Linear Inequalities

Question 1.
If $-2 < 2x - 1 < 2$ then the value of x lies in the interval
(a) (1/2, 3/2)
(b) (-1/2, 3/2)
(c) (3/2, 1/2)
(d) (3/2, -1/2)

Answer: (b) (-1/2, 3/2)
Given, $-2 < 2x - 1 < 2$
$\Rightarrow -2 + 1 < 2x < 2 + 1$
$\Rightarrow -1 < 2x < 3$
$\Rightarrow -1/2 < x < 3/2$
$\Rightarrow x \in (-1/2, 3/2)$

Question 2.
If $x^2 < -4$ then the value of x is
(a) (-2, 2)
(b) (2, ∞)
(c) (-2, ∞)
(d) No solution

Answer: (d) No solution
Given, $x^2 < -4$
$\Rightarrow x^2 + 4 < 0$
Which is not possible.
So, there is no solution.

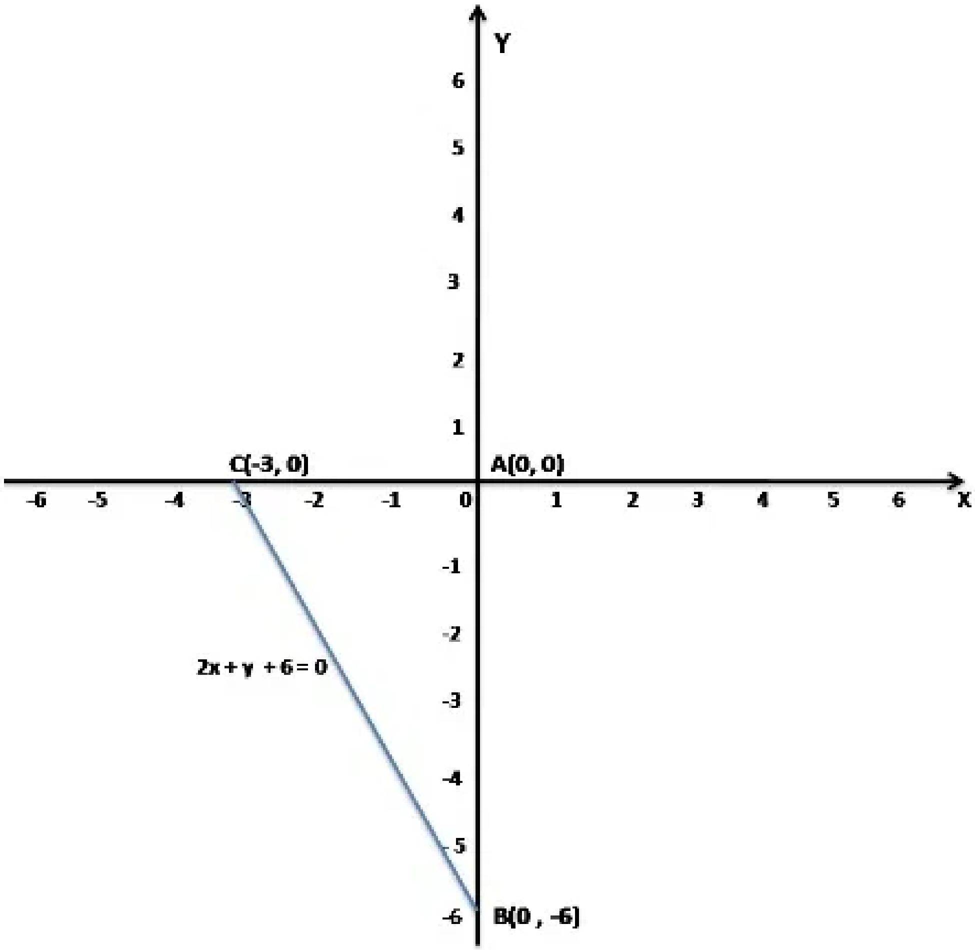
Question 3.
If $|x| < -5$ then the value of x lies in the interval
(a) (-∞, -5)
(b) (∞, 5)

