Chapter 3 Systems of Linear Equations and Inequalities

Ex 3.4

Answer 1e.

We know that a linear equation in three variables $x, y,$ and z is an equation of the form $ax + by + cz = d$ where a, b, and c are not all zero.
Thus, a linear equation in three variables can be $2x - 3y + z = 6$.
The graph of a linear equation in three variables is a plane in three-dimensional space.

Answer 1gp.

Number the equations.

$$3x + y - 2z = 10 \quad \text{Equation 1}$$
$$6x - 2y + z = -2 \quad \text{Equation 2}$$
$$x + 4y + 3z = 7 \quad \text{Equation 3}$$

STEP 1 Rewrite the system as a linear system in two variables.
Add 2 times Equation 1 to Equation 2.

$$2 \times \text{Equation 1} \Rightarrow 6x + 2y - 4z = 20$$
$$\underline{6x - 2y + z = -2}$$
$$12x - 3z = 18 \quad \text{New Equation 1}$$

Add 2 times Equation 2 to Equation 3.

$$2 \times \text{Equation 2} \Rightarrow 12x - 4y + 2z = -4$$
$$\underline{x + 4y + 3z = 7}$$
$$13x + 5z = 3 \quad \text{New Equation 2}$$

STEP 2 Solve the new linear system for both of its variables.
Add 5 times new equation 1 to 3 times new equation 2.

$$5 \times \text{New equation 1} \Rightarrow 60x - 15z = 90$$
$$3 \times \text{New equation 2} \Rightarrow \underline{39x + 15z = 9}$$
$$99x = 99$$

Divide both the sides by 99.

$$\frac{99x}{99} = \frac{99}{99}$$
$$x = 1$$

STEP 3 Substitute 1 for x in new equation 2 and solve for z.

$$\begin{aligned} 13(1) + 5z &= 3 \\ 13 + 5z &= 3 \\ 5z &= -10 \\ z &= -2 \end{aligned}$$

Substitute 1 for x and -2 for z in $3x + y - 2z = 10$ and solve for y.

$$\begin{aligned} 3(1) + y - 2(-2) &= 10 \\ 3 + y + 4 &= 10 \\ y + 7 &= 10 \\ y &= 3 \end{aligned}$$

Therefore, the ordered triple is (1, 3, –2).

Answer 1mr.

a. Let x be the number of necklaces and bond and y be the number of bracelets.

Write a verbal model for the total amount spent on materials.

Cost of materials to make x necklaces	+	Cost of materials to make y bracelets	=	Total amount spent on materials
⇓		⇓		⇓
$3.5x$	+	$2.5y$	=	121

The required equation is $3.5x + 2.5y = 121$.

Now, write a verbal model for the total amount spent on jewelry.

Amount spent on selling x necklaces	+	Amount spent on selling y bracelets	=	Total amount spent on jewelry
⇓		⇓		⇓
$9x$	+	$7.5y$	=	324

The required equation is $9x + 7.5y = 324$.

Thus, the system of equations that represents the given situation is

$3.5x + 2.5y = 121$

$9x + 7.5y = 324$.

b. Number the equations.

$$3.5x + 2.5y = 121 \quad \text{Equation 1}$$
$$9x + 7.5y = 324 \quad \text{Equation 2}$$

Solve the system using the substitution method. For this, subtract $3.5x$ from both sides of Equation 1.

$$3.5x + 2.5y - 3.5x = 121 - 3.5x$$
$$2.5y = 121 - 3.5x$$

Divide both the sides by 2.5.

$$\frac{2.5y}{2.5} = \frac{121 - 3.5x}{2.5}$$
$$y = \frac{121 - 3.5x}{2.5} \quad \text{Revised Equation 1}$$

Substitute for y in Equation 2 and evaluate.

$$9x + 7.5\left(\frac{121 - 3.5x}{2.5}\right) = 324$$
$$9x + 3(121 - 3.5x) = 324$$
$$9x + 363 - 10.5x = 324$$

Solve for x.

$$-1.5x = -39$$
$$x = 26$$

Substitute 26 for x in Revised Equation 1 and solve for y.

$$y = \frac{121 - 3.5(26)}{2.5}$$
$$= \frac{30}{2.5}$$
$$= 12$$

Therefore. 26 necklaces and 12 bracelets were sold.

Answer 2e.

To solve a system of three linear equations in three variables, by using substitution method as follows:

Step (1): Solve one of the three equations for one of its variables as a combination of other variables.

Step (2): Substitute the expression from step (1) in the other two equations.
Then the system converts into the system of two variables.
Now solve the system of two variables.

Step (3): Substitute the values two variables obtained in step (2) in the revised equation from step (1), to solve for the third variable.

Answer 2gp.

Consider the following system,

$$x + y - z = 2 \qquad \ldots\ldots (1)$$
$$2x + 2y - 2z = 6 \qquad \ldots\ldots (2)$$
$$5x + y - 3z = 8 \qquad \ldots\ldots (3)$$

Step 1: Use elimination method to convert the three variables linear system as a two variables linear system as follows:

First multiply the equation (1) by -2, and, add the result to equation (2), then,

Multiply Equation (1), by -2, then,

$$x + y - z = 2$$
$$(-2)(x + y - z) = (-2)(2)$$
$$-2x - 2y + 2z = -4$$

Add the new equation to equation (2), then,

$$-2x - 2y + 2z = -4$$
$$\underline{2x + 2y - 2z = 6}$$
$$0 = 2$$

Since, $0 = 2$ is a false statement, so, conclude that the original system has no solution.

Answer 3e.

Substitute 1 for x, 4 for y, and -3 for z in the first equation and check.

$$2(1) - 4 + (-3) \stackrel{?}{=} -5$$
$$2 - 4 - 3 \stackrel{?}{=} -5$$
$$-5 = -5 \quad \checkmark$$

The given ordered triple satisfies the first equation.

Similarly, check the ordered triple in the second and third equations.

$$5(1) + 2(4) - 2(-3) \stackrel{?}{=} 19 \qquad 1 - 3(4) + (-3) \stackrel{?}{=} -5$$
$$5 + 8 + 6 \stackrel{?}{=} 19 \qquad 1 - 12 - 3 \stackrel{?}{=} 19$$
$$19 = 19 \quad \checkmark \qquad -14 \stackrel{?}{=} 19 \quad \times$$

Since the ordered triple does not satisfy the three equations, it is not a solution of the system.

Answer 3gp.

Number the equations.

$$\begin{aligned} x + y + z &= 3 \quad \text{Equation 1} \\ x + y - z &= 3 \quad \text{Equation 2} \\ 2x + 2y + z &= 6 \quad \text{Equation 3} \end{aligned}$$

STEP 1 Rewrite the system as a linear system in two variables.
Add Equation 1 to Equation 2.

$$\begin{aligned} x + y + z &= 3 \\ \underline{x + y - z} &= \underline{3} \\ 2x + 2y &= 6 \quad \text{New Equation 1} \end{aligned}$$

Add Equation 2 to Equation 3.

$$\begin{aligned} x + y - z &= 3 \\ \underline{2x + 2y + z} &= \underline{6} \\ 3x + 3y &= 9 \quad \text{New Equation 2} \end{aligned}$$

STEP 2 Solve the new linear system for both of its variables.
Add 3 times new equation 1 to 2 times new equation 2.

$$\begin{aligned} 3 \times \text{New equation } 1 \Rightarrow \quad 6x + 6y &= 18 \\ -2 \times \text{New equation } 2 \Rightarrow \underline{-6x - 6y} &= \underline{-9} \\ 0 &= 9 \end{aligned}$$

The equation obtained is false. Therefore, the given system of equations has no solution.

Answer 3mr.

Assume that $x = 0$ and $y = 0$ represent two sides of a right triangle. We have to find the hypotenuse.

The hypotenuse or the third side of the right triangle can be found by selecting any two points and finding the equation of the line that passes through the points.

Now, assume that the two points (1, 2) and (0, 4) lie on a line. Find the slope of the line first.

$$\begin{aligned} m &= \frac{4-2}{0-1} \\ &= \frac{2}{-1} \\ &= -2 \end{aligned}$$

Now, find the equation of the line. For this, substitute −2 for m, 0 for x_1, and 4 for y_1 in $y - y_1 = m(x - x_1)$.

$$y - 4 = -2(x - 0)$$

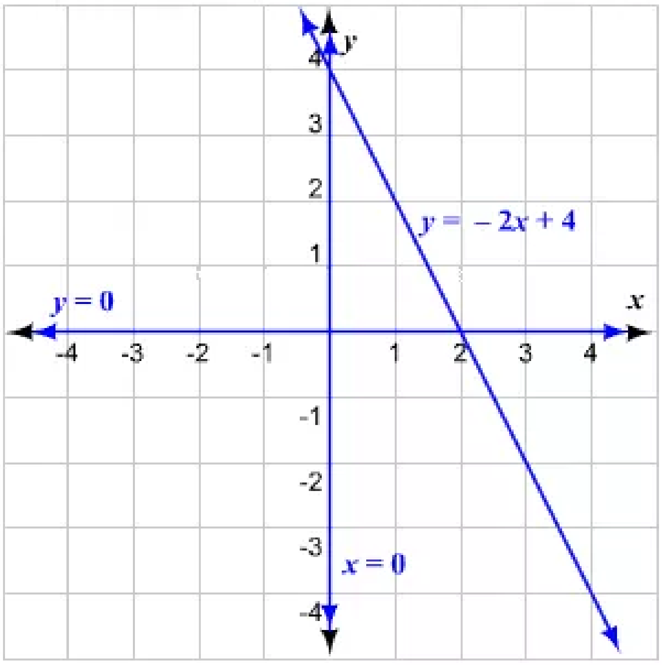
$$y - 4 = -2x$$

Add 4 to both the sides to rewrite the equation in slope-intercept form.

$$y - 4 + 4 = -2x + 4$$

$$y = -2x + 4$$

Graph the three equations on the same coordinate plane.



Choose a test point inside the right triangle, say, (1, 1). Now, substitute the point on the original equations to find the inequality symbol.

Consider the equation $x = 0$. Substitute 1 for x.

$1 = 0$

We know that 1 is always greater than or equal to zero. For the statement to be true, the inequality should be $\geq$. Thus, we get an inequality as $x \geq 0$.

Similarly, check for the other equations. We get the inequalities as $y \geq 0$ and $y \leq -2x + 4$.

Thus, a system of linear inequalities whose graph is the interior of a right triangle is

$x \geq 0$

$y \geq 0$

$y \leq -2x + 4$.

Answer 4e.

Consider the following system,

$$4x - y + 3z = 13 \quad \ldots\ldots (1)$$

$$x + y + z = 2 \quad \ldots\ldots (2)$$

$$x + 3y - 2z = -17 \quad \ldots\ldots (3)$$

And, the ordered triple $(-1,-2,5)$.

Next, check the ordered triple $(-1,-2,5)$ is a solution of the system or not.

To verify, substitute $(-1,-2,5)$ in each of the three equations, and, if the obtained result is true in each of the three equations, then, the ordered triple $(-1,-2,5)$ is a solution of the system.

Case 1:

Substitute $(x, y, z) = (-1,-2,5)$ in equation (1), then,

$$4x - y + 3z = 13$$

$$4(-1) - (-2) + 3(5) \overset{?}{=} 13 \qquad \text{Substitute } -1 \text{ for } x,\ -2 \text{ for } y, \text{ and, } 5 \text{ for } z$$

$$-4 + 2 + 15 \overset{?}{=} 13 \qquad \text{Multiply}$$

$$13 = 13 \qquad \text{True}$$

Case 2:

Substitute $(x, y, z) = (-1,-2,5)$ in equation (2), then,

$$x + y + z = 2$$

$$(-1) + (-2) + 5 \overset{?}{=} 2 \qquad \text{Substitute } -1 \text{ for } x,\ -2 \text{ for } y, \text{ and, } 5 \text{ for } z$$

$$-1 - 2 + 5 \overset{?}{=} 2 \qquad \text{Multiply}$$

$$2 = 2 \qquad \text{True}$$

Case 3:

Substitute $(x, y, z) = (-1,-2,5)$ in equation (3), then,

$$x + 3y - 2z = -17$$

$$(-1) + 3(-2) - 2(5) \overset{?}{=} -17 \qquad \text{Substitute } -1 \text{ for } x,\ -2 \text{ for } y, \text{ and, } 5 \text{ for } z$$

$$-1 - 6 - 10 \overset{?}{=} -17 \qquad \text{Simplify}$$

$$-17 = -17 \qquad \text{True}$$

Since, in the three cases the obtained result is true.

Therefore, the ordered triple $(-1,-2,5)$ is the solution of system.

Answer 4gp.

Consider the monthly budget of a marketing department of a company is \$25,000, for advertising 60 ads per month.

a) The cost for television ad is \$1000, radio ad is \$200, and, newspaper ad \$500.

b) Also, have as many radio ads as television and newspaper ads combined.

First write a system of equations.

Let,

x, be the number of TV ads,

y, be the number of radio ads, and,

z, be the number of newspaper ads.

From the data, form a system of equation as follows:

Since, the total number of ads per month is 60, so,

$$x+y+z=60 \quad \ldots\ldots (1)$$

And, from the condition (a), write an equation as,

$$1000x+200y+500z=25,000 \quad \ldots\ldots (2)$$

And, from the condition (b), write an equation as,

$$y=x+z \quad \ldots\ldots (3)$$

Now find the number of each type of ads that the marketing department runs each month.

Step 1: Use substitution method to convert the three variables linear system as a two variables linear system as follows:

Substitute the expression $x+z$, in equations (1), and, (2), to convert the system into a linear system in two variables.

Take equation (1):

$$x+y+z=60 \quad \text{Equation (1)}$$

$$x+(x+z)+z=60 \quad \text{Substitute } x+z \text{ for } y$$

$$2x+2z=60 \quad \ldots\ldots (4)$$

Next take equation (2):

$$1000x+200y+500z=25,000 \quad \text{Equation (2)}$$

$$1000x+200(x+z)+500z=25,000 \quad \text{Substitute } x+z \text{ for } y$$

$$1000x+200x+200z+500z=25,000$$

$$1200x+700z=25,000 \quad \ldots\ldots (5)$$

Step 2: Now solve the system in two variables:

First multiply equation (4) by -600,

$$2x+2z=60$$

$$(-600)(2x+2z)=(-600)(60)$$

$$-1200x-1200z=-36000 \quad \text{New equation 1}$$

Add the new equation 1, and, equation (5), then,

$$\begin{aligned} -1200x-1200z &= -36,000 \\ 1200x+\ \ 700z &= \ \ 25,000 \\ \hline -\ \ 500z &= -11,000 \\ z &= 22 \end{aligned} \quad \text{Divide both sides by } -500$$

Therefore,

$\boxed{z=22}$.

Substitute 22 for z in equation (4) or (5), and solve for x, as follows:

$2x+2z=60$	Equation (4)
$2x+2(22)=60$	Substitute 22 for z
$2x+44=60$	Multiply
$2x=16$	Subtract 44 from both sides
$\boxed{x=8}$	Divide both sides by 2

Therefore,

$\boxed{x=8}$.

