Chapter – 1

Matrices and Determinants

Ex 1.1

Question 1.
Find the minors and cofactors of all the elements of the following determinants.

(i)
$$\begin{vmatrix} 5 & 20 \\ 0 & -1 \end{vmatrix}$$

(ii)
$$\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$$

Solution:

(i)
$$\begin{vmatrix} 5 & 20 \\ 0 & -1 \end{vmatrix}$$

Minor of 5 = M_{11} = -1
Minor of 20 = M_{12} = 0

Minor of 0 = M_{21} = 20
Minor of -1 = M_{22} = 5

Cofactor of 5 = $A_{11} = (-1)^{1+1} M_{11} = 1 \times -1 = -1$
Cofactor of 20 = $A_{12} = (-1)^{1+2} M_{12} = -1 \times 0 = 0$

Cofactor of 0 = $A_{21} = (-1)^{2+1} M_{21} = -1 \times 20 = -20$
Cofactor of -1 = $A_{22} = (-1)^{2+2} M_{22} = 1 \times 5 = 5$

(ii) $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$

Minor of 1 is $M_{11} = \begin{vmatrix} -1 & 2 \\ 5 & 2 \end{vmatrix}$ = -2 – 10 = -12

Minor of -3 is $M_{12} = \begin{vmatrix} 4 & 2 \\ 3 & 2 \end{vmatrix}$ = 8 – 6 = 2

Minor of 2 is $M_{13} = \begin{vmatrix} 4 & -1 \\ 3 & 5 \end{vmatrix}$ = 20 + 3 = 23

Minor of 4 is $M_{21} = \begin{vmatrix} -3 & 2 \\ 5 & 2 \end{vmatrix}$ = -6 – 10 = -16

Minor of -1 is $M_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$ = 2 – 6 = -4

Minor of 2 is $M_{23} = \begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix}$ = 5 + 9 = 14

Minor of 3 is $M_{31} = \begin{vmatrix} -3 & 2 \\ -1 & 2 \end{vmatrix}$ = -6 + 2 = -4

Minor of 5 is $M_{32} = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$ = 2 – 8 = -6

Minor of 2 is $M_{33} = \begin{vmatrix} 1 & -3 \\ 4 & -1 \end{vmatrix}$ = -1 + 12 = 11

Cofactor of 1 is $A_{11} = (-1)^{1+1} M_{11}$ = -12

Cofactor of -3 is $A_{12} = (-1)^{1+2} M_{12}$ = -2

Cofactor of 2 is $A_{13} = (-1)^{1+3} M_{13}$ = 23

Cofactor of 4 is $A_{21} = (-1)^{2+1} M_{21}$ = -1 × -16 = 16

Cofactor of -1 is $A_{22} = (-1)^{2+2} M_{22}$ = -4

Cofactor of 2 is $A_{23} = (-1)^{2+3} M_{23}$ = -14

Cofactor of 3 is $A_{31} = (-1)^{3+1} M_{31} = -4$

Cofactor of 5 is $A_{32} = (-1)^{3+2} M_{32} = -1 \times -6 = 6$

Cofactor of 2 is $A_{33} = (-1)^{3+3} M_{33} = 11$

Question 2.

Evaluate $\begin{vmatrix} 3 & -2 & 4 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Solution:

$$\begin{vmatrix} 3 & -2 & 4 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 3\begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} - (-2)\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} + 4\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix}$$

= 3(0 – 2) + 2(6 – 1) + 4(4 – 0)
= -6 + 10 + 16
= 20

Question 3.

Solve: $\begin{vmatrix} 2 & x & 3 \\ 4 & 1 & 6 \\ 1 & 2 & 7 \end{vmatrix} = 0$

Solution:

$$\begin{vmatrix} 2 & x & 3 \\ 4 & 1 & 6 \\ 1 & 2 & 7 \end{vmatrix} = 0$$

$$2\begin{vmatrix} 1 & 6 \\ 2 & 7 \end{vmatrix} - x\begin{vmatrix} 4 & 6 \\ 1 & 7 \end{vmatrix} + 3\begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix} = 0$$

2(7 – 12) – x(28 – 6) + 3(8 – 1) = 0
2(-5) – x(22) + 3(7) = 0
-10 – 22x + 21 = 0
-22x + 11 = 0

$-22x = -11$

$x = \frac{-11}{-22} = \frac{1}{2}$

Question 4.

Find |AB| if A = $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$

Solution:

$$AB = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 0+2 \\ 6+1 & 0-2 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 7 & -2 \end{bmatrix}$$

$$\therefore |AB| = \begin{vmatrix} 8 & 2 \\ 7 & -2 \end{vmatrix} = -16 - 14 = -30$$

Question 5.

Solve: $\begin{vmatrix} 7 & 4 & 11 \\ -3 & 5 & x \\ -x & 3 & 1 \end{vmatrix} = 0$

Solution:

$$\begin{vmatrix} 7 & 4 & 11 \\ -3 & 5 & x \\ -x & 3 & 1 \end{vmatrix} = 0$$

$$7\begin{vmatrix} 5 & x \\ 3 & 1 \end{vmatrix} - 4\begin{vmatrix} -3 & x \\ -x & 1 \end{vmatrix} + 11\begin{vmatrix} -3 & 5 \\ -x & 3 \end{vmatrix} = 0$$

$7(5 - 3x) - 4(-3 + x^2) + 11(-9 + 5x) = 0$

$35 - 21x + 12 - 4x^2 - 99 + 55x = 0$

$-4x^2 - 21x + 55x + 35 + 12 - 99 = 0$

$-4x^2 + 34x - 52 = 0$

$2 \times 26 = 52$

$-13 \quad -4$

$\frac{-13}{2} \quad \frac{-4}{2}$

Divide throughout by -2 we get

$2x^2 - 17x + 26 = 0$

$(2x - 13)(x - 2) = 0$

$2x - 13 = 0$ (or) $x - 2 = 0$

$x = 13/2$ (or) $x = 2$

$\therefore x = 13/2, x = 2$

Question 6.

Evaluate: $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$

Solution:

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} 1 & a & -bc \\ 1 & b & -ca \\ 1 & c & -ab \end{vmatrix} \text{(By property 6)}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} \text{Take out } -1 \text{ from } C_3$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$$

Multiply R_1 by a, R_2 by b, R_3 by c and divide the determinant by abc

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{abc}{abc}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \text{ Take out } abc \text{ from } C_3$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} C_2 \leftrightarrow C_3$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad C_1 \leftrightarrow C_2$$

$= 0$

Question 7.

Prove that $\begin{vmatrix} \frac{1}{a} & bc & b+c \\ \frac{1}{b} & ca & c+a \\ \frac{1}{c} & ab & a+b \end{vmatrix} = 0$

Solution:

$$\begin{vmatrix} \frac{1}{a} & bc & b+c \\ \frac{1}{b} & ca & c+a \\ \frac{1}{c} & ab & a+b \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} \frac{a}{a} & abc & a(b+c) \\ \frac{b}{b} & abc & b(c+a) \\ \frac{c}{c} & abc & c(a+b) \end{vmatrix} \text{Multiply } R_1 \text{ by } a,\ R_2 \text{ by } b,\ R_3 \text{ by } c \text{ and divide the determinant by } abc$$

$$= \frac{abc}{abc}\begin{vmatrix} 1 & 1 & a(b+c) \\ 1 & 1 & b(c+a) \\ 1 & 1 & c(a+b) \end{vmatrix} \text{Take out } abc \text{ from } C_2 = 0\ [\because C_1 \equiv C_2]$$

Hence proved.

Question 8.

