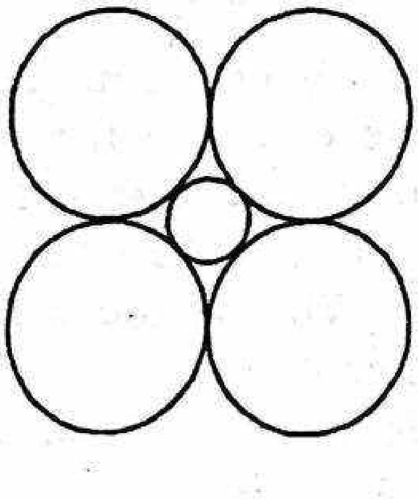
10. The area of a circular field is equal to the area of a rectangular field. The ratio of the length and the breadth of the rectangular field is 14 : 11 respectively and perimeter is 100 meters. What is the diameter of circular field ?
(a) 14m (b) 22m

(c) 24m (d) 28m

11. Area of circle is equal to the area of a rectangle having perimeter of 100cms and length is more than the breadth by 6cms. What is the diameter of the circle ?
(a) 14cm (b) 28cm
(c) 22cm (d) 24cm

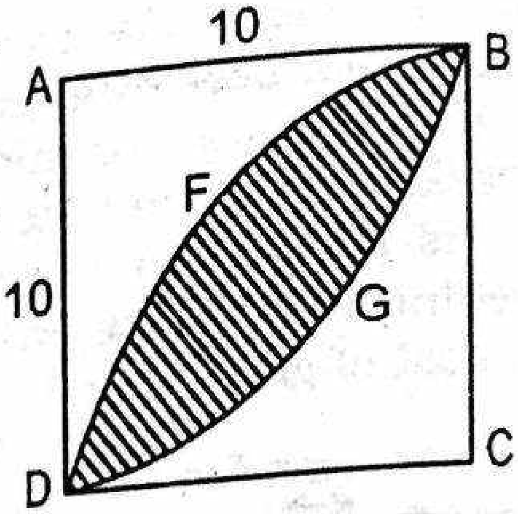
12. A room 8m long, 6m broad and 3m heigh has two windows $1\frac{1}{2}$ m × 1m and a door 2m × $1\frac{1}{2}$ m. Find the cost of papering the walls with paper 50cm wide at 25paise per meter :
(a) ₹ 39 (b) ₹ 37
(c) ₹ 33 (d) ₹ 35

13. In the given figure, when the outer circles all have radii 'R' then the radius of the inner circle will be :



(a) $\frac{2}{(\sqrt{2}+1)R}$ (c) $\frac{1}{\sqrt{2}}R$
(c) $(\sqrt{2}-1)R$ (d) $\sqrt{2}R$

14. In the figure, ABCD is a square with side 10. BFD is an arc of a circle with centre C. BGD is and arc of a circle with centre A. What is the area of the shaded region :

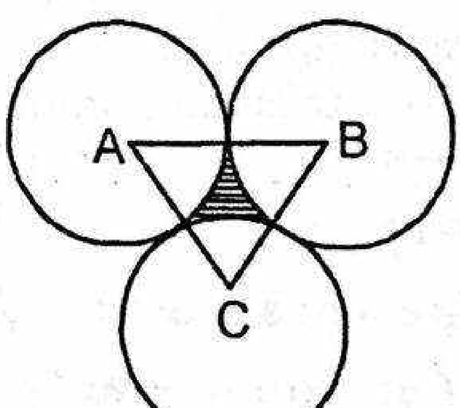


(a) $100 - 50\pi$ (c) $50\pi - 100$
(c) $25\pi - 100$
(d) None of these

15. Let ABCDEF be a regular hexagon. What is the ratio of the area of the triangle ACE to that of the hexagon ABCDEF ?
(a) 1 : 2 (c) 1 : 3
(c) 2 : 3 (d) 5 : 6

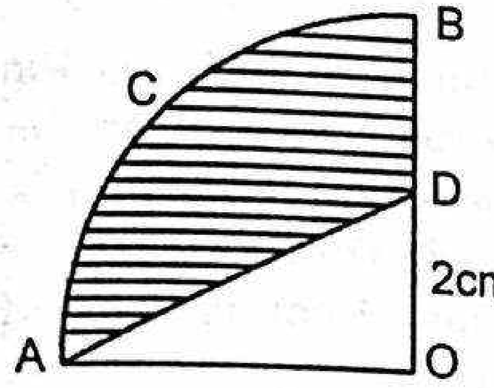
16. Find the ratio of the diameter of the circles inscribed in and circumscribed an equilateral triangle to its height.
(a) 1 : 2 : 3 (b) 2 : 4 : 3
(c) 1 : 3 : 4 (d) 3 : 2 : 1

17. Find the area of the shaded region if the radius of each of the circle is 1cm.



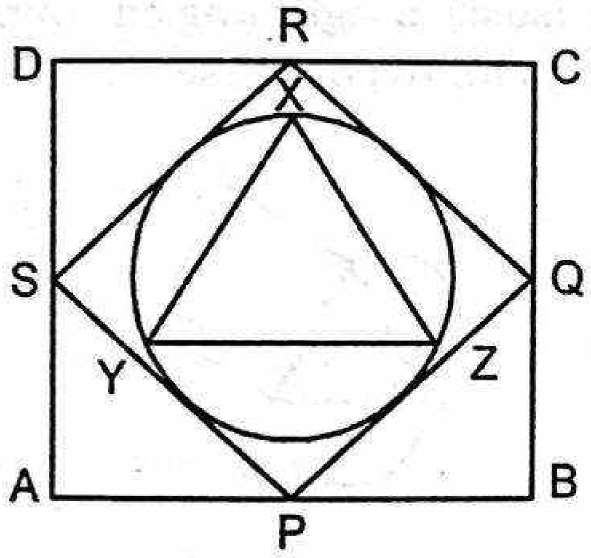
(a) $2-\frac{\pi}{3}$ (b) $\sqrt{3}-\pi$
(c) $\sqrt{3}-\frac{\pi}{2}$ (d) $\sqrt{3}-\frac{\pi}{4}$

18. In the adjoining figure, AOBCA represents a quadrant of a circle of radius 4cm with centre O. Calculate the area of the shaded portion.



(a) $8.56cm^2$ (b) $7.35cm^2$
(c) $8.45cm^2$ (d) $9cm^2$

19. In the given figure ABCD is a square and PQRS is also a square made by joining the mid-points of the sides of the larger square ABCD. There is a inscribed the circle. In □PQRS and an equilateral ΔXYZ inscribed in the circle.
Find the ratio of the side of the square ABCD to the side of the equilateral triangle XYZ.



(a) $\sqrt{2}:\sqrt{3}$ (b) $2\sqrt{2}:3$
(c) $2\sqrt{2}:\sqrt{3}$
(d) none of these

20. Find the area of the largest (or maximum sized) square that can be made inside a right angle triangle having sides 6cm, 8cm & 10cm when one of vertices of the square coincide with the vertex of right angle of the triangle ?

(a) $\frac{576}{49}cm^2$ (b) $24cm^2$
(c) $\frac{24}{7}cm^2$
(d) None of these

21. Area of the trapezium formed by x-axis; y-axis and the lines $3x + 4y = 12$ and $6x + 8y = 60$ is :
(a) 37.5sq.unit (b) 31.5sq.unit
(c) 48sq.unit (d) 36.5sq.unit

22. A square having area 200sq.m, is formed in such a way that the length ofits diagonal is $\sqrt{2}$ times of the diagonal of the given square. Then the area of the new square formed is :
(a) $200\sqrt{2}sq.m$ (b) $400\sqrt{2}sq.m$
(c) 400sq.m (d) 800sq.m

23. A metal wire when bent in the form of a square encloses an area $484cm^2$. If the same wire is bent in the form of a circle, then (talking $\pi = \frac{22}{7}$) its area is :
(a) $308cm^2$ (b) $506cm^2$
(c) $600cm^2$ (d) $616cm^2$

24. Sides of a parallelogram are in the ratio 5 : 4. Its area is 1000sq.units. Altitude on the greater side is 20units, Altittude on the smaller side is :
(a) 30 units (b) 25 units
(c) 10 units (d) 15 units

