

Combinational circuit

Date _____
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- Logic circuits can be divided into two types -



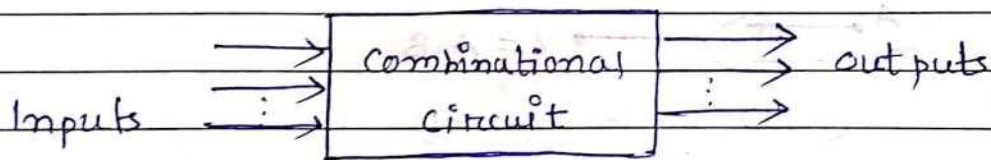
Combinational logic circuit.

→ Sequential logic circuit.

- Combinational Logic Circuit:

A combinational logic circuit consists of logic gates whose output is determined by the combination of current inputs.

- It consists of input variables, logic gate and output variable.
- No feedback is required.
- No memory is required.



Ex - Multiplexers, Encoders, Decoders, parallel adders, etc.

Combinational circuit• Analysis procedure —

→ The Analysis of combinational circuit requires that we determine the function that the circuit implements.

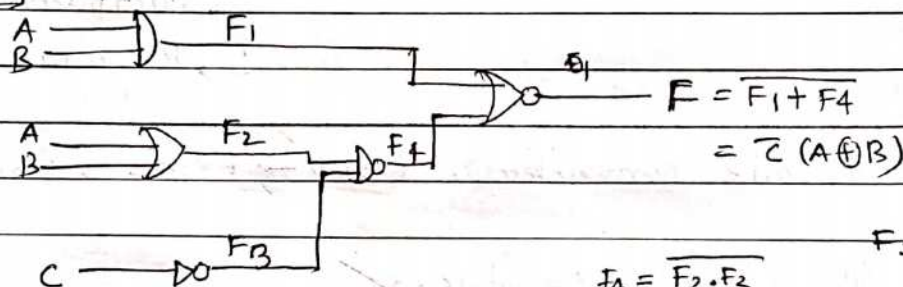
Logic diagram
↓

Boolean functions
↓

truth table.
↓

circuit operation.

Ex-1]



A	B	C	$\frac{A \cdot B}{F_1}$	$\frac{A+B}{F_2}$	$\frac{\bar{C}}{F_3}$	F_4	F
0	0	0	0	0	1	1	0
0	0	1	0	1	0	1	0
0	1	0	0	1	1	0	1 ✓
0	1	1	0	1	0	1	0
1	0	0	0	1	1	0	1 ✓
1	0	1	0	1	0	1	0
1	1	0	0	1	1	0	1 ✓
1	1	1	1	1	0	1	0

$$F = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}B\bar{C}$$

$$= \bar{C}(\bar{A}\bar{B} + \bar{A}B) + \bar{A}B\bar{C}$$

$$F = \bar{C}[(A \oplus B) + AB] = \bar{C}(A+B) \text{ (Final Analysis procedure)}$$

• Design procedure:



• Combinational circuits —

(I) Arithmetic circuits —

- Adders ✓
- Subtractors ✓
- Multiplier ✓
- Magnitude comparator ✓

(II) Code converters, (BCD → Excess-3 code)

(III) Encoders & Decoders ✓

(IV) Multiplexers & Demultiplexers.

(V) programmable Logic Devices (PLDs)

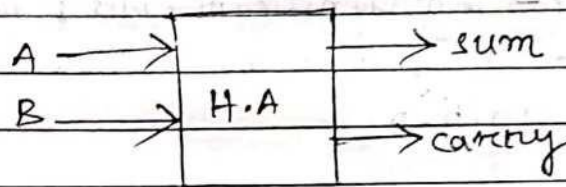
- PLA (programmable array logic)
- PAL (programmable logic array)

$$\begin{array}{r} 0 \\ 0 \\ 0 \end{array} \quad \begin{array}{r} 0 \\ 1 \\ 1 \end{array} \quad \begin{array}{r} 1 \\ 0 \\ 1 \end{array} \quad \begin{array}{r} 1 \\ 1 \\ 2 \\ 10 \end{array}$$

• ADDERS :

- Half-adder.
- Full-adder.
- Binary parallel Adder
(Ripple Carry adder)
- Carry Look-Ahead Adder.

• Half adder :



→ Half adder is a combinational circuit that performs arithmetic sum of Two bits.

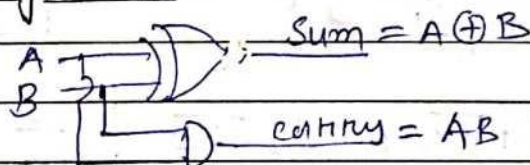
Truth table :

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$\text{Sum} = A \oplus B = \bar{A}B + A\bar{B}$$

$$\text{Carry} = A \cdot B$$

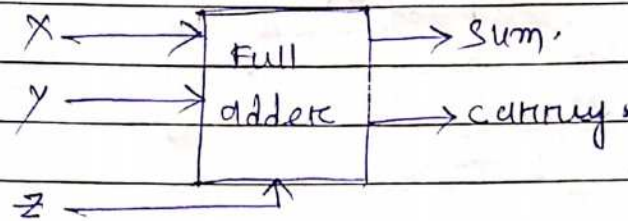
Logic diagram :



* → required minimum no. of NAND and NOR gate to implement H.A = 5.

• Full Adder:

→ A full adder is a combinational circuit that performs the arithmetic sum of three bits



here, x, y = Two significant bits to be added.
and z = Carry from the previous lower significant position.

Truth Table:

X	Y	Z	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

S

h \ yz	00	01	11	10
0	0	1	0	1
1	1	0	1	0

C

h \ yz	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$S = h\bar{y}\bar{z} + h\bar{y}z + h y\bar{z} + h yz$$

$$C = h\bar{y}z + h yz + h y$$

$$= h(\bar{y}\bar{z} + \bar{y}z + y\bar{z} + yz) + z(h\bar{y} + hy)$$

$$= \bar{z}(h(\bar{y}\bar{z} + \bar{y}z + y\bar{z} + yz)) + z(h(\bar{y} + y))$$

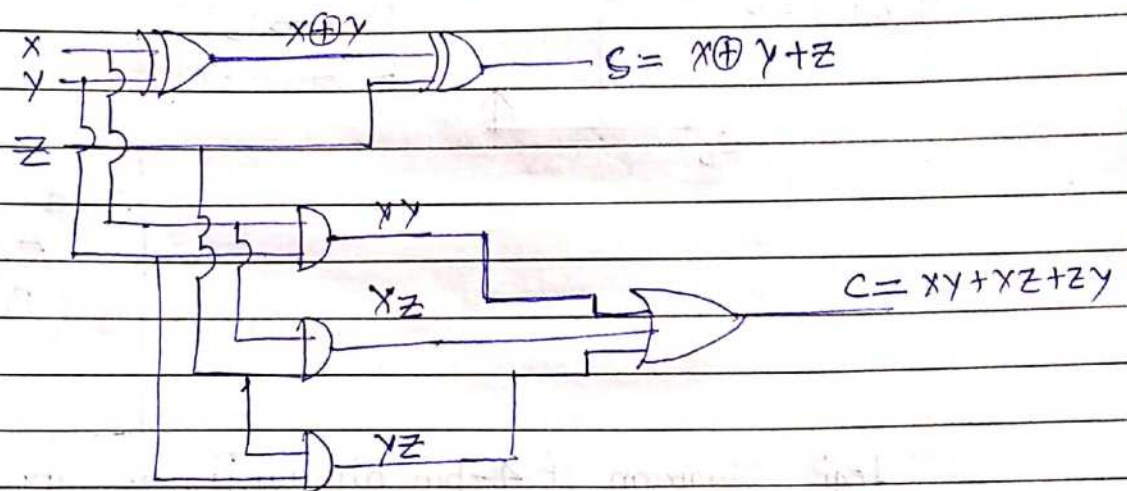
$$\Rightarrow \overline{z} \overline{(x \oplus y)} + \overline{z} (x \oplus y)$$