Prove that $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

Solution:

$$\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$$

$$= abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} \text{Take out } a,b,c \text{ respectively from } R_1, R_2 \text{ and } R_3$$

$$= a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \text{Take out } a,b,c \text{ respectively from } C_1, C_2 \text{ and } C_3$$

$$= a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix} \begin{matrix} R_2 \to R_2 + R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$= a^2b^2c^2\ [-(0 - 4) + 0 + 0]$
$= 4a^2b^2c^2$

Ex 1.2

Question 1.

Find the adjoint of the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

Question 2.

If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then verify that A(adj A) = |A| I and also find A^{-1}.

Solution:

$$\text{Given } A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix}$$

$$= 1\begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix}$$

= 1[16 – 9] – 3[4 – 3] + 3[3 – 4]

= 1(7) – 3(1) + 3(-1)

= 7 – 3 – 3

= 1

$$\text{Cofactor } [A_{ij}] = \begin{bmatrix} 7 & -1 & -1 \\ -\begin{vmatrix} 3 & 3 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} \\ \begin{vmatrix} 3 & 3 \\ 4 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -1 & -1 \\ -(12-9) & (4-3) & 0 \\ (9-12) & 0 & (4-3) \end{bmatrix} = \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Now } A(\text{adj } A) = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7-3-3 & -3+3+0 & -3+0+3 \\ 7-4-3 & -3+4 & -3+0+3 \\ 7-3-4 & -3+3+0 & -3+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{.........} \ (1)$$

$$|A|\, I = 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{........} \ (2)$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$$= \frac{1}{1} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

From (1) and (2), $A(\text{Adj } A) = |A|\, I$

Question 3.
Find the inverse of each of the following matrices:

(i) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

(iv) $\begin{bmatrix} -3 & -5 & 4 \\ -2 & 3 & -1 \\ 1 & -4 & -6 \end{bmatrix}$

Solution:

(i) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ $\therefore |A| = 3 + 2 = 5$

$$\text{adj } A = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

(ii) $\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}$

Let $A = \begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}$ $\therefore |A| = 9 + 1 = 10$

$$\text{adj } A = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{10}\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$$

(iii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} - 2\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} + 3\begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix}$$

$$= 1\,[10-0] - 2\,[0-0] + 3\,[0-0]$$

$$= 10 - 0 + 0$$

$$= 10$$

$$[A_{ij}] = \begin{bmatrix} 10 & 0 & 0 \\ -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & 0 \\ -(10-0) & (5-0) & 0 \\ (8-6) & -(4-0) & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$$= \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

(iv) $\begin{bmatrix} -3 & -5 & 4 \\ -2 & 3 & -1 \\ 1 & -4 & -6 \end{bmatrix}$

Let $A = \begin{bmatrix} -3 & -5 & 4 \\ -2 & 3 & -1 \\ 1 & -4 & -6 \end{bmatrix}$

$$|A| = \begin{vmatrix} -3 & -5 & 4 \\ -2 & 3 & -1 \\ 1 & -4 & -6 \end{vmatrix}$$

$$= -3\begin{vmatrix} 3 & -1 \\ -4 & -6 \end{vmatrix} - (-5)\begin{vmatrix} -2 & -1 \\ 1 & -6 \end{vmatrix} + 4\begin{vmatrix} -2 & 3 \\ 1 & -4 \end{vmatrix}$$

$$= -3\,[-18 - 4] - (-5)\,[12 + 1] + 4\,[8 - 3]$$

$$= -3\,[-22] - (-5)\,[13] + 4\,[5]$$

$$= 66 + 65 + 20 = 151$$

$$[A_{ij}] = \begin{bmatrix} -22 & -13 & 5 \\ -\begin{vmatrix} -5 & 4 \\ -4 & -6 \end{vmatrix} & \begin{vmatrix} -3 & 4 \\ 1 & -6 \end{vmatrix} & -\begin{vmatrix} -3 & -5 \\ 1 & -4 \end{vmatrix} \\ \begin{vmatrix} -5 & 4 \\ 3 & -1 \end{vmatrix} & -\begin{vmatrix} -3 & 4 \\ -2 & -1 \end{vmatrix} & \begin{vmatrix} -3 & -5 \\ -2 & 3 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -22 & -13 & 5 \\ -(30+16) & (18-4) & -(12+5) \\ 5-12 & -(3+8) & (-9-10) \end{bmatrix}$$

$$= \begin{bmatrix} -22 & -13 & 5 \\ -46 & 14 & -17 \\ -7 & -11 & -19 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$= \frac{1}{151}\begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

Question 4.

If $A = \begin{bmatrix} 2 & 3 \\ 1 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}$, then verify adj(AB) = (adj B) (adj A).

Solution:

$$AB = \begin{bmatrix} 2 & 3 \\ 1 & -6 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+3 & 8-6 \\ -1-6 & 4+12 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -7 & 16 \end{bmatrix}$$

$$\text{adj }(AB) = \begin{bmatrix} 16 & -2 \\ 7 & 1 \end{bmatrix} \quad \text{........ (1)}$$

$$B = \begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix} \therefore \text{adj } B = \begin{bmatrix} -2 & -4 \\ -1 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -6 \end{bmatrix} \therefore \text{adj } A = \begin{bmatrix} -6 & -3 \\ -1 & 2 \end{bmatrix}$$

$$\therefore (\text{adj } B)(\text{adj } A) = \begin{bmatrix} -2 & -4 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+4 & 6-8 \\ 6+1 & 3-2 \end{bmatrix} = \begin{bmatrix} 16 & -2 \\ 7 & 1 \end{bmatrix} \quad \text{...... (2)}$$

From (1) and (2), adj (AB) = (adj B) (adj A)

Question 5.

If A = $\begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & 0 \\ 9 & 1 & 5 \end{bmatrix}$ then, show that (adj A) A = O.

Solution:

$$\text{Given } A = \begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & 0 \\ 9 & 1 & 5 \end{bmatrix}$$

$$[A_{ij}] = \begin{bmatrix} \begin{vmatrix} 3 & 0 \\ 1 & 5 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 9 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 9 & 1 \end{vmatrix} \\ -\begin{vmatrix} -2 & 2 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 9 & 5 \end{vmatrix} & -\begin{vmatrix} 2 & -2 \\ 9 & 1 \end{vmatrix} \\ \begin{vmatrix} -2 & 2 \\ 3 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 2 & 0 \end{vmatrix} & \begin{vmatrix} 2 & -2 \\ 2 & 3 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 15-0 & -(10-0) & (2-27) \\ -(-10-2) & (10-18) & -(2+18) \\ 0-6 & -(0-4) & (6+4) \end{bmatrix}$$

$$= \begin{bmatrix} 15 & -10 & -25 \\ 12 & -8 & -20 \\ -6 & 4 & 10 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} 15 & 12 & -6 \\ -10 & -8 & 4 \\ -25 & -20 & 10 \end{bmatrix}$$

$$\text{Now (adj A) } A = \begin{bmatrix} 15 & 12 & -6 \\ -10 & -8 & 4 \\ -25 & -20 & 10 \end{bmatrix} \begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & 0 \\ 9 & 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 30+24-54 & -30+36-6 & 30+0-30 \\ -20-16+36 & 20-24+4 & -20+0+20 \\ -50-40+90 & 50-60+10 & -50+0+50 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Question 6.

If $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$ then, show that the inverse of A is A itself.

