JEE ADVANCED 2018 - PHYSICS

PAPER - 1

SECTION - I (MAXIMUM MARKS: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
  Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
  Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : **–2** In all other cases.
- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

1. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true?

(A) $v = \sqrt{\frac{k}{2m}}R$ (B) $v = \sqrt{\frac{k}{m}}R$

(C) $L = \sqrt{mk}R^2$ (D) $L = \sqrt{\frac{mk}{2}}R^2$

2. Consider a body of mass 1.0 kg at rest at the origin at time $t = 0$. A force $\vec{F} = (\alpha t\,\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0\ \text{Ns}^{-1}$ and $\beta = 1.0\ N$. The torque acting on the body about the origin at time $t = 1.0\,s$ is $\vec{\tau}$. Which of the following statements is (are) true?

(A) $|\vec{\tau}| = \frac{1}{3}\,Nm$

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at $t = 1\,s$ is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})ms^{-1}$

(D) The magnitude of displacement of the body at $t = 1s$ is $\frac{1}{6}m$

3. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

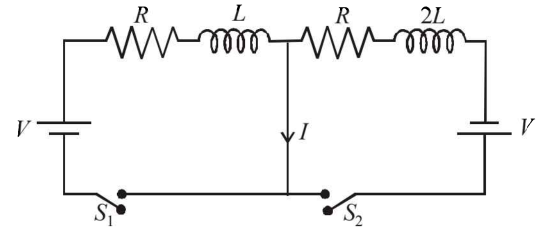
(A) For a given material of the capillary tube, h decreases with increase in r

(B) For a given material of the capillary tube, h is independent of σ

(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases

(D) h is proportional to contact angle θ

4. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statements is (are) true?



(A) $I_{max} = \frac{V}{2R}$ (B) $I_{max} = \frac{V}{4R}$

(C) $\tau = \frac{L}{R}\ln 2$ (D) $\tau = \frac{2L}{R}\ln 2$

5. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0,0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true?

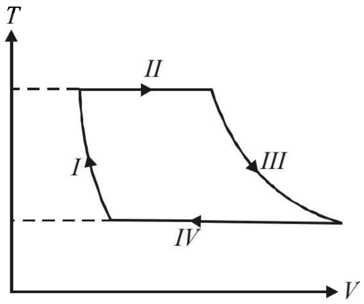
(A) If $I_1 = I_2$, then $\vec{B}$ **cannot** be equal to zero at the origin $(0, 0, 0)$

(B) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin $(0, 0, 0)$

(C) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin $(0, 0, 0)$

(D) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

6. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where ***V*** is the volume and ***T*** is the temperature). Which of the statements below is (are) true?



(A) Process I is an isochoric process

(B) In process II, gas absorbs heat

(C) In process IV, gas releases heat

(D) Processes I and III are **not** isobaric

SECTION - II (MAXIMUM MARKS: 24)

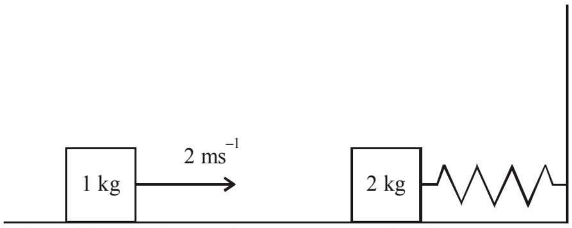
- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.
  Zero Marks : **0** In all other cases.

7. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos \omega t\, \hat{i} + \sin \omega t\, \hat{j})$, where a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}\,|\vec{A}-\vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in *seconds*, is __________.

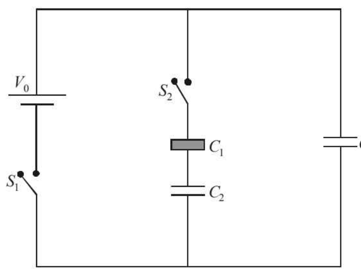
8. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is __________.

9. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is __________. Take g = 10 ms^{-2}.

10. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 Nm^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 ms^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is __________.

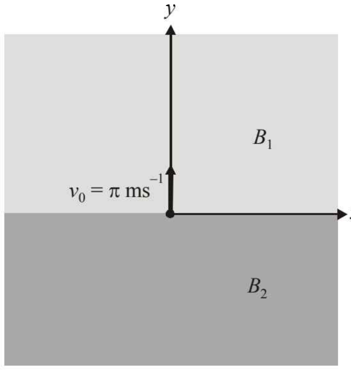


11. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 μC. The value of ε_r = ____________.



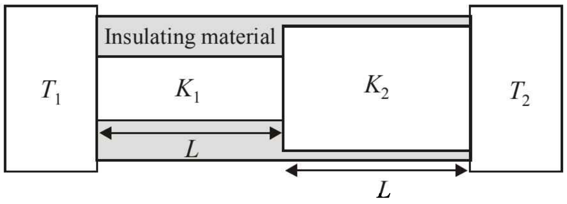
12. In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi$ ms^{-1} at $t = 0$, as shown in the figure. Neglect gravity in this problem.

Let $t = T$ be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is __________.



13. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is __________.

14. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300 K and T_2 = 100 K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then K_1/K_2 = __________.



SECTION - III (MAXIMUM MARKS: 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct option is chosen.
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PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, $[E]$ and $[B]$ stand for dimensions of electric and magnetic fields respectively, while $[\varepsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. $[L]$ and $[T]$ are dimensions of length and time respectively. All the quantities are given in SI units.