25. The perimeter of a rhombus is 40cm and the measure of an angle is 60°, then the area of it is :
(a) $1000\sqrt{3}cm^2$ (b) $50\sqrt{3}cm^2$
(c) $160\sqrt{3}cm^2$ (d) $100cm^2$

26. If the four equal circles of radius 3cm touch each other externally, then the area of the region bounded by the four circles is :

(a) $4(9-\pi)\,sq.cm$ (b) $9(4-\pi)\,sq.cm$
(c) $5(6-\pi)\,sq.cm$ (d) $6(5-\pi)\,sq.cm$

27. The length of a room floor exceeds its breadth by 20m. The area of the floor remains unaltered when the length is increased by 5m and breadth is increased by 10m. The area of the floor (in square metres) is:
(a) 280 (b) 325
(c) 300 (d) 420

28. A right angled isosceles triangle is inscribed ina semi-circle of radius 7cm. The area enclosed by the semi-circle but exterior to the triangle is :
(a) $14\,cm^2$ (b) $28cm^2$
(c) $44cm^2$ (d) $68cm^2$

28. A right angled isosceles triangle is inscribed ina semi-circle of radius 7cm. The area enclosed by the semi-circle but exterior to the triangle is :
(a) $14cm^2$ (b) $28cm^2$
(c) $44cm^2$ (d) $68cm^2$

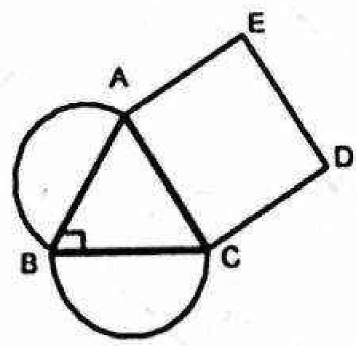
29. The perimeter of semi-circular area is 18cm, then the the radius is :
(using $\pi=\frac{22}{7}$)
(a) $5\frac{1}{3}cm$ (b) $3\frac{1}{2}cm$
(c) 6 cm (d) 4 cm

LEVEL - 3

1. A rectangular park $60\times40m^2$ has two croos roads running in the middle of the park and the rest park has been lawn. If the area of the lawn is $2109m^2$. What is the width of the road?
(a) 3m (b) 4m
(c) 5m (d) 7m

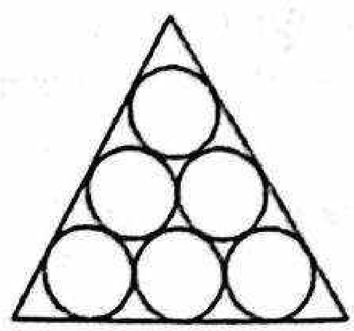
2. Find the perimeter of a square which is symmetrically inscribed in a semicircle of radius 10cm.
(a) $\sqrt{80}cm$ (b) 80cm
(c) $8\sqrt{24}cm$ (d) $16\sqrt{5}cm$

3. The area of the square on AC as a side is $128cm^2$. What is the sum of the areas of semicircles drawn on AB and AC as diameters, given ABC is an isoscles right angled triangle and AC is its hypotenuse.



(a) $32\pi\ cm^2$ (b) $16\pi\ cm^2$
(c) $16\ cm^2$ (d) $32\ cm^2$

4. An equilateral triangle circumscribes all the cricles, each with radius 10cm. What is the perimeter of the equilatral triangle ?

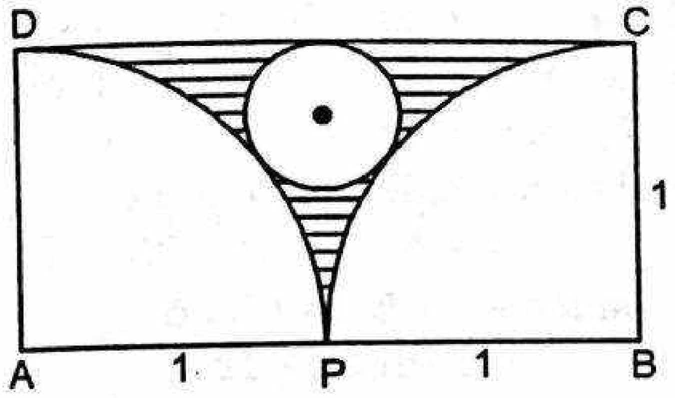


(a) $20\left(2+\sqrt{3}\right)$ cm (b) $30\left(2+\sqrt{3}\right)$ cm

(c) $60\left(2+\sqrt{3}\right)$ cm

(d) None of these

5. In the following figure ABCD is a rectangle with AD and DC equal to 1 and 2 units respectively. Two quarter circles are drawn with centres at B and A respectively. Now a circle is drawn touching both the quarter circles and done of the sides of the rectangle. Find the area of the shaded region :



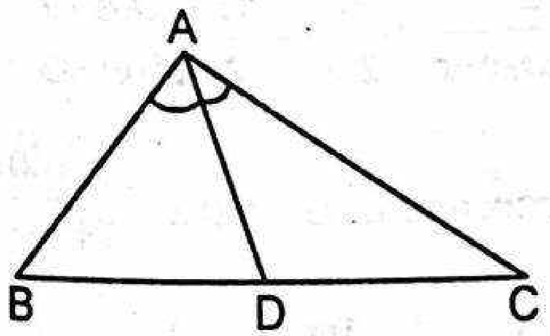
(a) $\frac{32}{115}$ square units

(b) $\frac{13}{56}$ square units

(c) $\frac{16}{83}$ square units

(d) $\frac{7}{20}$ square units

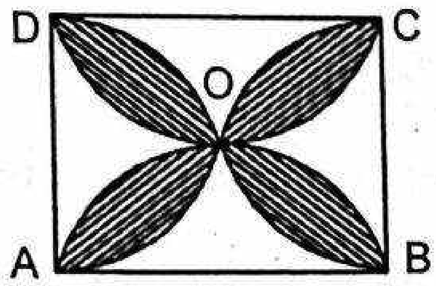
6. In the figure given below, AD bisects $\angle$BAC. If the area of ΔABD = 40 cm^2 and AC = 3AB, then the area of ΔABC :



(a) 150cm^2 (b) 100cm^2

(c) 120cm^2 (d) 160cm^2

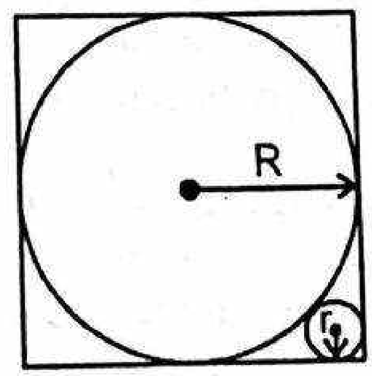
7. In the given figure ABCD is a square. Four equal semicircles are drawn in such a way that they meet each other at 'O'. Sides AB, BC, CD and DA are the repective diameters of the four simicircles. Each of the side of the square is 8cm. Find the area of the shaded region.



(a) $32(\pi-2)$cm^2 (b) $16(\pi-2)$cm^2

(c) $(2\pi-8)$cm^2 (d) $\left(\frac{3}{4}\pi-4\right)$cm^2

8. In the given figure, find the radius of smaller circle (r) :



(a) $\left(\sqrt{3}-2\sqrt{2}\right)R$ (b) $2\left(\sqrt{2}-1\right)R$

(c) $\left(3-2\sqrt{2}\right)R$ (d) None of these

9. Three circles of radius $\sqrt{2}+1, \sqrt{2}+1$ and 1unit, touch each other externally, then find the perimeter of the sorrounded part by three circles.