Solution:

$$\text{Given } A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$$

$$|A| = -1\begin{vmatrix} -3 & 4 \\ -4 & 5 \end{vmatrix} - 2\begin{vmatrix} 4 & 4 \\ 4 & 5 \end{vmatrix} - 2\begin{vmatrix} 4 & -3 \\ 4 & -4 \end{vmatrix}$$

$$= -1\,[-15 + 16] - 2\,[20 - 16] - 2\,[-16 + 12]$$

$$= -1\,[1] - 2\,[4] - 2\,[-4]$$

$$= -1 - 8 + 8 \Rightarrow -1 \neq 0$$

$$[A_{ij}] = \begin{bmatrix} 1 & -4 & -4 \\ -\begin{vmatrix} 2 & -2 \\ -4 & 5 \end{vmatrix} & \begin{vmatrix} -1 & -2 \\ 4 & 5 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ 4 & -4 \end{vmatrix} \\ \begin{vmatrix} 2 & -2 \\ -3 & 4 \end{vmatrix} & -\begin{vmatrix} -1 & -2 \\ 4 & 4 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 4 & -3 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 & -4 \\ -(10-8) & (-5+8) & -(4-8) \\ (8-6) & -(-4+8) & (3-8) \end{bmatrix} = \begin{bmatrix} 1 & -4 & -4 \\ -2 & 3 & 4 \\ 2 & -4 & -5 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} 1 & -2 & 2 \\ -4 & 3 & -4 \\ -4 & 4 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$$= \frac{1}{-1}\begin{bmatrix} 1 & -2 & 2 \\ -4 & 3 & -4 \\ -4 & 4 & -5 \end{bmatrix}$$

$$= -1\begin{bmatrix} 1 & -2 & 2 \\ -4 & 3 & -4 \\ -4 & 4 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$$

$\therefore A^{-1} = A$

Hence proved.

Question 7.

If $A^{-1} = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ then, find A.

Solution:

Given $A^{-1} = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

We know that $(A^{-1})^{-1} = A$

So we have to find inverse of A^{-1}

$$|A^{-1}| = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= 1(1-1) - 0(2+1) + 3(-2-1)$$
$$= 1(0) - 0(3) + 3(-3)$$
$$= 0 - 0 - 9 = -9 \neq 0$$

$$[A^{-1}{}_{ij}] = \begin{bmatrix} 0 & -3 & -3 \\ -\begin{vmatrix} 0 & 3 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} \\ \begin{vmatrix} 0 & 3 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -3 & -3 \\ -(0+3) & 1-3 & -(-1-0) \\ 0-3 & -(-1-6) & (1-0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -3 & -3 \\ -3 & -2 & 1 \\ -3 & 7 & 1 \end{bmatrix}$$

$$\text{adj } A^{-1} = [A^{-1}{}_{ij}]^T = \begin{bmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{bmatrix}$$

$$\therefore (A^{-1})^{-1} = \frac{1}{|A^{-1}|}(\text{adj } A^{-1})$$

$$= \frac{1}{-9}\begin{bmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{bmatrix}$$

$$\text{i.e., } A = \frac{1}{9}\begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -7 \\ 3 & -1 & -1 \end{bmatrix}$$

Question 8.

Show that the matrices A = $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} \frac{4}{5} & \frac{-2}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{3}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-2}{5} & \frac{4}{5} \end{bmatrix}$ are inverses of each other.

Solution:
To prove that A and B are inverses of each other.
We have to prove that AB = BA = I.

$$\text{Now } AB = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} \frac{4}{5} & \frac{-2}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{3}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-2}{5} & \frac{4}{5} \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & -2 & -1 \\ -1 & 3 & -1 \\ -1 & -2 & 4 \end{bmatrix} \quad \text{[Take out 5 common from the matrix]}$$

$$= \frac{1}{5} \begin{bmatrix} 8-2-1 & -4+6-2 & -2-2+4 \\ 4-3-1 & -2+9-2 & -1-3+4 \\ 4-2-2 & -2+6-4 & -1-2+8 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \frac{1}{5} \times 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$BA = \begin{bmatrix} \frac{4}{5} & \frac{-2}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{3}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-2}{5} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & -2 & -1 \\ -1 & 3 & -1 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 8-2-1 & 8-6-2 & 4-2-2 \\ -2+3-1 & -2+9-2 & -1+3-2 \\ -2-2+4 & -2-6+8 & -1-2+8 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \frac{1}{5} \times 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Thus AB = BA = I
Hence A and B are inverses of each other.

Question 9.
If $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$

Solution:

$$\text{Now } AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix} = 4087 - 4089 = -2$$

$$\text{adj }(AB) = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|}\ (\text{adj } AB)$$

$$= \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \quad \ldots\ldots (1)$$

Now we will find $B^{-1}A^{-1}$

$$B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}, |B| = 54 - 56 = -2$$

$$\text{adj } B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|}\ (\text{adj } B) = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}, |A| = 15 - 14 = 1$$

$$\text{adj } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ (\text{adj } A)$$

$$= \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \quad \text{.......... (2)}$$

From (1) and (2), $(AB)^{-1} = B^{-1}A^{-1}$

Question 10.

Find λ if the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 2 & \lambda & 4 \\ 9 & 7 & 11 \end{bmatrix}$ has no inverse.

Solution:

Since, $\begin{bmatrix} 1 & 1 & 3 \\ 2 & \lambda & 4 \\ 9 & 7 & 11 \end{bmatrix}$ is no inverse, $\begin{vmatrix} 1 & 1 & 3 \\ 2 & \lambda & 4 \\ 9 & 7 & 11 \end{vmatrix} = 0$

$$1\begin{vmatrix} \lambda & 4 \\ 7 & 11 \end{vmatrix} - 1\begin{vmatrix} 2 & 4 \\ 9 & 11 \end{vmatrix} + 3\begin{vmatrix} 2 & \lambda \\ 9 & 7 \end{vmatrix} = 0$$

$1[11\lambda - 28] - 1[22 - 36] + 3[14 - 9\lambda] = 0$

$11\lambda - 28 + 14 + 42 - 27\lambda = 0$

$-16\lambda + 28 = 0$

$-16\lambda = -28$

$\lambda = \frac{-28}{-16} = \frac{7}{4}$

Question 11.

If X = $\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$ and Y = $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & p & q \end{bmatrix}$ then, find p, q if $Y = X^{-1}$

Solution:

Given that Y is the inverse of X.

$\therefore XY = I$

$$\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & p & q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 16-0-15 & 8-2-3p & -8-1-3q \\ -10+0+10 & -5+2+2p & 5+1+2q \\ 20-0-20 & 10-2-4p & -10-1-4q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6-3p & -9-3q \\ 0 & -3+2p & 6+2q \\ 0 & 8-4p & -11-4q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6 – 3p = 0 and -9 – 3q = 0
6 = 3p and -9 = 3q
∴ p = 2; q = -3

Ex 1.3

Question 1.
Solve by matrix inversion method: 2x + 3y – 5 = 0; x – 2y + 1 = 0.