15. The relation between $[E]$ and $[B]$ is

(A) $[E] = [B][L][T]$ (B) $[E] = [B][L]^{-1}[T]$

(C) $[E] = [B][L][T]^{-1}$ (D) $[E] = [B][L]^{-1}[T]^{-1}$

16. The relation between $[\varepsilon_0]$ and $[\mu_0]$ is

(A) $[\mu_0] = [\varepsilon_0][L]^2[T]^{-2}$ (B) $[\mu_0] = [\varepsilon_0][L]^{-2}[T]^2$

(C) $[\mu_0] = [\varepsilon_0]^{-1}[L]^2[T]^{-2}$ (D) $[\mu_0] = [\varepsilon_0]^{-1}[L]^{-2}[T]^2$

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\Delta x/x \ll 1$, $\Delta y/y \ll 1$. Therefore, the higher powers of these quantities are neglected.

17. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa ($\Delta a/a \ll 1$, then what is the error Δr in determining r?

(A) $\frac{\Delta a}{(1+a)^2}$ (B) $\frac{2\Delta a}{(1+a)^2}$ (C) $\frac{2\Delta a}{(1-a^2)}$ (D) $\frac{2a\Delta a}{(1-a^2)}$

18. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| \ll 1$, $\ln(1 + x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

(A) 0.04 (B) 0.03 (C) 0.02 (D) 0.01

PAPER - 2

SECTION - I (MAXIMUM MARKS: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
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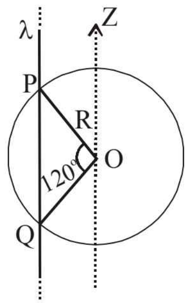
1. A particle of mass *m* is initially at rest at the origin. It is subjected to a force and starts moving along the *x*-axis. Its kinetic energy *K* changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?

(A) The force applied on the particle is constant
(B) The speed of the particle is proportional to time
(C) The distance of the particle from the origin increases linearly with time
(D) The force is conservative

2. Consider a thin square plate floating on a viscous liquid in a large tank. The height *h* of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity μ_0. Which of the following statements is (are) true?

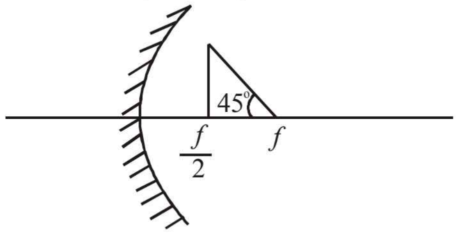
(A) The resistive force of liquid on the plate is inversely proportional to h
(B) The resistive force of liquid on the plate is independent of the area of the plate
(C) The tangential (shear) stress on the floor of the tank increases with μ_0
(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

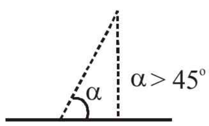
3. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ϵ_0. Which of the following statements is (are) true?

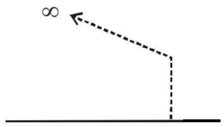


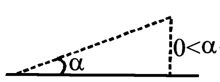
(A) The electric flux through the shell is $\sqrt{3}R\lambda/\epsilon_0$
(B) The z-component of the electric field is zero at all the points on the surface of the shell
(C) The electric flux through the shell is $\sqrt{2}R\lambda/\epsilon_0$
(D) The electric field is normal to the surface of the shell at all points

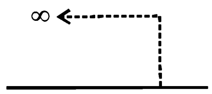
4. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length *f*, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)



(A) 

(B) 

(C) 

(D) 

5. In a radioactive decay chain, $^{232}_{90}Th$ nucleus decays to $^{212}_{82}Pb$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true?

(A) $N_\alpha = 5$ (B) $N_\alpha = 6$ (C) $N_\beta = 2$ (D) $N_\beta = 4$

6. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 *Hz* is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 *cm* and 83.9 *cm*. Which of the following statements is (are) true?
   (A) The speed of sound determined from this experiment is 332 ms^{-1}
   (B) The end correction in this experiment is 0.9 *cm*
   (C) The wavelength of the sound wave is 66.4 *cm*
   (D) The resonance at 50.7 *cm* corresponds to the fundamental harmonic

SECTION - II (MAXIMUM MARKS: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
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  Zero Marks : **0** In all other cases.

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7. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4$ *kg* is at rest on this surface. An impulse of 1.0 *Ns* is applied to the block at time $t = 0$ so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4s$. The displacement of the block, in *metres*, at $t = \tau$ is __________. Take $e^{-1} = 0.37$.
8. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 *m* and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in *metres*, is __________.
9. A particle, of mass 10^{-3} *kg* and charge 1.0 *C*, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t \hat{i}$, where $E_0 = 1.0$ NC^{-1} and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is __________.
10. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} $N\,m\,rad^{-1}$. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 –1.0 *A*. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in *ohms*, is __________.
11. A steel wire of diameter 0.5 *mm* and Young's modulus 2×10^{11} $N\,m^{-2}$ carries a load of mass *M*. The length of the wire with the load is 1.0 *m*. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 *mm*, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 *kg*, the vernier scale division which coincides with a main scale division is __________. Take $g = 10$ $m\,s^{-2}$ and $\pi = 3.2$.
12. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 *K* and the universal gas constant $R = 8.0$ $J\,mol^{-1}\,K^{-1}$, the decrease in its internal energy, in *Joule*, is __________.
13. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 *W* is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 *V* is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of *n* is __________. Mass of the electron $m_e = 9 \times 10^{-31}$ *kg* and $1.0\text{ eV} = 1.6 \times 10^{-19}$ *J*.
14. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of *Z* is __________.

