Sample Question Paper - 38
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. A point P is 10 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 8 cm, then find the radius of circle.

OR

O is the centre of two concentric circles of radii 12 cm and 13 cm. AB is a chord of outer circle which touches the inner circle. What is the length of chord AB.

2. A solid sphere of radius r is melted and recast into the shape of a solid cone of height r, find the radius of the base of the cone.

3. A sum of ₹ 2000 is invested at 6% simple interest per annum.

(i) Calculate the interests at the end of 1, 2, 3,... years.

(ii) Does the sequence of interests forms an A.P.?

4. If the mean of the following distribution is 6, then find the value of p.

$x:$	2	4	6	10	$p+5$
$f:$	3	2	3	1	2

5. The product of two consecutive even integers is 528. Represent the situation in the form of a quadratic equation.

OR

Find the value of a and b, if $x = 7$ and 5 are the solutions of the equation $ax^2 - bx + 35 = 0$.

6. Find the median of the collection of first seven whole numbers. If 9 is also included in the collection, find the difference of the median in two cases.

SECTION - B

7. In an A.P., the first term is 25, n^{th} term is -17 and sum of first n terms is 60. Find n and d, the common difference.

OR

Which term of the A.P.: –2, –7, –12,... will be –77? Find the sum of this A.P. upto the term –77.

8. In a class test, the sum of the marks obtained by Ankur in Mathematics and Science is 28. If he had got 3 more marks in Mathematics and 4 marks less in Science, then product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

9. Draw a circle of radius 5 cm. From a point *P*, 8 cm away from its centre, construct a pair of tangents to the circle. Measure the length of each one of the tangents.

10. The angle of elevation of the top of a chimney from the top of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40 m, then find the height of the chimney.

SECTION - C

11. Two circles with centres *A* and *B* of radii 6 cm and 8 cm respectively intersect at two points *C* and *D* such that *AC* and *BC* are tangents to the two circles. Find the length of common chord *CD*.

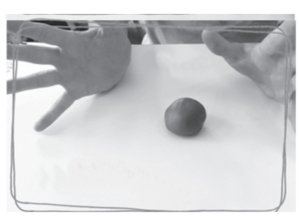
12. The angles of elevation of the top of a rock from the top and foot of 100 m high tower are 30° and 45° respectively. Find the height of the rock.

OR

Mr Anna Hazare Padyatra party wanted to go from Delhi to Dehradun. The walkers travelled 150 km straight and then took a 45° turn towards Varanasi and walked straight for another 120 km. Approximately how far was the party from the starting point? (Use $\sqrt{2} = 1.414$)

Case Study - 1

13. To make the learning process more interesting, creative and innovative, Amayra's class teacher brings clay in the classroom, to teach the topic - Surface Areas and Volumes. With clay, she forms a cylinder of radius 6 cm and height 8 cm. Then she moulds the cylinder into a sphere and asks some questions to students.

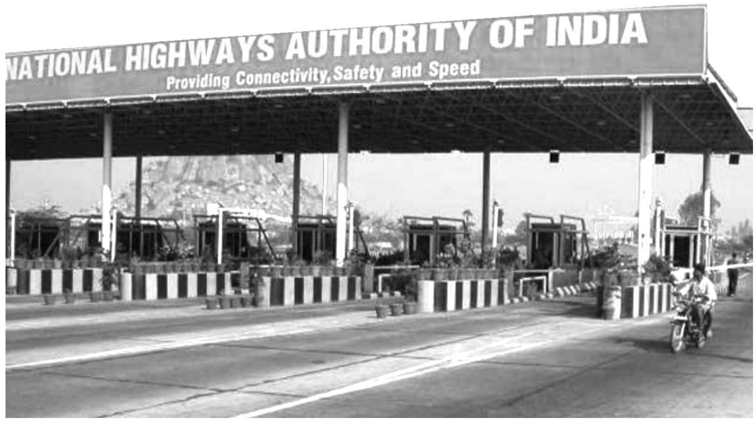



(i) Find the radius of the sphere so formed.