(a) $\frac{\pi}{2}\left(2\sqrt{2}+2\right)$ (b) $\frac{\pi}{2}\left(\sqrt{2}+2\right)$

(c) $\pi\left(\sqrt{2}+2\right)$

(d) None of these

Hints and Solutions:
LEVEL-1

1.(b) $\text{side} = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$= \frac{1}{2}\sqrt{3^2 + 4^2} = \frac{5}{2} = 2.5\text{m}$

2.(c) $\frac{a+b+c}{2} = \frac{15+16+17}{2} = 24\text{m}$

Aera (A) $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-15)(24-16)(24-17)}$

$= \sqrt{24 \times 9 \times 8 \times 7}$

$= 24\sqrt{21}\text{m}^2$

4.(a) $3a = 27 \Rightarrow a = 9\text{cm}$

ner

$\sqrt{3}$ 9

Area(A) $= 2x(l + b + 2x)$

$= 2 \times 2.5(116 + 68 + 5)$

$= 5(189) = 945\text{m}^2$

8.(c) Area(A) = length(l) $\times$ breadth(b)

$lb = 490 \Rightarrow b = \frac{490}{24.5} = 20\text{m}$

$\therefore$ Perimeter $= 2(l + b)$

$= 2(20 + 24.5)$

$= 89\text{m}$

9.(b) Ratio of sides = Ratio of diagonals

$= 2 : 5$

$\therefore$ Ratio of areas = (Ratio of sides)^2

$= 4 : 25$

10.(a) Let the breadth $= x$ $\therefore$ length $= 2x$

$\Rightarrow x.2x = 578 \Rightarrow x^2 = 289 \Rightarrow x = 17\text{m}$

$\therefore$ length $= 2x = 34\text{m}$

11.(c) Let the sides $= 4x$ & $3x$

$\therefore 4x \,.\, 3x = 432 \Rightarrow x^2 = 36 \Rightarrow x = 6\text{m}$

$\therefore$ sides $= 4 \times 6$ and 3×6

$= 24\text{m}$ and 18m

Alternatively :- Let length = x and breadth = y

New length $= x\left(\frac{150}{100}\right) = \frac{3x}{2}$

As the area remains the same, the new breadth of the rectangle - so,

$\frac{3x}{2} \times$ New breadth = xy

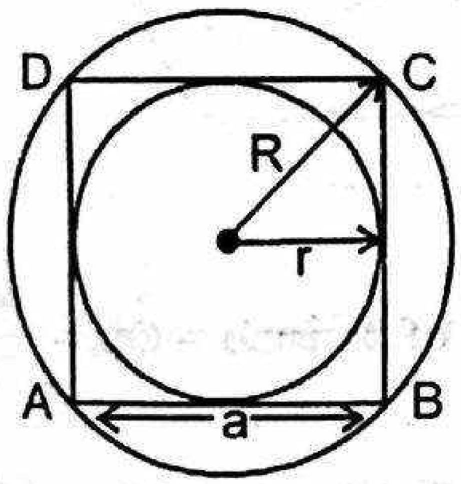
$\Rightarrow$ New breadth $= \frac{2y}{3}$

$\therefore$ decrease in breadth $= y - \frac{2y}{3} = \frac{y}{3}$

$\therefore$ % decrease in breadth $= \frac{y/3}{y} \times 100$

$= \frac{100}{3} = 33\frac{1}{3}\%$

17.(a)



Radius of incircle $= r = \frac{a}{2}$

and Radius of circum-circle $= \frac{a}{\sqrt{2}} = R$

$\therefore$ Ratio of area $= \frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2}$

$= \frac{a^2/4}{a^2/2} = \frac{2}{4} = \frac{1}{2} = 1:2$

18.(c) Area of equilateral $\Delta = \frac{\sqrt{3}}{4}(\text{side})^2$

$= 9\sqrt{3}$

$\Rightarrow$ side = 6cm

$\therefore$ circum-radius of equilateral Δ =

$\frac{side}{\sqrt{3}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$

So, area of circle = $\pi \times \left(2\sqrt{3}\right)^2 = 12\pi\ cm^2$

19.(b) By the question, we have -

$\pi(r-n)^2 = \frac{\pi r^2}{2}$

$\Rightarrow r^2 = 2(r - n)^2$

$\Rightarrow r^2 - \left\{\sqrt{2}(r-n)\right\}^2 = 0$

$\Rightarrow \left\{r - \sqrt{2}(r-n)\right\}\left\{r + \sqrt{2}(r-n)\right\} = 0$

Since, $r + \sqrt{2}(r-n) \neq 0$, we have

$r - \sqrt{2}(r-n) = 0$

$\Rightarrow r = \frac{\sqrt{2}n}{\sqrt{2}-1}$

20.(a) Let length = width = 100m
If length = 160m, then let width = x m
s.t. $160x = 10000$

$\Rightarrow x = \frac{10000}{160} = \frac{1000}{16} = 62\frac{1}{2}$

$\therefore$ width is reduced to $37\frac{1}{2}\%$

21.(a) Larger Radius (R) = 14 + 7
= 21cm
Smaller Radius (S) = 7cm
$\therefore$ Area of shaded portion

$= \pi R^2 \frac{\theta}{360^\circ} - \pi r^2 \frac{\theta}{360^\circ}$

$= \pi \frac{30^\circ}{360^\circ}(21\times 21 - 7\times 7)$

$= \frac{22}{7}\times\frac{1}{12}\times 7\times 7(8)$

$= 102.67cm^2$

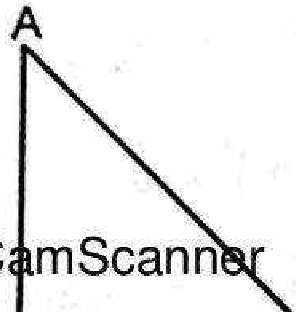
22.(d) $P_1 = \sqrt{3}, P_2 = 2\sqrt{3}, P_3 = 5\sqrt{3}$

$\therefore P = \frac{\sqrt{3}}{2}a = P_1 + P_2 + P_3 = 8\sqrt{3}$

$\Rightarrow a = 16$

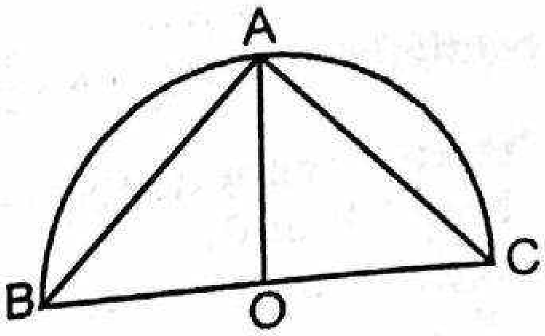
$\therefore$ Perimeter = 3a = 48m

23.(a)



$= (3-2\sqrt{2})p^2 sq.units$

24.(b)



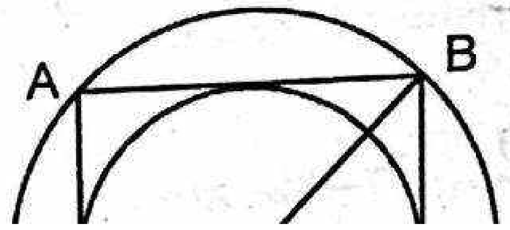
$OA = \frac{1}{2}BC = \text{radius}$

Area of the largest triangle

$= \frac{1}{2}\times BC\times OA = \frac{1}{2}\times 2x\times x$

$= x^2$

25. (b)



$= 2\sqrt{d_1^2 + d_2^2}$

$\Rightarrow 146 = 2\sqrt{55^2 + d_2^2}$

$\Rightarrow 73 = \sqrt{55^2 + d_2^2}$

$\Rightarrow 73^2 = 55^2 + d_2^2$

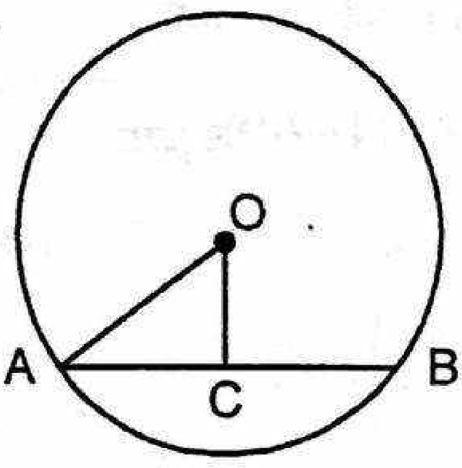
$\Rightarrow 73^2 - 55^2 = d_2^2$

$\Rightarrow d_2^2 = (73+55)\ (73 - 55)$

$= 128 \times 18$

$\Rightarrow d_2 = 48cm$

27. (c)



OC = 12cm AC = CB = 5cm

$\therefore$ Radius 'OA' = $\sqrt{OC^2 + AC^2}$

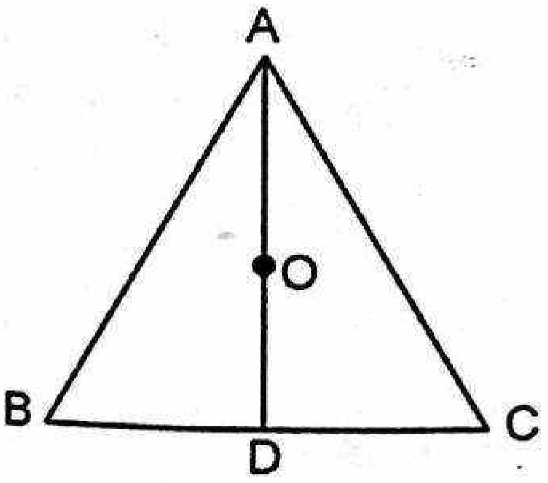
$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$

$= \sqrt{169} = 13cm$

$\therefore$ *Diameter* of circle

$= 2 \times 13 = 26$cm

28. (d)



DB = DC = 3cm.