Step 3: To solve for y, substitute 8 for x, and, 22 for z, in the revised equation (3), then,

$y=x+z$	
$=8+22$	Substitute 8 for x, 22 for z
$=30$	Add

Therefore,

$\boxed{y=30}$.

Hence, the department should run $\boxed{8}$ TV ads, $\boxed{30}$ radio ads, and, $\boxed{22}$ newspaper ads each month.

Answer 5e.

Substitute 6 for x, 0 for y, and -3 for z in the first equation and check.

$$6+4(0)-2(-3) \stackrel{?}{=} 12$$

$$6+0+6 \stackrel{?}{=} 12$$

$$12 = 12 \quad \checkmark$$

The given ordered triple satisfies the first equation.

Similarly, check the ordered triple in the second and third equations.

$$3(6) - 0 + 4(-3) \stackrel{?}{=} 6 \qquad -6 + 3(0) + (-3) \stackrel{?}{=} -9$$

$$18 - 0 - 12 \stackrel{?}{=} 6 \qquad -6 + 0 - 3 \stackrel{?}{=} -9$$

$$6 = 6 \checkmark \qquad -9 = -9 \checkmark$$

Since the ordered triple satisfies the three equations, it is a solution of the system.

Answer 5mr.

Write verbal models for the given situation.

Cost of one soda + Cost of one pretzel + Cost of two hot dogs = 7

Cost of two soda + Cost of one pretzel + Cost of two hot dogs = 8

Cost of one soda + Cost of four hot dogs = 10

Now, translate the verbal models to equations. Let x represent the cost of a soda, y represent the cost of a pretzel, and z represent the cost of a hot dog.

$$x + y + 2z = 7 \qquad \text{Equation 1}$$

$$2x + y + 2z = 8 \qquad \text{Equation 2}$$

$$x + 4z = 10 \qquad \text{Equation 3}$$

Solve the system of equations. Add -1 times Equations 1 to 2 to eliminate z.

$$\begin{array}{lcl} x + y + 2z = 7 & \xrightarrow{\times\,-1} & -x - y - 2z = -7 \\ 2x + y + 2z = 8 & & \underline{2x + y + 2z = 8} \\ & & x \qquad\qquad = 1 \end{array}$$

Substitute 1 for x in Equation 3 and solve for z.

$$4z = 9$$

$$z = \frac{9}{4}$$

$$= 2.25$$

Thus, the cost of one hot dog is 2.25 dollars.

Answer 6e.

Consider the following system,

$$3x+4y-2z=-11 \quad \ldots\ldots (1)$$

$$2x+y-z=11 \quad \ldots\ldots (2)$$

$$x+4y+3z=-1 \quad \ldots\ldots (3)$$

And, the ordered triple $(-5,1,0)$.

Next, check the ordered triple $(-5,1,0)$ is a solution of the system or not.

To verify, substitute $(-5,1,0)$ in each of the three equations, and, if the obtained result is true in each of the three equations, then, the ordered triple $(-5,1,0)$ is a solution of the system.

Case 1:

Substitute $(x,y,z)=(-5,1,0)$ in equation (1), then,

$$3x+4y-2z=-11$$

$$3(-5)+4(1)-2(0)\overset{?}{=}-11 \quad \text{Substitute } -5 \text{ for } x,\ 1 \text{ for } y, \text{ and, } 0 \text{ for } z$$

$$-15+4-0\overset{?}{=}-11 \quad \text{Multiply}$$

$$-11=-11 \quad \text{True}$$

Case 2:

Substitute $(x,y,z)=(-5,1,0)$ in equation (2), then,

$$2x+y-z=11$$

$$2(-5)+(1)-(0)\overset{?}{=}11 \quad \text{Substitute } -5 \text{ for } x,\ 1 \text{ for } y, \text{ and, } 0 \text{ for } z$$

$$-10+1-0\overset{?}{=}11 \quad \text{Multiply}$$

$$-9=11 \quad \text{True}$$

Since, in the case 2, the obtained result $(-9=11)$ is false.

So, the ordered triple $(-5,1,0)$ does not satisfy the equation (2).

Therefore, the ordered triple $(-5,1,0)$ is not the solution of system.

Answer 7e.

Substitute 2 for x, 8 for y, and 4 for z in the first equation and check.

$$3(2)-8+5(4)\overset{?}{=}34$$

$$6-8+20\overset{?}{=}34$$

$$18=34 \quad \times$$

The given ordered triple does not satisfy the first equation.

Similarly, check the ordered triple in the second and third equations.

$$2+3(8)-6(4)\stackrel{?}{=}2 \qquad -3(2)+8-2(4)\stackrel{?}{=}-6$$

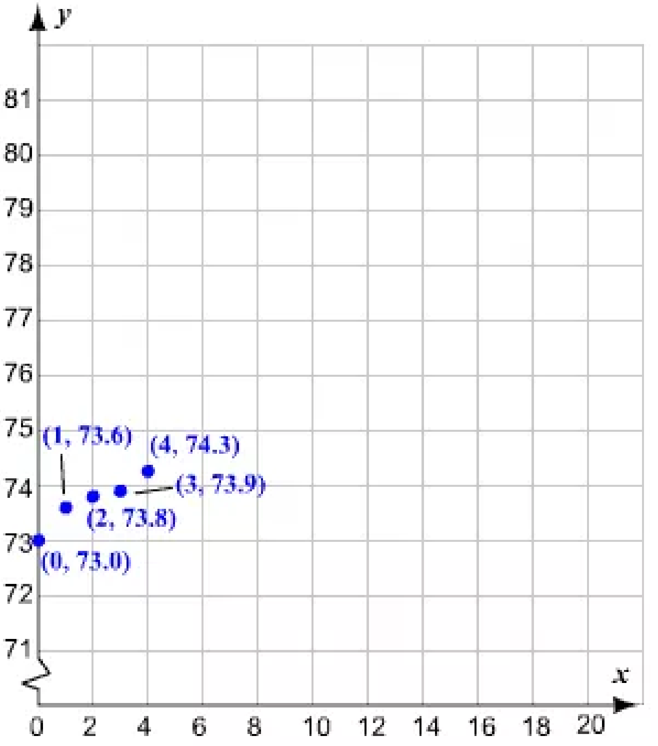
$$2+24-24\stackrel{?}{=}2 \qquad -6+8-8\stackrel{?}{=}-6$$

$$2\stackrel{?}{=}2 \quad \checkmark \qquad -6\stackrel{?}{=}-6 \quad \checkmark$$

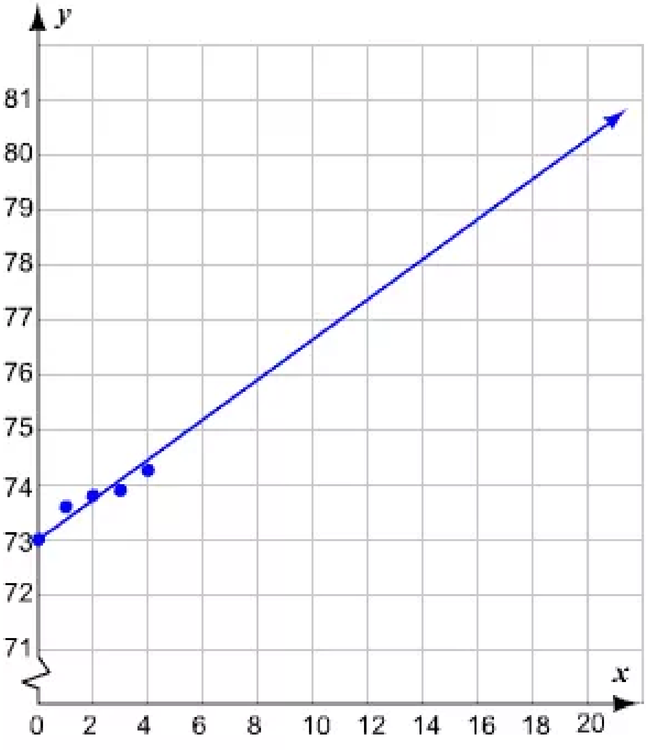
Since the ordered triple does not satisfy the three equations, it is not a solution of the system.

Answer 7mr.

a. The given data for (x, m)can be represented as (0, 73.0), (1, 73.6), (2, 73.8), (3, 73.9), and (4, 74.3). Plot the above points on a graph to get the scatter plot.



The line that follows the trends given by the data points most closely is called a best-fitting line. Sketch the line that best fits the data.



Choose two data points that appear to lie on the line. Let the points be (0, 73.0) and (2, 73.8). Find the slope, m, using these points.

$$m = \frac{73.8 - 73.0}{2 - 0}$$

$$= \frac{0.8}{2}$$

$$= 0.4$$

The point slope form of an equation is $y - y_1 = m(x - x_1)$. Choose (0, 73.0) as the point (x_1, y_1).

$y - 73.0 = 0.4(x - 0)$

Use the distributive property to open the parentheses.

$y - 73.0 = 0.4x$

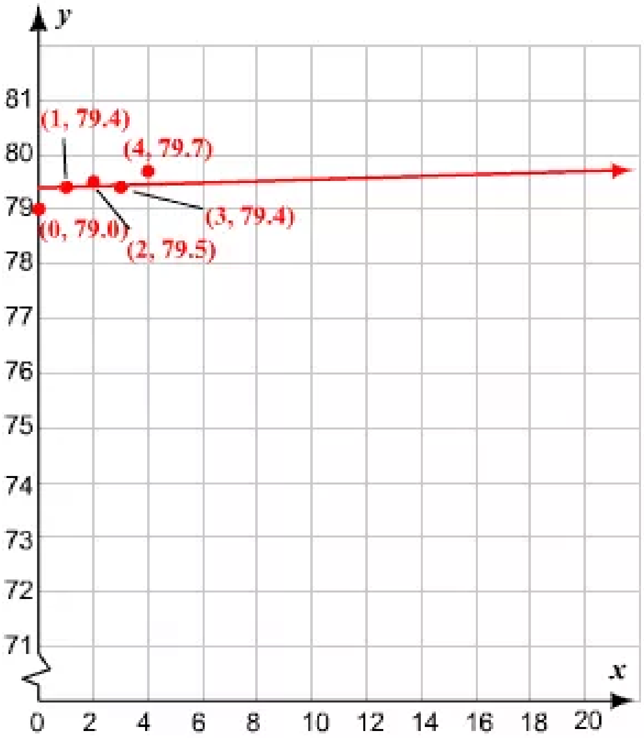
Add 73.0 to both sides of the equation.

$$y - 73.0 + 73.0 = 0.4x + 73.0$$

$$y = 0.4x + 73.0$$

An approximation of the best fitting line for the data pairs is $y = 0.4x + 73.0$.

b. The given data for (x, w) can be represented as (0, 79.0), (1, 79.4), (2, 79.5), (3, 79.4), and (4, 79.7). Plot the above points on a graph to get the scatter plot and sketch the line that best fits the data.



Choose two data points that appear to lie on the line. Let the points be (1, 79.4) and (3, 79.4). Find the slope, m, using these points.

$$m = \frac{79.4 - 79.4}{3 - 1}$$

$$= \frac{0}{2}$$

$$= 0$$

The point slope form of an equation is $y - y_1 = m(x - x_1)$. Choose (1, 79.4) as the point (x_1, y_1).

Substitute 0 for m, 1 for x_1, and 79.4 for y_1 in the above equation.

$y - 79.4 = 0(x - 1)$

Simplify.

$y - 79.4 = 0$

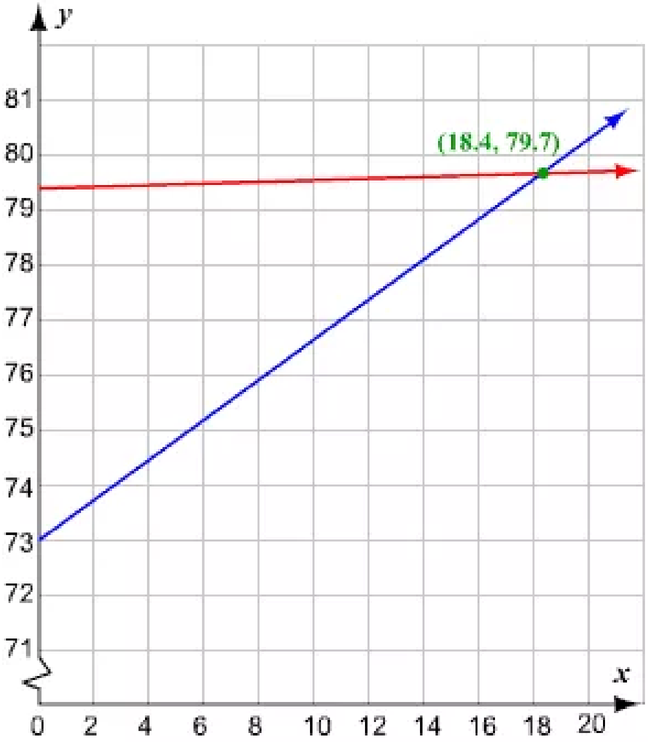
Add 79.4 to both the sides.

$$y - 79.4 + 79.4 = 79.4$$

$$y = 79.4$$

Thus, an approximation of the best fitting line for the data pairs (x, w) is $y = 79.4$.

c. Plot the best fit line for data pairs (x, w) and that for the data pairs (x, m) on the same coordinate plane and find the intersection point.



From the figure, we get the intersection point as (18.4, 79.7). Since $x = 0$ represents the year 1996, $x = 18.4$ represents the year 2014.

Thus, the expected life span at birth for men and women will be the same at the year 2014.

Answer 8e.

Consider the following system,

$$2x+4y-z=-23 \quad \ldots\ldots (1)$$
$$x-5y-3z=-1 \quad \ldots\ldots (2)$$
$$-x+y+4z=24 \quad \ldots\ldots (3)$$

And, the ordered triple $(0,-4,7)$.

Next, check the ordered triple $(0,-4,7)$ is a solution of the system or not.

To verify, substitute $(0,-4,7)$ in each of the three equations, and, if the obtained result is true in each of the three equations, then, the ordered triple $(0,-4,7)$ is a solution of the system.

Case 1:

Substitute $(x,y,z)=(0,-4,7)$ in equation (1), then,

$$2x+4y-z=-23$$
$$2(0)+4(-4)-7\overset{?}{=}-23 \quad \text{Substitute 0 for } x,\ -4 \text{ for } y, \text{ and, 7 for } z$$
$$0-16-7\overset{?}{=}-23 \quad \text{Multiply}$$
$$-23=-23 \quad \text{True}$$

Case 2:

Substitute $(x,y,z)=(0,-4,7)$ in equation (2), then,

$$x-5y-3z=-1$$

$(0)-5(-4)-3(7)\overset{?}{=}-1$	Substitute 0 for x, -4 for y, and, 7 for z
$0+20-21\overset{?}{=}-1$	Multiply
$-1=-1$	True

Case 3:

Substitute $(x,y,z)=(0,-4,7)$ in equation (3), then,

$$-x+y+4z=24$$

$-(0)+(-4)+4(7)\overset{?}{=}24$	Substitute 0 for x, -4 for y, and, 7 for z
$-0-4-28\overset{?}{=}24$	Simplify
$24=24$	True

Since, in the three cases the obtained result is true.