SECTION - III (MAXIMUM MARKS: 12)

- This section contains **FOUR (04)** questions.
- Each question has **TWO (02)** matching lists: **LIST-I** and **LIST-II**.
- **FOUR** options are given representing matching of elements from **LIST-I** and **LIST-II**. **ONLY ONE** of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme:
  Full Marks : **+3** If ONLY the option corresponding to the correct matching chosen.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : –1 In all other cases.

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15. The electric field E is measured at a point P (0, 0, d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

LIST–I	**LIST–II**
P. E is independent of d	**1.** A point charge Q at the origin
Q. $E \propto 1/d$	**2.** A small dipole with point charges Q at $(0, 0, l)$ and $-Q$ at $(0, 0, -l)$. Take $2l << d$
R. $E \propto 1/d^2$	**3.** An infinite line charge coincident with the x-axis, with uniform linear charge density λ
S. $E \propto 1/d^3$	**4.** Two infinite wires carrying uniform linear charge density parallel to the x - axis. The one along $(y = 0, z = l)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -l)$ has a charge density $-\lambda$ Take $2l << d$
	5. Infinite plane charge coincident with the xy-plane with uniform surface charge density

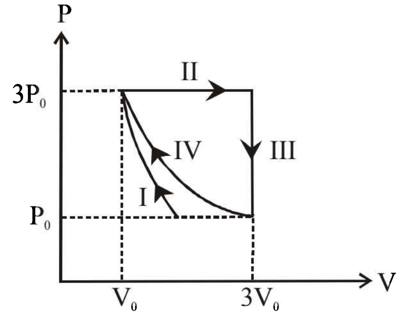
(A) $P \to 5; Q \to 3, 4; R \to 1; S \to 2$
(B) $P \to 5; Q \to 3; R \to 1, 4; S \to 2$
(C) $P \to 5; Q \to 3; R \to 1, 2; S \to 4$
(D) $P \to 4; Q \to 2, 3; R \to 1; S \to 5$

16. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively, Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2, and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

LIST–I	**LIST–II**
P. v_1/v_2	**1.** 1/8
Q. L_1/L_2	**2.** 1
R. K_1/K_2	**3.** 2
S. T_1/T_2	**4.** 8

(A) $P \to 4; Q \to 2; R \to 1; S \to 3$
(B) $P \to 3; Q \to 2; R \to 4; S \to 1$
(C) $P \to 2; Q \to 3; R \to 1; S \to 4$
(D) $P \to 2; Q \to 3; R \to 4; S \to 1$

17. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II.



LIST–I	**LIST–II**
P. In process I	**1.** Work done by the gas is zero
Q. In process II	**2.** Temperature of the gas remains unchanged
R. In process III	**3.** No heat is exchanged between the gas and its surroundings
S. In process IV	**4.** Work done by the gas is $6P_0V0$

(A) $P \to 4; Q \to 3; R \to 1; S \to 2$
(B) $P \to 1; Q \to 3; R \to 2; S \to 4$
(C) $P \to 3; Q \to 4; R \to 1; S \to 2$
(D) $P \to 3; Q \to 4; R \to 2; S \to 1$

18. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$ In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path.

LIST–I	**LIST–II**
P. $\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$	**1.** $\vec{p}$
Q. $\vec{r}(t) = \alpha\cos\omega t\ \hat{i} + \beta\sin\omega t\ \hat{j}$	**2.** $\vec{L}$
R. $\vec{r}(t) = \alpha(\cos\omega t\ \hat{i} + \sin\omega t\ \hat{j})$	**3.** K
S. $\vec{r}(t) = \alpha t\ \hat{i} + \frac{\beta}{2}t^2\ \hat{j}$	**4.** U
	5. E

(A) $P \to 1, 2, 3, 4, 5; Q \to 2, 5; R \to 2, 3, 4, 5; S \to 5$
(B) $P \to 1, 2, 3, 4, 5; Q \to 3, 5; R \to 2, 3, 4, 5; S \to 2, 5$
(C) $P \to 2, 3, 4; Q \to 5; R \to 1, 2, 4; S \to 2, 5$
(D) $P \to 1, 2, 3, 5; Q \to 2, 5; R \to 2, 3, 4, 5; S \to 2, 5$

SOLUTIONS

Paper - 1

1. **(B, C)** We know that, $|F| = \frac{dv}{dr} = \frac{d}{dr}\left[\frac{kr^2}{2}\right] = kr$

$\because$ Potential energy, $V(r) = Kr^2/2$ given

For $r = R$, $F = kR$

Also $F = \frac{mv^2}{R}$ (circular motion)

$\therefore \frac{mv^2}{R} = kR \quad \therefore v = \sqrt{\frac{k}{m}} \times R$

Angular momentum $L = mvR = m\left(\sqrt{\frac{k}{m}}R\right)R = \sqrt{km}R^2$

2. **(A, C)** Given $\vec{F} = t\hat{i} + \hat{j} \quad \therefore \frac{md\vec{v}}{dt} = t\,\hat{i} + \hat{j}$

$\therefore d\vec{v} = t\,dt\,\hat{i} + dt\,\hat{j} \quad [\because m = 1]$

$\therefore \int_0^v d\vec{v} = \int_0^t t\,dt\,\hat{i} + \int_0^t dt\,\hat{j}$

$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$

At $t = 1s$, $\vec{v} = \frac{1}{2}\hat{i} + \hat{j} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ms}^{-1}$

Further $\frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$

$\therefore d\vec{r} = \frac{t^2}{2}dt\,\hat{i} + t\,dt\,\hat{j}$

$\therefore \int_0^r d\vec{r} = \int_0^t \frac{t^2}{2}dt\,\hat{i} + \int_0^t t\,dt\,\hat{j}$