(c) (-5, ∞)
(d) No Solution

Answer: (d) No Solution
Given, $|x| < -5$
Now, LHS ≥ 0 and RHS < 0
Since LHS is non-negative and RHS is negative
So, $|x| < -5$ does not posses any solution

Question 4.
The graph of the inequations $x \leq 0$, $y \leq 0$, and $2x + y + 6 \geq 0$ is
(a) exterior of a triangle
(b) a triangular region in the 3rd quadrant
(c) in the 1st quadrant
(d) none of these

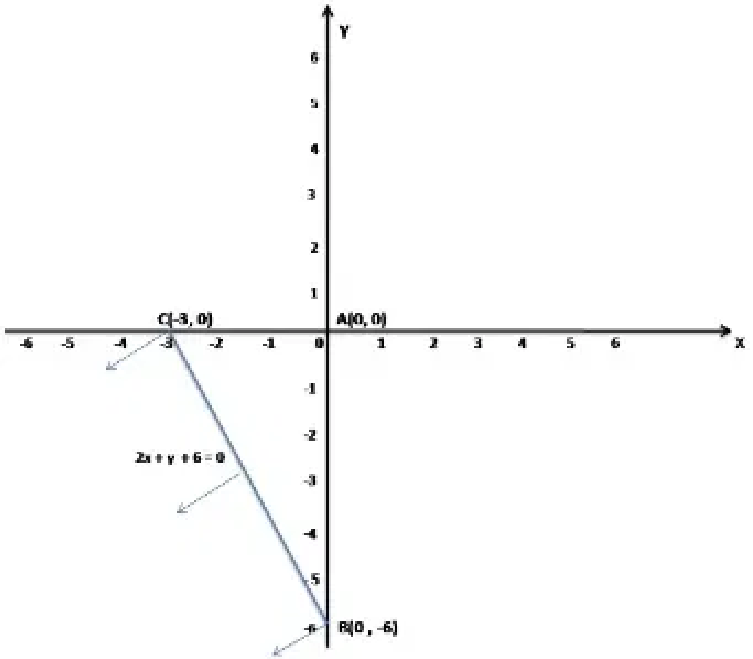
Answer: (b) a triangular region in the 3rd quadrant
Given inequalities $x \geq 0$, $y \geq 0$, $2x + y + 6 \geq 0$
Now take $x = 0$, $y = 0$ and $2x + y + 6 = 0$
when $x = 0$, $y = -6$
when $y = 0$, $x = -3$
So, the points are A(0, 0), B(0, -6) and C(-3, 0)



So, the graph of the inequations $x \leq 0$, $y \leq 0$, and $2x + y + 6 \geq 0$ is a triangular region in the 3rd quadrant.

Question 5.
The graph of the inequalities $x \geq 0$, $y \geq 0$, $2x + y + 6 \leq 0$ is
(a) a square
(b) a triangle
(c) { }
(d) none of these

Answer: (c) { }
Given inequalities $x \geq 0$, $y \geq 0$, $2x + y + 6 \leq 0$
Now take $x = 0$, $y = 0$ and $2x + y + 6 = 0$
when $x = 0$, $y = -6$
when $y = 0$, $x = -3$
So, the points are A(0, 0), B(0, -6) and C(-3, 0)



Since region is outside from the line 2x + y + 6 = 0
So, it does not represent any figure.