$AD = \sqrt{AB^2 - BD^2} = \sqrt{6^2 - 3^2}$

$= \sqrt{36 - 9} = \sqrt{27} = 3\sqrt{3}cm.$

$\therefore$ OD = In-radius

$= \frac{1}{3} \times 3\sqrt{3} = \sqrt{3}cm.$

$\therefore$ *Area* of circle $= \pi r^2$

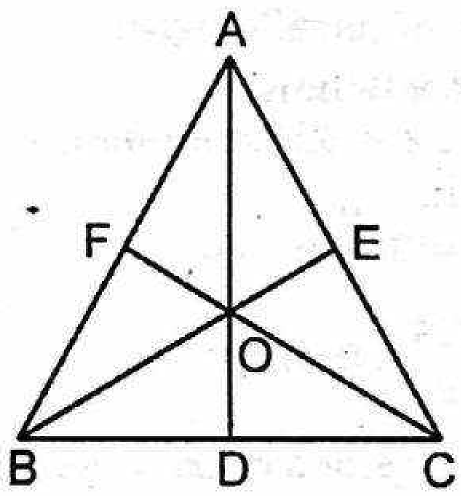
$= \pi \times \sqrt{3} \times \sqrt{3} = 3\pi$ sq.cm

Alternatively :

Inradius(r) = $\frac{a}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} = \sqrt{3}$

$\therefore$ Area = $\pi r^2 = \pi(\sqrt{3})^2 = 3\pi$

29. (a)



Let AB = x cm

$\therefore BD = \frac{X}{2}$

$AD = \sqrt{x^2 - \frac{x^2}{4}} = \frac{\sqrt{3}}{2} x\ cm.$

$\therefore OD = \frac{1}{3} \times \frac{\sqrt{3}}{2} x = \frac{x}{2\sqrt{3}} cm.$

$OB = \sqrt{BD^2 + OD^2}$

$= \sqrt{\frac{x^2}{4} + \frac{x^2}{12}} = \sqrt{\frac{4x^2}{12}} = \frac{x}{\sqrt{3}} cm.$

$\therefore \frac{x}{\sqrt{3}} = 10 \Rightarrow x = 10\sqrt{3}cm.$

$\therefore OD = \frac{x}{2\sqrt{3}} = \frac{10\sqrt{3}}{2\sqrt{3}} = 5cm.$

Alternatively :

circumradius(R) = $\frac{a}{\sqrt{3}}$

$\Rightarrow 10 = \frac{a}{\sqrt{3}}$

$\Rightarrow a = 10\sqrt{3}$

$\therefore$ Inradius = $\frac{a}{2\sqrt{3}} = \frac{10\sqrt{3}}{2\sqrt{3}} = 5$ cm

30.(d) Area of parallelogram
= base $\times$ height
= $27 \times 12 = 324$sq. cm.
Again,
$324 = 36 \times h$

$\Rightarrow h = \frac{324}{36} = 9cm$

31.(a) Let the sides of triangle be a, b and c respectively,

$\therefore 2s = a + b + c = 32$

$\Rightarrow 11 + b + c = 32$

$\Rightarrow b + c = 32 - 11 = 21$ ____(i)

and $b - c = 5$ ______(ii)

By equations (i) and (ii)

$2b = 26 \Rightarrow b = 13$

$\therefore c = 13 - 5 = 8$

$\therefore 2s = 32 \Rightarrow s = 16$

$a = 11, b = 13, c = 8$

$\therefore$ Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-1)(16-13)(16-8)}$

$= \sqrt{16 \times 5 \times 3 \times 8}$

$= 8\sqrt{30}$ sq cm

32.(d) $2\pi r = 2(18+26)$

$2 \times \frac{22}{7} \times r = 44 \times 2 \Rightarrow r = 14cm$

$\therefore$ *Area* of circle $= \pi r^2$

$= \frac{22}{7} \times 14 \times 14 = 616 sq.cm$

33.(d) $\pi r + 2r = 36$

$\Rightarrow r\left(\frac{22}{7} + 2\right) = 36$

$\Rightarrow r\left(\frac{22+14}{7}\right) = 36$

$\Rightarrow r = \frac{36 \times 7}{36} = 7$ *metre*

$Area = \frac{\pi r^2}{2} = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77 sq.metre$

LEVEL - 2

1.(a) Area of path = $x(l + b - x)$
= $5(60 + 40 - 5)$
= $5 \times 95 = 475m^2$

$\therefore$ Total cost = $475 \times \frac{60}{100} =$ ₹ 285

2.(c) Let length = $3x$, then width = $2x$
$\therefore$ $(3x+5)2x = 2600 \Rightarrow (3x+5)x = 1300$
we, go through the option
option (c) $2x = 40 \Rightarrow x = 20$ which satisfy the above equation
$\therefore$ width = $2x = 40m$

Note : you can also solve the above equation.

3.(b) In a ||gm, $d_1^2 + d_2^2 = 2(a^2 + b^2)$

$\Rightarrow (16)2 + d_2^2 = \left[(11)^2 + (13)^2\right] \times 2$

$\Rightarrow d_2^2 = 2(290) - 256 \Rightarrow d^2 = 324$

$\Rightarrow$ d = 18cm

4.(d) Let the side of square = x and the side of equilateral triangle = y

$\therefore$ $x^2 = 3\sqrt{3} \times \left(\frac{\sqrt{3}}{4}y^2\right) \Rightarrow x^2 = \frac{9}{4}y^2$

$\Rightarrow \frac{x}{y} = \frac{3}{2} \Rightarrow x : y = 3 : 2$

5.(b) Let the radius of circle be 'r'
$\Rightarrow 2\pi r = 24\pi \Rightarrow r = 12$
$\therefore$ Area of circle = $\pi(12)^2 = 144\pi$
$\therefore$ Area of the rectangle = area of circle = 144π

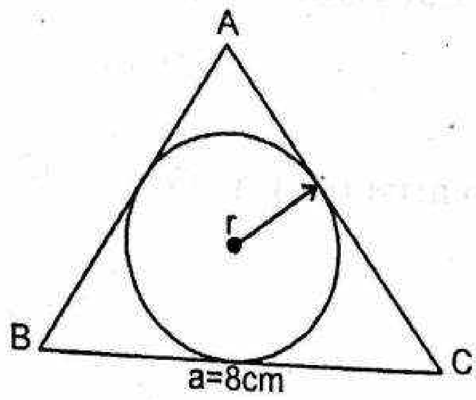
6.(a) Area of the circle = πr^2
= $\pi(2)^2$
= 4π
The circle is cut to make a square
$\therefore$ Perimeter of square = Perimeter of circle

$\Rightarrow 4a = 2\pi r \Rightarrow \Rightarrow a = \frac{2\pi \times 2}{4} = \pi$

$\therefore$ Area of the square = $a^2 = \pi^2$

$\therefore$ required ratio = $\frac{4\pi}{\pi^2} = 4 : \pi$

7.(b)