$\Rightarrow \vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$

At $t = 1$, $\vec{r} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j} \quad \therefore |\vec{r}| = \sqrt{\frac{1}{36} + \frac{1}{4}} = \sqrt{\frac{10}{36}}$

$\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j})$ (At $t = 1$s)

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{6} & \frac{1}{2} & 0 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}\left(\frac{1}{6} - \frac{1}{2}\right) = -\frac{1}{3}\hat{k}$$

$\therefore |\vec{\tau}| = \frac{1}{3}$ Nm

3. **(A, C)** We know that $h = \frac{2\sigma\cos\theta}{r\rho g_{eff}}$

As 'r' increases, h decreases

[all other parameter remaining constant]

Also $h \propto \sigma$

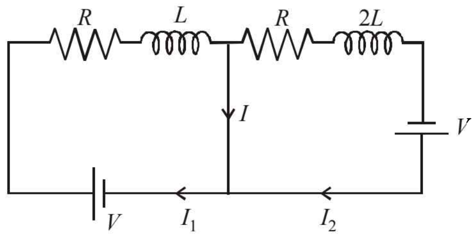
Further if lift is going up with an acceleration 'a' then $g_{eff} = g + a$. As g_{eff} increases, 'h' decreases.

Also h is not proportional to 'θ' but $h \propto \cos\theta$

4. **(B, D)** Here $I + I_2 = I_1 \quad \therefore I = I_1 - I_2$

$\therefore I = \frac{V}{R}\left[1 - e^{\frac{-Rt}{2L}}\right] - \frac{V}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$

$\Rightarrow I = \frac{V}{R}\left[e^{\frac{-Rt}{L}} - e^{\frac{-Rt}{2L}}\right]$



For I to be maximum, $\frac{dI}{dt} = 0$

$\therefore \frac{V}{R}\left[\frac{-R}{L}e^{\frac{-Rt}{L}} - \left(\frac{-R}{2L}\right)e^{\frac{-Rt}{2L}}\right] = 0$

$\therefore e^{\frac{-Rt}{2L}} = \frac{1}{2} \Rightarrow \left(\frac{R}{2L}\right)t = \ln 2$

$\therefore t = \frac{2L}{R}\ln 2$

This is the time when I is maximum

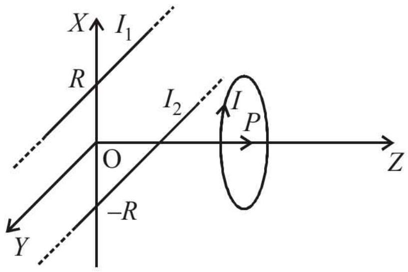
Further $I_{max} = \frac{V}{R}\left[e^{\frac{-R}{L}\left(\frac{2L}{R}\ell n 2\right)} - e^{\frac{-R}{2L}\left(\frac{2L}{R}\ell n 2\right)}\right]$

$\Rightarrow I_{max} = \frac{V}{R}\left[\frac{1}{4} - \frac{1}{2}\right]$

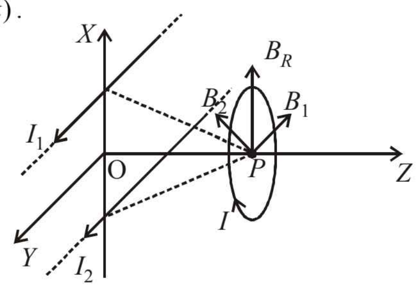
$\therefore I_{max} = \frac{V}{4R}$

5. **(A, B, D)** If $I_1 = I_2$, then the magnetic fields due to I_1 and I_2 at origin 'O' will cancel out each other. But the magnetic field at 'O' due to the circular loop will be present. Therefore 'A' is correct.

If $I_1 > 0$ and $I_2 < 0$, then the magnetic field due to both current will be in $+Z$ direction and add-up. The magnetic field due to current I will be in $-Z$ direction and if its magnitude is equal to the combined magnitudes of I_1 and I_2, then $\vec{B}$ can be zero at the origin. Therefore option 'B' is correct.



If $I_1 < 0$ and $I_2 > 0$ then their net magnetic field at origin will be in $-Z$ direction and the magnetic field due to I at origin will also be in $-Z$ direction. Therefore $\vec{B}$ at origin cannot be zero. Therefore 'C' is incorrect. If $I_1 = I_2$ then the resultant of the magnetic field B_R at P (the centre of the circular loop) is along $+X$ direction. Therefore the magnetic field at P is only due to the current I which is in $-Z$ direction and is equal to $\vec{B} = \frac{\mu_0 I}{2R}(-\hat{k})$.



Therefore option 'D' is correct.

6. **(B, C, D)** In process I, volume is changing. Therefore it is not isochoric. Therefore 'A' is incorrect.

In process II, $q = \Delta U + W$. $\Delta U = 0$ as temperature is constant. Therefore $q = W$. Here $W = P(V_f - V_i)$ is positive therefore q is positive i.e., gas absorbs heat. Therefore 'B' is correct.

For process IV, $q = \Delta U + W$. Here $\Delta U = 0$ and W is negative (volume decreases). Therefore q is negative i.e., gas releases heat. 'C' is correct.

For an isobaric process, $V \propto T$ i.e., we will get a straight inclined line in T-V graph. Therefore I and II are NOT isobaric. 'D' is correct.