Question 6.
Solve: 2x + 1 > 3
(a) [-1, ∞]
(b) (1, ∞)
(c) (∞, ∞)
(d) (∞, 1)

Answer: (b) (1, ∞)
Given, 2x + 1 > 3
⇒ 2x > 3 – 1
⇒ 2x > 2
⇒ x > 1
⇒ x ∈ (1, ∞)

Question 7.
The solution of the inequality 3(x – 2)/5 ≥ 5(2 – x)/3 is
(a) x ∈ (2, ∞)
(b) x ∈ [-2, ∞)
(c) x ∈ [∞, 2)
(d) x ∈ [2, ∞)

Answer: (d) $x \in [2, \infty)$
Given, $3(x - 2)/5 \geq 5(2 - x)/3$
$\Rightarrow 3(x - 2) \times 3 \geq 5(2 - x) \times 5$
$\Rightarrow 9(x - 2) \geq 25(2 - x)$
$\Rightarrow 9x - 18 \geq 50 - 25x$
$\Rightarrow 9x - 18 + 25x \geq 50$
$\Rightarrow 34x - 18 \geq 50$
$\Rightarrow 34x \geq 50 + 18$
$\Rightarrow 34x \geq 68$
$\Rightarrow x \geq 68/34$
$\Rightarrow x \geq 2$
$\Rightarrow x \in [2, \infty)$

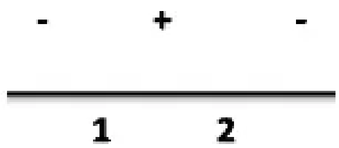
Question 8.
Solve: $1 \leq |x - 1| \leq 3$
(a) [-2, 0]
(b) [2, 4]
(c) $[-2, 0] \cup [2, 4]$
(d) None of these

Answer: (c) $[-2, 0] \cup [2, 4]$
Given, $1 \leq |x - 1| \leq 3$
$\Rightarrow -3 \leq (x - 1) \leq -1$ or $1 \leq (x - 1) \leq 3$
i.e. the distance covered is between 1 unit to 3 units
$\Rightarrow -2 \leq x \leq 0$ or $2 \leq x \leq 4$
Hence, the solution set of the given inequality is
$x \in [-2, 0] \cup [2, 4]$

Question 9.
Solve: $-1/(|x| - 2) \geq 1$ where $x \in R$, $x \neq \pm 2$
(a) (-2, -1)
(b) (-2, 2)
(c) $(-2, -1) \cup (1, 2)$
(d) None of these

Answer: (c) $(-2, -1) \cup (1, 2)$
Given, $-1/(|x| - 2) \geq 1$
$\Rightarrow -1/(|x| - 2) - 1 \geq 0$
$\Rightarrow \{-1 - (|x| - 2)\}/(|x| - 2) \geq 0$

⇒ $\{1 - |x|\}/(|x| - 2) \geq 0$
⇒ $-(|x| - 1)/(|x| - 2) \geq 0$



Using number line rule:
$1 \leq |x| < 2$
⇒ $x \in (-2, -1) \cup (1, 2)$

Question 10.
If $x^2 < 4$ then the value of x is
(a) (0, 2)
(b) (-2, 2)
(c) (-2, 0)
(d) None of these

Answer: (b) (-2, 2)
Given, $x^2 < 4$
⇒ $x^2 - 4 < 0$
⇒ $(x - 2) \times (x + 2) < 0$
⇒ $-2 < x < 2$
⇒ $x \in (-2, 2)$

Question 11.
Solve: $2x + 1 > 3$
(a) [1, 1)
(b) (1, ∞)
(c) (∞, ∞)
(d) (∞, 1)

Answer: (b) (1, ∞)
Given, $2x + 1 > 3$
⇒ $2x > 3 - 1$
⇒ $2x > 2$
⇒ $x > 1$
⇒ $x \in (1, \infty)$

Question 12.
If a is an irrational number which is divisible by b then the number b
(a) must be rational
(b) must be irrational
(c) may be rational or irrational
(d) None of these

Answer: (b) must be irrational
If a is an irrational number which is divisible by b then the number b must be irrational.
Ex: Let the two irrational numbers are $\sqrt{2}$ and $\sqrt{3}$
Now, $\sqrt{2}/\sqrt{3} = \sqrt{(2/3)}$

Question 13.
Sum of two rational numbers is _______ number.
(a) rational
(b) irrational
(c) Integer

Answer: (a) rational
The sum of two rational numbers is a rational number.
Ex: Let two rational numbers are 1/2 and 1/3
Now, 1/2 + 1/3 = 5/6 which is a rational number.

