DETERMINANTS

BASIC CONCEPTS

1. **Singular and Non-singular Matrix:** A square matrix is a singular matrix if its determinant is zero. Otherwise, it is a non-singular matrix.
2. **Some Important Properties of Determinants:**
   (*i*) Let $A = [a_{ij}]$ be a square matrix of order n *and* C_{ij} *be corresponding co-factors*, then
   $$\sum_{j=1}^{n} a_{ij}C_{ij} = |A| \text{ and } \sum_{i=1}^{n} a_{ij}C_{ij} = |A|$$
   (*ii*) Let $A = [a_{ij}]$ be a square matrix of order n *and* C_{ik} *and* C_{kj} *be corresponding co-factors*, then
   $$\sum_{j=1}^{n} a_{ij}C_{kj} = 0 \text{ and } \sum_{i=1}^{n} a_{ij}C_{ik} = 0, i \neq k \text{ or } j \neq k$$
   (*iii*) Let $A = [a_{ij}]$ be a square matrix of order n, then $|A| = |A^T|$.
   (*iv*) Let $A = [a_{ij}]$ be a square matrix of order $n(\geq 2)$ and B be a matrix obtained from A by interchanging any two rows (columns) of A, then $|B| = -|A|$.
   (*v*) If any two rows (columns) of a square matrix $A = [a_{ij}]$ of order n (≥ 2) are identical, then value of its determinant is zero *i.e.*, $|A| = 0$.
   (*vi*) Let $A = [a_{ij}]$ be a square matrix of order n, and let B be the matrix obtained from A by multiplying each element of a row (column) of A by a scalar k, then $|B| = k|A|$.
   (*vii*) Let A be a square matrix such that each element of a row (column) of A is expressed as the sum of two or more terms. Then the determinant of A can be expressed as the sum of the determinants of two or more matrices of the same order.
   (*viii*) Let A be a square matrix and B be a matrix obtained from A by adding to a row (column) of A a scalar multiple of another row (column) of A, then $|B| = |A|$.
   (*ix*) Let A be a square matrix of order n (≥ 2) such that each element in a row (column) of A is zero, then $|A| = 0$.
   (*x*) If $A = [a_{ij}]$ is a diagonal matrix of order n (≥ 2), then
   $|A| = a_{11} . a_{22} . a_{33} .... a_{nn}$.
   (*xi*) If A and B are square matrices of the same order, then
   $|AB| = |A||B|$.
   (*xii*) If $A = [a_{ij}]$ is a triangular matrix of order n, then
   $|A| = a_{11} . a_{22} . a_{33} .... a_{nn}$.
   (*xiii*) If $A = [a_{ij}]$ is a square matrix of order n, then $|kA| = k^n|A|$.
   (*xiv*) We can take out any common factor from any one row or any one column of a given determinant.
3. Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \text{Numerical value of } \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Note: Since area is positive quantity therefore we take absolute value of Δ.

4. (*i*) If A is a skew-symmetric matrix of odd order, then $|A| = 0$.

(*ii*) The determinant of a skew-symmetric matrix of even order is a perfect square.

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(*a*) 6 (*b*) ± 6 (*c*) -6 (*d*) 0

2. The area of a triangle with vertices $(-3, 0), (3, 0)$ and $(0, k)$ is 9 sq. units. The value of k will be

(*a*) 9 (*b*) 3 (*c*) -9 (*d*) 6

3. If A, B and C are angles of a triangle, then the determinant $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is equal to

(*a*) 0 (*b*) -1 (*c*) 1 (*d*) None of these

4. If $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then *[NCERT Exemplar]*

(*a*) $f(a) = 0$ (*b*) $f(b) = 0$ (*c*) $f(0) = 0$ (*d*) $f(1) = 0$

5. If x, y, z are all different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, then value of $x^{-1} + y^{-1} + z^{-1}$ is

(*a*) xyz (*b*) $x^{-1} y^{-1} z^{-1}$ (*c*) $-x - y - z$ (*d*) -1

6. There are two values of a which makes determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then sum of these numbers is

(*a*) 4 (*b*) 5 (*c*) -4 (*d*) 9

7. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then value of $|A|$ is *[CBSE 2020 (65/2/1)]*

(*a*) -3 (*b*) 3 (*c*) 9 (*d*) 27

8. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is *[CBSE 2020 (65/4/1)]*

(*a*) 3 (*b*) 0 (*c*) -1 (*d*) 1

9. The value of the determinant $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$ is *[NCERT Exemplar]*

(*a*) $9x^2(x + y)$ (*b*) $9y^2(x + y)$ (*c*) $3y^2(x + y)$ (*d*) $7x^2(x + y)$

10. The value of $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$ is

(*a*) a (*b*) b (*c*) 0 (*d*) None of these

11. **The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{vmatrix}$ is**

(*a*) 1 (*b*) –1 (*c*) 0 (*d*) ω

12. **If area of triangle is 35 sq units with vertices (2, –6) (5, 4) and (k, 4), then k is**

(*a*) 12 (*b*) –2 (*c*) –12, –2 (*d*) 12, –2

13. **If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is cofactors of a_{ij}, then value of Δ is given by**

(*a*) $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33}$ (*b*) $a_{11} A_{11} + a_{12} A_{21} + a_{13} A_{31}$

(*c*) $a_{21} A_{11} + a_{22} A_{12} + a_{23} A_{13}$ (*d*) $a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31}$

14. **The value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3A - B$ then the values of A and B are:**