7. **(2.00)**

$$|\vec{A} + \vec{B}| = \sqrt{3}\,|\vec{A} - \vec{B}|$$

$$\therefore \quad |a\,\hat{i} + a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}| = \sqrt{3}\,|a\,\hat{i} - a\cos\omega t\,\hat{i} - a\sin\omega t\,\hat{j}|$$

$$\Rightarrow \quad |(1+\cos\omega t)\hat{i} + \sin\omega t\,\hat{j}| = \sqrt{3}\,|(1-\cos\omega t)\hat{i} - \sin\omega t\,\hat{j}|$$

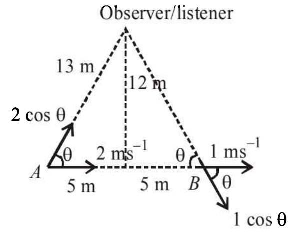
$$\sqrt{2+2\cos\omega t} = \sqrt{3}\,\sqrt{2-2\cos\omega t}$$

$$\therefore \quad 1+\cos\omega t = 3(1-\cos\omega t)$$

$$\Rightarrow \quad 4\cos\omega t = 2 \quad \therefore \cos\omega t = \frac{1}{2} \quad or,\ \omega t = \frac{\pi}{3}$$

$$\therefore \quad \frac{\pi}{6} \times \tau = \frac{\pi}{3} \quad \therefore \tau = 2.00 \text{ seconds}$$

8. **(5.00)**



$$\nu_A = \nu\left[\frac{v}{v - 2\cos\theta}\right]$$

$$\nu_B = \nu\left[\frac{v}{v + 1\cos\theta}\right]$$

$$\therefore \quad \text{Beat frequency} = \nu\left[\frac{v}{v-2\cos\theta}\right] - \nu\left[\frac{v}{v+\cos\theta}\right]$$

$$= \nu\, v\left[\frac{1}{v-2\cos\theta} - \frac{1}{v+\cos\theta}\right]$$

$$= 1430 \times 330\left[\frac{1}{330 - 2\times\frac{5}{13}} - \frac{1}{330 + \frac{5}{13}}\right]$$

$$= 1430 \times 330 \times 13\left[\frac{1}{330\times 13 - 10} - \frac{1}{330\times 13 + 5}\right]$$

$$= 1430 \times 330 \times 13\left[\frac{1}{4280} - \frac{1}{4295}\right] \approx 5 \text{ Hz}$$

9. **(0.75)**

The time taken to reach the ground is given by

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1 + \frac{I_C}{MR^2}\right)}$$

For ring $$t_1 = \frac{1}{\sin 60^\circ}\sqrt{\frac{2h}{g}\left(1+\frac{MR^2}{MR^2}\right)} = \frac{4}{\sqrt{3}}\sqrt{\frac{h}{g}}$$

For disc $$t_2 = \frac{1}{\sin 60^\circ}\sqrt{\frac{2h}{g}\left(1+\frac{\frac{1}{2}MR^2}{MR^2}\right)} = \frac{2}{\sqrt{3}}\sqrt{\frac{3h}{g}}$$

Given $$t_1 - t_2 = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\therefore \quad \frac{4}{\sqrt{3}}\sqrt{\frac{h}{g}} - \frac{2}{\sqrt{3}}\sqrt{\frac{3h}{g}} = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$2\sqrt{\frac{h}{10}} - \sqrt{\frac{3h}{10}} = \frac{(2-\sqrt{3})}{\sqrt{10}}\left(\frac{\sqrt{3}}{2}\right)$$

$$2\sqrt{h} - \sqrt{3h} = \sqrt{3} - \frac{3}{2}$$

$$\sqrt{h}\,(2 - 1.732) = 1.732 - 1.5 \qquad \therefore \sqrt{h} = \frac{0.232}{0.268}$$

$$\therefore h \approx 0.75 \text{ m}$$

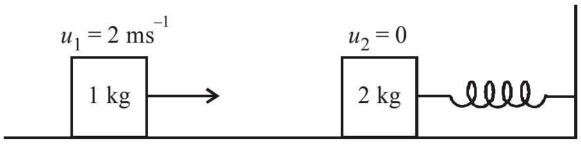
10. **(2.09)**

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2 u_2}{m_1 + m_2} = \frac{(1-2)2}{1+2} = \frac{-2}{3}\text{ ms}^{-1}$$

$$v_2 = \frac{(m_2 - m_1)u_2}{m_1 + m_2} + \frac{2m_1u_1}{m_1 + m_2} = \frac{2 \times 1 \times 2}{1+2} = \frac{4}{3}\,\text{ms}^{-1}$$

The time period of mass 2 kg after attaining velocity is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{2}} = 2\pi$$



Therefore the time taken to return the original position by 2 kg mass is π sec.

∴ Distance between the two blocks

$$= \frac{2}{3} \times \pi = \frac{2}{3} \times \frac{22}{7} = 2.09 \text{ m}$$

11. (1.50)

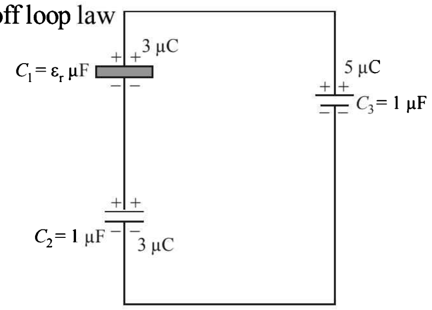
Initially

The charge on C_3 is $q_3 = C_3V = 1 \times 8\ \mu C = 8\ \mu C$

Finally

As the charge on C_3 is found to be 5 μC therefore charges on C_1 and C_2 are 3 μC each

Applying Kirchhoff loop law



$$\frac{5}{1} - \frac{3}{\varepsilon_r} - \frac{3}{1} = 0$$

$$\therefore 5 = 3\left[1 + \frac{1}{\varepsilon_r}\right]$$

$$\therefore \frac{1}{\varepsilon_r} = \frac{5}{3} - 1 = \frac{2}{3} \qquad \therefore \varepsilon_r = 1.5$$