Question 14.
If $|x| = -5$ then the value of x lies in the interval
(a) $(-5, \infty)$
(b) $(5, \infty)$
(c) $(\infty, -5)$
(d) No solution

Answer: (d) No solution
Given, $|x| = -5$
Since $|x|$ is always positive or zero
So, it can not be negative
Hence, given inequality has no solution.

Question 15.
The value of x for which $|x + 1| + \sqrt{(x - 1)} = 0$
(a) 0
(b) 1

(c) -1
(d) No value of x

Answer: (d) No value of x
Given, $|x + 1| + \sqrt{(x - 1)} = 0$, where each term is non-negative.
So, $|x + 1| = 0$ and $\sqrt{(x - 1)} = 0$ should be zero simultaneously.
i.e. $x = -1$ and $x = 1$, which is not possible.
So, there is no value of x for which each term is zero simultaneously.

Question 16.
If $x^2 < -4$ then the value of x is
(a) $(-2, 2)$
(b) $(2, \infty)$
(c) $(-2, \infty)$
(d) No solution

Answer: (d) No solution
Given, $x^2 < -4$
$\Rightarrow x^2 + 4 < 0$
Which is not possible.
So, there is no solution.

Question 17.
The solution of $|2/(x - 4)| > 1$ where $x \neq 4$ is
(a) $(2, 6)$
(b) $(2, 4) \cup (4, 6)$
(c) $(2, 4) \cup (4, \infty)$
(d) $(-\infty, 4) \cup (4, 6)$

Answer: (b) $(2, 4) \cup (4, 6)$
Given, $|2/(x - 4)| > 1$
$\Rightarrow 2/|x - 4| > 1$
$\Rightarrow 2 > |x - 4|$
$\Rightarrow |x - 4| < 2$
$\Rightarrow -2 < x - 4 < 2$
$\Rightarrow -2 + 4 < x < 2 + 4$
$\Rightarrow 2 < x < 6$
$\Rightarrow x \in (2, 6)$, where $x \neq 4$
$\Rightarrow x \in (2, 4) \cup (4, 6)$

Question 18.
The solution of the function $f(x) = |x| > 0$ is
(a) R
(b) R – {0}
(c) R – {1}
(d) R – {-1}

Answer: (b) R – {0}
Given, $f(x) = |x| > 0$
We know that modulus is non negative quantity.
So, $x \in R$ except that $x = 0$
$\Rightarrow x \in R - \{0\}$
This is the required solution

Question 19.
Solve: $|x - 1| \leq 5$, $|x| \geq 2$
(a) [2, 6]
(b) [-4, -2]
(c) [-4, -2] ∪ [2, 6]
(d) None of these

Answer: (c) [-4, -2] ∪ [2, 6]
Given, $|x - 1| \leq 5$, $|x| \geq 2$
$\Rightarrow -(5 \leq (x - 1) \leq 5), (x \leq -2 \text{ or } x \geq 2)$
$\Rightarrow -(4 \leq x \leq 6), (x \leq -2 \text{ or } x \geq 2)$
Now, required solution is
$x \in [-4, -2] \cup [2, 6]$

Question 20.
The solution of the $15 < 3(x - 2)/5 < 0$ is
(a) $27 < x < 2$
(b) $27 < x < -2$
(c) $-27 < x < 2$
(d) $-27 < x < -2$

Answer: (a) $27 < x < 2$
Given inequality is:
$15 < 3(x - 2)/5 < 0$
$\Rightarrow 15 \times 5 < 3(x - 2) < 0 \times 5$
$\Rightarrow 75 < 3(x - 2) < 0$
$\Rightarrow 75/3 < x - 2 < 0$

$\Rightarrow 25 < x - 2 < 0$
$\Rightarrow 25 + 2 < x < 0 + 2$
$\Rightarrow 27 < x < 2$