Solution:

2x + 3y = 5

x – 2y = -1

The given system can be written as

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

AX = B

where A = $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and B = $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -4-3 = -7 \neq 0$$

$\therefore A^{-1}$ Exists.

$$\text{adj } A = \begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{-7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$\Rightarrow -\frac{1}{7}\begin{bmatrix} -10+3 \\ -5-2 \end{bmatrix}$$

$$\Rightarrow -\frac{1}{7}\begin{bmatrix} -7 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\therefore x = 1, y = 1$

Question 2.
Solve by matrix inversion method:
(i) 3x – y + 2z = 13; 2x + y – z = 3; x + 3y – 5z = -8
(ii) x – y + 2z = 3; 2x + z = 1; 3x + 2y + z = 4
(iii) 2x – z = 0; 5x + y = 4; y + 3z = 5

Solution:
(i) The given system can be written as

$$\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 3 \\ -8 \end{bmatrix}$$

AX = B

Where A = $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 3 & -5 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 13 \\ 3 \\ -8 \end{bmatrix}$

|A| = $\begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 3 & -5 \end{vmatrix}$

= 3(-5 + 3) – (-1) (-10 + 1) + 2 (6 – 1)

= 3(-2) + 1(-9) + 2(5)

= -6 – 9 + 10

= -5

$$[A_{ij}] = \begin{bmatrix} \begin{vmatrix} 1 & -1 \\ 3 & -5 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 1 & -5 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \\ -2 & -(-9) & 5 \\ -\begin{vmatrix} -1 & 2 \\ 3 & -5 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 1 & -5 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} \\ \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 9 & 5 \\ -(5-6) & (-15-2) & -(9+1) \\ (1-2) & -(-3-4) & (3+2) \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 9 & 5 \\ 1 & -17 & -10 \\ -1 & 7 & 5 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$= \frac{1}{-5} \begin{bmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{-5}\begin{bmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{bmatrix}\begin{bmatrix} 13 \\ 3 \\ -8 \end{bmatrix} \Rightarrow \frac{1}{-5}\begin{bmatrix} -26+3+8 \\ 117-51-56 \\ 65-30-40 \end{bmatrix} \Rightarrow \frac{1}{-5}\begin{bmatrix} -15 \\ 10 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

∴ x = 3, y = -2, z = 1.

(ii) The given system can be written as

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

AX = B

where A = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

|A| = $\begin{vmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

= 1(0 – 2) – (-1)(2 – 3) + 2(4 – 0)

= -2 – (-1)(-1) + 2(4)

= -2 – 1 + 8

= 5

$$[A_{ij}] = \begin{bmatrix} -2 & -(-1) & 4 \\ -\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & 4 \\ -(-1-4) & (1-6) & -(2+3) \\ (-1-0) & -(1-4) & (0+2) \end{bmatrix} = \begin{bmatrix} -2 & 1 & 4 \\ 5 & -5 & -5 \\ -1 & 3 & 2 \end{bmatrix}$$

$$\therefore \text{adj } A = [A_{ij}]^T = \begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$= \frac{1}{5}\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{5}\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \Rightarrow \frac{1}{5}\begin{bmatrix} -6+5-4 \\ 3-5+12 \\ 12-5+8 \end{bmatrix} \Rightarrow \frac{1}{5}\begin{bmatrix} -5 \\ 10 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

x = -1, y = 2, z = 3.

(iii) The given system can be written as

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$$

AX = B

Where A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$$

= 2(3 – 0) – 0(15 – 0) – 1(5 – 0)

= 2(3) – 0(15) – 1(5)

= 6 – 0 – 5

= 1

$$[A_{ij}] = \begin{bmatrix} 3 & -15 & 5 \\ -\begin{vmatrix} 0 & -1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 5 & 0 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$= \frac{1}{1}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 0-4+5 \\ 0+24-25 \\ 0-8+10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

∴ x = 1, y = -1, z = 2.

Question 3.
A salesperson Ravi has the following record of sales for the month of January, February, and March 2009 for three products A, B, and C. He has been paid a commission at a fixed rate per unit but at varying rates for products A, B and C.

Months	Sales in Units			Commission
	A	*B*	*C*	
January	9	10	2	800
February	15	5	4	900
March	6	10	3	850

Find the rate of commission payable on A, B and C per unit sold using matrix inversion method.

Solution:
Let x, y and z be the rate of commission for the three products A, B and C respectively.
$9x + 10y + 2z = 800$
$15x + 5y + 4z = 900$
$6x + 10y + 3z = 850$
The given system can be written as

$$\begin{bmatrix} 9 & 10 & 2 \\ 15 & 5 & 4 \\ 6 & 10 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 800 \\ 900 \\ 850 \end{bmatrix}$$

AX = B

Where A = $\begin{bmatrix} 9 & 10 & 2 \\ 15 & 5 & 4 \\ 6 & 10 & 3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 800 \\ 900 \\ 850 \end{bmatrix}$

Now, |A| = $\begin{vmatrix} 9 & 10 & 2 \\ 15 & 5 & 4 \\ 6 & 10 & 3 \end{vmatrix}$

$= 9\begin{vmatrix} 5 & 4 \\ 10 & 3 \end{vmatrix} - 10\begin{vmatrix} 15 & 4 \\ 6 & 3 \end{vmatrix} + 2\begin{vmatrix} 15 & 5 \\ 6 & 10 \end{vmatrix}$

= 9[15 – 40] – 10(45 – 24) + 2(150 – 30)

= 9[-25] – 10[21] + 2[120]

= -225 – 210 + 240

= -195

$$[A_{ij}] = \begin{bmatrix} -25 & -21 & 120 \\ -\begin{vmatrix} 10 & 2 \\ 10 & 3 \end{vmatrix} & \begin{vmatrix} 9 & 2 \\ 6 & 3 \end{vmatrix} & -\begin{vmatrix} 9 & 10 \\ 6 & 10 \end{vmatrix} \\ \begin{vmatrix} 10 & 2 \\ 5 & 4 \end{vmatrix} & -\begin{vmatrix} 9 & 2 \\ 15 & 4 \end{vmatrix} & \begin{vmatrix} 9 & 10 \\ 15 & 5 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -25 & -21 & 120 \\ -(30-20) & (27-12) & -(90-60) \\ (40-10) & -(36-30) & (45-150) \end{bmatrix} = \begin{bmatrix} -25 & -21 & 120 \\ -10 & 15 & -30 \\ 30 & -6 & -105 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} -25 & -10 & 30 \\ -21 & 15 & -6 \\ 120 & -30 & -105 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-195}\begin{bmatrix} -25 & -10 & 30 \\ -21 & 15 & -6 \\ 120 & -30 & -105 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{195}\begin{bmatrix} -25 & -10 & 30 \\ -21 & 15 & -6 \\ 120 & -30 & -105 \end{bmatrix}\begin{bmatrix} 800 \\ 900 \\ 850 \end{bmatrix}$$

$$= \frac{-1}{195}\begin{bmatrix} -20000-9000+25500 \\ -16800+13500-5100 \\ 96000-27000-89250 \end{bmatrix} \Rightarrow \frac{-1}{195}\begin{bmatrix} -3500 \\ -8400 \\ -20250 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17.948 \\ 43.0769 \\ 103.846 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17.95 \\ 43.08 \\ 103.85 \end{bmatrix}$$

$\therefore$ x = 17.95, y = 43.08, z = 103.85

The rate of commission of A, B and C are 17.95, 43.08 and 103.85 respectively.

Question 4.
The prices of three commodities A, B, and C are ₹ x, y, and z per unit respectively. P purchases 4 units of C and sells 3 units of A and 5 units of B. Q purchases 3 units of B and sells 2 units of A and 1 unit of C. R purchases 1 unit of A and sells 4 units of B and 6 units of C. In the process P, Q and R earn ₹ 6,000, ₹ 5,000 and ₹ 13,000 respectively. By using the matrix inversion method, find the prices per unit of A, B, and C.