12. (2.00)

$$\text{Average speed along X-axis} = \frac{D_1 + D_2}{t_1 + t_2} = \frac{2(R_1 + R_2)}{t_1 + t_2}$$

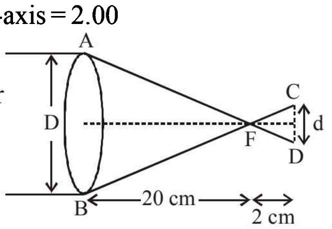
$$= 2\left[\frac{\dfrac{mv_0}{qB_1} + \dfrac{mv_0}{q(4B_1)}}{\dfrac{\pi m}{qB_1} + \dfrac{\pi m}{q(4B_1)}}\right]$$

But $V_0 = \pi$

∴ Average speed along X-axis = 2.00

13. (130.00)

ΔAFB and ΔCFD are similar



$$\frac{d}{D} = \frac{2}{20} = \frac{1}{10}$$

$$\therefore \text{ Ratio of area } = \frac{d^2}{D^2} = \frac{1}{100}$$

As there is no energy loss

∴ Average intensity of light at a distance 22 cm

$$= \frac{1.3 \times \pi D^2/4}{\pi d^2/4} = 1.3 \times 100 = 130.00\,\text{kWm}^{-2}$$

14. (4.00)

The intermediate temperature is given by the formula

$$T = \frac{\dfrac{k_1A_1T_1}{l_1} + \dfrac{k_2A_2T_2}{l_2}}{\dfrac{k_1A_1}{l_1} + \dfrac{k_2A_2}{l_2}}$$

Here, $T = 200$ k, $T_1 = 300$ k, $T_2 = 100$ k

$l_1 = l_2$ and $A_1 = \pi r^2, A_2 = 4\pi r^2$

$$\therefore 200 = \frac{k_1\pi r^2 \times 300 + k_2\pi(4r^2)100}{k_1\pi r^2 + k_2\pi(4r^2)}$$

$$\therefore 200 = \frac{300k_1 + 400k_2}{k_1 + 4k_2}$$

$$\therefore 200\,k_1 + 800\,k_2 = 300\,k_1 + 400\,k_2$$

$$\Rightarrow 400\,k_2 = 100\,k_1 \qquad \therefore \frac{k_1}{k_2} = 4.00$$

15. (C) We know that, $C = \dfrac{E}{B}$ where C = speed of light

∴ $E = CB = LT^{-1}B$

16. (D) We know that

$$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \therefore C^2 = \frac{1}{\mu_0\varepsilon_0}$$

$$\therefore \mu_0 = \varepsilon_0^{-1}L^{-2}T^2$$

17. (B) $r = \dfrac{1-a}{1+a} \quad \therefore \dfrac{dr}{da} = \dfrac{(1+a)(-1)-(1-a)}{(1+a)^2} = \dfrac{-2}{(1+a)^2}$

$$\therefore |\Delta r| = \frac{2\Delta a}{(1+a)^2}$$

18. (C) We know that, $N = N_0e^{-\lambda t}$

Taking *log* on both sides $\log_e N = \log_e N_0 - \lambda t$

differentiating with respect to 'λ' we get

$$\frac{1}{N}\frac{dN}{d\lambda} = 0 - t \qquad \therefore |d\lambda| = \frac{dN}{tN} = \frac{40}{1 \times 2000} = 0.02$$

[∵ N = 3000 – 1000 = 2000]

Paper - 2

1. **(A, B, D)** $\frac{dk}{dt}=\gamma t$ and $k=\frac{1}{2}mV^2 \quad \therefore \frac{d}{dt}\left(\frac{1}{2}mV^2\right)=\gamma t$

$\Rightarrow \frac{m}{2}\times 2V\frac{dV}{dt}=\gamma t \qquad \therefore mV\frac{dV}{dt}=\gamma t$

$\therefore m\int_0^V V dV=\gamma\int_0^t t dt \qquad \Rightarrow \frac{mV^2}{2}=\frac{\gamma t^2}{2}$

$\therefore V=\sqrt{\frac{\gamma}{m}}\times t.$ i.e., $V \propto t$

As V is proportional to 't', distance cannot be proportional to 't'.

Now $F=ma=m\frac{dV}{dt}=m\frac{d}{dt}\left[\sqrt{\frac{\gamma}{m}}\times t\right]=m\sqrt{\frac{\gamma}{m}}=\sqrt{\gamma m}=$ constant

2. **(A, C, D)** We know that $|F|=\eta A\frac{u_0}{h}$

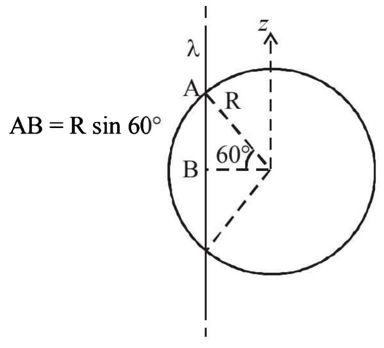
where $\frac{u_0}{h}$ = velocity gradient

Also $\frac{|F|}{A}=\eta\frac{u_0}{h}$

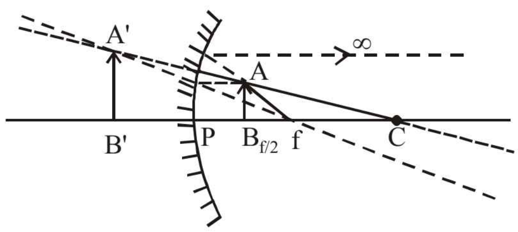
3. **(A, B)** According to Gauss's Law,

Electric flux, $\phi=\frac{1}{\epsilon_0}q_{in}=\frac{1}{\epsilon_0}[\lambda\times 2R\sin 60°]=\frac{\sqrt{3}\lambda R}{\epsilon_0}$

Further electric field is perpendicular to the wire therefore its z-component will be zero.