(*a*) $A = 2abc, B = a+b+c$ (*b*) $A = 0, B = a^2+b^2+c^2$

(*c*) $A = 3abc, B = a+b+c$ (*d*) $A = abc, B = a^3+b^3+c^3$

15. **If $x, y \in R$, then the determinant $\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ lies in the interval**

(*a*) $[-\sqrt{2}, \sqrt{2}]$ (*b*) $[-1, 1]$ (*c*) $[-\sqrt{2}, 1]$ (*d*) $[-1, -\sqrt{2}]$

16. **If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of k if $|2A| = k|A|$**

(*a*) 4 (*b*) –4 (*c*) 3 (*d*) 0

17. **Let A be a symmetric matrix such the $|A| = 5$ then $|A'|$ is**

(*a*) 5 (*b*) 5^2 (*c*) $\frac{1}{5}$ (*d*) none of these

18. **What is the maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$ where θ is real number.**

(*a*) $\frac{1}{2}$ (*b*) $\frac{\sqrt{3}}{2}$ (*c*) $\sqrt{2}$ (*d*) $\frac{2\sqrt{3}}{4}$

19. **If there are two values of a which makes determinant, $\Delta = \frac{1}{2}\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then the sum of these number is**

(*a*) 4 (*b*) 5 (*c*) – 4 (*d*) 9

20. **If $\cos 2\theta = 0$, then $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$ is equal to**

(*a*) $\frac{-1}{2}$ (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) –1

21. **If $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then other two roots are**

(*a*) $x = 2, x = 7$ (*b*) $x = -2, x = 7$ (*c*) $x = 1, x = 7$ (*d*) None of these

22. The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{-1}{2}$ (*c*) 0 (*d*) 4

23. The value of determinant $\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$ is

(*a*) $2\,xyz$ (*b*) $2x^3y^3z^3$ (*c*) 0 (*d*) $4\,xyz$

24. The value of the determinant $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$ is

(*a*) 15 (*b*) $\frac{15}{2}$ (*c*) 10 (*d*) 0

25. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then the value of $|2A|$ is

(*a*) $4|A|$ (*b*) $|A|$ (*c*) $2|A|$ (*d*) None of these

26. The value of the determinant $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$ is

(*a*) a (*b*) $-a$ (*c*) b (*d*) 0

27. For any two square matrices A & B, $|A| = 2$ & $|B| = 6$ then $|AB|$ is

(*a*) 2 (*b*) 6 (*c*) 12 (*d*) none of these

28. The value of $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$ is:

(*a*) a (*b*) b (*c*) 0 (*d*) None of these

29. Let the points A (1, 3) and B (0, 0) $D(k, 0)$ form a triangle, using determinants find the value of k such that area of $\Delta\, ABD$ is 3 sq. units.

(*a*) 2 (*b*) ± 2 (*c*) –2 (*d*) 4

30. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then find x.

(*a*) ± 3 (*b*) –3 (*c*) +3 (*d*) ± 2

31. The sum of products of elements of any row with the cofactors of corresponding elements is equal to

(*a*) $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ (*b*) $a_{11}A_{11} + a_{12}A_{13} + a_{13}A_{12}$

(*c*) $a_{11}A_{11} + a_{12}A_{12} + a_{21}A_{13}$ (*d*) None of these

32. The value of the determinant $\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$ is

(*a*) 0 (*b*) 5 (*c*) –5 (*d*) 4

33. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, find the determinant of the matrix $A^2 - 2A$

(*a*) 25 (*b*) –25 (*c*) 0 (*d*) 4

34. If the points (2, –3), (λ, –1) and (0, 4) are collinear, find the value of λ.

(*a*) $\frac{10}{7}$ (*b*) 7 (*c*) 10 (*d*) 0

35. **If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x is equal to**

(*a*) 6 (*b*) ± 6 (*c*) – 6 (*d*) 0

36. **Which of the following is correct?**

(*a*) Determinant is a square matrix.

(*b*) Determinant is a number associated to a matrix.

(*c*) Determinant is a number associated to a square matrix

(*d*) None of these

37. **Let $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$, where $0 \le \theta \le 2\pi$, then**

(*a*) $\Delta = 0$ (*b*) $\Delta \in (2, \infty)$ (*c*) $\Delta \in (2, 4)$ (*d*) $\Delta \in [2, 4]$

38. **If 1, ω, ω^2 are the cube roots of unity, then the value of $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to**

(*a*) ω (*b*) –1 (*c*) 0 (*d*) ω^2

39. **Let ω be a complex number such that $2\omega + 1 = z$, where $z = \sqrt{-3}$, if $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to**

(*a*) z (*b*) –1 (*c*) $-z$ (*d*) 1

40. **Let $f(z) = \begin{vmatrix} 5 & 3 & 8 \\ 2 & z & 1 \\ 1 & 2 & z \end{vmatrix}$, then $f(5)$ is equal to**

(*a*) 10 (*b*) –20 (*c*) 80 (*d*) none of these

41. **Let $f(x) = \begin{vmatrix} x & -4 & 5 \\ 1 & 1 & -2 \\ 2 & x & 1 \end{vmatrix}$, then $f'(5)$ is equal to**

(*a*) 1 (*b*) 26 (*c*) 40 (*d*) none of these

42. **If $f(x) = \begin{vmatrix} 0 & x-1 & x-2 \\ x+1 & 0 & x-c \\ x+2 & x+c & 0 \end{vmatrix}$, then**