$$\Rightarrow \overline{z} (\overline{x \oplus y}) + \overline{z} (x \oplus y)$$

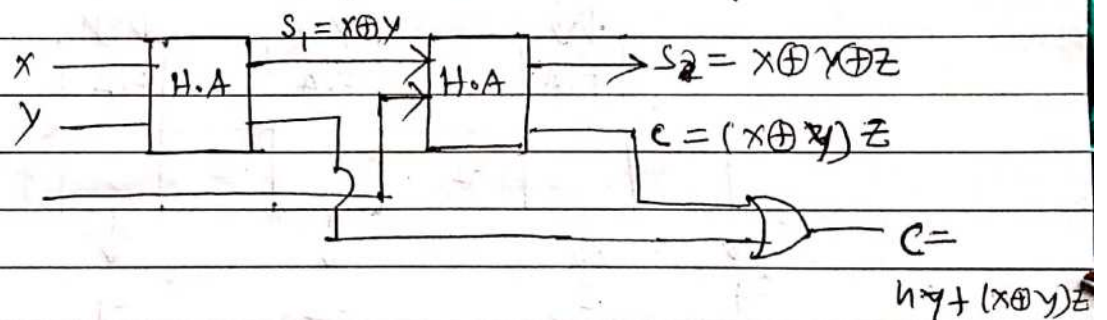
$$\Rightarrow (x \oplus y) \oplus z$$

$$S \Rightarrow x \oplus y \oplus z$$

Logic Diagram:



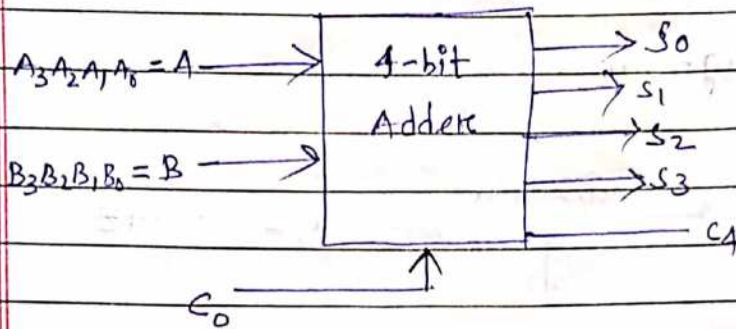
→ A full adder = 2 half adders + 1 OR gate.



→ * Required minimum number of NAND and NOR gate to implement F.A = 9.

• Binary adder:

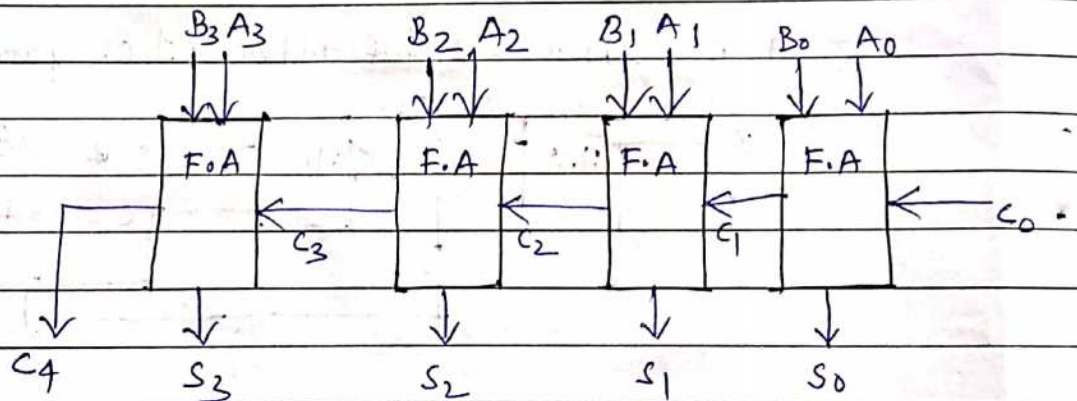
→ "Binary adder is a circuit that produces the arithmetic sum of two binary numbers."



EX =

$$\begin{array}{r}
 \begin{array}{c} \text{C}_3 \text{ C}_2 \text{ C}_1 \\ 0 \ 0 \ 1 \\ A = 1001 \\ B = 0101 \\ \hline 1110 \\ \text{C}_4 \end{array} \\
 \begin{array}{c} S_3 \ S_2 \ S_1 \ S_0 \\ 3 \end{array}
 \end{array}$$

Logic Diagram of 4-bit binary ripple carry adder:



→ with 'n'-bit ^{adder required} need 'n' no. of F.A.

(Ripple carry adder also called Binary parallel adder)

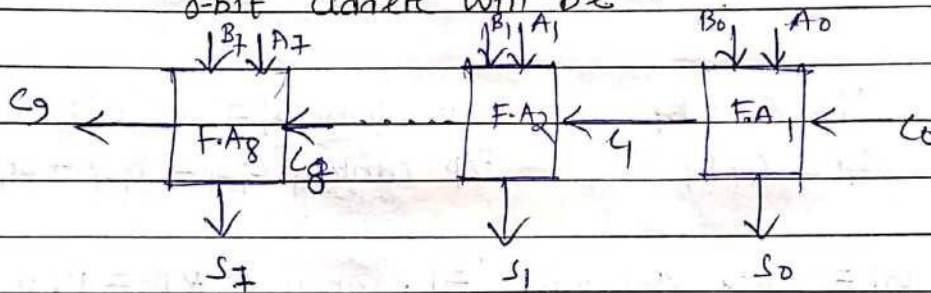
Draw back:

→ The longest propagation delay.

**→ In this adder, The longest propagation delay time in an adder is the time it takes the carry to propagate through the full adders.

Example: ①

A - 8 bit ripple carry adder. The carry propagation delay of each F.A is 10 ns & the sum propagation delay of each F.A is 15 ns. The worst case delay (in ns) of this 8-bit adder will be -



→ $8 \times 10 \text{ ns}$

→ 80 ns. (Ans)

② If there is no C_0 then,

→ $7 \times 10 \text{ ns} + (\text{time taken to produce } C_1)$

→ $70 \text{ ns} + 15 \text{ ns}$

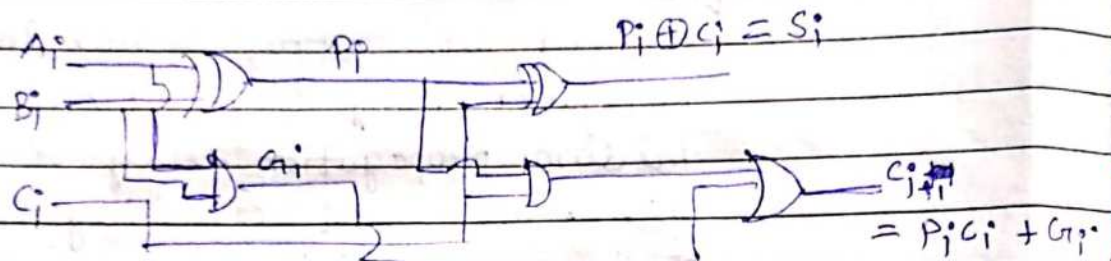
→ 85 ns.