4. **(D)** The image of AB will be A'B' as AB lies between pole and focus. Further as the object is moved towards the focus the image also moves away.



The object distance decreases from $\frac{f}{2}$ to f. Therefore the final result is (D).

5. **(A, C)** No. of α-particles $=\frac{232-212}{4}=\frac{20}{4}=5$

$\therefore {}^{232}_{90}Th \longrightarrow {}^{212}_{82}Pb + 5\,{}^{4}_{2}He + 2\,{}^{0}_{-1}\beta$

6. **(A, C)** Given $(2n+1)\frac{\lambda}{4}=50.7+e$

and $(2n+3)\frac{\lambda}{4}=83.9+e$

If $n=1$, $\frac{3\lambda/4}{5\lambda/4}=\frac{50.7+e}{83.9+e} \quad \therefore 3\times 83.9+3e=5\times 50.7+5e$

$\therefore \quad 2e=1.8 \qquad \Rightarrow e=0.9$ cm

$\therefore \quad \frac{3\lambda}{4}=50.7+0.9=51.6 \quad \therefore \lambda=66.4$ cm

Also $V=v\lambda=500\times 0.664\ ms^{-1}=332.0\ ms^{-1}$

7. **(6.30)** Impulse = Change in linear momentum

$\therefore \quad J=mV_0$ or $V_0=\frac{J}{m}=\frac{1}{0.4}=2.5\ ms^{-1}$

Also $V=v_0e^{-t/\tau} \quad \therefore \frac{ds}{dt}=v_0e^{-t/\tau} \Rightarrow ds=v_0e^{-t/\tau}dt$

$\therefore \quad s=v_0\int_0^\tau e^{-t/\tau}dt=v_0\tau(1-e^{-1})=2.5\times 4\times 0.63=6.30$ m

8. **(30.00)** $H=\frac{u^2\sin^2\theta}{2g} \quad \Rightarrow 120=\frac{u^2\left(\frac{1}{2}\right)}{2g}$

$\therefore \quad u^2=480$ g

$\therefore \quad K.E_{initial}=\frac{1}{2}mu^2=240$ mg

$$K.E_{final} = \frac{1}{2}(240\,mg) = 120\,mg$$

$$\therefore \quad \frac{1}{2}mv^2 = 120\,mg \qquad \therefore v^2 = 240\,g$$

$$\therefore \quad H' = \frac{v^2 \sin^2\theta}{2g} = \frac{240g \times \left(\frac{1}{4}\right)}{2g} = 30\,m$$

9. **(2.00)** Given $E = \sin 10^3 t\,\hat{i}$

$F = ma$

$$\therefore \quad qE = m\frac{dv}{dt} \qquad \therefore dv = \frac{qEdt}{m} = \frac{q\sin 1000t\,\hat{i}}{m}dt$$

$$\therefore \int_0^v dv = \frac{q}{m}\int_0^{\pi/\omega} \sin 1000t\,dt \quad \left[\text{max. speed is at } \frac{T}{2} = \frac{2\pi}{\omega \times 2}\right]$$

$$\therefore \quad V = -\frac{q}{m}\left[\frac{\cos 1000t}{1000}\right]_0^{\pi/\omega} = -\frac{1}{10^{-3}} \times \frac{[\cos 1000t]_0^{\pi/\omega}}{1000}$$

$$\therefore \quad V = -\left[\cos 1000 \times \frac{\pi}{1000} - \cos 0\right] = -[-1-1] = 2\,ms^{-1}$$

10. **(5.56)** We know that $C\theta = NBAI_g$

$$\therefore \quad I_g = \frac{C\theta}{NBA} = \frac{10^{-4} \times 0.2}{50 \times 2 \times 10^{-4} \times 0.02} = 0.1A$$

Further for a galvanometer

$$I_g \times G = (I - I_g)S$$

$$\therefore \quad S = \frac{I_g G}{I - I_g} = \frac{0.1 \times 50}{1 - 0.1} = \frac{50}{9}\,\Omega$$

11. **(3.00)** We know that $\Delta l = \dfrac{Fl}{AY}$

$$= \frac{1.2 \times 10 \times 1}{\pi\left(\frac{5 \times 10^{-4}}{2}\right)^2 \times 2 \times 10^{11}} \simeq 0.3\,mm$$

The third marking of vernier scale will coincide with the main scale because least count is 0.1 mm.