(*a*) $f(1) = 0$ (*b*) $f(2) = 0$ (*c*) $f(3) = 0$ (*d*) $f(0) = 0$

43. **There are two values of λ such that $\begin{vmatrix} 0 & 2 & 0 \\ \lambda & 3 & \lambda \\ \lambda & 5 & 6 \end{vmatrix} = -16$ then the sum of two values of λ is**

(*a*) 5 (*b*) 0 (*c*) 3 (*d*) 6

44. **If $\theta = \frac{\pi}{6}$, then $\begin{vmatrix} 0 & -1 & 1 \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}$ is equal to**

(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) $\frac{\sqrt{3}}{2}$ (*d*) none of these

45. **If the equation $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ possess a common root, then $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \cdot \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ is equal to**

(a) $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ (b) $2\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ (c) $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}^2$ (d) none of these

46. If $f = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$ and $g = (x-y)(y-z)(z-x)$, then $\frac{f}{g}$ is

(a) $x^2 + y^2 + z^2$ (b) $xy + yz + zx$

(c) $x^2 + y^2 + z^2 - xy - yz - zx$ (d) none of these

47. If $\Delta(x) = \begin{vmatrix} \sin\frac{x}{2} & 1 & 1 \\ 1 & \sin\frac{x}{2} & -\sin\frac{x}{2} \\ -\sin\frac{x}{2} & 1 & -1 \end{vmatrix}$ $\forall\, x \in [0, \pi]$ then

(a) $\Delta(x)$ will be maximum at $x = \pi$ (b) $\Delta(x)$ will be minimum at $x = 0$

(c) The range of $\Delta(x)$ is [2, 4]. (d) All of these

48. Let $\begin{vmatrix} x^2+x & 2x-1 & x+3 \\ 3x+1 & 2+x^2 & x^3-3 \\ x-3 & x^2+4 & 2x \end{vmatrix} = px^7 + qx^6 + rx^5 + sx^4 + tx^3 + ux^2 + vx + w$ then which of the following is not true?

(a) $w = 21, v = 75$ (b) $q = 0, s = -4$ (c) $p = -1, t = 8$ (d) $p = q = -1$

Answers

1. (b)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (c)
7. (d)	**8.** (c)	**9.** (b)	**10.** (c)	**11.** (c)	**12.** (d)
13. (d)	**14.** (d)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (a)
19. (c)	**20.** (b)	**21.** (a)	**22.** (a)	**23.** (b)	**24.** (b)
25. (a)	**26.** (d)	**27.** (c)	**28.** (c)	**29.** (b)	**30.** (a)
31. (a)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (b)	**36.** (c)
37. (d)	**38.** (c)	**39.** (c)	**40.** (c)	**41.** (b)	**42.** (d)
43. (d)	**44.** (b)	**45.** (d)	**46.** (b)	**47.** (b)	**48.** (b)

CASE-BASED QUESTIONS

Choose and write the correct option in the following questions.

1. Read the following and answer any four questions from (i) to (v).

Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same, but if length is decreased by 10m and breadth is decreased by 20m, then its area will decrease by 5300 m^2. [*CBSE Question Bank*]

Based on the above informations answer the following:

(i) The equations in terms of X and Y are

(a) $x - y = 50, 2x - y = 550$ (b) $x - y = 50, 2x + y = 550$

(c) $x + y = 50, 2x + y = 550$ (d) $x + y = 50, 2x - y = 550$

(*ii*) Which of the following matrix equation is represented by the given information

(*a*) $\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ (*b*) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$

(*c*) $\begin{bmatrix}1 & 1\\2 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ (*d*) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-50\\-550\end{bmatrix}$

(*iii*) The value of x(length of rectangular field) is

(*a*) 150 m (*b*) 400 m (*c*) 200 m (*d*) 320 m

(*iv*) The value of y (breadth of rectangular field) is

(*a*) 150 m. (*b*) 200m. (*c*) 430m. (*d*) 350m.

(*v*) How much is the area of rectangular field?

(*a*) 60000 sq. m. (*b*) 30000 sq.m.

(*c*) 30000 m (*d*) 3000 m

Sol. (*i*) We have,

$$(x-50)(y+50) = xy \;\Rightarrow\; 50x-50y = 2500$$

$$\Rightarrow\quad x-y = 50 \quad ...(i)$$

Also, $(x-10)(y-20) = xy-5300$

$$\Rightarrow\quad -20x-10y = -5500 \;\Rightarrow\; 2x+y = 550 \;...(ii)$$

$\therefore$ Equation in terms of x and y are

$x-y = 50$ and $2x+y = 550$

Option (*b*) is correct.

(*ii*) Now the equation $x-y = 50$

$\therefore$ $2x+y = 550$ will be represented by

$$\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$$

Option (*a*) is correct.

(*iii*) We have,

$$\underset{A}{\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}}\underset{X}{\begin{bmatrix}x\\y\end{bmatrix}}=\underset{B}{\begin{bmatrix}50\\550\end{bmatrix}}$$

$\Rightarrow AX = B \;\Rightarrow\; X = A^{-1}.B \quad ...(i)$

Co- factors of Metrix A

$C_{11} = 1, C_{21} = 1$

$C_{12} = -2, C_{22} = 1$

$\therefore$ adj $A = \begin{bmatrix}1 & 1\\-2 & 1\end{bmatrix}$ and $|A| = \begin{vmatrix}1 & -1\\2 & 1\end{vmatrix} = 1+2 = 3 \neq 0$

Therefore is A inverse exists.

$$\therefore\quad A^{-1} = \frac{adjA}{|A|} = \frac{1}{3}\begin{bmatrix}1 & 1\\-2 & 1\end{bmatrix}$$

From (*i*), we get

$$\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1\\-2 & 1\end{bmatrix}\begin{bmatrix}50\\550\end{bmatrix} = \frac{1}{3}\begin{bmatrix}600\\450\end{bmatrix} = \begin{bmatrix}200\\150\end{bmatrix}$$

$\Rightarrow\quad x = 200$ and $y = 150$

$\therefore\quad x = 200$ m

Option (*c*) is correct.