Solution:
Take selling the units is positive earning and buying the units is negative earning.
Given that

3x + 5y – 4z = 6000
2x – 3y + z = 5000
-1x + 4y + 6z = 13000

	A *x*	B *y*	C *z*
P	3	5	–4
Q	2	–3	1
R	–1	4	6

The given statement can be written as

$$\begin{pmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6000 \\ 5000 \\ 13000 \end{pmatrix}$$

AX = B

Where A = $\begin{pmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{pmatrix}$, X = $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and B = $\begin{pmatrix} 6000 \\ 5000 \\ 13000 \end{pmatrix}$

X = $A^{-1}B$

$$|A| = \begin{vmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{vmatrix}$$

= 3(-18 – 4) – 5(12 + 1) – 4(8 – 3)

= 3(-22) – 5(13) – 4(5)

= -66 – 65 – 20

= -151

$$(A_{ij}) = \begin{bmatrix} -22 & -13 & 5 \\ -(30+16) & 18-4 & -(12+5) \\ (5-12) & -(3+8) & (-9-10) \end{bmatrix}$$

$$= \begin{bmatrix} -22 & -13 & 5 \\ -46 & 14 & -17 \\ -7 & -11 & -19 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{-151}\begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{-151}\begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}\begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

$$= \frac{-1000}{151}\begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \\ 13 \end{bmatrix}$$

$$X = \frac{-1000}{151}\begin{bmatrix} -132-230-91 \\ -78+70-143 \\ 30-85-247 \end{bmatrix}$$

$$= \frac{-1000}{151}\begin{bmatrix} -453 \\ -151 \\ -302 \end{bmatrix} = -1000\begin{bmatrix} -3 \\ -1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3000 \\ 1000 \\ 2000 \end{bmatrix}$$

The prices per unit of A, B and C are ₹ 3000, ₹ 1000 and ₹ 2000.

Question 5.

The sum of three numbers is 20. If we multiply the first by 2 and add the second number and subtract the third we get 23. If we multiply the first by 3 and add second and third to it, we get 46. By using the matrix inversion method find the numbers.

Solution:

Let the three numbers be x, y, and z.

$x + y + z = 20$

$2x + y - z = 23$

$3x + y + z = 46$

The given system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix}$$

$$AX = B$$

$$\text{where } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= 1\,[1 + 1] - 1\,(2 + 3) + 1\,(2 - 3)$$

$$= 1\,(2) - 1\,(5) + 1\,(-1)$$

$$= 2 - 5 - 1 = -4$$

$$[A_{ij}] = \begin{bmatrix} 2 & -5 & -1 \\ -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -5 & -1 \\ -(0) & (1-3) & -(1-3) \\ (-1-1) & -(-1-2) & (1-2) \end{bmatrix} = \begin{bmatrix} 2 & -5 & -1 \\ 0 & -2 & 2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$= \frac{1}{-4} \begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-4} \begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix} = \frac{-1}{4} \begin{bmatrix} 40+0-92 \\ -100-46+138 \\ -20+46-46 \end{bmatrix} = \frac{-1}{4} \begin{bmatrix} -52 \\ -8 \\ -20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 2 \\ 5 \end{bmatrix}$$

The numbers are 13, 2, and 5.

Question 6.
Weekly expenditure in an office for three weeks is given as follows. Assuming that the salary in all three weeks of different categories of staff did not vary, calculate the salary for each type of staff, using the matrix inversion method.

Week	Number of employees			Total weekly salary (in ₹)
	A	B	C	
1st week	4	2	3	4900
2nd week	3	3	2	4500
3rd week	4	3	4	5800

Solution:

Let ₹ x, ₹ y, ₹ z be the salary for each type of staff A, B and C.

4x + 2y + 3z = 4900

3x + 3y + 2z = 4500

4x + 3y + 4z = 5800

The given system can be written as

$$\begin{bmatrix} 4 & 2 & 3 \\ 3 & 3 & 2 \\ 4 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4900 \\ 4500 \\ 5800 \end{bmatrix}$$

AX = B

where A = $\begin{bmatrix} 4 & 2 & 3 \\ 3 & 3 & 2 \\ 4 & 3 & 4 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 4900 \\ 4500 \\ 5800 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 2 & 3 \\ 3 & 3 & 2 \\ 4 & 3 & 4 \end{vmatrix}$$

= 4(12 – 6) – 2(12 – 8) + 3(9 – 12)

= 4(6) – 2(4) + 3(-3)

= 24 – 8 – 9

= 7

$$[A_{ij}] = \begin{bmatrix} 6 & -4 & -3 \\ -\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 4 & 3 \\ 4 & 4 \end{vmatrix} & -\begin{vmatrix} 4 & 2 \\ 4 & 3 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 4 & 2 \\ 3 & 3 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 & -3 \\ -(8-9) & 16-12 & -(12-8) \\ (4-9) & -(8-9) & (12-6) \end{bmatrix} = \begin{bmatrix} 6 & -4 & -3 \\ 1 & 4 & -4 \\ -5 & 1 & 6 \end{bmatrix}$$

$$\text{adj } A = [A_{ij}]^T = \begin{bmatrix} 6 & 1 & -5 \\ -4 & 4 & 1 \\ -3 & -4 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$= \frac{1}{7} \begin{bmatrix} 6 & 1 & -5 \\ -4 & 4 & 1 \\ -3 & -4 & 6 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 6 & 1 & -5 \\ -4 & 4 & 1 \\ -3 & -4 & 6 \end{bmatrix} \begin{bmatrix} 4900 \\ 4500 \\ 5800 \end{bmatrix}$$

$$= \frac{100}{7} \begin{bmatrix} 6 & 1 & -5 \\ -4 & 4 & 1 \\ -3 & -4 & 6 \end{bmatrix} \begin{bmatrix} 49 \\ 45 \\ 58 \end{bmatrix} = \frac{100}{7} \begin{bmatrix} 294+45-290 \\ -196+180+58 \\ -147-180+348 \end{bmatrix} = \frac{100}{7} \begin{bmatrix} 49 \\ 42 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 700 \\ 600 \\ 300 \end{bmatrix}$$

∴ Salary for each type of staff A, B and C are ₹ 700, ₹ 600 and ₹ 300.

Ex 1.4

Question 1.
The technology matrix of an economic system of two industries is

$\begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix}$ Test whether the system is viable as per Hawkins Simon conditions.

Solution:

Technology matrix B = $\begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix}$

I – B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix}$

= $\begin{bmatrix} 0.50 & -0.30 \\ -0.41 & 0.67 \end{bmatrix}$, the main diagonal elements are positive.

|I – B| = $\begin{bmatrix} 0.50 & -0.30 \\ -0.41 & 0.67 \end{bmatrix}$

= 0.335 – 0.123

= 0.212, positive

Since the main diagonal elements of I – B are positive and |I – B| is positive, Hawkins-Simon conditions are satisfied. Therefore, the given system is viable.

Question 2.
The technology matrix of ah economic system of two industries is $\begin{bmatrix} 0.6 & 0.9 \\ 0.20 & 0.80 \end{bmatrix}$. Test whether the system is viable as per Hawkins-Simon conditions.

Solution:

Technology matrix B = $\begin{bmatrix} 0.60 & 0.9 \\ 0.20 & 0.80 \end{bmatrix}$

I – B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.60 & 0.9 \\ 0.20 & 0.80 \end{bmatrix}$

$$= \begin{bmatrix} 0.4 & -0.9 \\ -0.20 & 0.20 \end{bmatrix}$$, the main diagonal elements are positive.

$$|I - B| = \begin{vmatrix} 0.4 & -0.9 \\ -0.20 & 0.20 \end{vmatrix}$$

= 0.4 × 0.20 – (-0.20) × (-0.9)

= 0.08 – 0.18

= -0.1, negative

Since |I – B| is negative one of the Hawkins-Simon condition is not satisfied. Therefore, the given system is not viable.

Question 3.
The technology matrix of an economic system of two industries is $\begin{bmatrix} 0.50 & 0.25 \\ 0.40 & 0.67 \end{bmatrix}$. Test whether the system is viable as per Hawkins-Simon conditions.