Therefore, the ordered triple $(0,-4,7)$ is the solution of system.

Answer 9e.

Number the equations.

$3x+y+z=14$	Equation 1
$-x+2y-3z=-9$	Equation 2
$5x-y+5z=30$	Equation 3

STEP 1 Rewrite the system as a linear system in two variables.

Add 3 times Equation 2 to Equation 1.

$$3x+y+z=14$$

$$3\times \text{Equation 2} \Rightarrow -3x+6y-9z=-27$$

$$7y-8z=-13 \quad \text{New Equation 1}$$

Add 5 times Equation 2 to Equation 3.

$$5\times \text{Equation 2} \Rightarrow -5x+10y-15z=-45$$

$$5x-y+5z=30$$

$$9y-10z=-15 \quad \text{New Equation 2}$$

STEP 2 Solve the new linear system for both of its variables.

Add 9 times new equation 1 to -7 times new equation 2.

$$63y-72z=-117$$

$$-63y+70z=105$$

$$-2z=-12$$

Divide both the sides by -2.

$$\frac{-2z}{-2}=\frac{-12}{-2}$$

$$z=6$$

Substitute 6 for z in any of the new equations, say, new equation 2 to find the value of y.

$$9y - 10(6) = -15$$
$$9y - 60 = -15$$
$$9y = 45$$
$$y = 5$$

STEP 3 Substitute 6 for z and 5 for y in any of the original equations, say, equation 1 to find the value of x.

$$3x + 5 + 6 = 14$$
$$3x + 11 = 14$$
$$3x = 3$$
$$x = 1$$

Check the values of x, y, and z in each of the original equations.

$$3(1) + 5 + 6 \stackrel{?}{=} 14 \qquad -1 + 2(5) - 3(6) \stackrel{?}{=} -9 \qquad 5(1) - 5 + 5(6) \stackrel{?}{=} 30$$
$$3 + 5 + 6 \stackrel{?}{=} 14 \qquad -1 + 10 - 18 \stackrel{?}{=} -9 \qquad 5 - 5 + 30 \stackrel{?}{=} 30$$
$$14 \stackrel{?}{=} 14 \checkmark \qquad -9 \stackrel{?}{=} -9 \checkmark \qquad 30 \stackrel{?}{=} 30 \checkmark$$

The solutions check.
Therefore, the ordered triple is (1, 5, 6).

Answer 10e.

Consider the following system,

$$2x - y + 2z = -7 \quad \ldots\ldots (1)$$
$$-x + 2y - 4z = 5 \quad \ldots\ldots (2)$$
$$x + 4y - 6z = -1 \quad \ldots\ldots (3)$$

Step 1: Use elimination method to convert the three variables linear system as a two variables linear system as follows:
First multiply equation (3) by -2, and, add the resultant new equation to the equation (1), as follows:
Multiply Equation (3), by -2, then,

$$x + 4y - 6z = -1$$
$$(-2)(x + 4y - 6z) = (-2)(-1)$$
$$-2x - 8y + 12z = 2$$

Add the resultant equation to equation (1), then,

$$2x - y + 2z = -7$$
$$-2x - 8y + 12z = 2$$
$$-9y + 14z = -5 \quad \ldots\ldots (4)$$

Next add the equations (2), and, (3), then,

$$-x + 2y - 4z = 5$$
$$x + 4y - 6z = -1$$
$$6y - 10z = 4$$

Divide both sides by 2, then,

$$3y - 5z = 2 \quad \ldots\ldots (5)$$

Step 2: Next solve the new linear system for both of its variables.
Multiply the equation (5) by 3, and, add the resultant new equation to the equation (4), as follows:
Multiply Equation (5), by 3, then,

$$3y-5z=2$$
$$(3)(3y-5z)=(3)(2)$$
$$9y-15z=6$$

Add the resultant equation to equation (4), then,

$$\begin{array}{r} -9y+14z=-5 \\ 9y-15z=6 \\ \hline -z=1 \end{array}$$

Therefore,

$$\boxed{z=-1}.$$

To solve for y, substitute $z=-1$ in the equation (5), then,

$3y-5z=2$	
$3y-5(-1)=2$	Substitute -1 for z
$3y+5=2$	
$3y=-3$	Subtract 5 from both sides
$\boxed{y=-1}$	Divide both sides by 3

Step 3:
Now substitute $y=-1$, and, $z=-1$ in an original equation, say (3), and solve for x.

$x+4y-6z=-1$	Equation (3)
$x+4(-1)-6(-1)=-1$	Substitute -1 for y, and, -1 for z
$x-4+6=-1$	Simplify
$x+2=-1$	Combine the like terms
$\boxed{x=-3}$	Subtract 2 from both sides

Therefore, the solution set as ordered triple is $\boxed{(-3,-1,-1)}$.

Answer 11e.

Number the equations.

$3x - y + 2z = 4$	Equation 1
$6x - 2y + 4z = -8$	Equation 2
$2x - y + 3z = 10$	Equation 3

STEP 1 Rewrite the system as a linear system in two variables.
Add −2 times Equation 1 to Equation 2.

$$
\begin{aligned}
-2 \times \text{Equation } 2 \Rightarrow -6x + 2y - 4z &= -8 \\
6x - 2y + 4z &= -8 \\
\hline
0 &= -16
\end{aligned}
$$

The equation obtained is false. Therefore, the given system of equations has no solution.

Answer 12e.

Consider the following system,

$$4x-y+2z=-18 \qquad \ldots\ldots (1)$$

$$-x+2y+z=11 \qquad \ldots\ldots (2)$$

$$3x+3y-4z=44 \qquad \ldots\ldots (3)$$

Step 1: Use elimination method to convert the three variables linear system as a two variables linear system as follows:

First multiply equation (2) by 4, and, add the resultant new equation to the equation (1), as follows:

Multiply Equation (2), by 4, then,

$$-x+2y+z=11$$

$$(4)(-x+2y+z)=(4)(11)$$

$$-4x+8y+4z=44$$

Add the resultant equation to equation (1), then,

$$
\begin{aligned}
4x-\ \ y+2z&=-18 \\
-4x+8y+4z&=44 \\
\hline
7y+6z&=26
\end{aligned}
$$

$$7y+6z=26 \qquad \ldots\ldots (4)$$

Next, multiply equation (2) by 3, and, add the resultant new equation to the equation (3), as follows:

Multiply Equation (2), by 3, then,

$$-x+2y+z=11$$

$$(3)(-x+2y+z)=(3)(11)$$

$$-3x+6y+3z=33$$

Add the resultant equation to equation (1), then,

$$
\begin{aligned}
-3x+6y+3z&=33 \\
3x+3y-4z&=44 \\
\hline
9y-\ \ z&=77
\end{aligned}
$$

$$9y-z=77 \qquad \ldots\ldots (5)$$

Step 2: Next solve the new linear system for both of its variables.
Multiply the equation (5) by 6, and, add the resultant new equation to the equation (4), as follows:

Multiply Equation (5), by 6, then,

$$9y - z = 77$$
$$(6)(9y - z) = (6)(77)$$
$$54y - 6z = 462$$

Add the resultant equation to equation (4), then,

$$\begin{array}{l} 7y + 6z = 26 \\ \underline{54y - 6z = 462} \\ 61y \qquad = 488 \\ \qquad y = 8 \end{array}$$ Divide both sides by 61

Therefore,

$$\boxed{y = 8}.$$

To solve for z, substitute $y = 8$ in the equation (5), then,

$$9y - z = 77$$
$$9(8) - z = 77$$ Substitute 8 for y
$$72 - z = 77$$
$$-z = 6$$ Subtract 72 from both sides
$$\boxed{z = -6}$$ Multiply both sides by -1

Step 3: Now substitute $y = 8$, and, $z = -5$ in an original equation, say (2), and solve for x.

$$-x + 2y + z = 11$$ Equation (2)
$$-x + 2(8) + (-5) = 11$$ Substitute 8 for y, and, -5 for z
$$-x + 16 - 5 = 11$$ Simplify
$$-x + 11 = 11$$
$$-x = 0$$ Combine the like terms
$$\boxed{x = 0}$$ Multiply both sides by -1

Therefore, the solution set as ordered triple is $\boxed{(0, 8, -5)}$.

Answer 13e.

Number the equations.

$5x + y - z = 6$ Equation 1

$x + y + z = 2$ Equation 2

$3x + y = 4$ Equation 3

STEP 1 Rewrite the system as a linear system in two variables.

Add −1 times Equation 1 to Equation 3.

$$
\begin{array}{rl}
-1 \times \text{Equation } 1 \Rightarrow & -5x - y + z = -6 \\
& \underline{3x + y \qquad = 4} \\
& -2x \qquad + z = -2 \quad \text{New Equation 1}
\end{array}
$$

Add −1 times Equation 2 to Equation 3.

$$
\begin{array}{rl}
-1 \times \text{Equation } 2 \Rightarrow & -x - y - z = -2 \\
& \underline{3x + y \qquad = 4} \\
& 2x \qquad - z = 2 \quad \text{New Equation 2}
\end{array}
$$

STEP 2 Solve the new linear system for both of its variables.

Add the new equation 1 to new equation 2.

$$
\begin{array}{r}
-2x + z = -2 \\
\underline{2x - z = 2} \\
0 = 0
\end{array}
$$

The equation obtained is an identity $0 = 0$. Therefore, the system has infinitely many solutions.

STEP 3 Describe the solutions of the system.

Solve new equation 1 for z.

$z = 2x - 2$

Substitute this value in equation 1 to get the value of y.

$$
\begin{aligned}
5x + y - (2x - 2) &= 6 \\
5x + y - 2x + 2 &= 6 \\
3x + y &= 4 \\
y &= 4 - 3x
\end{aligned}
$$

Therefore, any ordered triple of the form $(x, 4 - 3x, 2x - 2)$ is a solution of the given system.

Answer 14e.

Consider the following system,

$$2x+y-z=9 \qquad \ldots\ldots (1)$$
$$-x+6y+2z=-17 \qquad \ldots\ldots (2)$$
$$5x+7y+z=4 \qquad \ldots\ldots (3)$$

Step 1: Use elimination method to convert the three variables linear system as a two variables linear system as follows:

First multiply equation (1) by 2, and, add the resultant new equation to the equation (2), as follows:

Multiply Equation (1), by 2, then,

$$2x+y-z=9$$
$$(2)(2x+y-z)=(2)(9)$$
$$4x+2y-2z=18$$

Add the resultant equation to equation (2), then,

$$\begin{array}{r} 4x+2y-2z=\ 18 \\ -x+6y+2z=-17 \\ \hline 3x+8y \quad = \quad 1 \end{array}$$

$$3x+8y=1 \qquad \ldots\ldots (4)$$

Next, add equation (1), and, equation (3), as follows:

$$\begin{array}{r} 2x+\ y-\ z=9 \\ 5x+7y+\ z=4 \\ \hline 7x+8y \quad =13 \end{array}$$

$$7x+8y=13 \qquad \ldots\ldots (5)$$

Step 2: Next solve the new linear system for both of its variables.

Multiply the equation (4) by -1, and, add the resultant new equation to the equation (5), as follows:

Multiply Equation (4), by -1, then,

$$3x+8y=1$$
$$(-1)(3x+8y)=(-1)(1)$$
$$-3x-8y=-1$$

Add the resultant equation to equation (5), then,

$$\begin{array}{r} -3x-8y=-1 \\ 7x+8y=\ 13 \\ \hline 4x \quad = \ 12 \end{array}$$

$$x=3 \qquad \text{Divide both sides by 4}$$

Therefore,

$$\boxed{x=3}.$$

To solve for y, substitute $x=3$ in the equation (5), then,

$$3x+8y=1 \qquad \text{Substitute 8 for } y$$
$$3(3)+8y=1$$
$$9+8y=1 \qquad \text{Multiply}$$
$$8y=-8 \qquad \text{Subtract 9 from both sides}$$
$$\boxed{y=-1} \qquad \text{Divide both sides by 8}$$

Step 3: Now substitute $x=3$, and, $y=-1$ in an original equation, say (2), and solve for z.

$-x+6y+2z=-17$	Equation (2)
$-3+6(-1)+2z=-17$	Substitute 3 for x, and, -1 for y
$-3-6+2z=-17$	Simplify
$-9+2z=-17$	Combine the like terms
$2z=-8$	Add 9 on both sides
$\boxed{z=-4}$	Divide both sides by 2

Therefore, the solution set as ordered triple is $\boxed{(3,-1,-4)}$.

Answer 15e.

Number the equations.

$x + y - z = 4$ Equation 1

$3x + 2y + 4z = 17$ Equation 2

$-x + 5y + z = 8$ Equation 3

Solve equation 1 for any one variable, say, x.

$x = z - y + 4$ New equation 1

Substitute the expression for x in equation 2 and simplify to isolate y.

$$3(z - y + 4) + 2y + 4z = 17$$
$$3z - 3y + 12 + 2y + 4z = 17$$
$$12 - y + 7z = 17$$

Subtract 12 from both the sides.

$$12 - y + 7z - 12 = 17 - 12$$
$$7z - y = 5 \quad \text{New equation 2}$$

Substitute $z - y + 4$ for x in equation 3 and simplify.

$$-(z - y + 4) + 5y + z = 8$$
$$-z + y - 4 + 5y + z = 8$$
$$6y - 4 = 8$$

Add 4 to both the sides.

$$6y - 4 + 4 = 8 + 4$$
$$6y = 12$$

Divide both the sides by 6.

$$\frac{6y}{6} = \frac{12}{6}$$
$$y = 2$$

Substitute 2 for y in new equation 2 and solve for z.

$$7z - 2 = 5$$
$$7z = 7$$
$$z = 1$$

Substitute 2 for y and 1 for z in new equation 1 to find the value of x.

$$x = 1 - 2 + 4$$
$$= 3$$

Therefore, the ordered triple is (3, 2, 1).

Answer 16e.

Consider the following system,

$$2x - y - z = 15 \quad \text{...... (1)}$$
$$4x + 5y + 2z = 10 \quad \text{...... (2)}$$
$$-x - 4y + 3z = -20 \quad \text{...... (3)}$$

Step 1: Use substitution method to convert the three variables linear system as a two variables linear system as follows:

Because, the coefficient of y in equation (1) is -1, so, write,

$$2x - y - z = 15$$
$$y = 2x - z - 15. \quad \text{...... (4)}$$

Step 2: Substitute the expression (4) in equations (2), and, (3), to convert the system into a linear system in two variables.