(*iv*) From (*iii*) question, we get $y = 150$ m

Option (*a*) is correct.

(*v*) Area of the rectangular field $= xy = 200 \times 150$

$$= 30000 \text{ m}^2$$

Option (*b*) is correct.

2. Read the following and answer any four questions from (*i*) to (*v*).

Three friends Rahul, Ravi and Rakesh went to a vegetable market to purchase vegetable. From a vegetable shop Rahul purchased 1 kg each of Potato, Onion and Brinjal for a total of ₹21. Ravi purchased 4 kg of potato, 3 kg of onion and 2 kg of Brinjal for ₹60 while Rakesh purchased 6 kg potato, 2 kg onion and 3 kg brinjal for ₹70.

Based on above information answer the following.

(*i*) If the cost of potato, onion and brinjal, are ₹x, ₹y and ₹z per kg respectively, then algebraic representation of given situation of problem is

(*a*) $x + y + z = 6$, $x + y + 3z = 11$, $3x + 2y + z = 2$

(*b*) $x + y + z = 21$, $4x + 3y + 2z = 60$, $6x + 2y + 3z = 70$

(*c*) $2x + 3y + z = 21$, $x + y + z = 60$, $x + 2y + z = 70$

(*d*) $x + y + z = 70$, $4x + 2y + 2z = 21$, $6x + 2y + 3z = 6$

(*ii*) The algebraic representation obtained in question (*i*) is represented in matrix-system as

(*a*) $AX = B$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

(*b*) $AX = B$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}, B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(*c*) $A = BX$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

(*d*) $AB = X$, where $A = \begin{bmatrix} 1 & 1 & 21 \\ 4 & 3 & 60 \\ 6 & 2 & 70 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(*iii*) If $AX = B$, where A, X, B are matrix then X should be

(*a*) $X = AB$ (*b*) $X = BA$ (*c*) $X = A^{-1}B$ (*d*) $X = AB^{-1}$

(*iv*) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$ then A^{-1} is

(*a*) $-\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ 0 & 4 & 0 \end{bmatrix}$ (*b*) $-\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & 3 & 0 \\ -10 & 4 & -1 \end{bmatrix}$

(*c*) $\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$ (*d*) $-\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$

(*v*) The cost of potato, onion and brinjal are

(*a*) ₹5, ₹8 and ₹8 (*b*) ₹4, ₹8 and ₹11 (*c*) ₹4, ₹11 and ₹10 (*d*) ₹4, ₹8 and ₹15

Sol. (*i*) From question

For Rahul $x + y + z = 21$

For Ravi $4x + 3y + 2z = 60$

For Rakesh $6x + 2y + 3z = 70$

Therefore, Algebraical representation is

$$x + y + z = 21$$
$$4x + 3y + 2z = 60$$
$$6x + 2y + 3z = 70$$

Option (*b*) is correct.

(*ii*) The given Algebraical system of Linear equation can be written in matrix system as

$$AX = B \quad ...(i)$$

Where, A is co-efficient matrix

$$\therefore \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

X is variable matrix

$$\therefore \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and B constant matrix

$$\therefore \quad B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

Option (*a*) is correct.

(*iii*) We have $AX = B$

Pre multiplying by A^{-1} on both sides, we have

$$A^{-1} AX = A^{-1} B$$

$\Rightarrow \quad (A^{-1} A)X = A^{-1} B$

$\Rightarrow \quad IX = A^{-1} B \quad [A^{-1} A = I \text{ (Identity matrix]}$

$\Rightarrow \quad X = A^{-1} B \quad [IX = X]$

Option (*c*) is correct.

(*iv*) We have $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix} = 1(9-4) - 1(12-12) + (8-18)$

$= 5 - 0 - 10 = -5 \neq 0$

Therefore , A^{-1} exists.

Now, $A_{11} = 9 - 4 = 5; A_{21} = -(3-2) = -1; A_{31} = 2 - 3 = -1$

$A_{12} = -(12-12) = 0; A_{22} = (3-6) = -3; A_{32} = -(2-4) = 2$

$A_{13} = (8-18) = -10; A_{23} = -(2-6) = 4; A_{33} = (3-4) = -1$

$\therefore \quad \text{Adj } A = \begin{bmatrix} 5 & 0 & -10 \\ -1 & -3 & 4 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$

$\because \quad A^{-1} = \frac{1}{|A|} \text{ Adj A}$

$= -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$

Option (*d*) is correct.

(*v*) We have $X = A^{-1}B$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$

$\Rightarrow \quad x = 5, y = 8, z = 8$

$\Rightarrow$ Cost of potato,onion and brinjal are ₹5, ₹ 8 and ₹8

Option (*a*) is correct.