• CARRY LOOKAHEAD ADDER: (CLA)

→ It reduces carry propagation delay time compared to ripple carry adder.

→ i.e. it is a high speed adder.

→ But hardware complexity increases.



$$P_i = A_i \oplus B_i$$

$$G_i = A_i B_i$$

$$\text{o/p sum } S_i = P_i \oplus C_i$$

$$\text{o/p carry } C_{i+1} = P_i C_i + G_i$$

G_i = carry generate (=1, when A_i & $B_i = 1$, regardless of i/p carry C_i)

P_i = Carry propagate, it determines whether a carry into stage 'i' will propagate into stage 'i+1'.

Boolean function for the carry outputs of each stage.

$$C_0 = \text{i/p carry} \quad [C_{i+1} = P_i C_i + G_i]$$

$$C_1 = G_0 + P_0 C_0 \quad [G_0 = A_0 B_0, P_0 = A_0 \oplus B_0]$$

$$C_2 = G_1 + P_1 C_1 \Rightarrow G_1 + P_1 (G_0 + P_0 C_0) \Rightarrow G_1 + P_1 G_0 + P_1 P_0 C_0$$

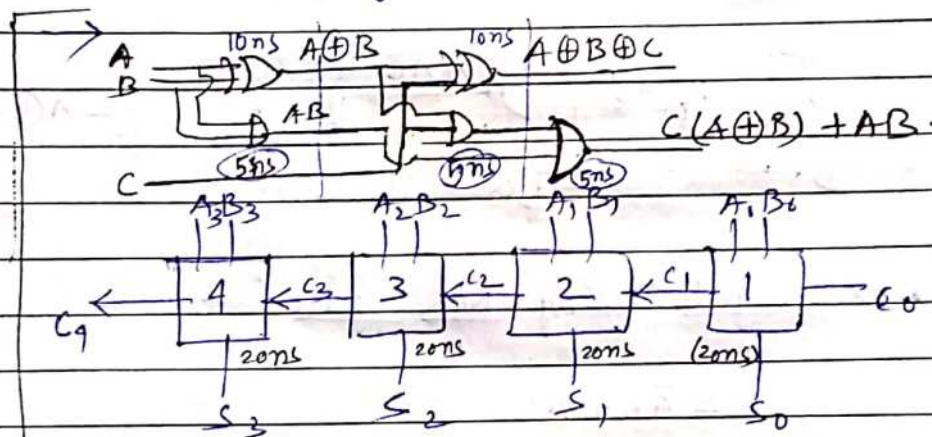
$$C_3 = G_2 + P_2 C_2 \Rightarrow G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_0 P_1 C_0$$

$$C_4 = G_3 + P_3 C_3 \Rightarrow G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_0 P_1 C_0$$

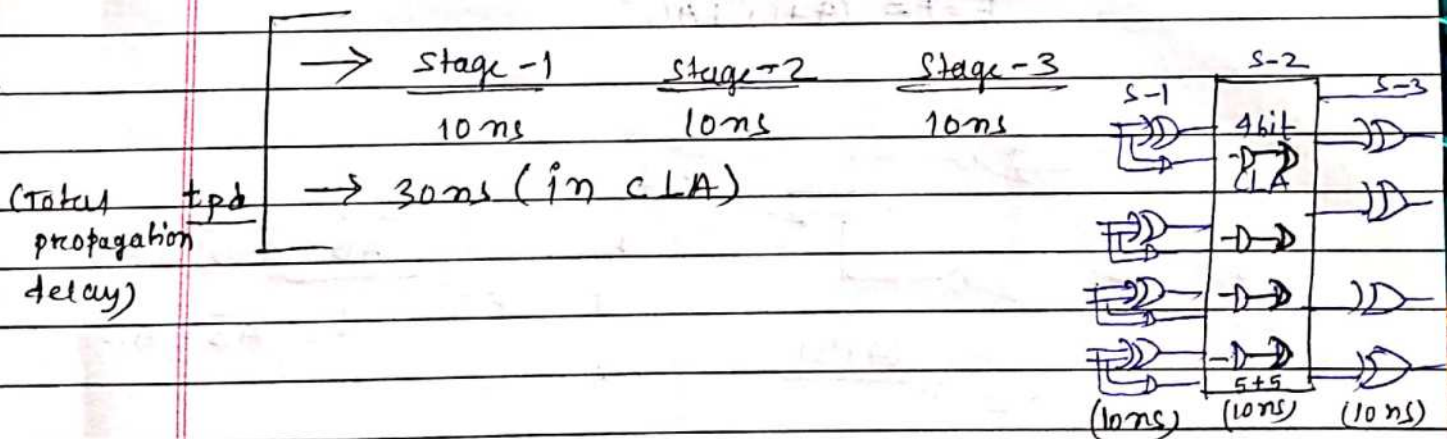
Comparison of Hardware: 4 bit

	4 bit RCA	4 bit CLA
ex-OR \rightarrow	8	8
AND \rightarrow	8	14
OR \rightarrow	4	4
	20 gate req	26 gate req.

Example: Assume that the ex-OR gate has a propagation delay of 10 ns & that the AND (or) OR gates have a prop delay of 5 ns what is the total propagation delay time in the 4-bit RCA & CLA.



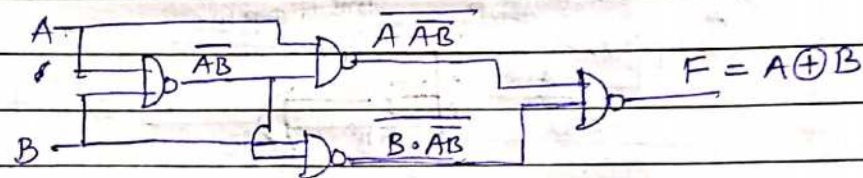
$t_{pd} \rightarrow 20\text{ ns} \times 4 \rightarrow 80\text{ ns}$ (in RCA)



• ADDERS using universal gates:

• EX-OR gate using NAND gates

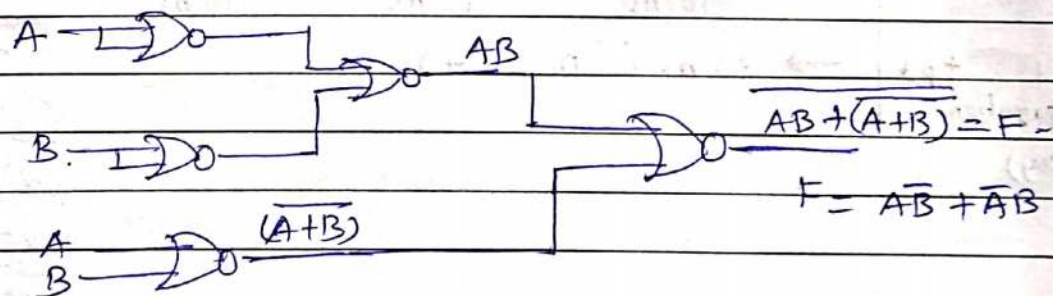
$$\begin{aligned}
 F &= \bar{A}B + A\bar{B} \\
 &= (A+B)(\bar{A}+\bar{B}) \\
 &= (A+B) \cdot \overline{AB} \\
 &= A\bar{A}\bar{B} + B\bar{A}\bar{B} \\
 \bar{F} &= \overline{A\bar{A}\bar{B} + B\bar{A}\bar{B}} \\
 &= \overline{A\bar{A}\bar{B}} \cdot \overline{B\bar{A}\bar{B}} \\
 F = \bar{\bar{F}} &= \overline{A\bar{A}\bar{B} \cdot B\bar{A}\bar{B}}
 \end{aligned}$$



• EX-OR gate using NOR gate

$$\begin{aligned}
 F &= \bar{A}B + A\bar{B} \\
 &= (A+B)(\bar{A}+\bar{B}) \\
 &= (A+B) \cdot \overline{AB} \\
 \bar{F} &= \overline{(A+B) + AB} \\
 \bar{\bar{F}} = F &= \overline{(A+B) + AB}
 \end{aligned}$$

$$\begin{aligned}
 \bar{\bar{F}} &= \overline{\overline{AB}} \\
 &= \overline{\bar{A} + \bar{B}}
 \end{aligned}$$

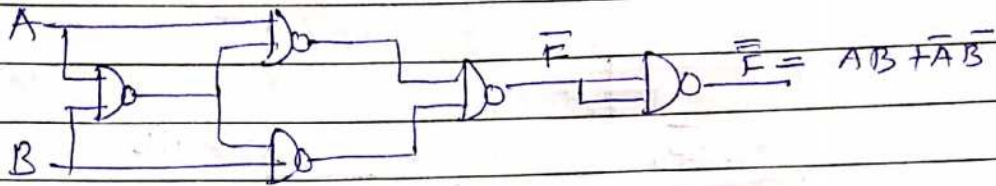


• EX-NOR using NAND gates =

$$F = AB + \bar{A}\bar{B}$$

$$F = \overline{A \oplus B}$$

$$F = \overline{A\bar{B} + \bar{A}B}$$



• EX-NOR using NOR gates =

$$F = AB + \bar{A}\bar{B}$$

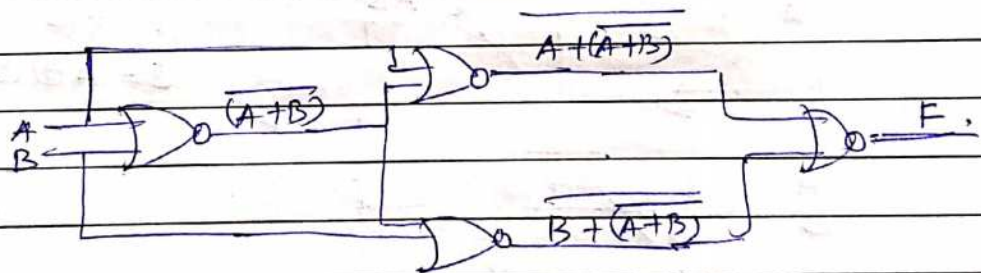
$$= (A+B)(\bar{A}+\bar{B})$$

$$= (A+\bar{A}\bar{B})(B+\bar{A}\bar{B})$$

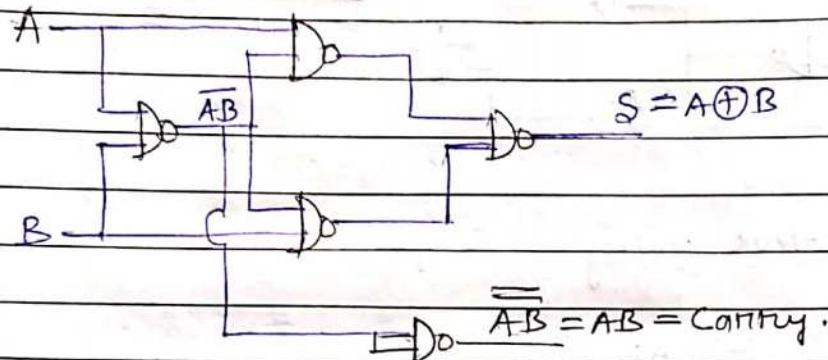
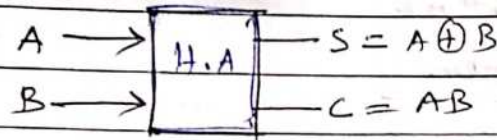
$$= (A+(\bar{A}+\bar{B}))(\bar{B}+(\bar{A}+\bar{B}))$$

$$\bar{F} = \overline{A+(\bar{A}+\bar{B})} + \overline{\bar{B}+(\bar{A}+\bar{B})}$$

$$\bar{F} = F = \overline{A+(\bar{A}+\bar{B})} + \overline{\bar{B}+(\bar{A}+\bar{B})}$$



• Half adder using NAND gates =



✓ (5 - NAND gate required to implement H.A.)

• Half adder using NOR gate =

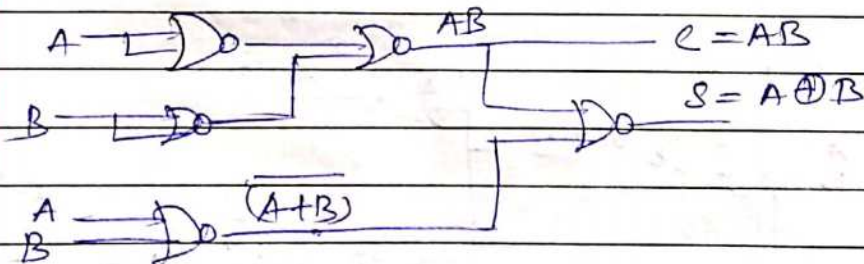
$$S = A\bar{B} + \bar{A}B$$

$$C = AB$$

$$S = \overline{A\bar{B}} \cdot \overline{\bar{A}B}$$

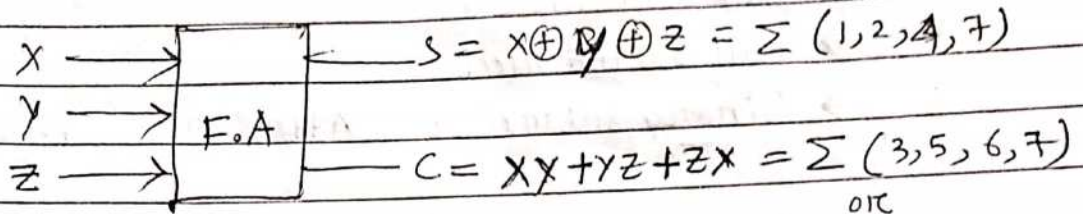
$$S = \overline{(A+B)} \cdot \overline{(A+B)}$$

$$= \overline{(A+B)} + \overline{(A+B)}$$



✓ (5 - ~~NOR~~ ^{NOR} gates required to implement H.A.)