(ii) Find the ratio of the volume of sphere to the volume of cylinder.

Case Study - 2

14. On a particular day, National Highway Authority of India (NHAI) checked the toll tax collection of a particular toll plaza in Rajasthan.



The following table shows the toll tax paid by drivers and the number of vehicles on that particular day.

Toll tax (in ₹)	30-40	40-50	50-60	60-70	70-80
Number of vehicles	80	110	120	70	40

Based on the above information, answer the following questions.

(i) If x_i's denotes the class marks and d_i's denotes the deviation of assumed mean (A) from x_i's, then find the minimum value of $|d_i|$.

(ii) Find the mean of toll tax received by NHAI by assumed mean method.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Since PT is a tangent of circle

$\therefore\ \angle T = 90°$

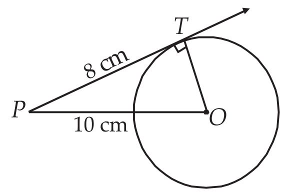
In right ΔOPT,

$OT^2 = OP^2 - PT^2$

$\Rightarrow\ OT^2 = 10^2 - 8^2$

$\Rightarrow\ OT^2 = 100 - 64 = 36$

$\Rightarrow\ OT = 6$ cm



OR

BM is a tangent of inner circle

$\therefore\ \angle M = 90°$

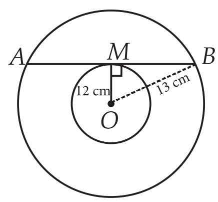
In right ΔOMB,

$BM^2 = OB^2 - OM^2$

$\Rightarrow\ BM^2 = 13^2 - 12^2$

$\Rightarrow\ BM^2 = 169 - 144 = 25$

$\Rightarrow\ BM = 5$ cm and $AB = 2BM = 2 \times 5 = 10$ cm



2. Volume of the sphere $= \frac{4}{3}\pi r^3$

Since the sphere is melted and recast into a cone.

$\therefore$ Volume of the cone = Volume of the sphere

$\therefore\ \frac{1}{3}\pi R^2 h = \frac{4}{3}\pi r^3$ [R is radius of the cone]

But $h = r$

$\therefore\ \frac{1}{3}\pi R^2 \cdot r = \frac{4}{3}\pi r^3 \Rightarrow R^2 = 4r^2 \Rightarrow\ R = 2r$

$\therefore$ Base radius of the cone $= 2r$.

3. Here, $P =$ ₹ 2000, $R = 6\%$ per annum

We know that, Simple interest (S.I.) $= PRT/100$, …(i)

where T is the time in years

(i) Putting $T = 1, 2, 3, \ldots$ in (i)

Interest at the end of first year $= \frac{2000 \times 6 \times 1}{100} =$ ₹ 120

Interest at the end of second year $= \frac{2000 \times 6 \times 2}{100} =$ ₹ 240

Interest at the end of third year $= \frac{2000 \times 6 \times 3}{100} =$ ₹ 360

So, the sequence of interest (in ₹) is 120, 240, 360, ...

(ii) In the above sequence,

Since, $a_1 = 120$, $a_2 = 240$, $a_3 = 360$,...

Now, $a_2 - a_1 = 240 - 120 = 120$,

$a_3 - a_2 = 360 - 240 = 120$

$\because\ a_2 - a_1 = a_3 - a_2 = 120$ (in each case)

$\therefore$ The sequence of interests forms an A.P.

4. We construct the following table :

x_i	f_i	$f_i x_i$
2	3	6
4	2	8
6	3	18
10	1	10
$p + 5$	2	$2p + 10$
	$n = \Sigma f_i = 11$	$\Sigma f_i x_i = 2p + 52$

We have, $n = \Sigma f_i = 11$, $\Sigma f_i x_i = 2p + 52$

$\therefore\ \text{Mean} = \frac{\Sigma f_i x_i}{n} \Rightarrow 6 = \frac{2p + 52}{11}$

$\Rightarrow\ 66 = 2p + 52 \Rightarrow 2p = 14 \Rightarrow p = 7$

5. Let the two consecutive even integers are $2x$ and $2x + 2$.

According to the condition, $2x(2x + 2) = 528$

$\Rightarrow\ 4x^2 + 4x - 528 = 0\ \Rightarrow\ x^2 + x - 132 = 0$

This is the required quadratic equation.