Solution:

Technology matrix $B = \begin{bmatrix} 0.50 & 0.25 \\ 0.40 & 0.67 \end{bmatrix}$

$$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.50 & 0.25 \\ 0.40 & 0.67 \end{bmatrix}$$

$$= \begin{bmatrix} 0.50 & -0.25 \\ -0.40 & 0.33 \end{bmatrix}$$, the main diagonal elements are positive.

$$|I - B| = \begin{vmatrix} 0.50 & -0.25 \\ -0.40 & 0.33 \end{vmatrix}$$

= (0.50) (0.33) – (-0.40) (-0.25)

= 0.165 – 0.1

= 0.065 (positive)

Since the main diagonal elements of I – B are positive and |I – B| is positive, Hawkins-Simon conditions are satisfied. Therefore, the given system is viable.

Question 4.
Two commodities A and B are produced such that 0.4 tonne of A and 0.7 tonnes of B are required to produce a tonnes of A. Similarly 0.1 tonne of A and 0.7 tonne of B are needed to produce a tonnes of B. Write down the technology matrix. If 6.8 tonnes of A and 10.2 tones of B are required, find the gross production of both of them.

Solution:
Here the technology matrix is given under

	A	B	Final demand
A	0.4	0.1	6.8
B	0.7	0.7	10.2

The technology matrix is B = $\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.7 \end{bmatrix}$

I – B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.7 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.3 \end{bmatrix}$

|I – B| = $\begin{vmatrix} 0.6 & -0.1 \\ -0.7 & 0.3 \end{vmatrix}$

= (0.6) (0.3) – (-0.1) (-0.7)

= 0.18 – 0.07

= 0.11

Since the main diagonal elements of I – B are positive and the value of |I – B| is positive, the system is viable.

$$\text{adj}(I-B) = \begin{bmatrix} 0.3 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$$

$$(I-B)^{-1} = \frac{1}{|I-B|} \text{adj}(I-B)$$

$$= \frac{1}{0.11} \begin{bmatrix} 0.3 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$$

$$X = (I-B)^{-1}D \quad \text{where } D = \begin{bmatrix} 6.8 \\ 10.2 \end{bmatrix}$$

$$= \frac{1}{0.11} \begin{bmatrix} 0.3 & 0.1 \\ 0.7 & 0.6 \end{bmatrix} \begin{bmatrix} 6.8 \\ 10.2 \end{bmatrix}$$

$$= \frac{1}{0.11} \begin{bmatrix} 2.04 + 1.02 \\ 4.76 + 6.12 \end{bmatrix} = \begin{bmatrix} 27.8181 \\ 98.9090 \end{bmatrix} = \begin{bmatrix} 27.82 \\ 98.91 \end{bmatrix}$$

Production of A is 27.82 tonnes and the production of B is 98.91 tonnes.

Question 5.
Suppose the inter-industry flow of the product of two industries is given as under.

Production sector	Consumption sector		Domestic demand	Total output
	X	*Y*		
X	30	40	50	120
Y	20	10	30	60

Determine the technology matrix and test Hawkin's-Simon conditions for the viability of the system. If the domestic changes to 80 and 40 units respectively, what should be the gross output of each sector in order to meet the new demands.

Solution:

$a_{11} = 30, a_{12} = 40, x_1 = 120$

$a_{21} = 20, a_{22} = 10, x_2 = 60$

$$b_{11} = \frac{a_{11}}{x_1} = \frac{30}{120} = \frac{1}{4}, b_{12} = \frac{a_{12}}{x_2} = \frac{40}{60} = \frac{2}{3}$$

$$b_{21} = \frac{a_{21}}{x_1} = \frac{20}{120} = \frac{1}{6}, b_{22} = \frac{a_{22}}{x_2} = \frac{10}{60} = \frac{1}{6}$$

The technology matrix B = $\begin{bmatrix} \frac{1}{4} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$

$$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{4} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

$= \begin{bmatrix} \frac{3}{4} & -\frac{2}{3} \\ -\frac{1}{6} & \frac{5}{6} \end{bmatrix}$, elements of main diagonal are positive.

$$|I - B| = \begin{bmatrix} \frac{3}{4} & -\frac{2}{3} \\ -\frac{1}{6} & \frac{5}{6} \end{bmatrix}$$

$$= \frac{3}{4} \times \frac{5}{6} - \left(-\frac{1}{6}\right) \times \left(\frac{-2}{3}\right)$$

$$= \frac{1}{4} \times \frac{5}{2} - \left(\frac{1}{3}\right) \times \left(\frac{1}{3}\right)$$

$$= \frac{5}{8} - \frac{1}{9} = \frac{5 \times 9 - 1 \times 8}{8 \times 9} = \frac{45 - 8}{72} = \frac{37}{72}$$

The main diagonal elements of I – B are positive and |I – B| is positive. Therefore the system is viable.

$$\text{adj}(I-B) = \begin{bmatrix} \frac{5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{3}{4} \end{bmatrix}$$

$$(I-B)^{-1} = \frac{1}{|I-B|}\text{adj}(I-B)$$

$$= \frac{1}{\left(\frac{37}{72}\right)}\begin{bmatrix} \frac{5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{3}{4} \end{bmatrix} = \frac{72}{37}\begin{bmatrix} \frac{5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{3}{4} \end{bmatrix}$$

$$X = (I-B)^{-1}D, \text{ where } D = \begin{bmatrix} 80 \\ 40 \end{bmatrix}$$

$$= \frac{72}{37}\begin{bmatrix} \frac{5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{3}{4} \end{bmatrix}\begin{bmatrix} 80 \\ 40 \end{bmatrix} \Rightarrow \frac{1}{37}\begin{bmatrix} 72\times\frac{5}{6} & 72\times\frac{2}{3} \\ 72\times\frac{1}{6} & 72\times\frac{3}{4} \end{bmatrix}\begin{bmatrix} 80 \\ 40 \end{bmatrix}$$

$$= \frac{1}{37}\begin{bmatrix} 60 & 48 \\ 12 & 54 \end{bmatrix}\begin{bmatrix} 80 \\ 40 \end{bmatrix} \Rightarrow \frac{1}{37}\begin{bmatrix} 60\times80+48\times40 \\ 12\times80+54\times40 \end{bmatrix}$$

$$= \frac{1}{37}\begin{bmatrix} 6720 \\ 3120 \end{bmatrix} = \begin{bmatrix} 181.62 \\ 84.32 \end{bmatrix}$$

The output of industry X should be 181.62 and Y should be 84.32.

Question 6.
You are given the following transaction matrix for a two-sector economy.

Sector	Sales		Final demand	Gross output
	1	2		
1	4	3	13	20
2	5	4	3	12

(i) Write the technology matrix?
(ii) Determine the output when the final demand for the output sector 1 alone increases to 23 units.

Solution:

$a_{11} = 4, a_{12} = 3, x_1 = 20$

$a_{21} = 5, a_{22} = 4, x_2 = 12$

$$b_{11} = \frac{a_{11}}{x_1} = \frac{4}{20} = \frac{1}{5}, b_{12} = \frac{a_{12}}{x_2} = \frac{3}{12} = \frac{1}{4}$$

$$b_{21} = \frac{a_{21}}{x_1} = \frac{5}{20} = \frac{1}{4}, b_{22} = \frac{a_{22}}{x_2} = \frac{4}{12} = \frac{1}{3}$$

The technology matrix is $B = \begin{bmatrix} \frac{1}{5} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{3} \end{bmatrix}$

$$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{5} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{5} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{2}{3} \end{bmatrix}$$, elements of main diagonal are positive

$$|I - B| = \frac{4}{5} \times \frac{2}{3} - \left(-\frac{1}{4}\right) \times \left(-\frac{1}{4}\right)$$

$$= \frac{8}{15} - \frac{1}{16} = \frac{8 \times 16 - 1 \times 15}{15 \times 16} = \frac{128 - 15}{15 \times 16} = \frac{113}{240}$$

The main diagonal elements are positive and |I – B| is positive. Therefore the system is viable.