ASSERTION-REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(*a*) *Both A and R are true and R is the correct explanation of A.*

(*b*) *Both A and R are true but R is not the correct explanation of A.*

(*c*) *A is true but R is false.*

(*d*) *A is false and R is also false.*

1. Assertion (A) : Determinant is a number associated with a square matrix.

Reason (R) : Determinant is a square matrix.

2. Assertion (A) : If $A = \begin{bmatrix} 5 - x & x+1 \\ 2 & 4 \end{bmatrix}$, then the matrix A is singular if $x = 3$.

Reason (R) : A square matrix is a singular matrix if its determinant is zero.

3. **Assertion (A) :** If A is a 3×3 matrix, $|A| \neq 0$ and $|5A| = K|A|$, then the value of $K = 125$.

Reason (R) : If A be any square matrix of order $n \times n$ and k be any scalar then $|KA| = K^n|A|$.

4. **Assertion (A) :** If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then $x = \pm 6$.

Reason (R) : If A is a skew-symmetric matrix of odd order, then $|A| = 0$.

Answers

1. (*c*) **2.** (*a*) **3.** (*a*) **4.** (*b*)

HINTS/SOLUTIONS OF SELECTED MCQS

1. We have,

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$\Rightarrow \quad x^2 - 36 = 36 - 36$

$\Rightarrow \quad x^2 - 36 = 0$

$\Rightarrow \quad x^2 = 36$

$\Rightarrow \quad x = \pm 6$

Option (*b*) is correct.

2. We know that, area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right| = \frac{1}{2}\left|\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}\right|$$

Expanding along R_1, we get

$$9 = \frac{1}{2}|[-3(-k) - 0 + 1(3k)]| \qquad \Rightarrow \qquad 18 = |3k + 3k| = |6k|$$

$\therefore \quad k = \pm \frac{18}{6} = \pm 3 = 3, -3$

Option (*b*) is correct.

3. We have, $\Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$

Expanding long c_1, we get

$$\begin{aligned} \Delta &= (-1)(1 - \cos^2 A) - \cos C(-\cos C - \cos A \cos B) + \cos B(\cos A \cos C + \cos B) \\ &= -1 + \cos^2 A + \cos^2 C + \cos A \cos B \cos C + \cos A \cos B \cos C + \cos^2 B \\ &= -1 + \cos^2 A + \cos^2 B + \cos^2 C + 2\cos A \cos B \cos C \end{aligned}$$

As A, B, C, are angles of ΔABC

$\therefore \; A + B + C = \pi \Rightarrow A + B = \pi - C$

$\Rightarrow \cos(A + B) = \cos(\pi - C)$

$\because \; 2(\cos^2 A + \cos^2 B + \cos^2 C) = 2\cos^2 A + 2\cos^2 B + 2\cos^2 C$

$\quad = 1 + \cos 2A + 1 + \cos 2B + 1 + \cos 2C$

$= 3 + \cos 2A + \cos 2B + \cos 2C$

$= 3 + (\cos 2A + \cos 2B) + \cos 2C$

$= 3 + \cos (A + B) . \cos (A - B) + \cos 2 C$

$= 3 + \cos (\pi - C) . \cos (A - B) + \cos 2 C$

$= 3 - \cos C \cos (A - B) + 2 \cos^2 C - 1$

$= 2 - \cos C \{\cos (A - B) - \cos C\}$

$= 2 - \cos C \{\cos (A - B) - \cos \{\pi - (A + B)\}$

$= 2 - 2 \cos C \{\cos (A - B) + \cos (A + B)\}$

$= 2 - 2 \cos C \left\{2 \cos \frac{A - B + A + B}{2} \cos \frac{A - B - A - B}{2}\right\}$

$= 2 - 2 \cos C \{2 \cos A . \cos B\}$

$= 2\{1 - 2 \cos A \cos B \cos C\}$

$\Rightarrow \cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$

$\Rightarrow -1 + \cos^2 A + \cos^2 B + \cos^2 C + 2 \cos A \cos B \cos C = 0$

$(i) \Rightarrow \Delta = 0$

Option (a) is correct.

4. We have, $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$

$\Rightarrow \quad f(a) = \begin{vmatrix} 0 & 0 & a-b \\ 2a & 0 & a-c \\ a+b & a+c & 0 \end{vmatrix} = [(a - b)\{2a. (a + c)\}] \neq 0$

and $\quad f(b) = \begin{vmatrix} 0 & b-a & 0 \\ b+a & 0 & b-c \\ 2b & b+c & 0 \end{vmatrix} = -(b - a)[-2b(b - c)] = 2b(b - a)(b - c) \neq 0$

and $\quad f(0) = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = a(bc) - b(ac) = abc - abc = 0$

Option (c) is correct.

6. We have, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

$\Rightarrow 1(2a^2 + 4) - 2(-4a - 20) + 0 = 86$ [Expanding along first column]

$\Rightarrow 2a^2 + 4 + 8a + 40 = 86$

$\Rightarrow 2a^2 + 8a + 44 - 86 = 0$

$\Rightarrow a^2 + 4a - 21 = 0$

$\Rightarrow a^2 + 7a - 3a - 21 = 0$

$\Rightarrow (a + 7)(a - 3) = 0 \quad \Rightarrow \quad a = -7$ and 3

$\therefore$ Required sum $= -7 + 3 = -4$

Option (c) is correct.