• Implementation of Full adder using NAND gates =



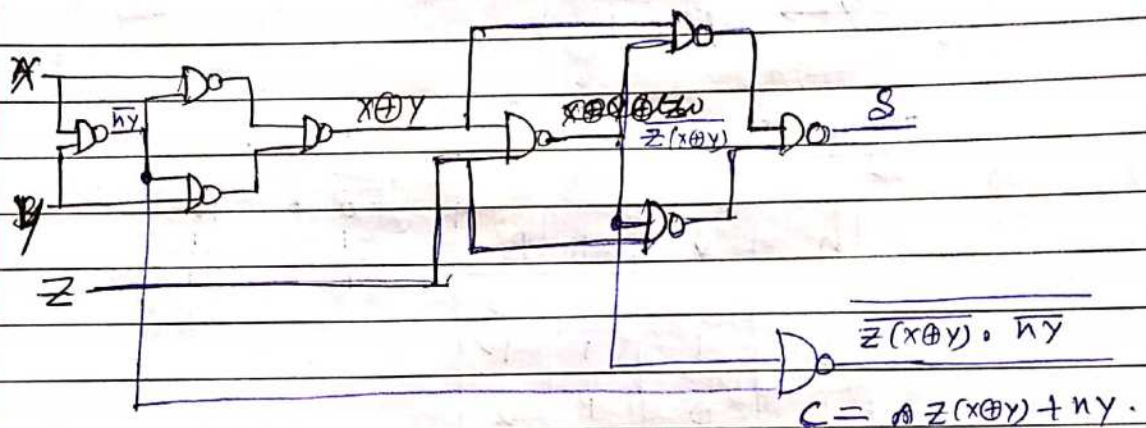
$$\text{OR}$$

$$= \overline{n}yz + n\overline{y}z + n\overline{y}\overline{z} + n\overline{y}z$$

$$= n\overline{y}(z + \overline{z}) + z(x \oplus y)$$

$$C = n\overline{y} + z(x \oplus y)$$

$$S = X \oplus Y \oplus Z \quad C = n\overline{y} + z(x \oplus y)$$



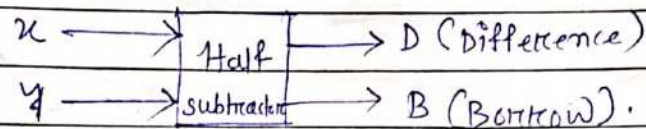
✓ (9-NAND gate required to implement one full adder)

✓ (5-NOR gate required to implement one full adder) → (every thing is same just change NAND by NOR)

Subtractors:

- Half Subtractor.
- Full Subtractor.
- Binary Subtractor, Adders cum subtractor.

• Half Subtractor: It performs subtraction of two bits.



$X \rightarrow$ minueend

$Y \rightarrow$ subtrahend

Truth Table:

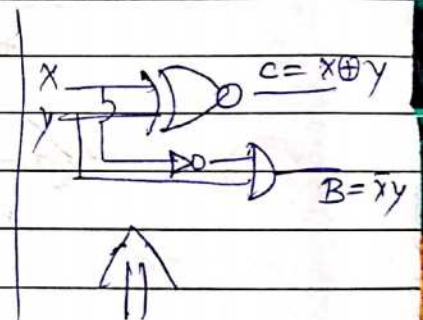
x	y	D	B
0	0	0	0
0	1	1	1 ✓
1	0	1	0
1	1	0	0

$$\begin{array}{r} 0 \ 2 \ 10^{-2} \\ -1 \ -0 \\ \hline 1 \ 0 \ 1 \end{array}$$

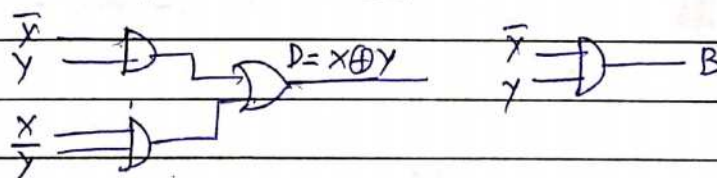
$$D = x \oplus y$$

$$B = \bar{x}y$$

Logic diagram



(a) Sum-of-products form =

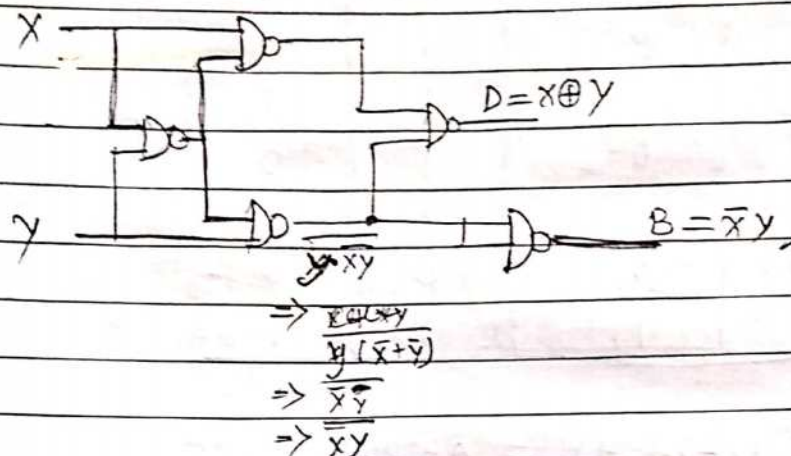


(H.S)

• using only NAND gates - (5-NAND gate required)

$$D = x \oplus y$$

$$B = \bar{x}y$$



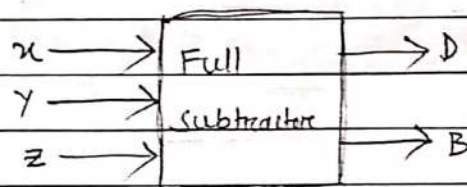
→ ~~5 NAND and NOR gate minimum req~~

minimum 5-NAND and ~~NOR~~ gate required to implement one Half adder.

→ also 5-NOR gate required to implement one H.A.

• Full Subtractor =

→ It performs the subtraction of '3' bits.



$x, y \rightarrow$ two significant bits to be subtract

$z \rightarrow$ Borrow from next stage.

Truth table of full Subtractor =

	x	y	z	D	B
0	0	0	0	0	0
1	0	0	1	1	1
2	0	1	0	1	1
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	0
6	1	1	0	0	0
7	1	1	1	1	1

$$\begin{array}{r} 10-2 \\ 01 \\ \hline 11 \end{array}$$

$$D = \Sigma(1, 2, 4, 7) = A \oplus B \oplus C$$

$$B = \Sigma(1, 2, 3, 7) = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$$