$$\text{adj } (I - B) = \begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix}$$

$$(I - B)^{-1} = \frac{1}{|I - B|} \text{adj } (I - B)$$

$$= \frac{1}{\left(\frac{113}{240}\right)} \begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix} = \frac{240}{113} \begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix}$$

$X = (I - B)^{-1}D$, where

$$D = \frac{16\times15}{113}\begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix}\begin{bmatrix} 23 \\ 3 \end{bmatrix} \Rightarrow \frac{1}{113}\begin{bmatrix} 16\times15\times\frac{2}{3} & 16\times15\times\frac{1}{4} \\ 16\times15\times\frac{1}{4} & 16\times15\times\frac{4}{5} \end{bmatrix}\begin{bmatrix} 23 \\ 3 \end{bmatrix}$$

$$= \frac{1}{113}\begin{bmatrix} 16\times5\times2 & 4\times15\times1 \\ 4\times15\times1 & 16\times3\times4 \end{bmatrix}\begin{bmatrix} 23 \\ 3 \end{bmatrix} \Rightarrow \frac{1}{113}\begin{bmatrix} 160 & 60 \\ 60 & 192 \end{bmatrix}\begin{bmatrix} 23 \\ 3 \end{bmatrix}$$

$$= \frac{1}{113}\begin{bmatrix} 160\times23+60\times3 \\ 60\times23+192\times3 \end{bmatrix}$$

$$= \frac{1}{113}\begin{bmatrix} 3680+180 \\ 1380+576 \end{bmatrix}$$

$$= \frac{1}{113}\begin{bmatrix} 3860 \\ 1956 \end{bmatrix} = \begin{bmatrix} 34.159 \\ 17.3097 \end{bmatrix}$$

$$X = \begin{bmatrix} 34.16 \\ 17.31 \end{bmatrix}$$

The output of sector 1 should be 34.16 and sector 2 should be 17.31.

Question 7.
Suppose the inter-industry flow of the product of two Sectors X and Y are given as under.

Production sector	Consumption Sector		Domestic demand	Gross output
	X	*Y*		
X	15	10	10	35
Y	20	30	15	65

Find the gross output when the domestic demand changes to 12 for X and 18 for Y.

Solution:

$$a_{11} = 15, a_{12} = 10, x_1 = 35$$
$$a_{21} = 20, a_{22} = 30, x_2 = 65$$
$$b_{11} = \frac{a_{11}}{x_1} = \frac{15}{35} = \frac{3}{7}, b_{12} = \frac{a_{12}}{x_2} = \frac{10}{65} = \frac{2}{13}$$

$$b_{21} = \frac{a_{21}}{x_1} = \frac{20}{35} = \frac{4}{7},\ b_{22} = \frac{a_{22}}{x_2} = \frac{30}{65} = \frac{6}{13}$$

The technology matrix $B = \begin{bmatrix} \frac{3}{7} & \frac{2}{13} \\ \frac{4}{7} & \frac{6}{13} \end{bmatrix}$

$$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{3}{7} & \frac{2}{13} \\ \frac{4}{7} & \frac{6}{13} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{7} & -\frac{2}{13} \\ -\frac{4}{7} & \frac{7}{13} \end{bmatrix}$$, The main diagonal elements are positive

$$|I - B| = \begin{vmatrix} \frac{4}{7} & -\frac{2}{13} \\ -\frac{4}{7} & \frac{7}{13} \end{vmatrix}$$

$$= \frac{4}{7} \times \frac{7}{13} - \left(-\frac{4}{7}\right) \times \left(\frac{-2}{13}\right) \Rightarrow \frac{28}{91} - \frac{8}{91} = \frac{20}{91} > 0$$

Since the main diagonal elements of I – B are positive and |I – B| is positive the problem has a solution.

$$\text{adj}\ (I - B) = \begin{bmatrix} \frac{7}{13} & \frac{2}{13} \\ \frac{4}{7} & \frac{4}{7} \end{bmatrix}$$

$$(I - B)^{-1} = \frac{1}{|I - B|} \text{adj}\ (I - B)$$

$$= \frac{1}{\left(\frac{20}{91}\right)} \begin{bmatrix} \frac{7}{13} & \frac{2}{13} \\ \frac{4}{7} & \frac{4}{7} \end{bmatrix} = \frac{91}{20} \begin{bmatrix} \frac{7}{13} & \frac{2}{13} \\ \frac{4}{7} & \frac{4}{7} \end{bmatrix}$$

$$= \frac{1}{20} \begin{bmatrix} 91 \times \frac{7}{13} & 91 \times \frac{2}{13} \\ 91 \times \frac{4}{7} & 91 \times \frac{4}{7} \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 49 & 14 \\ 52 & 52 \end{bmatrix}$$

$$X = (I - B)^{-1} D, \text{ where } D = \begin{bmatrix} 12 \\ 18 \end{bmatrix}$$

$$= \frac{1}{20} \begin{bmatrix} 49 & 14 \\ 52 & 52 \end{bmatrix} \begin{bmatrix} 12 \\ 18 \end{bmatrix} \Rightarrow \frac{1}{20} \begin{bmatrix} 588 + 252 \\ 624 + 936 \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 840 \\ 1560 \end{bmatrix} = \begin{bmatrix} 42 \\ 78 \end{bmatrix}$$

Ex 1.5

Question 1.

The value of x if $\begin{vmatrix} 0 & 1 & 0 \\ x & 2 & x \\ 1 & 3 & x \end{vmatrix} = 0$ is

(a) 0, -1
(b) 0, 1
(c) -1, 1
(d) -1, -1

Answer:
(b) 0, 1
Hint:
$0 - 1[x^2 - x] + 0 = 0$
$\Rightarrow x^2 - x = 0$
$\Rightarrow x(x - 1) = 0$
$\Rightarrow x = 0$ (or) $x = 1$

Question 2.

The value of $\begin{vmatrix} 2x+y & x & y \\ 2y+z & y & z \\ 2z+x & z & x \end{vmatrix}$ is

(a) xyz
(b) x + y + z
(c) 2x + 2y + 2z
(d) 0

Answer:
(d) 0

Hint:

$$= \begin{vmatrix} 2x & x & y \\ 2y & y & z \\ 2z & z & x \end{vmatrix} C_1 \rightarrow C_1 - C_3$$

= 0 (C_1 and C_2 are proportional)

Question 3.

The cofactor of -7 in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ is

(a) -18
(b) 18
(c) -7
(d) 7

Answer:
(b) 18
Hint:

A cofactor of -7 = $\begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix}$

= 0 + 18

= 18

Question 4.

If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}$ then $\begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{vmatrix}$ is

(a) Δ

(b) -Δ

(c) 3Δ

(d) -3Δ

Answer:
(b) -Δ
Hint:

$$\begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} R_1 \leftrightarrow R_2$$

= -Δ

Question 5.

The value of the determinant $\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}^2$ is

(a) abc
(b) 0
(c) $a^2b^2c^2$
(d) -abc

Answer:
(c) $a^2b^2c^2$
Hint:

$$a^2b^2c^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= a^2b^2c^2 \times 1^2$

$= a^2b^2c^2$

Question 6.
If A is square matrix of order 3 then |kA| is:
(a) k|A|
(b) -k|A|
(c) $k^3|A|$
(d) $-k^3|A|$

Answer:
(c) $k^3|A|$

Question 7.
adj (AB) is equal to:
(a) adj A adj B
(b) adj A^T adj B^T
(c) adj B adj A
(d) adj B^T adj A^T

Answer:
(c) adj B adj A

Question 8.
The inverse matrix of $\begin{pmatrix} \frac{4}{5} & \frac{5}{12} \\ \frac{2}{5} & \frac{1}{2} \end{pmatrix}$ is

(a) $\frac{7}{30}\begin{pmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$
(b) $\frac{7}{30}\begin{pmatrix} \frac{1}{2} & \frac{-5}{12} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix}$
(c) $\frac{30}{7}\begin{pmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$
(d) $\frac{30}{7}\begin{pmatrix} \frac{1}{2} & \frac{-5}{12} \\ \frac{-2}{5} & \frac{4}{5} \end{pmatrix}$

Answer:
(c) $\frac{30}{7}\begin{pmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$

Question 9.