7. We have,

$A^2 = 3A \quad \Rightarrow \quad |A^2| = |3A|$

$\Rightarrow \quad |A|.|A| = 3^3\,|A| \qquad$ ($\because$ order of matrix A is 3 and $|A|$ is not equal to zero)

$\Rightarrow \quad |A| = 3^3 = 27 \quad \Rightarrow \quad |A| = 27$

Option (*d*) is correct.

10. $$\Delta = \begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$$

$= (b-a)\{0-(a-c)(c-b)\} + (c-a)\{(a-b)(b-c)-0\}$

$= +(b-a)(a-c)(c-b) + (c-a)(a-b)(b-c)$

$= -(a-b)(b-c)(c-a) + (c-a)(a-b)(b-c)$

$= 0$

Option (*c*) is correct.

11. $$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{vmatrix} = 1(\omega^2-\omega) - \omega(\omega-\omega^3) - \omega^2(\omega-\omega^4)$$

$= \omega^2 - \omega - \omega^2 + \omega^4 - \omega^3 + \omega^6$

$= \omega^2 - \omega - \omega^2 + \omega - 1 + 1$

$= 0$

Option (*c*) is correct.

12. We have

$$\frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \perp 35$$

$$\Rightarrow \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 70$$

$\Rightarrow 2(4-4) - 5(-(-4) + k(-6-4) = \pm 70$

$\Rightarrow 0 + 50 - 10k = \pm 70$

$\Rightarrow 50 \mp 70 = 10k$

$\Rightarrow -20, 120 = 10k$

$\Rightarrow k = -2, 12$

Option (*d*) is correct.

13. If A_{ij} is cofactor of a_{ij}, then the value of Δ is given by, $\Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$.

Option (*d*) is correct.

14. $$\Delta\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc-a^2) - b(b^2-ac) + c(ab-c^2)$$

$= abc - a^3 - b^3 + abc + abc - c^3$

$= 3abc - (a^3 + b^3 + c^3)$

$$\because \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - (a^3+b^3+c^3)$$

$\therefore A = abc, B = a^3 + b^3 + c^3.$

Option (d) is correct.

16. $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}, 2A = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

$|2A| = 8 - 32 = -24$

$|A| = 2 - 8 = 6,$

$\because |2A| = 4 \times (-6) = 4 \mid A \mid$

$\Rightarrow \quad k = 4$

Option (a) is correct.

17. For any matrix A, $|A| = |A'|$ $\therefore |A'| = |A| = 5$

$\because$ A is symmetric so $A' = A$

Option (a) is correct.

19. $\Delta = \frac{1}{2}\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = \frac{1}{2}[1(2a^2 + 4) - 2(-4a - 20)]$

$= \frac{1}{2}[2a^2 + 4 + 8a + 40] = \frac{1}{2}[2a^2 + 8a + 44]$

$= a^2 + 4a + 22$

$\because \quad \Delta = 86 \text{ (Given)} \Rightarrow a^2 + 4a + 22 = 86$

$\Rightarrow \quad a^2 + 4a - 64 = 0$

$\Rightarrow \quad a = \frac{-4 \pm \sqrt{16 + 256}}{2} = \frac{-4 \pm \sqrt{272}}{2} \Rightarrow a = \frac{-4 + \sqrt{272}}{2}$ and $\frac{-4 - \sqrt{272}}{2}$

Sum of two values of $a = \frac{-4 + 8\sqrt{272} - 4 - \sqrt{272}}{2} = -4$

Option (c) is correct.

20. $\Delta = \begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$

$= (-\cos\theta(\cos^2\theta - 0) + \sin\theta(0 - \sin^2\theta))^2$

$= (-\cos^3\theta - \sin^3\theta)^2 = (\cos^3\theta + \sin^3\theta)^2$

Now $\cos 2\theta = 0 \Rightarrow 2\theta = (2n + 1)\frac{\pi}{2} \Rightarrow \theta = (2n + 1)\frac{\pi}{4}$

$\therefore \Delta = \left(\cos^3\frac{(2n + 1)\pi}{4} + \sin^3\frac{(2n + 1)\pi}{4}\right)^2$

$= \left(\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

Option (b) is correct.

21. $\Delta = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$\Rightarrow \quad x(x^2 - 12) - 2(3x-42) + 7(6 - 7x) = 0$

$\Rightarrow \quad x^3 - 12x - 6x + 84 + 42 - 49x = 0$

$\Rightarrow \quad x^3 - 67x + 126 = 0$

$\because \quad x = -9$ is a root of the above equation.

$(x + 9)$ is a factor of this equation.

$\Rightarrow \quad x^3 + 9x^2 - 9x^2 - 81x + 14x + 126 = 0$

$\Rightarrow \quad x^2(x+9) - 9x(x+9) + 14(x+9) = 0 \Rightarrow (x+9)[x^2 - 9x + 14] = 0$

$\Rightarrow \quad (x+9)(x^2 - 2x - 7x + 14) = 0$

$\Rightarrow \quad (x+9)(x(x-2) - 7(x-2)) = 0$

$\Rightarrow \quad (x+9)(x-2)(x-7) = 0$

$\Rightarrow \quad x = -9, 2, 7$

Other zeros are 2, 7

Option (*a*) is correct.

23. $\Delta = \begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix} = -x^2y(0 - xy^2z^3) + x^2z(xy^3z^2)$

$= x^3y^3z^3 + x^3y^3z^3 = 2x^3y^3z^3$

Option (*b*) is correct.