Area of triangle $= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4} \times 8 \times 8 = 16\sqrt{3}cm^2$

radius of incircle = r = $\frac{a}{2\sqrt{3}} = \frac{8}{2\sqrt{3}}$

$= \frac{4}{\sqrt{3}}$

Area of inscribed circle = πr^2

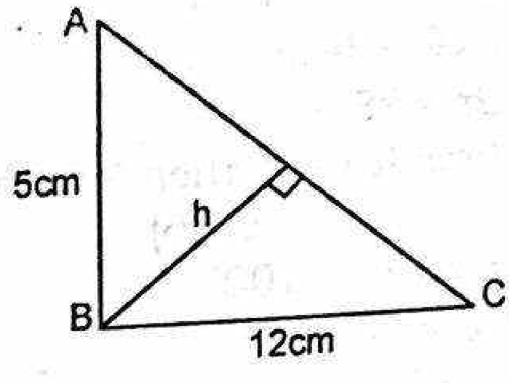
$\pi\left(\frac{4}{\sqrt{3}}\right)^2 = \frac{22}{7} \times \frac{16}{3}$

$\therefore$ Required area = $\left(16\sqrt{3} - \frac{22 \times 16}{21}\right)$

$= \frac{16}{21}(21 \times 1.732 - 22)$

$= \frac{16}{21}(17.372) = 10.95cm^2$

8.(c)



$AC=\sqrt{(12)^2+(5)^2}=13cm$

Area of $\Delta ABC=\frac{1}{2}\times 12\times 5$

$= 30cm^2$

also area of $\Delta ABC=\frac{1}{2}\times (AC)\times h$

$=\frac{13}{2}h\ cm^2$

$\therefore \frac{13}{2}h=30 \Rightarrow h=\frac{60}{13}=4\frac{8}{13}cm$

9.(d) Area of square = $(side)^2$
$= (24)^2$
$= 576cm^2$

Area of rectangle = length × breadth = 576 - 176
$= 400cm^2$

$\Rightarrow$ Breadth of rectangle $=\frac{400}{24}=\frac{50}{3}$

$=16\frac{2}{3}cm$

10.(d) Let length = $14x$, then breadth
$= 11x$

$\therefore 2(14x + 11x) = 100$

$\Rightarrow 50x = 100 \Rightarrow x = 2$

$\therefore$ Area of rectangular field $= 28\times 22m^2$

$\therefore$ Area of circular field $= 28\times 22m^2$

$\Rightarrow \pi r^2 = 28\times 22m^2$

$r^2=\frac{28\times 22}{22}\times 7$

$=28\times 7 = 7\times 7\times 4$

$\Rightarrow r = 7\times 2 = 14m$

$\therefore d = 2r = 28m$

11.(b) Let breadth = x, then length
$= (x + 6)$

$\therefore 2(x + x + 6) = 100$

$\Rightarrow 2x + 6 = 50$

$\Rightarrow x = 22cm$

$\therefore$ breadth $= x = 22cm$ & length
$= 22 + 6 = 28cm$

$\therefore$ Area of circle = Area of rectangle

$\Rightarrow \pi r^2 = 22\times 28$

$\Rightarrow r^2=\frac{22\times 28}{22}\times 7=7\times 4\times 7$

$\Rightarrow r = 7\times 2 = 14cm$

$\therefore$ Diameter = $2r$ = 28cm

12.(a) Area of walls = 2(length + breadth) × height

$= 2(8 + 6)\times 3 = 84m^2$

Area of two windows and a door

$=2\left(1\frac{1}{2}\times 1\right)+\left(2\times 1\frac{1}{2}\right)=6m^2$

$\therefore$ Area to be covered = 84 - 6
$= 78m^2$

$\therefore$ Area of paper = Area to be covered
= 78

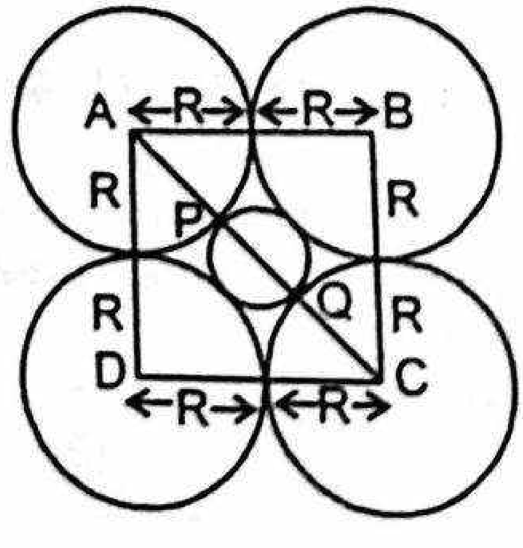
$\Rightarrow$ (length × breadth) of paper = 78

$\Rightarrow$ length of paper $=\frac{78}{50}\times 100m$

= 156cm

$\therefore$ cost $=\frac{156\times 25}{100}$ = ₹ 39

13.(c)



let radius of inner circle = r

A, B, C, D are the centres of the four outer circles

$\therefore$ ABCD is a square of side 2R

$\therefore AC=\sqrt{2}(side)=\sqrt{2}(2R)=2\sqrt{2}R$

$\therefore$ PQ = AC - AP - AQ = $2\sqrt{2}R - R - R$

$= 2R(\sqrt{2}-1)$

$\Rightarrow 2r = 2R(\sqrt{2}-1)$

$\Rightarrow r = R(\sqrt{2}-1)$

14.(b) Area of portion DFBOD = Area of portion DFBC area of DBCD

$= \frac{1}{4}\pi(10)^2 - \frac{1}{2}\times 10\times 10$

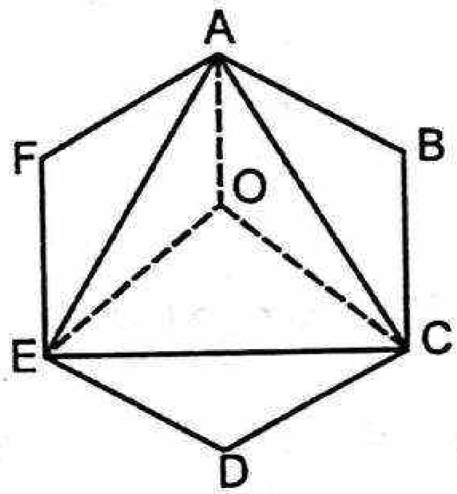
$= 25\pi - 50$

= Area of portion DFBGD

$\therefore$ Area of portion DFBGD = $(25\pi - 50) + (25\pi - 50)$

$= 50\pi - 100$

15. (a)



ABCDEF is a regular hexagon. Joining the centre O with verices A,C and E, we get,

$\Delta AFE = \Delta AOE$

similarly, $\Delta OAC = \Delta BAC$

also, $\Delta OEC = \Delta DEC$

$\therefore$ $\Delta ACE = \frac{1}{2}$ the area of regular hexagon.

16.(b) $height = h = \frac{\sqrt{3}}{2}a$ (a = side of Δ)

inradius = $\frac{a}{2\sqrt{3}}$

incirle's diameter (d) $= \frac{2a}{2\sqrt{3}} = \frac{a}{\sqrt{3}}$

circum-radius =

$\frac{a}{\sqrt{3}} \Rightarrow$ circumcircle's diamter

(D) $= \frac{2a}{\sqrt{3}}$

$\therefore$ d : D : H $= \frac{a}{\sqrt{3}} : \frac{2a}{\sqrt{3}} : \frac{\sqrt{3}}{2}a = 2:4:3$

17. (c) ABC is an equilateral triangle with sides = 2cm

$\therefore$ Area of shaded region = Area of ΔABC - Area of 3 quadrants.