OR

Since $x = 7$ and $x = 5$ are the solutions of equation $p(x) = 0$ where $p(x) = ax^2 - bx + 35$

$\therefore\ p(7) = 0$ and $p(5) = 0$

$p(7) = 0 \Rightarrow a(7)^2 - b(7) + 35 = 0$

$\Rightarrow\ 49a - 7b + 35 = 0$

$\Rightarrow\ 7a - b + 5 = 0\ \Rightarrow b = 7a + 5$ ...(i)

Now, $p(5) = 0 \Rightarrow a(5)^2 - b(5) + 35 = 0$

$\Rightarrow\ 25a - 5b + 35 = 0$

$\Rightarrow\ 5a - b + 7 = 0$ ...(ii)

Using (i) in (ii), we get

$5a - (7a + 5) + 7 = 0$

$\Rightarrow\ -2a - 5 + 7 = 0\ \Rightarrow\ -2a = -2\ \Rightarrow\ a = 1$.

From (i), we have, $b = 7(1) + 5 = 12$

Hence, $a = 1$ and $b = 12$.

6. The first seven whole numbers arranged in ascending order are 0, 1, 2, 3, 4, 5, 6

Here, $n = 7$

$\therefore$ Median $= 4^{th}$ observation $= 3$

If 9 is included, then observations are

0, 1, 2, 3, 4, 5, 6, 9

Now, $n = 8$

$\therefore\ \text{Median} = \frac{(4^{th} + 5^{th})\text{ observation}}{2}$

$= \dfrac{3+4}{2} = \dfrac{7}{2} = 3.5$

$\therefore$ Difference of medians = 3.5 – 3 = 0.5

7. Given, first term $(a) = 25$, $a_n = -17$ and $S_n = 60$

We have, $a_n = a + (n-1)d \Rightarrow 25 + (n-1)d = -17$

$\Rightarrow (n-1)d = -42$ ...(i)

$S_n = \dfrac{n}{2}[2a+(n-1)d] \Rightarrow \dfrac{n}{2}[2(25)+(n-1)d] = 60$ ...(ii)

From (i) and (ii), we get

$\dfrac{n}{2}[50-42] = 60 \Rightarrow n = \dfrac{60}{4} \Rightarrow n = 15$

Now, substituting $n = 15$ in (i), we get

$(15-1)d = -42 \Rightarrow d = \dfrac{-42}{14} = -3$

OR

Given A.P. is –2, –7, –12, ...

Let the n^{th} term of the A.P. is –77.

Then, first term, $a = -2$ and

common difference, $d = -7 - (-2) = -7 + 2 = -5$

$\because$ n^{th} term of an A.P., $a_n = a + (n-1)d$

$\Rightarrow -77 = -2 + (n-1)(-5)$

$\Rightarrow -75 = -(n-1) \times 5$

$\Rightarrow (n-1) = 15 \Rightarrow n = 16$

So, the 16^{th} term of the given A.P. will be –77.

Now, the sum of n terms of an A.P. is

$S_n = \dfrac{n}{2}[2a+(n-1)d]$

So, sum of 16 terms, $S_{16} = \dfrac{16}{2}[2\times(-2)+(n-1)(-5)]$

$= 8[-4 + (16-1)(-5)] = 8(-4 - 75) = 8 \times (-79) = -632$

Hence, the sum of this A.P. upto the term –77 is –632.