Take equation (2):

$$4x + 5y + 2z = 10 \quad \text{Equation (2)}$$
$$4x + 5(2x - z - 15) + 2z = 10 \quad \text{Substitute } 2x - z - 15 \text{ for } y \text{, from (4)}$$
$$4x + 10x - 5z - 75 + 2z = 10 \quad \text{Use distributive property}$$
$$14x - 3z - 75 = 10 \quad \text{Combine like terms}$$
$$14x - 3z = 85 \quad \text{Add 75 on both sides}$$
$$14x - 3z = 85 \quad \text{...... (5)}$$

Next take equation (3):

$$-x - 4y + 3z = -20 \quad \text{Equation (3)}$$
$$-x - 4(2x - z - 15) + 3z = -20 \quad \text{Substitute } 2x - z - 15 \text{ for } y \text{, from (4)}$$
$$-x - 8x + 4z + 60 + 3z = -20 \quad \text{Use distributive property}$$
$$-9x + 7z + 60 = -20 \quad \text{Combine like terms}$$
$$-9x + 7z = -80 \quad \text{Subtract 60 from both sides}$$
$$-9x + 7z = -80 \quad \text{...... (6)}$$

Now solve the system in two variables:

Multiply equation (5) by 7,

$$14x - 3z = 85$$
$$(7)14x - 3z = (7)85$$
$$98x - 21z = 595 \quad \text{New equation 1}$$

Multiply equation (6) by 3,

$$-9x + 7z = -80$$
$$(3)(-9x + 7z) = (3)(-80)$$
$$-27x + 21z = -240 \quad \text{New equation 2}$$

Add the two new equations, then,

$$\begin{array}{r} 98x-21z= \quad 595 \\ \underline{-27x+21z=-240} \\ 71x \qquad =355 \end{array}$$

$x=5$ Divide both sides by 71

Therefore,

$\boxed{x=5}$.

Substitute 5 for x in equation (4) or (5), and solve for z, as follows:

$-9x+7z=-80$ Equation (5)

$-9(5)+7z=-80$ Substitute 5 for x

$-45+7z=-80$ Multiply

$7z=-35$ Add 45 on both sides

$\boxed{z=-5}$ Divide both sides by 7

Step 3: To solve for y, substitute 5 for x, and, -5 for z, in the revised equation (1), then,

$y=2x-z-15$

$=2(5)-(-5)-15$ Substitute 5 for x, -5 for z

$=10+5-15$ Multiply and then combine the like terms

$=0$

Therefore,

$\boxed{y=0}$.

Hence, the solution set as ordered triple is $\boxed{(5,0,-5)}$.

Answer 17e.

Number the equations.

$4x+y+5z = -40$ Equation 1

$-3x+2y+4z = 10$ Equation 2

$x-y-2z = -2$ Equation 3

Solve equation 3 for any one variable, say, y.

$y = 2-2z+x$ New equation 1

Substitute the expression for y in equation 1 and simplify.

$4x+2-2z+x+5z = -40$

$5x+2+3z = -40$

Subtract 2 from both the sides.

$$5x + 2 + 3z - 2 = -40 - 2$$

$$5x + 3z = -42 \qquad \text{New equation 2}$$

Substitute $2 - 2z + x$ for y in equation 2 and simplify.

$$-3x + 2(2 - 2z + x) + 4z = 10$$

$$-3x + 4 - 4z + 2x + 4z = 10$$

$$-x + 4 = 10$$

Subtract 4 from both the sides.

$$-x + 4 - 4 = 10 - 4$$

$$-x = 6$$

Divide both the sides by -1.

$$\frac{-x}{-1} = \frac{6}{-1}$$

$$x = -6$$

Substitute -6 for x in new equation 2 and solve for z.

$$5(-6) + 3z = -42$$

$$-30 + 3z = -42$$

$$3z = -12$$

$$z = -4$$

Substitute -6 for x, and -4 for z in new equation 1 to isolate y.

$$y = 2 - 2(-4) + (-6)$$

$$= 2 + 8 - 6$$

$$= 4$$

Therefore, the ordered triple is $(-6, 4, -4)$.

Answer 18e.

Consider the system

$x+3y-z=12$ ...... (1)

$2x+4y-2z=6$ ...... (2)

$-x-2y+z=-6$ ...... (3)

Because the coefficient of x in equation (1) is 1, we write,

$x=-3y+z+12$

Substitute the expression in equations (2) and (3), to convert the system into a linear system in two variables.

$2x+4y-2z=6$	Equation (2)
$2(-3y+z+12)+4y-2z=6$	Substitute $-3y+z+12$ for x
$-6y+2z+24+4y-2z=6$	Simplify
$-2y+24=6$	
$-2y=-18$	
$\boxed{y=9}$	

$-x-2y+z=-6$	Equation (3)
$-(-3y+z+12)-2y+z=-6$	Substitute $-3y+z+12$ for x
$3y-z-12-2y+z=-6$	Simplify
$y-12=-6$	
$\boxed{y=6}$	

Since $9=6$, is not true,

Thus, the given system has no solution.

Answer 19e.

Number the equations.

$2x - y + z = -2$ Equation 1

$6x + 3y - 4z = 8$ Equation 2

$-3x + 2y + 3z = -6$ Equation 3

Solve equation 1 for any one variable, say, z.

$z = -2 - 2x + y$ New equation 1

Substitute the expression for z in equation 2 and simplify.

$$6x + 3y - 4(-2 - 2x + y) = 8$$

$$6x + 3y + 8 + 8x - 4y = 8$$

$$14x - y + 8 = 8$$

Subtract 8 from both the sides.

$$14x - y + 8 - 8 = 8 - 8$$

$$14x - y = 0 \qquad \text{New equation 2}$$

Substitute $2 - 2z + x$ for y in equation 3 and simplify.

$$-3x + 2y + 3(-2 - 2x + y) = -6$$

$$-3x + 2y - 6 - 6x + 3y = -6$$

$$-9x + 5y - 6 = -6$$

Add 6 to both the sides.

$$-9x + 5y - 6 + 6 = -6 + 6$$

$$-9x + 5y = 0 \qquad \text{New equation 3}$$

Solve new equations 1 and 2 to eliminate any one of the variables, say, y.
Add 5 times new equation 2 to new equation 3.

$$-5 \times \text{New equation 2} \Rightarrow 70x - 5y = 0$$

$$\begin{array}{r} -9x + 5y = 0 \\ \hline 61x \qquad = 0 \end{array}$$

Divide both the sides by 61.

$$\frac{61x}{61} = \frac{0}{61}$$

$$x = 0$$

Substitute 0 for x in new equation 3 to isolate y.

$$-9(0) + 5y = 0$$

$$5y = 0$$

$$y = 0$$

Substitute 0 for x and y in equation 1 and solve for z.

$$2(0) - 0 + z = -2$$

$$z = -2$$

Therefore, the ordered triple is $(0, 0, -2)$.

Answer 20e.

Consider the system

$3x+5y-z=12$ ...... (1)

$x+y+z=0$ ...... (2)

$-x+2y+2z=-27$ ...... (3)

Because the coefficient of x in equation (2) is 1, we write,

$x=-y-z$

Substitute the expression in equations (1) and (3), to convert the system into a linear system in two variables.

$3x+5y-z=12$	Equation (1)
$3(-y-z)+5y-z=12$	Substitute $-y-z$ for x
$-3y-3z+5y-z=12$	
$2y-4z=12$	
$y-2z=6$	...... (4)

Substitute $-y-z$ for x in (3)

$-x+2y+2z=-27$	Equation (3)
$-(-y-z)+2y+2z=-27$	Substitute $-y-z$ for x
$y+z+2y+2z=-27$	
$3y+3z=-27$	
$y+z=-9$	...... (5)

Now solve the system in two variables

Multiply equation (5) with 2 and add to the equation (4)

$$\begin{array}{r} y-2z=6 \\ \underline{2y+2z=-18} \\ 3y=-12 \quad \text{Add} \end{array}$$

Therefore, $\boxed{y=-4}$

Substitute -4 for y in equations (4) or (5)

$y-2z=6$	Equation (4)
$-4-2z=6$	Substitute -4 for y
$-2z=10$	
$\boxed{z=-5}$	

Again substitute -4 for y and -5 for t into the revised equation of (2).

$$\begin{aligned} x&=-y-z \\ &=-(-4)-(-5) \\ &=4+5 \\ &=9 \end{aligned}$$

$\boxed{x=9}$

Therefore the solution is $\boxed{(9,-4,-5)}$.

Answer 21e.

The second equation has been multiplied by 2 and added to the first equation.
By multiplying the second equation by 2, the new equation obtained should be
$6x + 4y + 2z = 22$
The error is that $2y$ is written instead of $4y$.

To correct the error, add 2 times the second equation to the first equation.

$$\begin{aligned} 2x + y - 2z &= 23 \\ 6x + 4y + 2z &= 22 \\ \hline 8x + 5y &= 45 \end{aligned}$$

Answer 22e.

Consider the system

$$2x + y - 2z = 23 \qquad \ldots\ldots (1)$$
$$3x + 2y + z = 11 \qquad \ldots\ldots (2)$$
$$x - y + z = -2 \qquad \ldots\ldots (3)$$

Use elimination method to solve the system
Because the coefficient of z in equation (2) is 1,
From (2), $z = -3x - 2y + 11$
Substitute the expression in equation (1) , to covert the system into a linear system in two variables.

$$2x + y - 2z = 23 \qquad \text{Equation (1)}$$
$$2x + y - 2(-3x - 2y + 11) = 23 \qquad \text{Substitute } -3x - 2y + 11 \text{ for } z$$
$$2x + y + 6x + 4y - 22 = 23$$
$$8x + 5y - 22 = 23$$
$$8x + 5y = 45$$

This is the first equation in two variables.

Answer 23e.

We can solve the three equations.
Number the given equations.

$2x + 5y + 3z = 10$ Equation 1
$3x - y + 4z = 8$ Equation 2
$5x - 2y + 7z = 12$ Equation 3

Solve equation 2 for any of the variable, say, y.
$y = 3x + 4z - 8$

Substitute $3x + 4z - 8$ for y in equation 1 and simplify.

$$2x + 5(3x + 4z - 8) + 3z = 10$$
$$2x + 15x + 20z - 40 + 3z = 10$$
$$17x + 23z - 40 = 10$$

Add 40 to both the sides.

$$17x + 23z - 40 + 40 = 10 + 40$$
$$17x + 23z = 50 \qquad \text{New Equation 1}$$

Substitute $3x + 4z - 8$ for y in equation (3) and simplify.

$$5x - 2(3x + 4z - 8) + 7z = 12$$
$$5x - 6x - 8z + 16 + 7z = 12$$
$$-x + 16 - z = 12$$

Subtract 16 from both the sides.

$$-x + 16 - z - 16 = 12 - 16$$

$$-x - z = -4 \qquad \text{New Equation 2}$$

Solve new equations 1 and 2 to eliminate any one of the variables, say, x.
Add 17 times new equation 2 to new equation 1.

$$\begin{aligned} 17x + 23z &= 50 \\ 17 \times \text{New Equation 2} \Rightarrow -17x - 17z &= -68 \\ \hline 6z &= -18 \end{aligned}$$

Divide each side by 6.

$$\frac{6z}{6} = \frac{-18}{6}$$

$$z = -3$$

Substitute the value of z in new equation 2 and isolate the value of x.

$$\begin{aligned} -x - (-3) &= -4 \\ -x + 3 &= -4 \\ -x &= -7 \\ x &= 7 \end{aligned}$$

Substitute the values of x and z in $y = 3x + 4z - 8$ and solve for y.

$$\begin{aligned} y &= 3(7) + 4(-3) - 8 \\ &= 21 - 12 - 8 \\ &= 1 \end{aligned}$$

The solution of the given system of equations is (7, 1, −3).
Therefore, the correct answer is choice A.

Answer 24e.

Consider the system

$$2x - 2y - z = 6 \qquad \ldots\ldots (1)$$

$$-x + y + 3z = -3 \qquad \ldots\ldots (2)$$

$$3x - 3y + 2z = 9 \qquad \ldots\ldots (3)$$

(a)

Consider the solution $(-x, x+2, 0)$

Check:

Equation (1): $2x-2y-z=6$

$2(-x)-2(x+2)-0=6$ Substitute $(x, y, z)=(-x, x+2, 0)$

$-2x-2x-4=6$

$-4x-4=6$ False

Therefore $(-x, x+2, 0)$ is not a solution of the given system

(b)

Consider the solution $(x, x-3, 0)$

Check:

Equation (1): $2x-2y-z=6$

$2x-2(x-3)-0=6$ Substitute $(x, y, z)=(x, x-3, 0)$

$2x-2x+6=6$

$6=6$ True

Equation (2): $-x+y+3z=-3$

$-x+x-3+3(0)=-3$ Substitute $(x, y, z)=(x, x-3, 0)$

$-3=-3$ True

Equation (3): $3x-3y+2z=9$

$3x-3(x-3)+2(0)=9$ Substitute $(x, y, z)=(x, x-3, 0)$

$3x-3x+9+0=9$

$9=9$ True

Therefore $(x, x-3, 0)$ is the solution of the given system,

Hence the correct answer is (B).

Answer 25e.

Number the equations.

$x + 5y - 2z = -1$ Equation 1

$-x - 2y + z = 6$ Equation 2

$-2x - 7y + 3z = 7$ Equation 3

STEP 1 Rewrite the system as a linear system in two variables.
Add Equation 2 to Equation 1.

$$\begin{array}{r} x + 5y - 2z = -1 \\ -x - 2y + z = 6 \\ \hline 3y - z = 5 \end{array} \quad \text{New Equation 1}$$

Add 2 times Equation 1 to Equation 3.

$$\begin{array}{rr} 2 \times \text{Equation } 1 \Rightarrow & 2x + 10y - 4z = -2 \\ & -2x - 7y + 3z = 7 \\ \hline & 3y - z = 5 \end{array} \quad \text{New Equation 2}$$

STEP 2 Solve the new linear system for both of its variables.
Add -1 times new equation 2 to new equation 1.

$$\begin{array}{r} 3y - z = 5 \\ -3y + z = -5 \\ \hline 0 = 0 \end{array}$$

The equation obtained is an identity $0 = 0$. Therefore, the system has infinitely many solutions.

STEP 3 Describe the solutions of the system.
Solve new equation 1 for z.
$z = 3y - 5$

Substitute this value in equation 1 to get the value of y.

$$\begin{aligned} x + 5y - 2(3y - 5) &= -1 \\ x + 5y - 6y + 10 &= -1 \\ x - y + 10 &= -1 \\ x &= y - 11 \end{aligned}$$

Therefore, any ordered triple of the form $(y - 11, y, 3y - 2)$ is a solution of the given system.

Answer 26e.