$\frac{\sqrt{3}}{4}(2)^2 - 3\left(\pi r^2 \frac{\theta}{360°}\right)$

$[\theta = 60° \because \Delta ABC$ is an equilateral triangle$]$

$= \frac{\sqrt{3}}{4}\times 4 - 3\left(\pi\times 1\times\frac{1}{6}\right)$

$= \sqrt{3} - \frac{\pi}{2}$

18.(a) Area of shaded region = Area of quadrant - Area of ΔAOD

$= \frac{\pi r^2}{4} - \frac{1}{2}\times 4\times 2$

$= \frac{\pi\times 4\times 4}{4} - 4 = 4\pi - 4$

$= 4(\pi - 1)cm^2$

$= 4(3.14-1) = 4 \times 2.14$

$= 8.56\ cm^2$

19.(c) Let side of □ABCD = 2a

$\therefore$ side of □PQRS =

$= \sqrt{AP^2 + AS^2} = \sqrt{a^2 + a^2} = \sqrt{2}a$

$\therefore$ radius of circle $= \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

Let side of $\Delta XYZ = b$

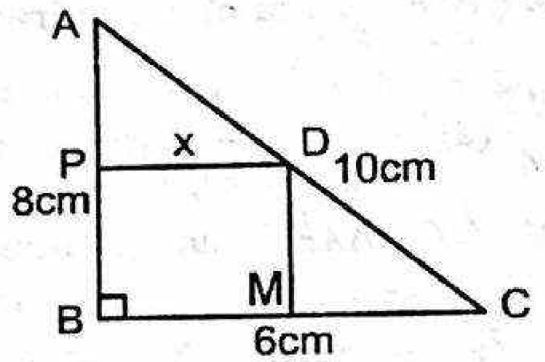
$\therefore$ radius of circumcircle of ΔXYZ

$= \frac{b}{\sqrt{3}}$

$\therefore \frac{b}{\sqrt{3}} = \frac{a}{\sqrt{2}} \Rightarrow \frac{a}{b} = \frac{\sqrt{2}}{\sqrt{3}}$

$\therefore \frac{2a}{b} = \frac{2\sqrt{2}}{\sqrt{3}}$

20. (a)



Side of maximum sized square

$\therefore$ area of square $= \left(\frac{24}{7}\right)^2 = \frac{576}{49} cm^2$

Alternatively, let side of largest square = x

$\therefore$ AP = $(8 - x)$cm and MC = $(6 - x)$cm
in D ABC and D DMC,
$\angle B = \angle M = 90°$
$\angle C = \angle C$ = (common)
D ABC ~ D DMC,

$\Rightarrow \frac{BC}{MC} = \frac{AB}{DM} \Rightarrow \frac{6}{6-x} = \frac{8}{x}$

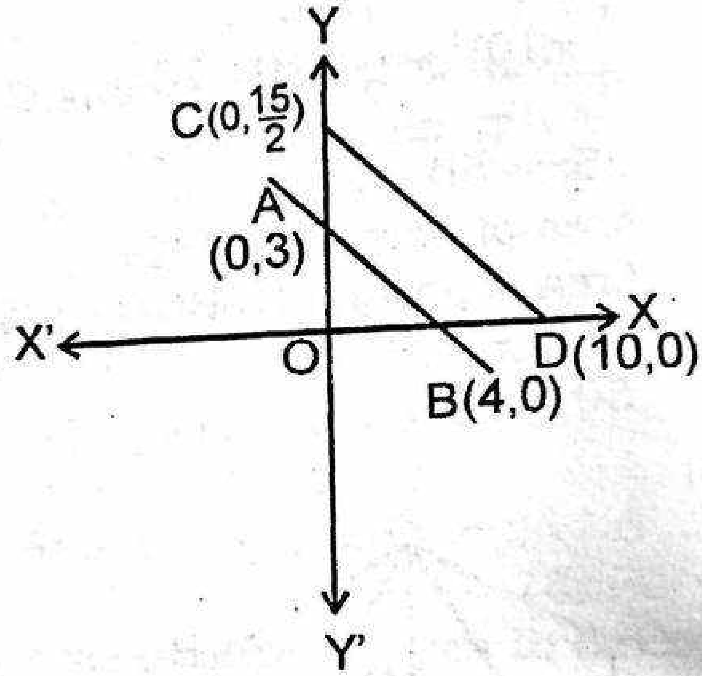
$\Rightarrow 3x = 24 - 4x \Rightarrow x = \frac{24}{7} cm$

$\therefore$ area of square $= x^2 = \frac{576}{49} cm^2$

21.(b) For $3x + 4y = 12$
By putting $x = 0$, $y = 3$
By puttting, $y = 0$, $x = 4$
For $6x + 8y = 60$,

By putting $x = 0$, $y = \frac{15}{2}$

By putting $y = 0$, $x = 10$



$\therefore$ Area of Δ OCD $= \frac{1}{2} \times OD \times OC$

$= \frac{1}{2} \times 10 \times \frac{15}{2} = \frac{75}{2}$

$\therefore$ Area of ΔOAB

$= \frac{1}{2} \times OB \times OA$

$= \frac{1}{2} \times 4 \times 3 = 6$

$\therefore$ Area of trapezium $= \frac{75}{2} - 6$

$= \frac{75-12}{2} = \frac{63}{2} = 31.5 sq.units$

22.(c) Side of the first square $= \sqrt{Area}$

$= \sqrt{200} = 10\sqrt{2} metre$

Its diagonal $= \sqrt{2} \times side$

$=10\sqrt{2}\times\sqrt{2}$

$=20 metre$

$\therefore$ Diagonal of new square

$=\sqrt{2}\times 20=20\sqrt{2}$ metre

$\therefore$ Its area $=\frac{1}{2}\times(diagonal)^2$

$=\frac{1}{2}\times 20\sqrt{2}\times 20\sqrt{2}$

$=400 sq.metre$

23.(d) Side of square $=\sqrt{484}=22$ cm

$\therefore$ length of wire = 22 × 4 = 88cm

$\therefore 2\pi r = 88 \Rightarrow 2\times\frac{22}{7}\times r=88$

$\Rightarrow r=\frac{88\times 7}{2\times 22}=14cm$

$\therefore$ Area $=\pi r^2$

$=\frac{22}{7}\times 14\times 14=616 sq.cm.$

24.(b) Let the side of parallelogram be $5x$ and $4x$

Base × height

= Area of parallelogra = $5x\times 20$

= 1000

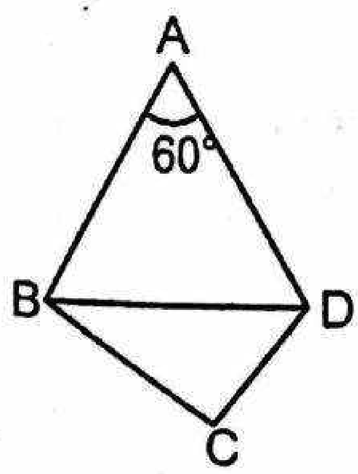
$\Rightarrow x=\frac{1000}{5\times 20}=10$

$\therefore$ Sides = 50 and 40units

$\therefore$ 40 × h = 1000

$\Rightarrow h=\frac{1000}{40}=25$ units

25. (b)



Side $=\frac{40}{4}=10$cm

AB = AD = 10cm

$\angle ABD=\angle ADB=60°$

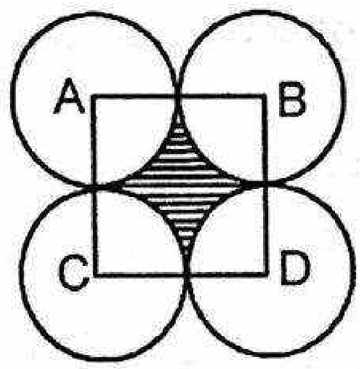
$\therefore$ Area of the rhombus

$=2\times\frac{\sqrt{3}}{4}\times(AB)^2$

$=2\times\frac{\sqrt{3}}{4}\times 10\times 10$

$=50\sqrt{3}cm^2$

26. (b)



Area of the shaded region

= Area of square of side 6cm - 4 × a right angled sector

$=36-4\times\frac{\pi\times 3^2}{4}$

$=36-9\pi=9(4-\pi)$ sq. cm.

27.(c) Let the breadth of floor be x metre.