8. Let marks obtained by Ankur in Mathematics be x, then marks obtained in Science is $28 - x$.

According to question, $(x+3)(28-x-4) = 180$

$\Rightarrow (x+3)(24-x) = 180$

$\Rightarrow 24x - x^2 + 72 - 3x = 180 \Rightarrow x^2 - 21x + 108 = 0$

$\Rightarrow x^2 - 12x - 9x + 108 = 0$

$\Rightarrow x(x-12) - 9(x-12) = 0$

$\Rightarrow (x-12)(x-9) = 0 \Rightarrow x = 12$ or $x = 9$

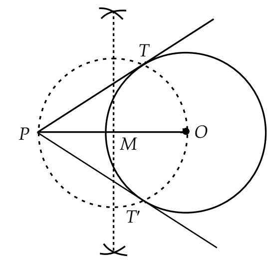
If marks obtained in Mathematics = 12, then marks obtained in Science = 28 – 12 = 16

If marks obtained in Mathematics = 9, then marks obtained in Science = 28 – 9 = 19

9. Steps of construction :

Step-I : Draw a circle with O as centre and radius 5 cm.

Step-II : Mark a point P outside the circle such that $OP = 8$ cm.



Step-III : Join OP and draw its perpendicular bisector, which cuts OP at M.

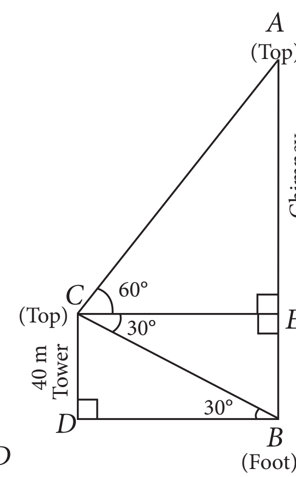
Step-IV : Draw a circle with M as centre and radius equal to MP to intersect the given circle at the point T and T'. Join PT and PT'.

Hence, PT and PT' are the required tangents.

10. In right Δ CDB,

$\tan 30° = \dfrac{CD}{DB} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{40}{DB}$

$\Rightarrow DB = 40\sqrt{3}$ m ...(i)



In right ΔAEC,

$\tan 60° = \dfrac{AE}{CE} \Rightarrow \sqrt{3} = \dfrac{AE}{DB}$

$\Rightarrow \sqrt{3} = \dfrac{AE}{40\sqrt{3}}$

$\Rightarrow AE = 40\sqrt{3}\sqrt{3} = 40 \times 3$

$\Rightarrow AE = 120$ m

$\therefore$ Height of the chimney

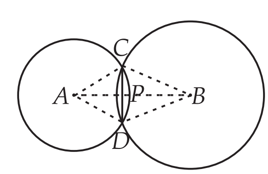
$= AB = AE + EB = AE + CD$

$= 120 + 40 = 160$ m

11. Since tangent at a point to a circle is perpendicular to the radius through the point of contact.

$\therefore$ $\angle ACB = 90°$

In ΔACB,



$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = (6)^2 + (8)^2$

$= 36 + 64 = 100$

$\therefore AB = 10$ cm

Since the line joining the centres of two intersecting circles is perpendicular bisector of their common chord.