Consider the system

$$4x+5y+3z=15 \quad \ldots\ldots (1)$$
$$x-3y+2z=-6 \quad \ldots\ldots (2)$$
$$-x+2y-z=3 \quad \ldots\ldots (3)$$

To solve the system, use elimination method.
Rewrite the system as linear system in two variables
Multiply equation (2) with -4 and add to the equation (1)

$$\begin{array}{r} 4x+5y+3z=15 \\ -4x+12y-8z=24 \\ \hline 17y-5z=39 \end{array} \quad \ldots\ldots(4) \text{ Add}$$

Again multiply equation (3) with 4 and add to the equation (1).

$$\begin{array}{r} 4x+5y+3z=15 \\ -4x+8y-4z=12 \\ \hline 13y-z=27 \end{array} \quad \ldots\ldots(5) \text{ Add}$$

Now solve the linear system in two variables
Multiply equation (5) with 5 and add to the equation (4).

$$\begin{array}{r} 17y-5z=39 \\ -65y+5z=-135 \\ \hline -48y=-96 \end{array} \quad \text{Add}$$

Therefore, $\boxed{y=2}$

Substitute 2 for y in equation (5), and solve for z.

$$13y-z=27 \quad \text{Equation (5)}$$
$$13(2)-z=27 \quad \text{Substitute for 2 for } y$$
$$26-z=27$$
$$-z=1$$

Therefore, $\boxed{z=-1}$

Again substitute 2 for y and -1 for z into one of the given equations.

$$x-3y+2z=-6 \quad \text{Equation (2)}$$
$$x-3(2)+2(-1)=-6 \quad \text{Substitute 2 for } y,\ -1 \text{ for } z$$
$$x-6-2=-6$$
$$x-8=-6$$

Therefore, $\boxed{x=2}$

Therefore the solution is $\boxed{(2,2,-1)}$.

Answer 27e.

Number the equations.

$$6x + y - z = -2 \qquad \text{Equation 1}$$

$$x + 6y + 3z = 23 \qquad \text{Equation 2}$$

$$-x + y + 2z = 5 \qquad \text{Equation 3}$$

STEP 1 Rewrite the system as a linear system in two variables. Solve equation (3) for y.

$$y = 5 + x - 2z \qquad \text{New Equation 1}$$

Substitute $5 + x - 2z$ for y in Equation 1.

$$6x + 5 + x - 2z - z = -2$$

Combine the like terms.

$$7x + 5 - 3z = -2$$

Subtract 5 from each side.

$$7x + 5 - 3z - 5 = -2 - 5$$

$$7x - 3z = -7 \qquad \text{New Equation 2}$$

Substitute $5 + x - 2z$ for y in Equation 2.

$$x + 6(5 + x - 2z) + 3z = 23$$

Apply the distributive property.

$$x + 30 + 6x - 12z + 3z = 23$$

Combine the like terms.

$$7x + 30 - 9z = 23$$

Subtract 30 from each side.

$$7x + 30 - 9z - 30 = 23 - 30$$

$$7x - 9z = -7 \qquad \text{New Equation 3}$$

STEP 2 Solve the new linear system for both of its variables.

Add -1 times new equation 2 to new equation 3.

$$7x - 3z = -7$$

$$\underline{-7x + 9z = 7}$$

$$6z = 0$$

Solve for z. Divide each side by 6.

$$\frac{6z}{6} = \frac{0}{6}$$

$$z = 0$$

STEP 3 Substitute 0 for z in new equation 3 to get the value of x and evaluate.

$$7x - 9(0) = -7$$
$$7x = -7$$
$$x = -1$$

Substitute 0 for z, and -1 for x in new equation 1 to get the value of y and evaluate.

$$y = 5 + (-1) - 2(0)$$
$$= 5 - 1 - 0$$
$$= 4$$

Check the values of $x, y,$ and z in each of the original equations.

$$6(-1) + 4 - 0 \stackrel{?}{=} -2 \qquad -1 + 6(4) + 3(0) \stackrel{?}{=} 23 \qquad -(-1) + 4 + 2(0) \stackrel{?}{=} 5$$
$$-6 + 4 \stackrel{?}{=} -2 \qquad -1 + 24 + 0 \stackrel{?}{=} 23 \qquad 1 + 4 + 0 \stackrel{?}{=} 5$$
$$-2 \stackrel{?}{=} -2 \checkmark \qquad 23 \stackrel{?}{=} 23 \checkmark \qquad 5 \stackrel{?}{=} 5 \checkmark$$

The solutions check.
Therefore, the ordered triple is $(-1, 4, 0)$.

Answer 28e.

Consider the system

$$x + 2y = -1 \qquad \ldots\ldots (1)$$
$$3x - y + 4z = 17 \qquad \ldots\ldots (2)$$
$$-4x + 2y - 3z = -30 \qquad \ldots\ldots (3)$$

Because the coefficient of x in equation (1) is 1 ,
We use substitution method to solve the system
From equation (1)

$$x = -2y - 1$$

Substitute the expression in equation (2) and (3)

$$3x - y + 4z = 17 \qquad \text{Write Equation (2)}$$
$$3(-2y - 1) - y + 4z = 17 \qquad \text{Substitute } -2y - 1 \text{ for } x$$
$$-6y - 3 - y + 4z = 17$$
$$-7y + 4z = 20 \qquad \ldots\ldots (4)$$

$$-4x + 2y - 3z = -30 \qquad \text{Write Equation (3)}$$
$$-4(-2y - 1) + 2y - 3z = -30 \qquad \text{Substitute } -2y - 1 \text{ for } x$$
$$8y + 4 + 2y - 3z = -30$$
$$10y - 3z = -34 \qquad \ldots\ldots (5)$$

Now solve the system in two variables
Multiply equation (4) with 3 and add to multiply the equation (5) with 4

$$(4) \times 3 \quad \Rightarrow -21y + 12z = 60$$
$$(5) \times 4 \quad \Rightarrow \underline{40y - 12z = -136}$$
$$19z = -76$$

Therefore, $\boxed{y = -4}$

Substitute -4 for y into equation (4) or (5)

$$10y-3z=-34 \qquad \text{Equation (5)}$$
$$10(-4)-3z=-34 \qquad \text{Substitute } -4 \text{ for } y$$
$$-40-3z=-34$$
$$-3z=6$$

Therefore, $\boxed{z=-2}$

Now substitute -4 for y and -2 for z in the revised equation (1), and solve for x

$$\begin{aligned} x&=-2y-1\\ &=-2(-4)-1\\ &=8-1\\ &=7 \end{aligned}$$

$\boxed{x=7}$

Therefore the solution is $\boxed{(7,-4,-2)}$.

Answer 29e.

Number the equations.

$$2x - y + 2z = -21 \qquad \text{Equation 1}$$
$$x + 5y - z = 25 \qquad \text{Equation 2}$$
$$-3x + 2y + 4z = 6 \qquad \text{Equation 3}$$

STEP 1 Rewrite the system as a linear system in two variables.

Add -2 times Equation 2 to Equation 1.

$$\begin{aligned} &2x - y + 2z = -21\\ -2\times\text{Equation 2} \Rightarrow\ &\underline{-2x - 10y + 2z = 50}\\ &-11y + 4z = -71 \qquad \text{New Equation 1} \end{aligned}$$

Add 3 times Equation 2 to Equation 3.

$$\begin{aligned} 3\times\text{Equation 2} \Rightarrow\ &3x + 15y - 3z = 75\\ &\underline{-3x + 2y + 4z = 6}\\ &17y + z = 81 \qquad \text{New Equation 2} \end{aligned}$$

STEP 2 Solve the new linear system for both of its variables.

Add –4 times new equation 2 to new equation 1.

$$\begin{array}{rl} -11y + 4z &= -71 \\ -68y - 4z &= -324 \\ \hline -79y \quad\quad &= -395 \end{array}$$

Divide both the sides by –79.

$$\frac{-79y}{-79} = \frac{-395}{-79}$$

$$y = 5$$

Substitute 5 for y in any of the new equations, say, new equation 2 to find the value of z.

$$17(5) + z = 81$$

$$85 + z = 81$$

$$z = -4$$

STEP 3 Substitute –4 for z, and 5 for y in any of the original equations, say, equation 1 to find the value of x.

$$2x - 5 + 2(-4) = -21$$

$$2x - 5 - 8 = -21$$

$$2x - 13 = -21$$

$$2x = -8$$

$$x = -4$$

Check the values of $x, y,$ and z in each of the original equations.

$$2(-4) - 5 + 2(-4) \stackrel{?}{=} -21 \qquad -4 + 5(5) - (-4) \stackrel{?}{=} 25 \qquad -3(-4) + 2(5) + 4(-4) \stackrel{?}{=} 6$$

$$-8 - 5 - 8 \stackrel{?}{=} -21 \qquad -4 + 25 + 4 \stackrel{?}{=} 25 \qquad 12 + 10 - 16 \stackrel{?}{=} 6$$

$$-21 \stackrel{?}{=} -21 \checkmark \qquad 25 \stackrel{?}{=} 25 \checkmark \qquad 6 \stackrel{?}{=} 6 \checkmark$$

The solutions check.

Therefore, the ordered triple is (–4, 5, –4).

Answer 30e.

Consider the system

$$4x-8y+2z=10 \qquad \ldots\ldots (1)$$

$$-3x+y-2z=6 \qquad \ldots\ldots (2)$$

$$2x-4y+z=8 \qquad \ldots\ldots (3)$$

Multiply equation (3) with -2 and add to the equation (1),

$$\begin{array}{r} 4x-8y+2z=10 \\ \underline{-4x+8y-2z=-16} \\ 0=-6 \quad \text{Add} \end{array}$$

Because $0=-6$, is false, Therefore, the original system has no solution.

Answer 31e.

Number the equations.

$$\begin{array}{rl} -x+5y-z=-16 & \text{Equation 1} \\ 2x+3y+4z=18 & \text{Equation 2} \\ x+y-z=-8 & \text{Equation 3} \end{array}$$

STEP 1 Rewrite the system as a linear system in two variables.

Add Equation 3 to Equation 1.

$$\begin{array}{r} -x+5y-z=-16 \\ \underline{x+y-z=-8} \\ 6y-2z=-24 \end{array} \quad \text{New Equation 1}$$

Add 2 times Equation 1 to Equation 2.

$$\begin{array}{rr} & 2x+3y+4z=18 \\ 2\times\text{Equation 1}\Rightarrow & \underline{-2x+10y-2z=-32} \\ & 13y+2z=-14 \end{array} \quad \text{New Equation 2}$$

STEP 2 Solve the new linear system for both of its variables.

Add new equation 2 to new equation 1.

$$\begin{array}{r} 6y-2z=-24 \\ \underline{13y+2z=-14} \\ 19y=-38 \end{array}$$

Divide both the sides by 19.

$$\frac{19y}{19}=\frac{-38}{19}$$

$$y=-2$$

Substitute −2 for y in any of the new equations, say, new equation 2 to find the value of z.

$$\begin{aligned} 13(-2) + 2z &= -14 \\ -26 + 2z &= -14 \\ 2z &= 12 \\ z &= 6 \end{aligned}$$

STEP 3 Substitute 6 for z, and −2 for y in any of the original equations, say, equation 1 to find the value of x.

$$\begin{aligned} -x + 5(-2) - 6 &= -16 \\ -x - 10 - 6 &= -16 \\ -x - 16 &= -16 \\ -x &= 0 \\ x &= 0 \end{aligned}$$

Check the values of x, y, and z in each of the original equations.

$$\begin{aligned} -0 + 5(-2) - 6 &\stackrel{?}{=} -16 \\ -0 - 10 - 6 &\stackrel{?}{=} -16 \\ -16 &\stackrel{?}{=} -16 \checkmark \end{aligned}$$

$$\begin{aligned} 2(0) + 3(-2) + 4(6) &\stackrel{?}{=} 18 \\ 0 - 6 + 24 &\stackrel{?}{=} 18 \\ 18 &\stackrel{?}{=} 18 \checkmark \end{aligned}$$

$$\begin{aligned} 0 + (-2) - 6 &\stackrel{?}{=} -8 \\ -2 - 6 &\stackrel{?}{=} -8 \\ -8 &\stackrel{?}{=} -8 \checkmark \end{aligned}$$

The solutions check.
Therefore, the ordered triple is (0, −2, 6).

Answer 32e.

Consider the system

$$\begin{aligned} 2x - y + 4z &= 19 \quad \ldots\ldots (1) \\ -x + 3y - 2z &= -7 \quad \ldots\ldots (2) \\ 4x + 2y + 3z &= 37 \quad \ldots\ldots (3) \end{aligned}$$

To solve the system, use elimination method
Multiply the equation (2) by 2 and add the result to equation (1),

$$\begin{aligned} 2x - y + 4z &= 19 \\ -2x + 6y - 4z &= -14 \\ \hline 5y &= 5 \end{aligned}$$

Therefore, $\boxed{y = 1}$

Multiply the equation (1) by -2 and add the result to equation (3),

$$-4x+2y-8z=-38$$
$$4x+2y+3z=37$$

$$4y-5z=-1 \qquad \text{New equation (1)}$$

Substitute $y=1$ in New equation (1)

$$4(1)-5z=-1$$
$$4-5z=-1$$
$$-5z=-5$$

Therefore, $\boxed{z=1}$

Substitute 1 for y and 1 for z in one of the original equations.

$$2x-y+4z=19 \qquad \text{Equation (1)}$$
$$2x-1+4(1)=19 \qquad \text{Substitute 1 for } y \text{ and 1 for } z$$
$$2x-1+4=19$$
$$2x+3=19$$
$$2x=16$$
$$\boxed{x=8}$$

Therefore, the solution is $\boxed{(8,1,1)}$.

Answer 33e.

Number the equations.

$$x+y+z=3 \qquad \text{Equation 1}$$
$$3x-4y+2z=-28 \qquad \text{Equation 2}$$
$$-x+5y+z=23 \qquad \text{Equation 3}$$

STEP 1 Rewrite the system as a linear system in two variables. Solve equation (1) for x.

$$x=3-y-z \qquad \text{New Equation 1}$$

Substitute $3-y-z$ for x in equation (2).

$$3(3-y-z)-4y+2z=-28$$

Apply the distributive property.

$$9-3y-3z-4y+2z=-28$$

Combine the like terms.

$$9-7y-z=-28$$

Subtract 9 from each side.

$$9-7y-z-9=-28-9$$
$$-7y-z=-37 \qquad \text{New Equation 2}$$

Substitute $3 - y - z$ for x in equation (3).
$-(3 - y - z) + 5y + z = 23$

Apply the distributive property.
$-3 + y + z + 5y + z = 23$

Simplify.
$6y + 2z = 26$ New Equation 3

STEP 2 Solve the new linear system for both of its variables.
Add 2 times new equation 2 to new equation 3.

$$\begin{array}{r} 6y + 2z = 26 \\ -14y - 2z = -74 \\ \hline -8y \qquad = -48 \end{array}$$

Solve for y. Divide each side by -8.

$$\frac{-8y}{-8} = \frac{-48}{-8}$$
$$y = 6$$

STEP 3 Substitute 6 for y in new equation 3 to get the value of y and evaluate.

$$\begin{aligned} 6(6) + 2z &= 26 \\ 36 + 2z &= 26 \\ 2z &= -10 \\ z &= -5 \end{aligned}$$

Substitute -5 for z, and 6 for y in new equation 1 to get the value of x and evaluate.

$$\begin{aligned} x &= 3 - 6 - (-5) \\ &= 3 - 6 + 5 \\ &= 2 \end{aligned}$$

Therefore, the ordered triple is (2, 6, −5).

Answer 34e.