$\therefore$ Length = $(x + 20)$metre

$\therefore$ Area of the floor = $(x+20)x$ sq.metre

In case II,

$(x+10)(x+5)=x(x+20)$

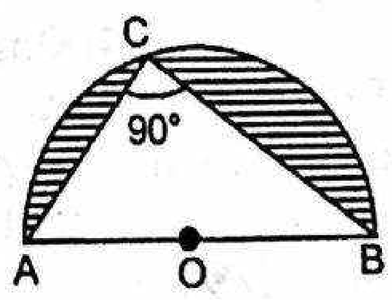
$\Rightarrow x^2+15x+50=x^2+20x$

$\Rightarrow 20x=15x+50$

$\Rightarrow 5x=50$

$\Rightarrow x=10$ metre

28. (b)



$\angle ACB=90°$

AC = CB = X cm
AB = 14 cm
From ΔABC
$AC^2 + BC^2 = AB^2$

$\Rightarrow x^2 + x^2 = 14^2$

$\Rightarrow 2x^2 = 14 \times 14$

$\Rightarrow x^2 = 14 \times 7$

$\Rightarrow x = \sqrt{14 \times 7} = 7\sqrt{2}cm$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AC \times BC$

$= \frac{1}{2} \times 7\sqrt{2} \times 7\sqrt{2} = 49 sq.cm$

29.(b) Perimeter of semi-circular region = 18cm

$\Rightarrow \pi r = 2r = 18$

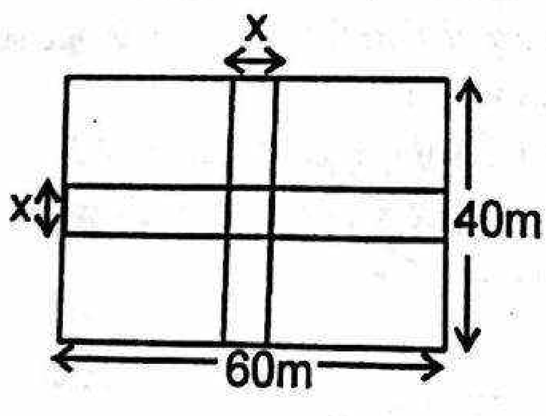
$\Rightarrow r(\pi + 2) = 18$

$\Rightarrow r\left(\frac{22}{7} + 2\right) = 18 \Rightarrow r\left(\frac{36}{7}\right) = 18$

$\Rightarrow r = \frac{18 \times 7}{36} = \frac{7}{2} = 3\frac{1}{2} cm$

LEVEL - 3

1.(a)



Total area of park = 60 × 40
= 2400m²

and area of lawn = 2109m² (given)
area of the cross roads = 2400-2109
= 291m²

$\Rightarrow x(60 + 40 - x) = 291$

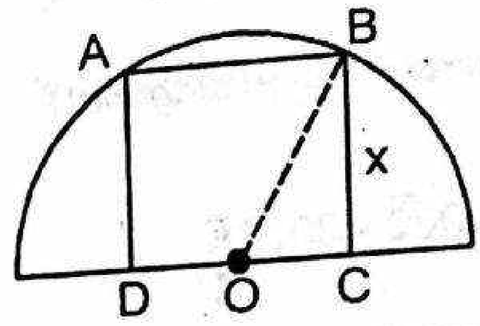
$\Rightarrow x^2 - 100x + 291 = 0$

$\Rightarrow (x - 97)(x - 3) = 0$

$\Rightarrow x = 3$ or 97

$\Rightarrow x = 3$ [$\because x = 97$ is not possible]

2.(d)



Here ABCD is a square of side x.

$\therefore OC = \frac{x}{2}$, $BC = x$, and OB = radius of circle = 10cm

In ΔOCB, $OB^2 = OC^2 + BC^2$

$\Rightarrow \left(\frac{x}{2}\right)^2 + x^2 = (10)^2$

$\Rightarrow \frac{5x^2}{4} = 100 \Rightarrow x^2 = 80$

$\Rightarrow x = 4\sqrt{5}cm$

Hence, perimeter of the square ABCD = $4x$

$= 16\sqrt{5}$

3.(b) Let AB = BC = x, then $AC = \sqrt{2}x$

But AC = $\sqrt{128} = 8\sqrt{2}cm$

$\sqrt{2}x = 8\sqrt{2} \Rightarrow x = 8cm$

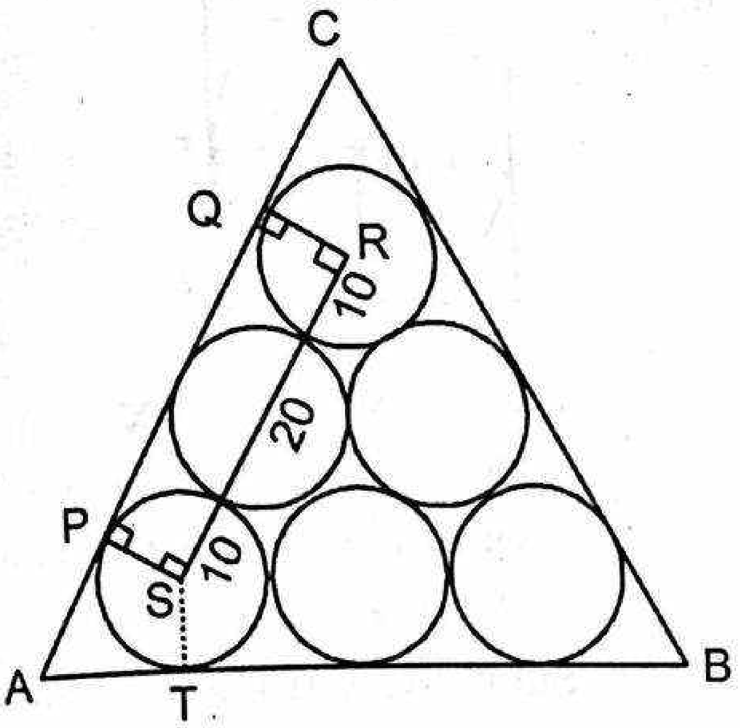
Area of semicircls

$= \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$= \frac{1}{2}\pi\,[2 \times 16]$

$= 16\,\pi\,cm^2$

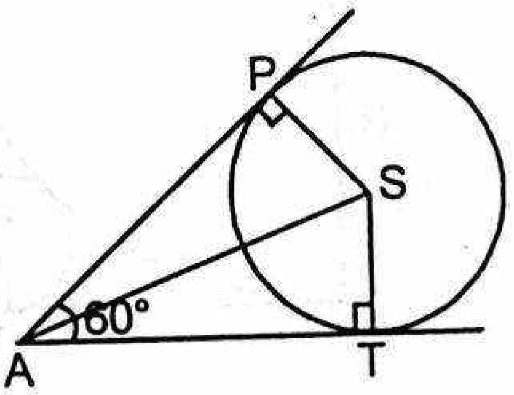
4.(c)



PQRS is a rectangle.

$\therefore$ PQ = 10 + 20 + 10 = 40cm

for AP,



$\angle A = 60°,\ \angle PST = 120°$

$\therefore \angle PSA = \angle AST = \frac{120°}{2} = 60°$

$and\ \angle PAS = \angle SAT = 30°$

$\therefore$ In Δ PSA , $\tan 30° = \frac{PS}{AP} = \frac{10}{AP}$

$\Rightarrow AP = \frac{10}{\tan 30°}$

$\Rightarrow AP = 10\sqrt{3}$

Similarly ;

$QC = 10\sqrt{3}$

$\therefore$ AC = PQ +AP + QC

$= 40 + 10\sqrt{3} + 10\sqrt{3}$

$= 20\left(2+\sqrt{3}\right)$cm

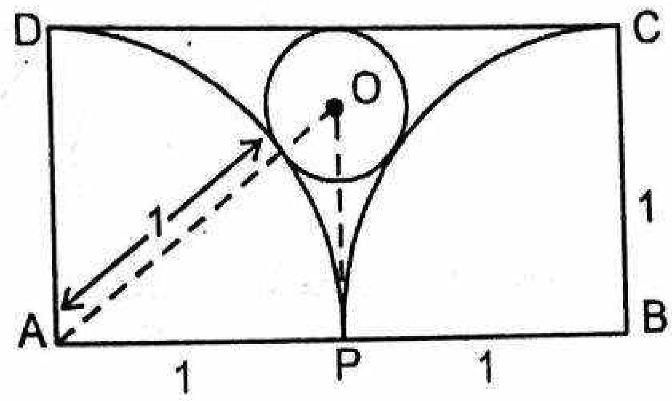
$\therefore$ AB = BC =AC = $20\left(2\sqrt{3}\right)$

$\therefore$ Perimeter of $\Delta ABC = 60\left(2+\sqrt{3}\right)$cm

5.(b) Let radius fo the circle is 'r' units

OP = (1- r), OA = (1+ r) and AP = 1

In ΔAOP; $OA^2 = AP^2 + OP^2$



$\Rightarrow (1+r)^2 = 1^2 + (1-r)^2$

$\Rightarrow r = \frac{1}{4}$ units

$\therefore$ Area of smaller circle $= \pi\left(\frac{1}{4}\right)^2$

$= \frac{\pi}{16}$ square units

Sum of the area of the quarter circes $= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ square units

Area of shaded region = $2 - \left(\frac{\pi}{16} + \frac{\pi}{2}\right)$

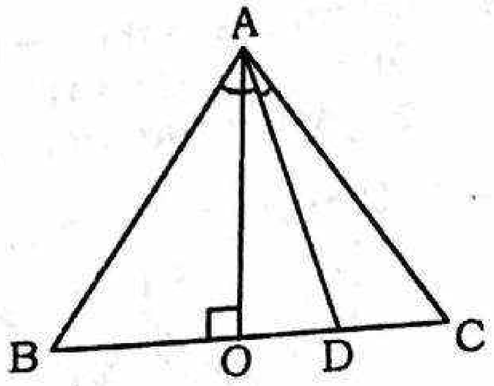
$= 2 - \frac{9}{16}\pi \cong 0.23$ square units

Among the option choices, option (b) is closest.