$\therefore$ $AP \perp CD$ and $CP = PD$

Let $AP = x$, then $BP = 10 - x$

Let $CP = DP = y$ cm

In ΔAPC and ΔBPC, applying pythagoras theorem,

$AC^2 = AP^2 + PC^2$ and $BC^2 = BP^2 + PC^2$

$\Rightarrow (6)^2 = x^2 + y^2$ and $(8)^2 = (10 - x)^2 + y^2$

$\Rightarrow 36 = x^2 + y^2$ and $64 = 100 + x^2 - 20x + y^2$

$\therefore 64 = 100 - 20x + 36 \Rightarrow 20x = 100 + 36 - 64$

$\Rightarrow 20x = 72 \Rightarrow x = \frac{72}{20} = 3.6$ cm

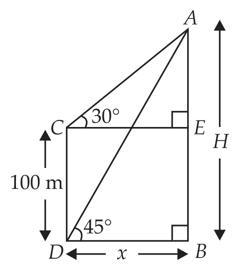
Also $36 = x^2 + y^2$

$\Rightarrow 36 = (3.6)^2 + y^2 \Rightarrow 36 - 12.96 = y^2$

$\Rightarrow 23.04 = y^2 \Rightarrow y = 4.8$ cm

$\therefore CD = 2CP = 2y = 2 \times 4.8$ cm $= 9.6$ cm

12. Let AB be the height of the rock and CD be the height of tower.



$CD = BE = 100$ m,

$AB = H$

$\therefore AE = AB - BE = H - 100,$

$CE = BD = x$

In ΔACE, $\tan 30° = \frac{AE}{CE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{H-100}{x} \Rightarrow x = \sqrt{3}\,(H-100)$ ...(1)

In ΔABD, $\tan 45° = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{H}{x} \Rightarrow x = H$ ...(2)

From (1) and (2), we have

$H = \sqrt{3}(H-100) \Rightarrow H = \sqrt{3}H - 100\sqrt{3}$

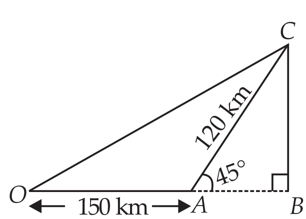
$\Rightarrow \sqrt{3}H - H = 100\sqrt{3} \Rightarrow H = \frac{100\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow H = \frac{100\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{100\sqrt{3}(\sqrt{3}+1)}{2}$

$\Rightarrow H = 50\sqrt{3}(\sqrt{3}+1) \Rightarrow H = 50(3+\sqrt{3})$ m

OR

Let O be the starting point of Mr Anna Hazare Padyatra party.



$OA = 150$ km,

$AC = 120$ km

In ΔABC, $\sin 45° = \frac{BC}{AC}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{120} \Rightarrow BC = 60\sqrt{2}$ km

and $\cos 45° = \frac{AB}{AC} \Rightarrow \frac{1}{\sqrt{2}} = \frac{AB}{120} \Rightarrow AB = 60\sqrt{2}$ km

$\therefore OB = OA + AB = (150 + 60\sqrt{2})$ km

$\Rightarrow OB = 150 + 60(1.414) = 234.84$ km

and $BC = 60\sqrt{2} = 60(1.414) = 84.84$ km

In ΔOBC, $OC^2 = OB^2 + BC^2$

$\Rightarrow OC^2 = (234.84)^2 + (84.84)^2$

$\Rightarrow OC^2 = 55149.82 + 7197.82$

$\Rightarrow OC^2 = 62347.64$

$\Rightarrow OC = \sqrt{62347.64} = 249.69$

$\therefore OC = 250$ km (approx.)

Thus, the distance between starting point to the final point is 250 km approx.

13. (i) Since, volume of sphere = volume of cylinder

$\Rightarrow \frac{4}{3}\pi R^3 = \pi r^2 h$, where R, r are the radii of sphere and cylinder respectively.

$\Rightarrow R^3 = \frac{6 \times 6 \times 8 \times 3}{4} = (6)^3 \Rightarrow R = 6$ cm

$\therefore$ Radius of sphere = 6 cm

(ii) Volume of sphere $= \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6 = 905.14 \text{ cm}^3$

$\because$ Volume of sphere = Volume of cylinder

$\therefore$ Required ratio = 1 : 1

14. Let us consider the following table :

Class	Class marks (x_i)	$d_i = x_i - A$	Frequency (f_i)	$f_i d_i$
30-40	35	–20	80	–1600
40-50	45	–10	110	–1100
50-60	55 = A	0	120	0
60-70	65	10	70	700
70-80	75	20	40	800
Total			$\sum f_i = 420$	$\sum f_i d_i = -1200$

(i) The values of $|d_i|$ are 0, 10, 20

Thus, the minimum value of $|d_i|$ is 0.

(ii) Required mean $= A + \frac{\sum f_i d_i}{\sum f_i} = 55 - \frac{1200}{420}$

$= ₹\ 52.14$