(a)

Consider system

$$x+4y-2z=12 \quad \text{...... (1)}$$
$$3x-y+4z=6 \quad \text{...... (2)}$$
$$-x+3y+z=-9 \quad \text{...... (3)}$$

Use elimination method to solve the system from equation (1) and (2)

$$-3x-12y+6z=-36$$
$$3x-y+4z=6$$
$$-13y+10z=-30 \quad \text{......(4)} \qquad \text{Add}$$

From equations (1) and (3)

Add

$$x+4y-2z=12$$
$$-x+3y+z=-9$$
$$7y-z=3 \quad \text{......(5)} \qquad \text{Add}$$

Now solve the system in two variables

Multiply equation (5) with 10 and add to the equation (4),

$$-13y+10z=-30$$
$$70y-10z=30$$
$$57y=0 \qquad \text{Add}$$
$$\boxed{y=0}$$

Substitute 0 for y in equation (5),

$$7y-z=3 \qquad \text{Equation (5)}$$
$$7(0)-z=3 \qquad \text{Substitute 0 for } y$$
$$0-z=3$$
$$\boxed{z=-3}$$

Now substitute 0 for y and -3 for z into one of the original equations and solve for x.

$$x+4y-2z=12 \qquad \text{Equation (1)}$$
$$x+4(0)-2(-3)=12 \qquad \text{Substitute 0 for } y \text{ and } -3 \text{ for } z$$
$$x+0+6=12$$
$$\boxed{x=6}$$

Therefore the solution is $\boxed{(6,0,-3)}$

(b)

Consider the system,

$3x+y-2z=10 \quad \text{......} \ (1)$

$6x+2y-4z=-1 \quad \text{......} \ (2)$

$x+4y+3z=7 \quad \text{......} \ (3)$

When we multiply equation (1) with -2 and add to equation (2), we get

$$\begin{array}{r} -6x-2y+4z=-20 \\ 6x+2y-4z=-1 \\ \hline 0=-21 \quad \text{Add} \end{array}$$

Because we obtain $0=-21$, a false equation.

We can conclude that the original system has no solution.

(c)

Consider the system

$x+y+z=3 \quad \text{......} \ (1)$

$x+y-z=3 \quad \text{......} \ (2)$

$2x+2y+z=6 \quad \text{......} \ (3)$

From equation (1) and (2)

$$\begin{array}{r} x+y+z=3 \\ x+y-z=3 \\ \hline 2x+2y=6 \quad \text{Add} \end{array}$$

$x+y=3 \quad \text{......} \ (4)$

From equations (2) and (3)

$$\begin{array}{r} x+y-z=3 \\ 2x+2y+z=6 \\ \hline 3x+3y=9 \quad \text{Add} \end{array}$$

$x+y=3 \quad \text{......} \ (5)$

Again on solving equation (4) and (5)

Multiply equation (4) with -1 and add to the equation (5),

$$\begin{array}{r} -x-y=-3 \\ x+y=3 \\ \hline 0=0 \quad \text{Add} \end{array}$$

Because we obtain the identity $0=0$, the system has infinitely many solutions.

Answer 35e.

Clear the fractions in the equations. Multiply each side of equation 1 by 2, equation 2 by 4, and equation 3 by 6.

$2x + y + z = 5$

$3x + y + 6z = 7$

$2x + 9y + 4z = 13$

Number the equations.

$2x + y + z = 5$ Equation 1

$3x + y + 6z = 7$ Equation 2

$2x + 9y + 4z = 13$ Equation 3

STEP 1 Rewrite the system as a linear system in two variables.

Add -1 times Equation 2 to Equation 1.

$$\begin{array}{rr} & 2x + y + z = 5 \\ -1 \times \text{Equation } 2 \Rightarrow & \underline{-3x - y - 6z = -7} \\ & -x \quad - 5z = -2 \quad \text{New Equation 1} \end{array}$$

Add 9 times Equation 1 to -1 times Equation 3.

$$\begin{array}{rr} 9 \times \text{Equation } 1 \Rightarrow & 18x + 9y + 9z = 45 \\ -1 \times \text{Equation } 3 \Rightarrow & \underline{-2x - 9y - 4z = -13} \\ & 16x \quad + 5z = 32 \quad \text{New Equation 2} \end{array}$$

STEP 2 Solve the new linear system for both of its variables.

Add new equation 2 to new equation 1.

$$\begin{array}{r} -x - 5z = -2 \\ \underline{16x + 5z = 32} \\ 15x \quad = 30 \end{array}$$

Divide both the sides by 15.

$$\frac{15x}{15} = \frac{30}{15}$$

$$x = 2$$

Substitute 2 for x in any of the new equations, say, new equation 1 to find the value of z.

$$-2 - 5z = -2$$
$$5z = 0$$
$$z = 0$$

STEP 3 Substitute 0 for z, and 2 for x in any of the original equations, say, equation 1 to find the value of y.

$$2(2) + y + 0 = 5$$
$$4 + y = 5$$
$$y = 1$$

Therefore, the ordered triple is (2, 1, 0).

Answer 36e.

Consider the system

$$\frac{1}{3}x+\frac{5}{6}y+\frac{2}{3}z=\frac{4}{3}$$
$$\frac{1}{6}x+\frac{2}{3}y+\frac{1}{4}z=\frac{5}{6}$$
$$\frac{2}{3}x+\frac{1}{6}y+\frac{3}{2}z=\frac{4}{3}$$

On multiplying the first and last equation by 6, and the second equation by 12, we get the new system

$$2x+5y+4z=8 \qquad \ldots\ldots (1)$$
$$2x+8y+3z=10 \qquad \ldots\ldots (2)$$
$$4x+y+9z=8 \qquad \ldots\ldots (3)$$

Multiply the equation (1) by -1 and add the result to equation (2),

$$-2x-5y-4z=-8$$
$$2x+8y+3z=10$$
$$\overline{\qquad 3y-z=2 \qquad} \qquad \ldots\ldots (4) \text{ Add}$$

Multiply the equation (1) by -2 and add the result to equation (3),

$$-4x-10y-8z=-16$$
$$4x+y+9z=8$$
$$\overline{\qquad -9y+z=-8 \qquad} \qquad \ldots\ldots (5) \text{ Add}$$

Again solve the equations (4) and (5) for the variables y and z.

$$\begin{array}{r} 3y-z=2 \\ -9y+z=-8 \\ \hline -6y=-6 \end{array} \quad \text{Add}$$

Therefore, $\boxed{y=1}$

Substitute 1 for y in equation (4) or (5)

$3y-z=2$	Equation (4)
$3(1)-z=2$	Substitute 1 for y
$3-z=2$	
$-z=-1$	

Therefore, $\boxed{z=1}$

To get the variable x, substitute 1 for y and 1 for z in one the revised equation

$2x+5y+4z=8$	Equation (1)
$2x+5(1)+z(1)=8$	Substitute 1 for y and 1 for z
$2x+5+4=8$	
$2x+9=8$	
$2x=-1$	

Therefore, $\boxed{x=\frac{-1}{2}}$

Therefore the solution is $\boxed{\left(\frac{-1}{2},1,1\right)}$.

Answer 37e.

Number the equations.

$x+2y-3z=a$	Equation 1
$-x-y+z=b$	Equation 2
$2x+3y-2z=c$	Equation 3

Replace x with -1, y with 2, and z with -3 in Equation 1.

$-1+2(2)-3(-3)=a$

Simplify.

$$-1+4+9=a$$

$$12=a$$

Substitute −1 for x, 2 for y, and −3 for z in Equation 2 and evaluate.

$$-(-1) - 2 + (-3) = b$$
$$1 - 2 - 3 = b$$
$$-4 = b$$

Now, put the values for x, y, and z in Equation 3 to get the value of c.

$$2(-1) + 3(2) - 2(-3) = c$$

Evaluate.

$$-2 + 6 + 6 = c$$
$$-2 + 12 = c$$
$$10 = c$$

Since (−1, 2, −3) satisfies all the three equation, when a is 12, b is −4, and c is 10, the given linear system of equations has (−1, 2, −3) as its only solution.

Answer 38e.

Consider the following system,

$$w + x + y + z = 2 \quad \ldots\ldots (1)$$
$$2w - x + 2y - z = 1 \quad \ldots\ldots (2)$$
$$-w + 2x - y + 2z = -2 \quad \ldots\ldots (3)$$
$$3w + x + y - z = -5 \quad \ldots\ldots (4)$$

First convert the system into linear system of three variables.

Solve equations (1), and, (2), then,

$$w + x + y + z = 2$$
$$\underline{2w - x + 2y - z = 1} \quad \text{Add}$$
$$3w + 3y = 3$$
$$w + y = 1 \quad \text{Divide both sides by 3} \quad \ldots\ldots (a)$$

Next solve equations (1), and, (3), then,

Multiply equation (1) by -2, get,

$$w + x + y + z = 2$$
$$(-2)(w + x + y + z) = (-2)(2)$$
$$-2w - 2x - 2y - 2z = -4$$

Add the above resultant equation, and, equation (3), then,

$$-2w - 2x - 2y - 2z = -4$$
$$\underline{-w + 2x - y + 2z = -2} \quad \text{Add}$$
$$-3w - 3y = -6$$
$$w + y = 2 \quad \text{Divide both sides by } -3 \quad \ldots\ldots (b)$$

Solve equations (a), and, (b), then,

$$\begin{aligned} w+y &= 1 \\ -w-y &= -2 \\ \hline 0 &= -3 \end{aligned} \qquad \text{Add}$$

Because, the condition $0=-3$ is not true, so, conclude that the given system has no solution.

Answer 39e.

Number the equations.

$$\begin{aligned} 2w + x - 3y + z &= 4 \quad \text{Equation 1} \\ w - 3x + y + z &= 32 \quad \text{Equation 2} \\ -w + 2x + 2y - z &= -10 \quad \text{Equation 3} \\ w + x - y + 3z &= 14 \quad \text{Equation 4} \end{aligned}$$

STEP 1 Rewrite the system as a linear system in three variables.

Add Equation 1 to Equation 3.

$$\begin{aligned} 2w + x - 3y + z &= 4 \\ -w + 2x + 2y - z &= -10 \\ \hline w + 3x - y &= -6 \quad \text{New Equation 1} \end{aligned}$$

Add -3 times Equation 2 to Equation 4.

$$\begin{aligned} -3 \times \text{Equation 2} \Rightarrow -3w + 9x - 3y - 3z &= -96 \\ w + x - y + 3z &= 14 \\ \hline -2w + 10x - 4y &= -82 \quad \text{New Equation 2} \end{aligned}$$

Add 2 times new equation 1 to new equation 2.

$$\begin{aligned} 2w + 6x - 2y &= -12 \\ -2w + 10x - 4y &= -82 \\ \hline 16x - 6y &= -96 \quad \text{New Equation 3} \end{aligned}$$

Add Equation 2 to Equation 3.

$$\begin{aligned} w - 3x + y + z &= 32 \\ -w + 2x + 2y - z &= -10 \\ \hline -x + 3y &= 22 \quad \text{New Equation 4} \end{aligned}$$

STEP 2 Solve the new linear system for both of its variables.

Add 2 times new equation 4 to new equation 3.

$$\begin{array}{rl} -2x + 6y &= 44 \\ 16x - 6y &= -94 \\ \hline 14x \quad\quad &= -50 \end{array}$$

Divide both the sides by 14.

$$\frac{14x}{14} = \frac{-50}{14}$$

$$x = -\frac{25}{7}$$

Substitute $-\frac{25}{7}$ for x in new equation 4 to find the value of y.

$$-\left(-\frac{25}{7}\right) + 3y = 22$$

$$\frac{25}{7} + 3y = 22$$

$$3y = \frac{129}{7}$$

$$y = \frac{43}{7}$$

STEP 3 Substitute $-\frac{25}{7}$ for x, and $\frac{43}{7}$ for y in new equation 1.

$$w + 3\left(-\frac{25}{7}\right) - \frac{43}{7} = -6$$

$$w - \frac{75}{7} - \frac{43}{7} = -6$$

$$w - \frac{118}{7} = -6$$

$$w = \frac{76}{7}$$

Substitute $-\frac{25}{7}$ for x, $\frac{43}{7}$ for y, and $\frac{76}{7}$ in any one of the original equations, say, Equation 1.

$$2\left(\frac{76}{7}\right) + \left(-\frac{25}{7}\right) - 3\left(\frac{43}{7}\right) + z = 4$$

$$\frac{152}{7} - \frac{25}{7} - \frac{129}{7} + z = 4$$

$$-\frac{2}{7} + z = 4$$

$$z = \frac{30}{7}$$

Therefore, the solution is $\left(\frac{76}{7}, -\frac{25}{7}, \frac{43}{7}, \frac{30}{7}\right)$.

Answer 40e.

Consider the following system,

$$w+2x+5y=11 \quad \ldots\ldots (1)$$
$$-2w+x+4y+2z=-7 \quad \ldots\ldots (2)$$
$$w+2x-2y+5z=3 \quad \ldots\ldots (3)$$
$$-3w+x=-1 \quad \ldots\ldots (4)$$

Step 1: Because the coefficient of x in equation (4) is 1, use substitution method to solve the system.

From equation (4), get,

$$x=3w-1$$

Now substitute the value of x in the remaining equations.

Take equation (1):

$w+2x+5y=11$	Equation (1)
$w+2(3w-1)+5y=11$	Substitute $3w-1$ for x
$w+6w-2+5y=11$	Apply distributive property
$7w+5y=13$	Add 2 on both sides, and, collect the like terms

$$7w+5y=13 \quad \ldots\ldots (a)$$

Take equation (2):

$-2w+x+4y+2z=-7$	Equation (2)
$-2w+3w-1+4y+2z=-7$	Substitute $3w-1$ for x
$w+4y+2z=-6$	Add 1 on both sides, and, collect the like terms

$$w+4y+2z=-6 \quad \ldots\ldots (b)$$

Also, take equation (3):

$w+2x-2y+5z=3$	Equation (3)
$w+2(3w-1)-2y+5z=3$	Substitute $3w-1$ for x
$w+6w-2-2y+5z=3$	Apply distributive property
$7w-2y+5z=5$	Add 2 on both sides, and, collect the like terms

$$7w-2y+5z=5 \quad \ldots\ldots (c)$$

Step 2: To, eliminate z, from equations (b), and, (c), multiply equation (b) by -5, and, equation (c), by 2, then, add the resultant equations.

Multiply equation (b) by -5,

$$w+4y+2z=-6$$
$$(-5)(w+4y+2z)=(-5)(-6)$$
$$-5w-20y-10z=30 \quad \text{New equation 1}$$

Multiply equation (c) by 2,

$$7w-2y+5z=5$$
$$(2)(7w-2y+5z)=(2)(5)$$
$$14w-4y+10z=10 \quad \text{New equation 2}$$

Next add the resultant new equations 1 and, 2 as follows:

$$\begin{array}{r} -5w-20y-10z=30 \\ \underline{14w-\ 4y+10z=10} \\ 9w-24y=40 \end{array}$$

$$9w-24y=40 \quad \ldots\ldots (d)$$

Again, solve (a), and, (d) to get the values of variables w, and, y.