6.(d) $\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ADC} = \frac{\frac{1}{2} \times (BD) \times (OA)}{\frac{1}{2} \times (DC) \times (OA)}$

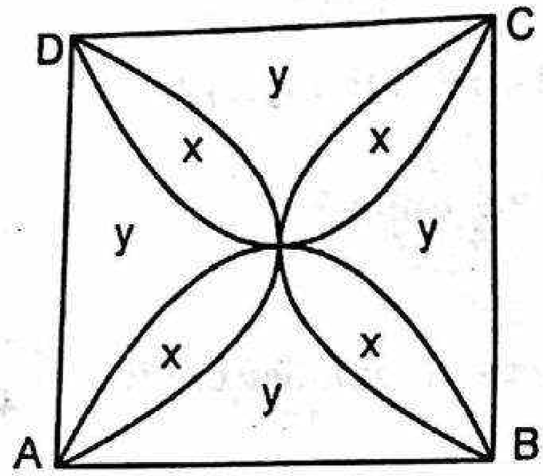
$= \frac{BD}{DC} = \frac{AB}{AC} = \frac{1}{3}$ ($\because AD$ is the angle bisector of $\angle BAC$)

$\Rightarrow$ Area of $\Delta ADC = 3 \times 40 = 120cm^2$



$\therefore$ Area of $\Delta ABC = 120 + 40$
$= 160cm^2$

7.(a)



Let area of each shaded portion = x and area of each unshaded portion = y

total area of square = $(8)^2$
$= 64$ cm^2

$\therefore$ $4(x + y) = 64$

$\Rightarrow$ $x + y = 16$ ---------(i)

Again in a semicircle,

$$AOB = x + y + x = \frac{1}{2}\pi \times (4)^2$$

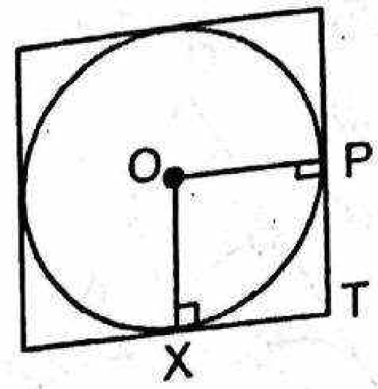
$\Rightarrow$ $2x + y = 8\pi$ ______(ii)

From (i) & (ii) we get.

$x = 8\pi - 16 = 8(\pi - 2)$

$\therefore$ Total area of shaded region
$= 32(\pi - 2)cm^2$

8.(c)



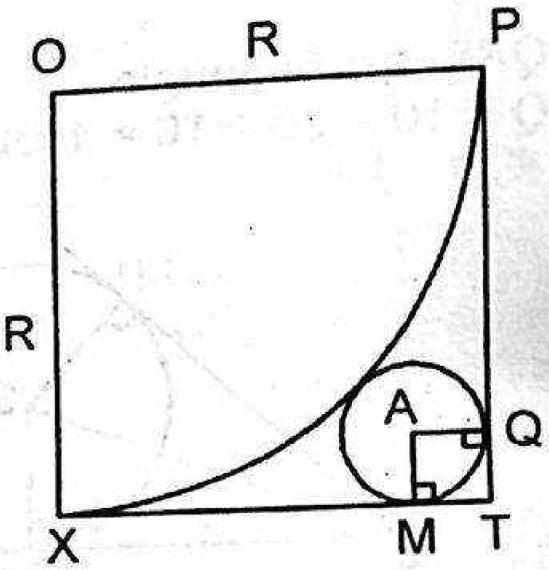
in $\square$OPTX, $\angle P = \angle X = \angle T = 90°$

$\angle O = 90°$ and $OP = OX = R$

$\therefore$ $\square$OPTX, is a square of side R

$\Rightarrow$ $OT = \sqrt{2}R$

$\therefore$ Similarly, AQTM is a square of side r.



$\therefore$ $AT = \sqrt{2}\,r$

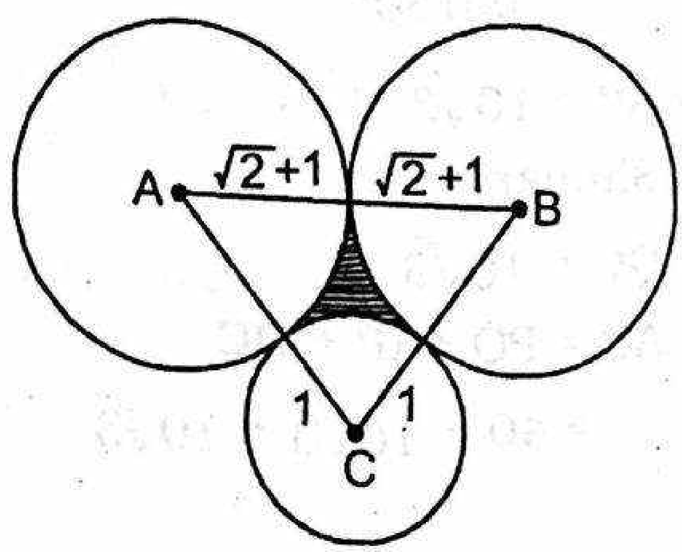
$\because$ $OT = OA + AT$

$\Rightarrow$ $\sqrt{2}R = (R + r) + \sqrt{2}\,r$

$\Rightarrow$ $(\sqrt{2} - 1)R = (1 + \sqrt{2})\,r$

$\Rightarrow$ $r = \left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)R = (3 - 2\sqrt{2})R$

9.(b)



$AB = 2(\sqrt{2}+1)$ and AC = BC

$= 2 + \sqrt{2}$

$\therefore\ AB^2 = 4(\sqrt{2}+1)^2$

and $AC^2 = BC^2 = (2+\sqrt{2})^2$

So, it is clear

$AB^2 = AC^2 + BC^2$

i.e. ABC is an isosceles right angled triangle.

$\therefore\ \angle ACB = 90°$ and $\angle CAB$

$= \angle ABC = 45°$

$\therefore$ required perimete $= 2\pi(\sqrt{2}+1)\ \dfrac{45°}{360°}$

$+2\pi(\sqrt{2}+1)\ \dfrac{45°}{360°} + 2\pi \times 1 \times \dfrac{90°}{360°}$

$= \ 2 \times 2\pi(\sqrt{2}+1)\dfrac{1}{8} + 2\pi\dfrac{1}{4}$

$= \ \dfrac{2\pi}{4}(\sqrt{2}+1+1)$

$= \ \dfrac{\pi}{2}(\sqrt{2}+2)$

Answer -Key

LEVEL - 1

1. (b)	2. (c)	3. (d)
4. (a)	5. (d)	6. (a)
7. (a)	8. (c)	9. (b)
10. (a)	11. (c)	12. (d)
13. (a)	14. (b)	15. (d)
16. (c)	17. (a)	18. (c)
19. (b)	20. (a)	21. (a)
22. (d)	23. (a)	24. (b)
25. (b)	26. (c)	27. (c)
28. (d)	29. (a)	30. (d)
31. (a)	32. (d)	33. (d)

LEVEL - 2

1. (a)	2. (c)	3. (b)
4. (d)	5. (b)	6. (a)
7. (b)	8. (c)	9. (d)
10. (d)	11. (b)	12. (a)
13. (c)	14. (b)	15. (a)
16. (b)	17. (c)	18. (a)
19. (c)	20. (a)	

LEVEL - 3

1. (a)	2. (d)	3. (b)
4. (c)	5. (b)	6. (d)
7. (a)	8. (c)	9. (b)