12. **(900.00)** For an adiabatic process $TV^{\gamma-1} = T_2(8V)^{\gamma-1}$

where $\gamma = \dfrac{5}{3} \quad \therefore T_2 = \dfrac{T}{4}$

Further $\Delta V = nC_v\Delta T = n\left(\frac{f}{2}R\right)\Delta T = \frac{nfR}{2}\left(\frac{-3T}{4}\right)$

$$\therefore \quad \Delta V = -\frac{1 \times 3 \times 8}{2} \times \frac{3}{4} \times 100 = -900J$$

13. **(24.00)** Number of electrons emitted per second

$$= \frac{200\,W}{6.25 \times 1.6 \times 10^{-19}J}$$

Force = Rate of change of linear momentum = $N\sqrt{2mk}$

$$= \frac{200}{6.25 \times 1.6 \times 10^{-19}} \times \sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500}$$

$[\because K = eV: e = 1.6 \times 10^{-19} = V = 500]$

$= 24.00$

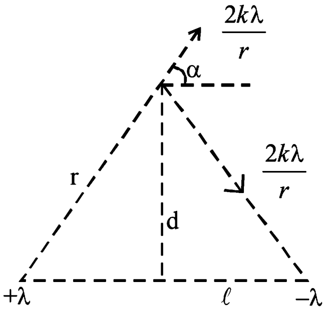
14. **(3.00)** $\Delta E_{2-1} = 74.8 + \Delta E_{3-2}$

$$13.6z^2\left[1 - \frac{1}{4}\right] = 74.8 + 13.6z^2\left[\frac{1}{4} - \frac{1}{9}\right]$$

$\therefore \quad z = 3$

15. **(B)** For a point charge $E = \dfrac{kQ}{d^2}$ and for a dipole $E = \dfrac{kp}{d^3}$

Further for an infinite long line charge $E = \dfrac{2k\lambda}{d}$ and for infinite plane charge $E = \dfrac{\sigma}{2\epsilon_0}$



Also for two infinite wires carrying uniform linear charge density.

$$E = \frac{2k\lambda}{r}\cos\alpha = \frac{2k\lambda}{\sqrt{d^2 + \ell^2}} \times \frac{\ell}{\sqrt{d^2 + \ell^2}} = \frac{2k\lambda\ell}{d^2 + \ell^2}$$

16. **(B)** $V_o = \sqrt{\dfrac{GM}{R}}, \quad \therefore \dfrac{V_1}{V_2} = \sqrt{\dfrac{R_2}{R_1}} = \dfrac{2}{1}$

Further $\dfrac{L_1}{L_2} = \dfrac{m_1 v_1 R_1}{m_1 v_2 R_2} = \dfrac{2 \times 2 \times 1}{1 \times 1 \times 4} = \dfrac{1}{1}$

Also K.E. $= \dfrac{GMm}{R}$. Therefore $\dfrac{k_1}{k_2} = \dfrac{m_1}{m_2} \times \dfrac{R_1}{R_2} = \dfrac{2 \times 4}{1 \times 1} = \dfrac{8}{1}$

Further $T^2 \propto R^3 \qquad \therefore \dfrac{T_1}{T_2} = \left(\dfrac{R_1}{R_2}\right)^{3/2} = \dfrac{1}{8}$

17. **(C)** Process 1 is adiabatic therefore $\Delta Q = 0$

Process 2 is isobaric therefore $W = P(V_2 - V_1) = 3P_0(3V_0 - V_0) = 6P_0V_0$

Process 3 is isochoric therefore $W = P(V_2 - V_1) = 0$

Process 4 is isothermal therefore temperature is constant, $\Delta u = 0$

18. **(A)** P $\rightarrow \vec{v} = \frac{d\vec{r}}{dt} = \alpha\hat{i} + \beta\hat{j}$ which is constant

$\therefore \quad \vec{a} = 0$

Further $\vec{P} = m\vec{v}$ is constant

and K = $\frac{1}{2}mv^2$ is constant

$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right) = 0 \qquad (\because \vec{a}$ is constant)

$\Rightarrow$ U = constant

Also E = K + U

$\therefore \quad \frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r}\times\vec{F} = 0 \qquad \therefore \vec{L}$ = constant

Q $\rightarrow \vec{v} = \frac{d\vec{r}}{dt} = -\alpha\omega(\sin\ \omega t)\hat{i} + \beta\omega(\cos\ \omega t)\hat{j}$

$\therefore \quad \vec{a} = \frac{d\vec{v}}{dt} = -\omega^2[\alpha\cos\omega t\ \hat{i} + \beta\sin\ \omega t\ \hat{j}] = -\omega^2\vec{r}$

Also $\vec{\tau} = \vec{r}\times\vec{F} = 0 \qquad (\therefore \vec{r}$ and $\vec{F}$ are parallel)

$\Delta U = -\int \vec{F}.\vec{dr} = +\int_0^r m\omega^2 r dr = \frac{m\omega^2 r^2}{2} \qquad \therefore U \propto r^2$

Also r $= \sqrt{\alpha^2\cos^2\omega t + \beta^2\sin^2\omega t} \qquad \therefore r = f(t)$

As the force is central therefore total energy remains constant.

R $\rightarrow \vec{v} = \frac{d\vec{r}}{dt} = \alpha[-\omega\sin\omega t\,\hat{i} + \omega\cos\omega t\,\hat{j}]$

$\therefore \quad$ v = $\alpha\omega$ i.e., speed is constant

$\vec{a} = \frac{d\vec{v}}{dt} = -\alpha\omega^2[\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}]$

$\therefore \quad \vec{a} = -\omega^2\vec{r}$

$\vec{\tau} = \vec{r}\times\vec{F} = 0$

Force is central in nature

U and K are also constant.

S $\rightarrow \vec{v} = \frac{d\vec{r}}{dt} = \alpha t\,\hat{i} + \beta t\,\hat{j} \qquad \therefore$ V = f(t)

$\vec{a} = \beta\hat{j}$ i.e., constant

$\vec{F} = m\vec{a}$ constant

$\Delta U = -\int \vec{F}\cdot d\vec{r} = -m\int_0^t \beta\hat{j}\cdot(\alpha\hat{i} + \beta t\hat{j})dt = \frac{-m\beta^2 t^2}{2}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\alpha^2 + \beta^2 t^2)$

Also E = K + U = $\frac{1}{2}m\alpha^2$ which is constant.