Multiply equation (a) by 9,

$$7w+5y=13$$

$$(9)(7w+5y)=(9)(13)$$

$$63w+45y=117$$ New equation 3

Multiply equation (d) by -7,

$$9w-24y=40$$

$$(-7)(9w-24y)=(-7)(40)$$

$$63w+168y=-280$$ New equation 4

Next add the resultant new equations 3 and, 4 as follows:

$$\begin{array}{r} 63w+45y=117 \\ -63w+168y=-280 \\ \hline 213y=-163 \end{array}$$

$$y=\frac{-163}{213}$$ Divide both sides by 213.

Therefore,

$$\boxed{y=-\frac{163}{213}}.$$

Substitute the value of y in equation (a), to solve for w:

$$7w+5y=13$$

$$7w+5\left(-\frac{163}{213}\right)=13$$ Substitute $-\frac{163}{213}$ for y.

$$7w-\frac{815}{213}=13$$ Apply distributive property

$$7w=\frac{3584}{213}$$ Add $\frac{815}{213}$ on both sides, and, simplify

$$w=\frac{512}{213}$$ Divide both sides by 7

Therefore,

$$\boxed{w=\frac{512}{213}}.$$

Substitute y, and, w values in equation (c), to solve for z:

$$7w-2y+5z=5$$

$$7\left(\frac{512}{213}\right)-2\left(-\frac{163}{213}\right)+5z=5$$ Substitute $-\frac{163}{213}$ for y, and, $\frac{512}{213}$ for w

$$\frac{3584}{213}+\frac{326}{213}+5z=5$$ Multiply

$$\frac{3910}{213}+5z=5$$ Combine the like terms

$$5z=-\frac{2845}{213}$$ Simplify

$$z=-\frac{569}{213}$$

Therefore,

$$\boxed{z=-\frac{569}{213}}.$$

Step 3: To solve for x, substitute $\frac{512}{213}$ for w, in the revised equation (4), then,

$$x=3w-1$$

$$x=3\left(\frac{512}{213}\right)-1$$

$$x=\frac{1323}{213}$$

Thus the solution set is $\boxed{\left(\frac{512}{213},\frac{1323}{213},-\frac{163}{213},-\frac{569}{213}\right)}$.

STEP 1 Rewrite the system as a linear system in three variables.

Add Equation 2 to -1 times Equation 4.

$$-w-2x+y=-13$$

$-1\times$ Equation 2 $\Rightarrow$ $w+x-y=8$

$$-x=-5$$

Divide each side by -1.

$$\frac{-x}{-1}=\frac{-5}{-1}$$

$$x=5$$

STEP 2 Rewrite the linear system in two variables. Substitute 5 for x in Equation 1 and simplify.

$$2w + 7(5) - 3y = 41$$
$$2w + 35 - 3y = 41$$
$$2w - 3y = 6 \quad \text{New equation 1}$$

Substitute 5 for x in Equation 2 and simplify.

$$-w - 2(5) + y = -13$$
$$-w - 10 + y = -13$$
$$-w + y = -3 \quad \text{New equation 2}$$

STEP 3 Solve the new linear system for both of its variables.
Add new equation 1 to 3 times new equation 2.

$$\begin{array}{r} 2w - 3y = 6 \\ -3w + 3y = -9 \\ \hline -w \quad\quad = -3 \end{array}$$

Divide both the sides by -1.

$$\frac{-w}{-1} = \frac{-3}{-1}$$
$$w = 3$$

Substitute 5 for x, and 3 for w in Equation 1 to find the value of y.

$$2(3) + 7(5) - 3y = 41$$
$$41 - 3y = 41$$
$$-3y = 0$$
$$y = 0$$

Substitute 5 for x, and 3 for w in Equation 3 to find the value of z.

$$-2(3) + 4(5) + z = 12$$
$$-6 + 20 + z = 12$$
$$14 + z = 12$$
$$z = -2$$

Therefore, the solution is (3, 5, 0, –2).

Answer 42e.

Consider the following costs at a pizza shop,
The cost of two small pizzas, a liter of soda, and, a salad is \$14.
The cost of one small pizza, a liter of soda, and, three salads is \$15.
The cost of three small pizzas and, a liter of soda is \$16.
Next, find the cost of one small pizza, cost of one liter of soda, and, cost of one salad.
Let x, y and z be the cost of pizza, a liter of soda, and, a salad respectively.
From the given data, write the system as,

$$2x+y+z=14 \quad \ldots\ldots (1)$$
$$x+y+3z=15 \quad \ldots\ldots (2)$$
$$3x+y=16 \quad \ldots\ldots (3)$$

Solve the system as follows:
Step 1: Use substitution method to convert the three variables linear system as a two variables linear system as follows:
Because, the coefficient of y in equation (3) is 1, so, from equation (3),

$$y=-3x+16.$$

Substitute the expression $y=-3x+16$, in equations (1), and, (2).
Take equation (1):

$2x+y+z=14$	Equation (1)
$2x-3x+16+z=14$	Substitute $-3x+16$ for y
$-x+16+z=14$	Combine the like terms
$-x+z=-2$	Subtract 16 from both sides
$-x+z=-2$	...... (4)

Next take equation (2):

$x+y+3z=15$	Equation (2)
$x-3x+16+3z=15$	Substitute $-3x+16$ for y
$-2x+16+3z=15$	Combine the like terms
$-2x+3z=-1$	Subtract 16 from both sides
$-2x+3z=-1$	...... (5)

Step 2: Now solve the new linear system in x, and, z.
First multiply equation (4) by -2,

$$-x+z=-2$$
$$(-2)(-x+z)=(-2)(-2)$$
$$2x-2z=4 \quad \text{New equation 1}$$

Add the new equation 1, and, equation (5), then,

$$\begin{array}{r} 2x-2z=\ \ 4 \\ \underline{-2x+3z=-1} \\ z=3 \end{array}$$

Therefore,

$$\boxed{z=3}.$$

To solve for x, substitute 3 for z, in equation (4) or (5), then,

$-x+z=-2$	Equation (4)
$-x+3=-2$	Substitute 3 for z
$-x=-5$	Subtract 3 from both sides
$\boxed{x=5}$	Divide both sides by -1

Step 3: To solve for y, substitute 5 for x, in the revised equation (3), then,

$y = -3x + 16$	Equation (3)
$= -3(5) + 16$	Substitute 5 for x
$= -15 + 16$	Multiply and then add
$= 1$	

Therefore,

$\boxed{y = 1}$.

Therefore, the cost of one small pizza is $\boxed{\$5}$, the cost of one liter of soda is $\boxed{\$1}$, and, the cost of one salad is $\boxed{\$3}$.

Answer 43e.

STEP 1 Write verbal models for the given situation.

$0.7 \cdot$ 1st delivery of Orange juice + $0.5 \cdot$ 2nd delivery of Orange juice + $0.3 \cdot$ 3rd delivery of Orange juice = Total gallons of Orange juice delivered

$0.2 \cdot$ 1st delivery of Pineapple juice + $0.3 \cdot$ 2nd delivery of Pineapple juice + $0.3 \cdot$ 3rd delivery of Pineapple juice = Total gallons of Pineapple juice delivered

$0.1 \cdot$ 1st delivery of Grapefruit juice + $0.2 \cdot$ 2nd delivery of Grapefruit juice + $0.4 \cdot$ 3rd delivery of Grapefruit juice = Total gallons of Grapefruit juice delivered

STEP 2 Write a system of equations.

Let x be the number of gallons of juice received in the first delivery, y be the number of gallons of juice received in the second delivery, and z be the number of gallons of juice received in the third delivery.

$0.7x + 0.5y + 0.3z = 1200$ Equation 1

$0.2x + 0.3y + 0.3z = 900$ Equation 2

$0.1x + 0.2y + 0.4z = 1000$ Equation 3

STEP 3 Rewrite the system as a linear system in two variables.

Add Equation 1 to −1 times Equation 2.

$$\begin{array}{rrcl} & 0.7x + 0.5y + 0.3z &=& 1200 \\ -1 \times \text{Equation 2} \Rightarrow & -0.2x - 0.3y - 0.3z &=& -900 \\ \hline & 0.5x + 0.2y &=& 300 \quad \text{New Equation 1} \end{array}$$

Add 4 times Equation 2 to −3 times Equation 3.

$$\begin{array}{rcl} 0.8x + 1.2y + 1.2z &=& 3600 \\ -0.3x - 0.6y - 1.2z &=& -3000 \\ \hline 0.5x + 0.6y &=& 600 \quad \text{New Equation 2} \end{array}$$

STEP 4 Solve the new linear system for both of its variables.

Add new equation 1 to −1 times new equation 2.

$$\begin{array}{rcl} 0.5x + 0.2y &=& 300 \\ -0.5x - 0.6y &=& -600 \\ \hline -0.4y &=& -300 \end{array}$$

Divide both the sides by −0.4.

$$\frac{-0.4y}{-0.4} = \frac{-300}{-0.4}$$

$$y = 750$$

Substitute 750 for x in new equation 2 to find the value of x.

$$\begin{aligned} 0.5x + 0.6(750) &= 600 \\ 0.5x + 450 &= 600 \\ 0.5x &= 150 \\ x &= 300 \end{aligned}$$

Substitute 300 for x and 750 for y in any one of the original equations, say, Equation 1 and evaluate.

$$\begin{aligned} 0.7(300) + 0.5(750) + 0.3z &= 1200 \\ 210 + 375 + 0.3z &= 1200 \\ 585 + 0.3z &= 1200 \\ 0.3z &= 615 \\ z &= 2050 \end{aligned}$$

Therefore, the health club received 300 gallons of juice in the first delivery, 750 gallons in the second delivery, and 2050 gallons of juice in the third delivery.

Answer 45e.

a. ***STEP 1*** Write verbal models for the given situation.

$$\text{Number of first-place finishers} + \text{Number of second-place finishers} + \text{Number of third-place finishers} = \text{Total number of events}$$

$$5 \cdot \text{Number of first-place finishers} + 3 \cdot \text{Number of second-place finishers} + \text{Number of third-place finishers} = \text{Combined number of points}$$

$$\text{Number of second-place finishers} = \text{Number of first-place finishers} + \text{Number of third-place finishers}$$

STEP 2 Write a system of equations.

Let f represents the number of athletes who finished in first place, s represents number of athletes who finished in second place, and t represents the number of athletes who finished in third place.

$f + s + t = 20$ Equation 1

$5f + 3s + t = 68$ Equation 2

$s = f + t$ Equation 3

STEP 3 Rewrite the system as a linear system in two variables.

Substitute $f + t$ for s in Equation 1.

$f + f + t + t = 20$

Combine the like terms.

$2f + 2t = 20$ New Equation 1

Substitute $f + t$ for s in Equation 2.

$5f + 3(f + t) + t = 68$

Apply the distributive property.

$5f + 3f + 3t + t = 68$

Combine the like terms.

$8f + 4t = 68$ New Equation 2

STEP 3 Solve the new linear system for both of its variables.

Add new equation 2 to −2 times new equation 1.

$$\begin{aligned} 8f + 4t &= 68 \\ -4f - 4t &= -40 \\ \hline 4f \qquad &= 28 \end{aligned}$$

Solve for f. Divide each side by 4.

$$\frac{4f}{4} = \frac{28}{4}$$
$$f = 7$$

Substitute 7 for f in new equation 2 to get the value of t.

$$\begin{aligned} 8(7) + 4t &= 68 \\ 56 + 4t &= 68 \\ 4t &= 12 \\ t &= 3 \end{aligned}$$

Substitute 7 for f, and 3 for t in Equation 3 and evaluate.

$$\begin{aligned} s &= 7 + 3 \\ &= 10 \end{aligned}$$

There are 7 athletes who finished in first place, 10 athletes finished in second place, and 3 athletes finished in third place.

b. ***STEP 1*** Write the system of equations for the given situation.

It is given that announcement had claimed that the athletes scored a total of 70 points instead of 68 points. The second equation becomes

$5f + 3s + t = 70$.

The system of equations for the given situation is

$$\begin{aligned} f + s + t &= 20 && \text{Equation 1} \\ 5f + 3s + t &= 70 && \text{Equation 2} \\ s &= f + t && \text{Equation 3} \end{aligned}$$

STEP 2 We have $2f + 2t = 20$ New Equation 1.

Substitute $f + t$ for s in Equation 2.

$5f + 3(f + t) + t = 70$

Apply the distributive property.
$5f + 3f + 3t + t = 70$

Combine the like terms.
$8f + 4t = 70$ New Equation 2

STEP 3 Solve the new linear system for both of its variables.
Add new equation 2 to −2 times new equation 1.

$$\begin{array}{r} 8f + 4t = 70 \\ -4f - 4t = -40 \\ \hline 4f \quad = 30 \end{array}$$

Solve for f. Divide each side by 4.

$$\frac{4f}{4} = \frac{30}{4}$$

$$f = 7.5$$

Since we cannot have fractions of a person, the claim is false.

Answer 46e.

Consider the following table which gives the amounts of mixed nuts, granola, and, dried fruit that the three persons (You, Jeff, and, Curtis) purchased,

	Mixed Nuts	Granola	Dried fruit
You	1 lb	0.5 lb	1 lb
Jeff	2 lb	0.5 lb	0.5 lb
Curtis	1 lb	2 lb	0.5 lb

Here, You spend a total of $8, Jeff spends $9, and, Curtis spends $9.
Now find the price per pound of each type of snacks.
Let x, y and z be the price per pound of mixed nuts, granola, and, dried fruit.
From the table, form a linear system as,

$$x + 0.5y + z = 8 \quad \text{...... (1)}$$

$$2x + 0.5y + 0.5z = 9 \quad \text{...... (2)}$$

$$x + 2y + 0.5z = 9 \quad \text{...... (3)}$$

Step 1: Use elimination method to convert the three variables linear system as a two variables linear system as follows:
First multiply equation (2) by -1, and, add the resultant new equation to the equation (1), as follows:
Multiply Equation (2), by -1, then,

$$2x+0.5y+0.5z=9$$
$$(-1)(2x+0.5y+0.5z)=(-1)(9)$$
$$-2x-0.5y-0.5z=-9$$

Add the resultant equation to equation (1), then,

$$\begin{array}{r} x+0.5y+\quad z=\quad 8 \\ \underline{-2x-0.5y-0.5z=-9} \\ -x\qquad\quad +0.5z=-1 \end{array}$$
$$-x+0.5z=-1 \qquad \ldots\ldots (4)$$

Next, multiply equation (1) by -4, and, add the resultant new equation to the equation (3), as follows:
Multiply Equation (1), by -4, then,

$$x+0.5y+z=8$$
$$(-4)(x+0.5y+z)=(-4)(8)$$
$$-4x-2y-4z=-32$$

Add the resultant equation to equation (3), then,

$$\begin{array}{r} -4x-2y-\ 4z=-32 \\ \underline{x+2y+0.5z=\quad 9} \\ -3x\qquad -3.5z=-23 \end{array}$$
$$3x+3.5z=23 \qquad \ldots\ldots (5)$$

Step 2: Next solve the new linear system for both of its variables.
Multiply the equation (4) by 3, and, add the resultant new equation to the equation (5), as follows:
Multiply Equation (4), by 3, then,

$$-x+0.5z=-1$$
$$(3)(-x+0.5z)=(3)(-1)$$
$$-3x+1.5z=-3$$

Add the resultant equation to equation (5), then,

$$\begin{array}{r} -3x+1.5z=-\ 3 \\ \underline{3x+3.5z=\ 23} \\ 5z=20 \\ z=4 \end{array} \qquad \text{Divide both sides by 5}$$

Therefore,

$$\boxed{z=4}.$$

To solve for x, substitute $z=4$ in the equation (4), then,

$-x+0.5z=-1$	Equation (4)
$-x+0.5(4)=-1$	Substitute 4 for z
$-x+2=-1$	Multiply
$-x=-3$	Subtract 2 from both sides
$x=3$	Multiply both sides by -1

Therefore,

$\boxed{x=3}$.