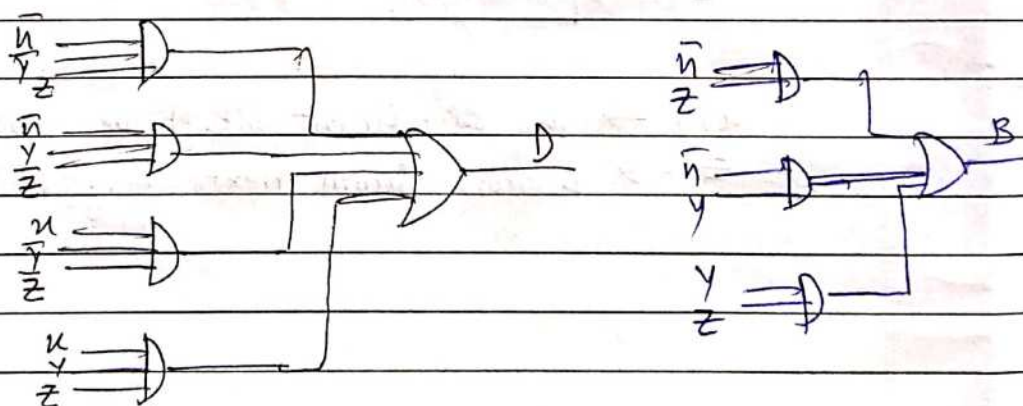
B	x \ yz	00	01	11	10
0	0 ₀	0	1 ₁	1 ₂	1 ₃
1	0 ₄	0	0 ₅	1 ₆	0 ₇

$$B = \bar{x}z + \bar{x}y + yz$$

$$= \bar{x}y + (x \oplus y) \cdot z$$

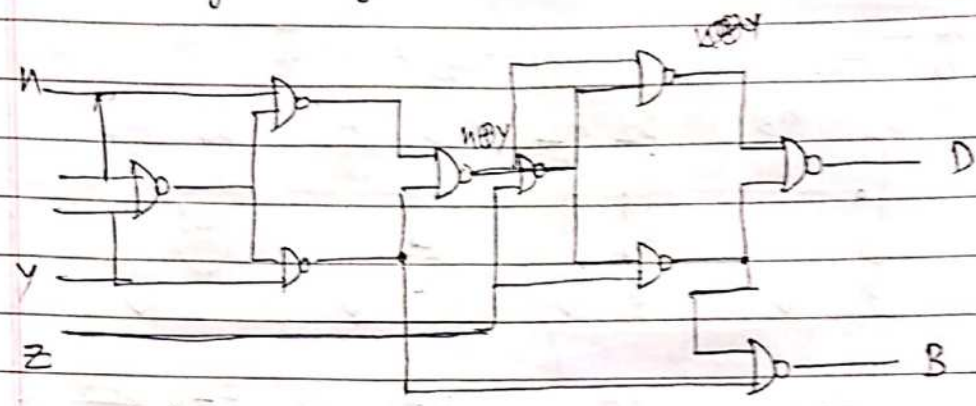
Logic diagram =

(a) Implementation in SOP form =



→ 9 NAND and NOR gate require to implement full subtractor.
 → 2 Half sub + 1 OR gate = full sub.

⑥ using only NAND gate =



(2-NAND gate required) .

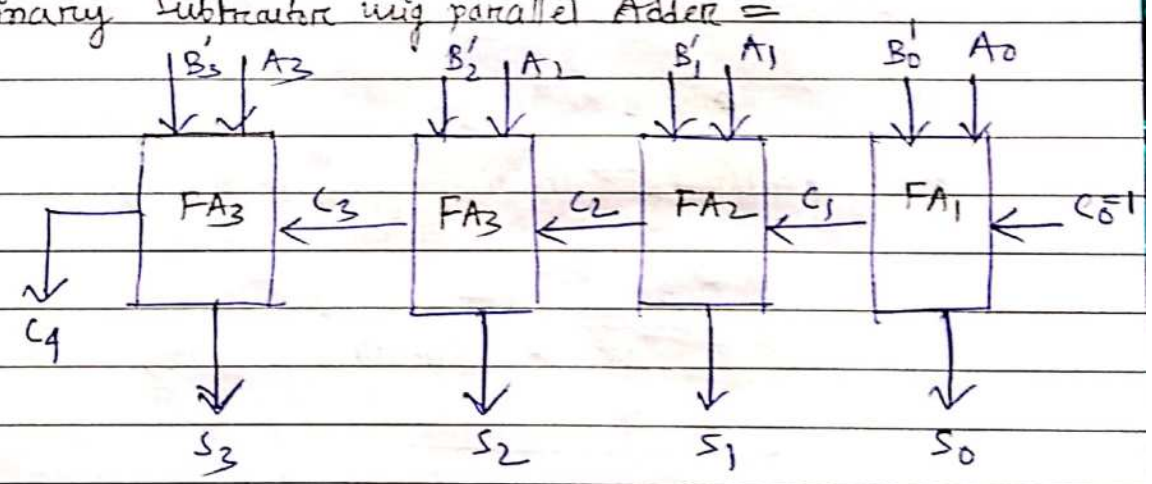
• Binary Subtractor =

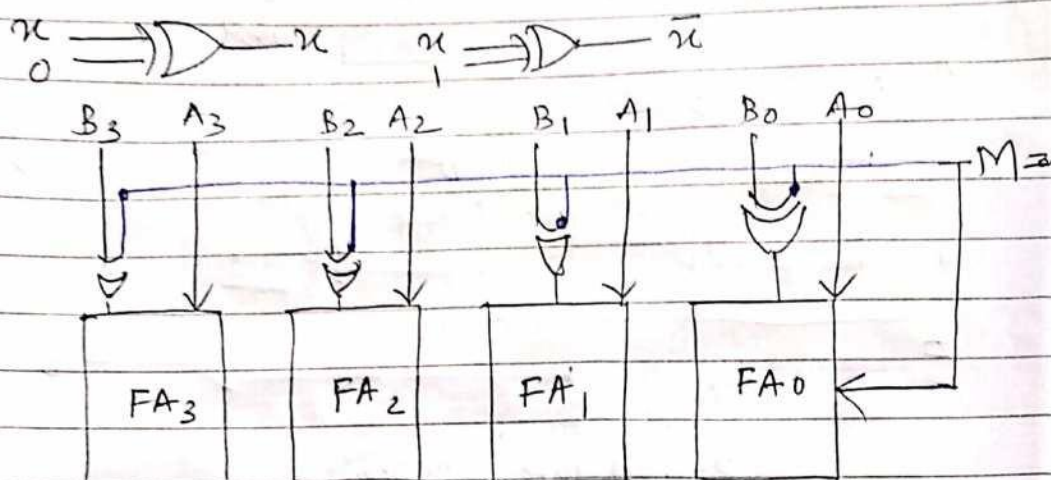
$A + [2's \text{ complement of } B]$

$A_3 \quad A_2 \quad A_1 \quad A_0$

$B'_3 \quad B'_2 \quad B'_1 \quad B'_0 \rightarrow (1's \text{ complement of } B)$
 $+ 1$

Binary Subtractor using parallel Adder =



4-bit Adder - Subtractor =

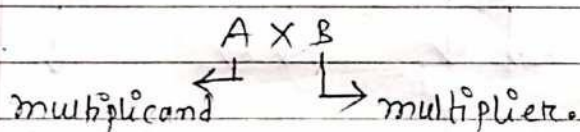
M → Mode i/p controls the operation.

M = 0 → adder.

M = 1 → subtractor.