24. $\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} = \begin{vmatrix} \log_3 2^9 & \log_4 3 \\ \log_3 2^3 & \log_4 3^2 \end{vmatrix}$

$= \begin{vmatrix} 9\log_3 2 & \log_4 3 \\ 3\log_3 2 & 2\log_4 3 \end{vmatrix} = 18\log_3 2 \times \log_4 3 - 3\log_3 2 \times \log_4 3$

$= \log_3 2 \times \log_4 3 (18 - 3) = \dfrac{\ln 2}{\ln 3} \times \dfrac{\ln 3}{\ln 4} \times 15 \quad \left[\because \log_a b = \dfrac{\ln b}{\ln a}\right]$

$= \dfrac{\ln 2}{\ln 4} \times 15 = \dfrac{\ln 2}{\ln 2^2} \times 15 = \dfrac{15 \ln 2}{2 \ln 2} = \dfrac{15}{2}$

Option (*b*) is correct.

25. $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}, 2A = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

$|2A| = 2^2|A| = 4|A|$

Option (*a*) is correct.

26. $\Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0 + a(0 + bc) + b(-ac - 0)$

$= abc - abc = 0$

Option (*d*) is correct.

27. $|AB| = |A|\ |B| = 2 \times 6 = 12$

Option (*c*) is correct.

28. $\Delta = \begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix} = \begin{vmatrix} 0 & a-b & a-c \\ -(a-b) & 0 & b-c \\ -(a-c) & -(b-c) & 0 \end{vmatrix}$

$$= 0 \qquad \left[\because A = \begin{bmatrix} 0 & a-b & a-c \\ -(a-b) & 0 & b-c \\ -(a-c) & -(b-c) & 0 \end{bmatrix} \text{ is a skew symmetric matrix of order 3.} \atop \therefore |A| = 0 \right]$$

Option (*c*) is correct.

29. We are given points

$A(1, 3), B(0, 0)$ and $D(k, 0)$

and ar(ΔABD) = 3 square units.

i.e., $\quad \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = 3$

Expanding along R_2, we get

$\Rightarrow \quad \frac{1}{2} \mid (-1)(0 - 3k) \mid = 3 \Rightarrow \mid 3k \mid = 6 \Rightarrow 3k = \pm 6$

$\Rightarrow \quad k = \pm 2$

Option (*b*) is correct.

30. $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix} \Rightarrow 2x^2 - 40 = 18 - 40$

$\Rightarrow \quad 2x^2 = 18 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

Option (*a*) is correct.

31. By the definition of expansion of determinant, the required relation is

$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

Option (*a*) is correct.

32. $\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Taking $3x$ common from R_3, we get

$= (3x)\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix} = 3x \times 0 \, [\because R_1 = R_3] = 0$

Option (*a*) is correct.

33. $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix}$

$\therefore \quad A^2 - 2A = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$\therefore \quad | A^2 - 2A | = 25$

Option (*a*) is correct.

34. Let $A(2, -3), B(\lambda, -1)$ and $C(0, 4)$ are given points.

Given points are collinear

$\therefore \quad ar(\Delta ABC) = 0$

$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ \lambda & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0 \Rightarrow |2(-1-4) - \lambda(-3-4) + 0| = 0$

$\Rightarrow \quad |-10+7\lambda| = 0 \Rightarrow 7\lambda - 10 = 0 \Rightarrow \lambda = \frac{10}{7}$

Option (*a*) is correct.

35. $x^2 - 36 = 36 - 36$

$\Rightarrow \quad x^2 - 36 = 0$

$\Rightarrow \quad x^2 = 36$

$\Rightarrow \quad x = \pm 6$

Option (*b*) is correct.

36. (*c*) Determinant is a number associated to a square matrix

37. $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

$= 1(1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$

$\Delta = 2(1+\sin^2\theta) \quad \ldots(i)$

Now, $\because \ -1 \le \sin\theta \le 1$

$\rightarrow \quad 0 \le \sin^2\theta \le 1$

$\Rightarrow \quad 0+1 \le 1+\sin^2\theta \le 1+1$

$\Rightarrow \quad 1 \le 1+\sin^2\theta \le 2$

$\Rightarrow \quad 2 \le 2(1+\sin^2\theta) \le 4$

$\Rightarrow \quad 2 \le \Delta \le 4$ [From (*i*)]

$\Rightarrow \quad \Delta \in [2, 4]$

Option (*d*) is correct.

40. $f(z) = \begin{vmatrix} 5 & 3 & 8 \\ 2 & z & 1 \\ 1 & 2 & z \end{vmatrix} = 5(z^2-2) - 2(3z-16) + 1(3-8z)$

$= 5z^2 - 10 - 6z + 32 + 3 - 8z = 5z^2 - 14z + 25$

$f(5) = 5 \times 5^2 - 14 \times 5 + 25 = 125 - 70 + 25$

$= 150 - 70 = 80$

Option (*c*) is correct.

41. $f(x) = \begin{vmatrix} x & -4 & 5 \\ 1 & 1 & -2 \\ 2 & x & 1 \end{vmatrix} = x(1+2x) - 1(-4-5x) + 2(8-5)$

$= x + 2x^2 + 4 + 5x + 6 = 2x^2 + 6x + 10$

$f'(x) = 4x + 6$

$f'(5) = 20 + 6 = 26$

Option (*b*) is correct.

42. $f(x) = \begin{vmatrix} 0 & x-1 & x-2 \\ x+1 & 0 & x-c \\ x+2 & x+c & 0 \end{vmatrix}$

$= -(x+1)\{-(x-2)(x+c)\} + (x+2)\{(x-1)(x-c) - 0\}$

$= +(x+1)(x-2)(x+c) + (x+2)(x-1)(x-c)$

$f(0) = -2c + 2c = 0$

Option (d) is correct.