Step 3: Now substitute $x=3$, and, $z=4$ in an original equation, say (3), and solve for y.

$x+2y+0.5z=9$	Equation (3)
$3+2(y)+0.5(4)=9$	Substitute 3 for x, and, 4 for z
$3+2y+2=9$	Multiply
$5+2y=9$	Combine the like terms
$2y=4$	Subtract 5 from both sides
$\boxed{y=2}$	Divide both sides by 2

Therefore, the price of mixed nuts is $\boxed{\$3}$ per pound, the price of granola is $\boxed{\$2}$ per pound, and the price of dried fruit is $\boxed{\$4}$ per pound.

Answer 47e.

a. Write verbal models for the given situation.

$$\text{Number of roses} + \text{Number of lilies} + \text{Number of irises} = \text{Total flowers in each bouquet}$$

$$2.5 \cdot \text{Number of roses} + 4 \cdot \text{Number of lilies} + 2 \cdot \text{Number of irises} = \text{Cost of a bouquet}$$

$$\text{Number of roses} = 2\left(\text{Number of lilies} + \text{Number of irises}\right)$$

Write a system of equations.

Let r represent the number of roses, l represent the number of lilies, and l represent the number of irises.

$r + l + i = 12$	Equation 1
$2.5r + 4l + 2i = 32$	Equation 2
$r = 2(l + i)$	Equation 3

b. Use the distributive property and rewrite Equation 3.

$r = 2l + 2i$ New Equation 1

Substitute $2l + 2i$ for r in Equation 1.

$2l + 2i + l + i = 12$

Combine the like terms.

$3l + 3i = 12$ New Equation 2

Substitute $2l + 2i$ for r in Equation 2.

$2.5(2l + 2i) + 4l + 2i = 32$

Use the distributive Property.

$5l + 5i + 4l + 2i = 32$

Combine the like terms.

$9l + 7i = 32$ New Equation 3

Add new equation 3 to -3 times new equation 2.

$$\begin{array}{r} 9l + 7i = 32 \\ -9l - 9i = -36 \\ \hline -2i = -4 \end{array}$$

Solve for i. Divide each side by -2.

$$\frac{-2i}{-2} = \frac{-4}{-2}$$
$$i = 2$$

Substitute 2 for i in new equation 3 to find the value of l.

$$9l + 7(2) = 32$$
$$9l + 14 = 32$$
$$9l = 18$$
$$l = 2$$

Substitute 2 for l, and 2 for i in new equation 1 to solve for r.

$$r = 2(2) + 2(2)$$
$$= 4 + 4$$
$$= 8$$

Therefore, there are 8 roses, 2 lilies, and 2 irises in each bouquet.

c. Write the system of equations for the given situation. It is given that there is no limitation on the total cost. Thus, we cannot form an equation for the total cost. The system of equations for the given situation is

$r + l + i = 12$ Equation 1

$r = 2(l + i)$ Equation 2

Rewrite Equation 2.

$\frac{r}{2} = l + i$ New equation 1

Substitute $\frac{r}{2}$ for $l + i$ in Equation 1.

$$r + \frac{r}{2} = 12$$

Multiply each side by 2.

$$2\left(r + \frac{r}{2}\right) = 2(12)$$

$$2r + r = 24$$

Combine the like terms.

$3r = 24$.

Divide each side by 3.

$$\frac{3r}{3} = \frac{24}{3}$$

$$r = 8$$

Substitute 8 for r in new equation 1 and simplify.

$$\frac{8}{2} = l + i$$

$$4 = l + i$$

We know that the possible whole number values for l and i are

$l = 4, i = 0$

$l = 0, i = 4$

$l = 3, i = 1$

$l = 1, i = 3$

$l = 2, i = 2$.

Since it is given that each bouquet should contain all the three flowers, the number of flowers cannot be zero. Thus, the possible values for l and i are

$l = 3, i = 1$

$l = 1, i = 3$

$l = 2, i = 2$.

The system of equations does not have unique solution and the three possible solutions are (8, 3, 1), (8, 1, 3), and (8, 2, 2).

Answer 48e.

Consider the following statements,

(a)

One tangerine and, one apple balance one grape fruit.

(b)

One tangerine and, one banana balance one apple.

(c)

Two grape fruits balance two bananas.

Let w, x, y, and, z denote the tangerine, apple, grape fruit, and, banana respectively.

From the given data, write the linear system as follows,

Since, one tangerine, and, one apple balance one grape fruit, so,

$$w+x=y \qquad \text{...... (1)}$$

Since, one tangerine, and, one banana balance one apple, so,

$$w+z=x \qquad \text{...... (2)}$$

Since, two grape fruits balance two bananas.

$$2y=3z \qquad \text{...... (3)}$$

Now find how many tangerines will balance the one apple.

First convert the system into a linear system of two variables by using elimination method.

First multiply equation (1) by 2, then,

$$w+x=y$$

$$2w+2x=2y \qquad \text{Multiply both sides by 2}$$

$$2w+2x=3z \qquad \text{Substitute } 2y=3z \text{, from (3)}$$

$$2w-3z=-2x \qquad \text{...... (4)}$$

Next multiply equation (2) by 3, then,

$$w+z=x$$

$$3(w+z)=3(x)$$

$$3w+3z=3x \qquad \text{...... (5)}$$

Add equations (4), and, (5),

$$3w+3z=3x$$

$$\underline{2w-3z=-2x}$$

$$5w=x$$

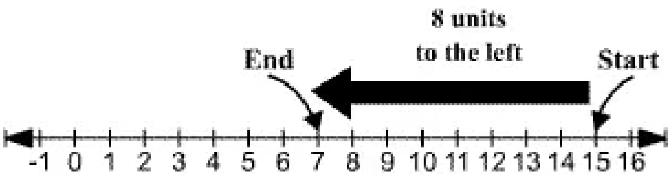
Therefore,

$$x=5w.$$

That means one apple balances $\boxed{5}$ tangerines.

Answer 49e.

Draw a number line. Start at 15 and move 8 units to the left.



We reach the final position. Therefore, $15 + (-8) = 7$.

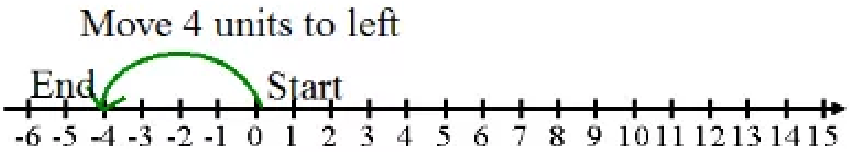
Answer 50e.

Consider the expression,

$$-4-(-13).$$

Now perform the indicated operation as follows:

Since, the first addend, the 4, is negative, so, the first arrow starts at 0, and, move 4 units to the left as shown in below figure:



Recall,

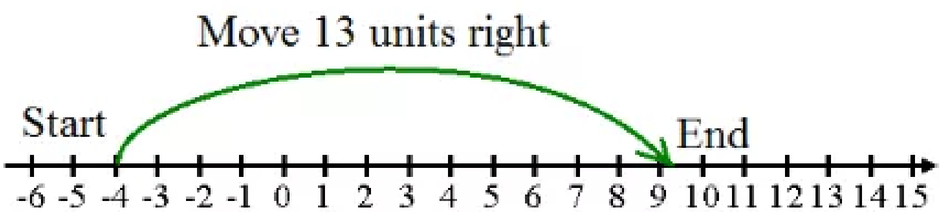
To subtract a negative number, add its opposite number.

The opposite number of -13, is 13.

So,

$$-4+13.$$

Since, the second addend, the 13, is positive, so, the second arrow start at -4, and, moves 13 units to the right as shown in below figure:



Since, the second arrow ends at 9.

Therefore,

$$-4-(-13)=\boxed{9}.$$

We know that the product of two numbers with the same sign is positive.

Therefore, $15 \cdot 7 = 105$.

Answer 52e.

Consider the expression,

$$-4(-8).$$

Now perform the indicated operation as follows:

Recall,

The product or division of two numbers, with same sign is positive.

Here, 4 and 8 have same sign (negative), so, the product of 4 and 8 is positive.

That is,

$$-4(-8)$$
$$= 32 \qquad \text{Multiply with } -4$$

Therefore,

$$-4(-8) = \boxed{32}.$$

Answer 53e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (1, –4) for (x_1, y_1), and (2, 6) for (x_2, y_2) and evaluate.

$$m = \frac{6 - (-4)}{2 - 1}$$
$$= \frac{6 + 4}{1}$$
$$= \frac{10}{1}$$
$$= 10$$

The slope of the line that passes through (1, –4) and (2, 6) is 10.

The slope of the given line is positive.

Therefore, the line passing through the given points rises from left to right.

Answer 54e.

Consider the following points,

$(4,2)$, and, $(-18,1)$.

Find the slope of the line passes through the points $(4,2)$, and, $(-18,1)$.

The formula to find the slope of a line passing through two points is,

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \ldots\ldots (1)$$

Here,

$(x_1, y_1) = (4,2)$, and,

$(x_2, y_2) = (-18,1)$.

Substitute the values in (1), then,

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-2}{-18-4} && \text{Substitute } 1 \text{ for } y_2,\ 2 \text{ for } y_1,\ -18 \text{ for } x_2 \text{ and } 4 \text{ for } x_1 \\ &= \frac{-1}{-22} && \text{Simplify} \\ &= \frac{1}{22} \\ &> 0 \end{aligned}$$

Since, the slope m is positive, that is, $m > 0$, so, the line rises.

Answer 55e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute $(6, -6)$ for (x_1, y_1), and $(-6, 6)$ for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{6 - (-6)}{-6 - (-6)} \\ &= \frac{6 + 6}{-6 + 6} \\ &= \frac{12}{0} \\ &= \text{undefined} \end{aligned}$$

The slope of the given line is undefined.
Therefore, the line passing through the given points is vertical.

Answer 56e.

Consider the following points,

$(-5,2)$, and, $(-5,10)$.

Find the slope of the line passes through the points $(-5,2)$, and, $(-5,10)$.

The formula to find the slope of a line passing through two points is,

$$m=\frac{y_2-y_1}{x_2-x_1} \qquad \ldots\ldots (1)$$

Here,

$(x_1,y_1)=(-5,2)$, and,

$(x_2,y_2)=(-5,10)$.

Substitute the values in (1), then,

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{10-2}{-5-(-5)} && \text{Substitute } 10 \text{ for } y_2,\ 2 \text{ for } y_1,\ -5 \text{ for } x_2 \text{ and } -5 \text{ for } x_1 \\ &= \frac{8}{-5+5} && \text{Simplify} \\ &= \frac{8}{0} \end{aligned}$$

Since, the slope m is undefined, so, the line is vertical.

Answer 57e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (−2, 4) for (x_1, y_1), and (−6, 8) for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{8-4}{-6-(-2)} \\ &= \frac{8-4}{-6+2} \\ &= \frac{4}{-4} \\ &= -1 \end{aligned}$$

The slope of the given line is negative.

Therefore, the line passing through the given points falls from left to right.

Answer 58e.

Consider the following points,

$(-7,3)$, and, $(5,3)$.

Find the slope of the line passes through the points $(-7,3)$, and, $(5,3)$.

The formula to find the slope of a line passing through two points is,

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \ldots\ldots (1)$$

Here,

$(x_1, y_1) = (-7, 3)$, and,

$(x_2, y_2) = (5, 3)$.

Substitute the values in (1), then,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-3}{5-(-7)} \qquad \text{Substitute } 3 \text{ for } y_2,\ 3 \text{ for } y_1,\ 5 \text{ for } x_2 \text{ and } -7 \text{ for } x_1$$

$$= \frac{0}{5+7} \qquad \text{Simplify}$$

$$= \frac{0}{12}$$

$$= 0$$

Since, the slope m is zero, that is, $m = 0$, so, the line is Horizontal.

Answer 59e.

Number the equations.

$3x - y = -7$ Equation 1

$2x + 3y = 21$ Equation 2

STEP 1 Solve Equation 2 for y. Subtract $3x$ from both the sides.

$$3x - y - 3x = -7 - 3x$$

$$-y = -7 - 3x$$

Divide both the sides by -1.

$$\frac{-y}{-1} = \frac{-7 - 3x}{-1}$$

$$y = 7 + 3x \qquad \text{Revised Equation 2}$$

STEP 2 Substitute $7 + 3x$ for y in Equation 1.

$2x + 3(7 + 3x) = 21$

Clear the parentheses using the distributive property and simplify.

$$2x + 21 + 9x = 21$$
$$11x + 21 = 21$$

Subtract 21 from both the sides.

$$11x + 21 - 21 = 21 - 21$$
$$11x = 0$$

Divide both the sides by 11.

$$\frac{11x}{11} = \frac{0}{11}$$
$$x = 0$$

STEP 3 Substitute 0 for x in Revised Equation 2 and solve for y.

$$y = 7 + 3(0)$$
$$y = 7$$

The solution is (0, 7).

Answer 60e.

Consider the following system of equations

$3x+2y=-3$ ...... (1)

$4x-3y=-38$ ...... (2)

Now solve the above system by the elimination method as follows:

Step 1: Multiply equation (1) by 3 and multiply equation (2) by 2, so that the coefficients of y differ only in sign.

$$3x+2y=-3 \xrightarrow{\text{Multiply (1) by 3}} 9x+6y=-6$$
$$4x-3y=-38 \xrightarrow{\text{Multiply (2) by 2}} 8x-6y=-76$$

Step2: Add

$$17x = -85$$
$$x = -5 \quad \text{Divide both sides by 17.}$$

Step3: Substitute the value of x into the original equations. Solve for y

Substitute -5 for x in equation (1),

$3x+2y=-3$

$3(-5)+2y=-3$ Substitute -5 for x

$-15+2y=-3$ Multiply.

$2y=12$ Add 12 to both sides.

$y=6$ Divide both sides by 2.

Thus, the ordered pair is $\boxed{(-5,6)}$

Answer 61e.

Number the equations.

$5x + y = 11$ Equation 1

$2x + 3y = -19$ Equation 2

STEP 1 Solve Equation 2 for y. Subtract $5x$ from both the sides.

$5x + y - 5x = 11 - 5x$

$y = 11 - 5x$ Revised Equation 1

STEP 2 Substitute $11 - 5x$ for y in Equation 1.

$5x + 11 - 5x = 11$

Simplify.

$11 = 11$

This is true for any value of x and y.

Therefore, the given system of equations has infinitely many solutions.