- Binary Multiplier =

→ It is a combinational circuit that performs multiplication of two binary numbers.



$$\begin{array}{r}
 23 \times 12 \\
 \hline
 46 \text{ (partial product)} \\
 23 \\
 \hline
 276 \text{ (final product)}
 \end{array}$$

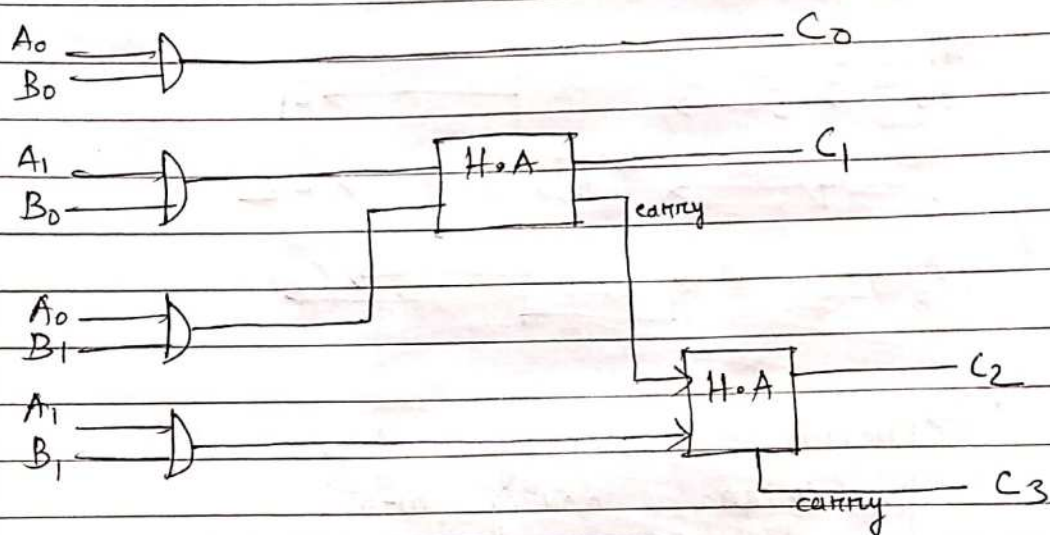
2-bit by 2-bit Binary multiplier =

$$A = A_1 A_0$$

$$B = B_1 B_0$$

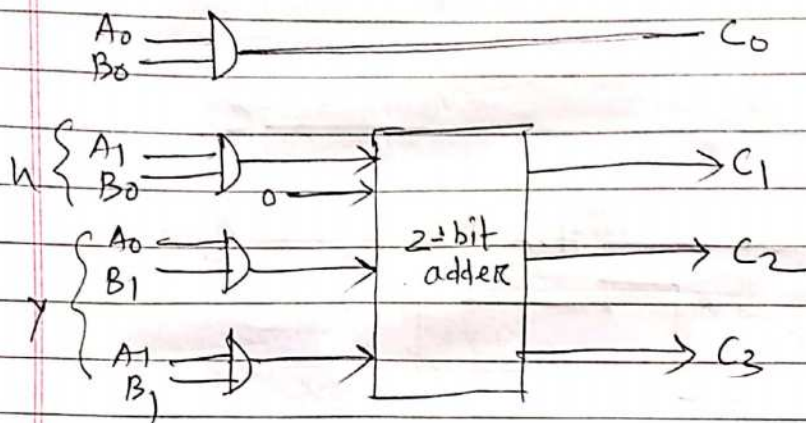
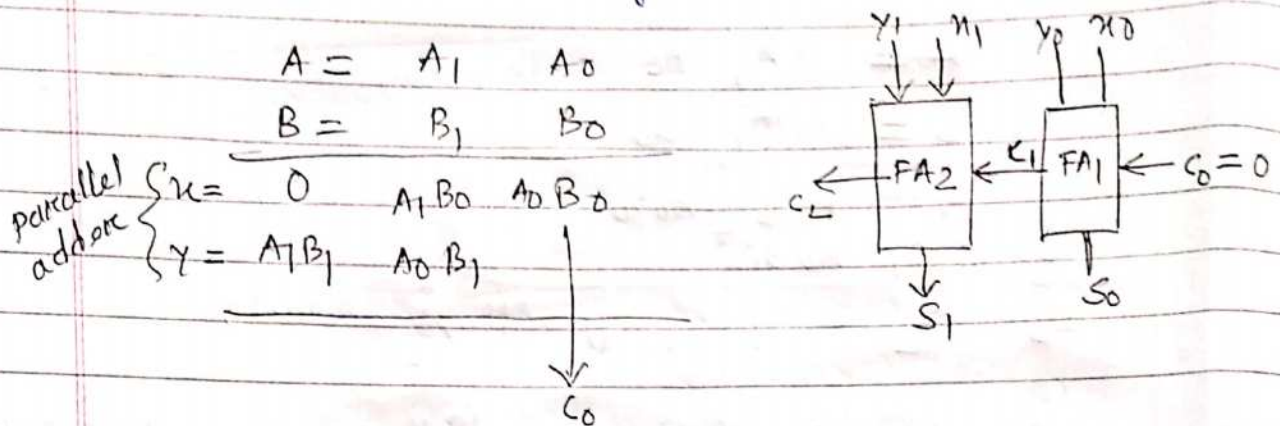
$$\begin{array}{r} \text{no. (carry)} \quad A_1 B_0 \quad A_0 B_0 \quad (\text{partial product}) \\ \text{C}_3 \quad A_1 B_1 \quad A_0 B_1 \\ \hline \text{C}_3 \quad \text{C}_2 \quad \text{C}_1 \quad \text{C}_0 \quad (\text{final product}) \end{array}$$

Implementation of Binary multiplier using Combinational circuit =



- ** \rightarrow If multiplier having n bits
 multiplicand having m bits
 then it produces a product of $(n+m)$ bits.
 $2 + 2 = 4 \text{ bit}$
- ** \rightarrow AND gate required $= (n \times m)$
 $= 2 \times 2 = 4 \text{ NAND gate.}$

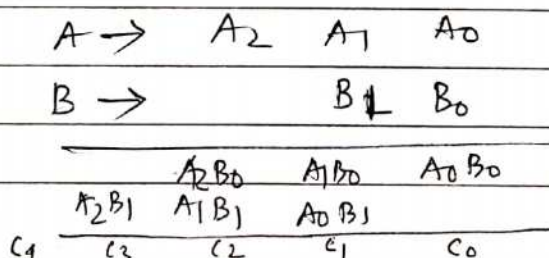
• Binary multiplier using Parallel Adder



** → multiplier is having n -bits
 multiplicand is having m -bits
 Then, $(m-1)$ ~~at~~ m -bit adders.
 $(2-1)$ 2-bit
 = 2-bit adders required

(3) how many
 binary parallel
 adder required
 $(n-1)$ m -bit
 = $(2-1)2 = 2$ bit

→ 3-bit by 2-bit Binary multiplier



(1) Final product contains 5 bits.