43. $\begin{vmatrix} 0 & 2 & 0 \\ \lambda & 3 & \lambda \\ \lambda & 5 & 6 \end{vmatrix} = -16$ then the sum of two values of $\boldsymbol{\lambda}$ is

$$\begin{vmatrix} 0 & 2 & 0 \\ \lambda & 3 & \lambda \\ \lambda & 5 & 6 \end{vmatrix} = -16 \Rightarrow -\lambda(12-0) + \lambda(2\lambda - 0) = -16$$

$\Rightarrow \quad -12\lambda + 2\lambda^2 = -16 \Rightarrow 2\lambda^2 - 12\lambda + 16 = 0$

$\Rightarrow \quad \lambda^2 - 6\lambda + 8 = 0 \Rightarrow (\lambda - 2)(\lambda - 4) = 0$

$\Rightarrow \quad \lambda = 2, 4$

$\therefore$ Sum of two values of $\lambda = 2 + 4 = 6$

Option (d) is correct.

44. Let $\Delta = \begin{vmatrix} 0 & -1 & 1 \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}$

Applying $C_2 \to C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & \cos\theta \end{vmatrix} = 1(\cos^2\theta - \sin^2\theta) = \cos 2\theta$$

If $\theta = \dfrac{\pi}{6}$, then $\Delta = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

Option (b) is correct.

45. Let α, β are common roots of the equations

$a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$

$\Rightarrow \quad \alpha + \beta = -\dfrac{b_1}{a_1}$, and $\alpha + \beta = -\dfrac{b_2}{a_2}$, $\alpha\beta = \dfrac{c_1}{a_1}$, $\alpha\beta = \dfrac{c_2}{a_2}$

$\Rightarrow \quad -\dfrac{b_1}{a_1} = -\dfrac{b_2}{a_2}$ and $\dfrac{c_1}{a_1} = \dfrac{c_2}{a_2}$

$\Rightarrow \quad \dfrac{b_1}{a_1} = \dfrac{b_2}{a_2}$ and $c_1a_2 = a_1c_2$

$\Rightarrow \quad b_1a_2 - b_2a_1 = 0$ and $c_1a_2 - a_1c_2 = 0$

$\therefore \quad \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \cdot \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$

$= 0 \times (b_1c_2 - b_2c_1) = 0$

Option (d) is correct.

47. $\Delta(x) = \begin{vmatrix} \sin\dfrac{x}{2} & 1 & 1 \\ 1 & \sin\dfrac{x}{2} & -\sin\dfrac{x}{2} \\ -\sin\dfrac{x}{2} & 1 & -1 \end{vmatrix}$

$= \sin\frac{x}{2}\left(-\sin\frac{x}{2}+\sin\frac{x}{2}\right)-1(-1-1)-\sin\frac{x}{2}\left(-\sin\frac{x}{2}-\sin\frac{x}{2}\right)$

$= \sin\frac{x}{2}\times 0+2+2\sin^2\frac{x}{2}$

$= 2\sin^2\frac{x}{2}+2 = 2+2\sin^2\frac{x}{2} = 2+1-\cos x \quad \left[\because 1-\cos x = 2\sin^2\frac{x}{2}\right]$

$= 3 - \cos x$

Now $-1 \le \cos x \le 1 \Rightarrow 1 \ge -\cos x \ge -1$

$\Rightarrow 3+1 \ge 3-\cos x \ge 3-1 \Rightarrow 4 \ge 3-\cos x \ge 2$

$\Rightarrow 2 \le 3-\cos x \le 4 \Rightarrow 2 \le \Delta(x) \le 4$

Also $\Delta(x) = 3-\cos x$

$\Rightarrow \Delta'(x) = \sin x$

$\therefore \Delta'(x) = 0 \Rightarrow \sin x = 0 \Rightarrow x = 0, \pi$

$\Delta''(x) = \cos x$

$\therefore \Delta''(0) = 1 > 0 \Rightarrow$ At $x = 0$, $\Delta(x)$ has minimum value.

$\Delta''(\pi) = -1 < 0 \Rightarrow$ At $x = \pi$, $\Delta(x)$ has maximum value.

So all the statements are true.

Option (*b*) is correct.

48. We have

$$\Delta = \begin{vmatrix} x^2+x & 2x-1 & x+3 \\ 3x+1 & 2+x^2 & x^3-3 \\ x-3 & x^2+4 & 2x \end{vmatrix} = px^7+qx^6+rx^5+sx^4+tx^3+ux^2+vx+w$$

$\Rightarrow (x^2+x)\{(4x+2x^3)-(x^5+4x^3-3x^2-12)\}-(3x+1)\{(4x^2-2x)-(x^3+3x^2+4x+12)\}$

$+(x-3)\{(2x^4-x^3-6x+3)-(x^3+3x^2+2x+6)\}$

$= px^7+qx^6+rx^5+sx^4+tx^3+ux^2+vx+w$

$\Rightarrow \quad -x^7-x^6+0x^5-4x^4+8x^3+34x^2+75x+21 = px^7+qx^6+rx^5+sx^4+tx^3+ux^2+vx+w$

$\Rightarrow \quad p=-1, q=-1, r=0, s=-4, t=8, u=34, v=75, w=21$

Option (*b*) is correct.