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that ad – bc ≠ 0 then A^{-1} is:

(a) $\frac{1}{ad-bc}\begin{bmatrix} d & b \\ -c & a \end{bmatrix}$

(b) $\frac{1}{ad-bc}\begin{bmatrix} d & b \\ c & a \end{bmatrix}$

(c) $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(d) $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ c & a \end{bmatrix}$

Answer:

(c) $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Hint:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = ad - bc$$

$$\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Question 10.

The number of Hawkins-Simon conditions for the viability of input-output analysis is:

(a) 1

(b) 3

(c) 4
(d) 2

Answer:
(d) 2

Question 11.
The inventor of input-output analysis is:
(a) Sir Francis Galton
(b) Fisher
(c) Prof. Wassily W. Leontief
(d) Arthur Cayley

Answer:
(c) Prof. Wassily W. Leontief

Question 12.
Which of the following matrix has no inverse?
(a) $\begin{pmatrix} -1 & 1 \\ 1 & -4 \end{pmatrix}$
(b) $\begin{pmatrix} 2 & -1 \\ -4 & 2 \end{pmatrix}$
(c) $\begin{pmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{pmatrix}$
(d) $\begin{pmatrix} \sin a & \sin a \\ -\cos a & \cos a \end{pmatrix}$

Answer:
(b) $\begin{pmatrix} 2 & -1 \\ -4 & 2 \end{pmatrix}$
Hint:
So $\begin{pmatrix} 2 & -1 \\ -4 & 2 \end{pmatrix}$ has no inverse.

Question 13.

Inverse of $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ is:

(a) $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} -2 & 5 \\ 1 & -3 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & -1 \\ -5 & -3 \end{pmatrix}$

(d) $\begin{pmatrix} -3 & 5 \\ 1 & -2 \end{pmatrix}$

Answer:

(a) $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

Hint:

Let A = $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

|A| = [6 – 5] = 1

adj A = $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

∴ A^{-1} = $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

Question 14.

If A = $\begin{pmatrix} -1 & 2 \\ 1 & -4 \end{pmatrix}$ then A (adj A) is:

(a) $\begin{pmatrix} -4 & -2 \\ -1 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & -2 \\ -1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

Answer:

(c) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Hint:

A = $\begin{pmatrix} -1 & 2 \\ 1 & -4 \end{pmatrix}$

|A| = 4 – 2 = 2

We know that A (adj A) = |A| I

$\Rightarrow 2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Question 15.

If A and B non-singular matrix then, which of the following is incorrect?

(a) $A^2 = I$ implies $A^{-1} = A$

(b) $I^{-1} = I$

(c) If AX = B then $X = B^{-1}A$

(d) If A is square matrix of order 3 then $|adj\ A| = |A|^2$

Answer:

(c) If AX = B then $X = B^{-1}A$

Hint:

If AX = B then $X = A^{-1}B$ so, $X = B^{-1}A$ is incorrect.

Question 16.

The value of $\begin{vmatrix} 5 & 5 & 5 \\ 4x & 4y & 4z \\ -3x & -3y & -3z \end{vmatrix}$ is:

(a) 5
(b) 4
(c) 0
(d) -3

Answer:
(c) 0
Hint:

$= 4 \times (-3) \begin{vmatrix} 5 & 5 & 5 \\ x & y & z \\ x & y & z \end{vmatrix}$

[Take out 4 from R_2 and -3 from R_3]

$= 0$ ($\because R_2 \equiv R_3$)

Question 17.
If A is an invertible matrix of order 2 then det (A^{-1}) be equal

(a) det (A)

(b) $\frac{1}{det(A)}$

(c) 1

(d) 0

Answer:

(b) $\frac{1}{det(A)}$

Hint:

$AA^{-1} = I$

$|AA^{-1}| = |I|$

$|A|\,|A^{-1}| = 1$

$|A^{-1}| = \frac{1}{|A|}$

$\det A^{-1} = \frac{1}{\det(A)}$

Question 18.
If A is 3 × 3 matrix and |A| = 4 then $|A^{-1}|$ is equal to:

(a) $\frac{1}{4}$
(b) $\frac{1}{16}$
(c) 2
(d) 4

Answer:
(a) $\frac{1}{4}$
Hint:
$|A^{-1}| = \frac{1}{|A|} = \frac{1}{4}$

Question 19.
If A is a square matrix of order 3 and |A| = 3 then |adj A| is equal to:
(a) 81
(b) 27
(c) 3
(d) 9

Answer:
(d) 9
Hint:
$|\text{adj } A| = |A|^2 = 3^2 = 9$

Question 20.

The value of $\begin{vmatrix} x & x^2 - yz & 1 \\ y & y^2 - zx & 1 \\ z & z^2 - xy & 1 \end{vmatrix}$ is:

(a) 1
(b) 0
(c) -1
(d) -xyz

Answer:
(b) 0
Hint:

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & -yz & 1 \\ y & -zx & 1 \\ z & -xy & 1 \end{vmatrix} \Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} - \begin{vmatrix} x & yz & 1 \\ y & zx & 1 \\ z & xy & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} - \frac{1}{xyz}\begin{vmatrix} x^2 & xyz & x \\ y^2 & xyz & y \\ z^2 & xyz & z \end{vmatrix} \Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} - \frac{xyz}{xyz}\begin{vmatrix} x^2 & 1 & x \\ y^2 & 1 & y \\ z^2 & 1 & z \end{vmatrix}$$

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} \Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} - \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}$$

$= 0$

Question 21.

If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then |2A| is equal to:

(a) 4 cos 2θ
(b) 4
(c) 2
(d) 1

Answer:
(b) 4
Hint:
$|2A| = 2^2 |A|$

$$= 4 \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$

$= 4 [\cos^2\theta + \sin^2\theta]$

$= 4 \times 1$

$= 4$

Question 22.

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is cofactor of a_{ij}, then value of Δ is given by:

(a) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$
(b) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
(c) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$
(d) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$
Answer:
(d) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Question 23.

If $\begin{vmatrix} x & 2 \\ 8 & 5 \end{vmatrix} = 0$ then the value of x is:

(a) $\frac{-5}{6}$
(b) $\frac{5}{6}$
(c) $\frac{-16}{5}$
(d) $\frac{16}{5}$

Answer:

(d) $\frac{16}{5}$

Hint:

$$\begin{vmatrix} x & 2 \\ 8 & 5 \end{vmatrix} = 0$$

5x – 16 = 0

⇒ x = $\frac{16}{5}$

Question 24.

If $\begin{vmatrix} 4 & 3 \\ 3 & 1 \end{vmatrix}$ = -5 then the value of $\begin{vmatrix} 20 & 15 \\ 15 & 5 \end{vmatrix}$ is:

(a) -5

(b) -125

(c) -25

(4) 0

Answer:

(b) -125

Hint:

$$\begin{vmatrix} 20 & 15 \\ 15 & 5 \end{vmatrix}$$

= 5 × 5 $\begin{vmatrix} 4 & 3 \\ 3 & 1 \end{vmatrix}$

= 5 × 5 × (-5)

= -125

Question 25.

If any three rows or columns of a determinant are identical then the value of the determinant is:

(a) 0

(b) 2

(c) 1
(d) 3

Answer:
(a) 0