→ $(3+2) = 5$

(2) AND gate required 6

→ $3 \times 2 = 6$

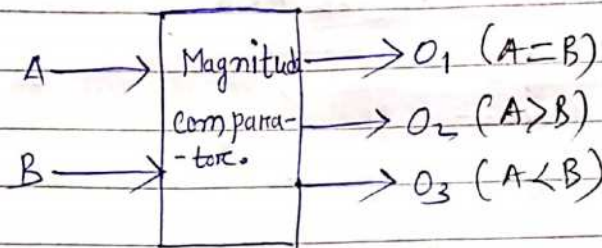
not important for gate.

classmate

Date _____
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Magnitude Comparator

"A magnitude Comparator is a Combinational circuit that compare two binary numbers A & B."



$$\begin{matrix} A = 1100 \\ B = 1011 \end{matrix}$$

2-bit magnitude comparator

$$A = A_1 A_0$$

$$B = B_1 B_0$$

$A=B$: all pairs of significant digits are equal.
 $A_1=B_1 \quad A_0=B_0$

Ex-NOR \rightarrow (Both i/p's same)

$$x_i = A_i B_i + \bar{A}_i \bar{B}_i$$

$x_i = 1$, only if the pair of digits are equal in i th position.

$$O_1 (A=B) = x_1 \cdot x_0 \quad \text{--- (1)}$$

$$= 1 \cdot 1 = 1 \quad (\text{yes both are equal})$$

$$\begin{matrix} A_1 & A_0 \\ A = & 1 & 1 \\ B = & 1 & 1 \end{matrix}$$

$A>B$

$$O_2 (A>B) = A_1 \bar{B}_1 + x_1 A_0 \bar{B}_0$$

$$1 \cdot 1 + ()$$

$$= 1 + ()$$

$$= 1 \quad (\text{satisfy that } A>B)$$

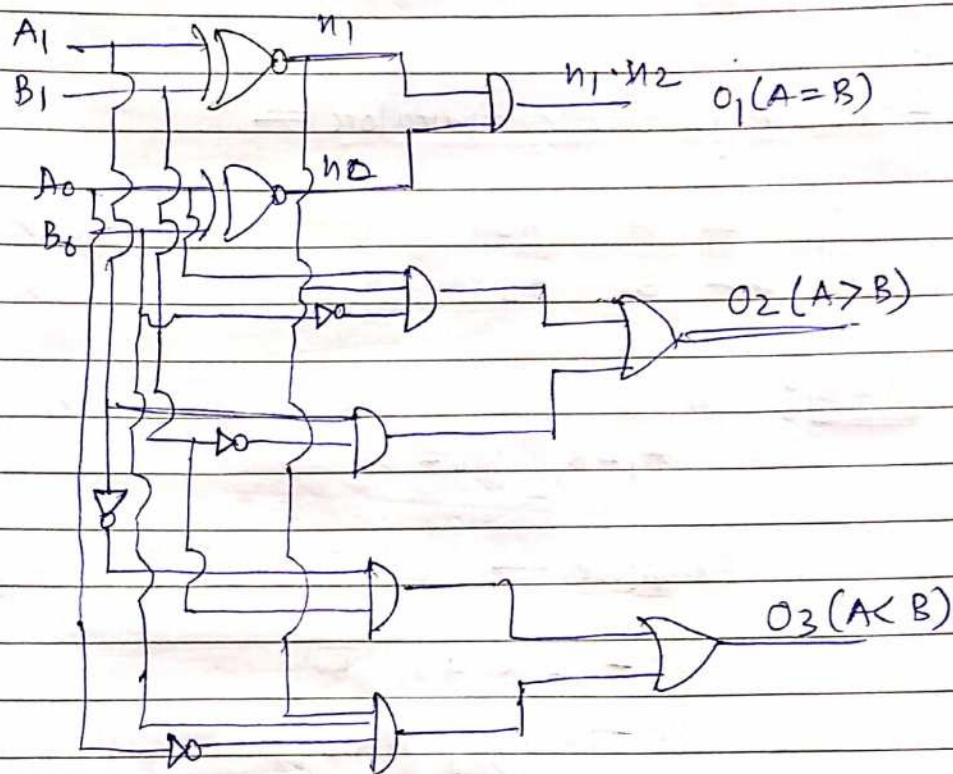
$$\begin{matrix} A_1 & A_0 \\ A = & 1 & 0 \\ B = & 0 & 1 \\ & B_1 & B_0 \end{matrix}$$

$$[A < B]_0$$

$$\begin{array}{cc} A_1 & A_0 \\ A = & 0 \ 1 \\ B_1 & B_0 \\ B = & 1 \ 0 \end{array}$$

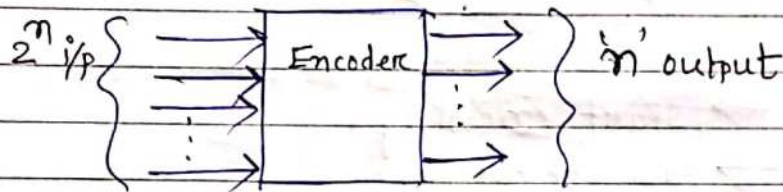
$$\begin{aligned} O_3(A < B) &= \bar{A}_1 B_1 + A_1 \bar{A}_0 B_0 \\ &= 1 + () \\ &= 1 \text{ (yes } A < B) \end{aligned}$$

logic diagram —



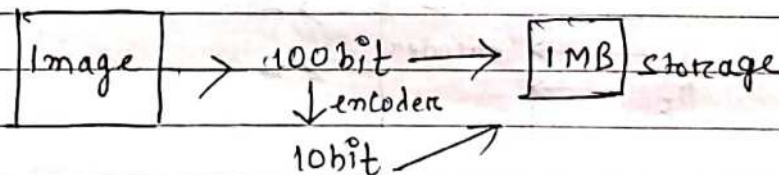
• Encoders :

→ "It is a combinational circuit that takes multiple i/p and converts into a single binary code."

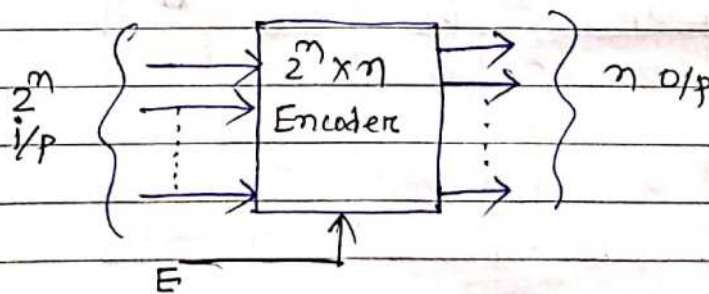


Advantages :-

- To store more data in a given space.
- To send more data from source to destination.
- To detect the errors easily.



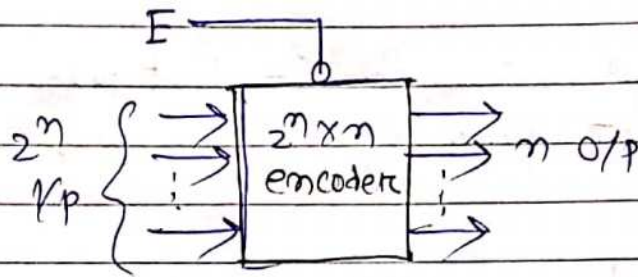
Encoder with Enable input =



Active-high

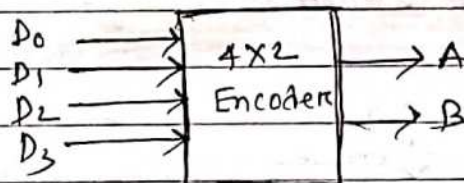
$E = 1 \rightarrow$ start encoding (enable the circuit)

$E = 0 \rightarrow$ no operation (Disable the circuit)

Active-low

$E=0 \rightarrow$ start encoding.

$E=1 \rightarrow$ No operation.

Example - 1① 4 by 2 Encoder:4x2 Encoder

Truth Table =

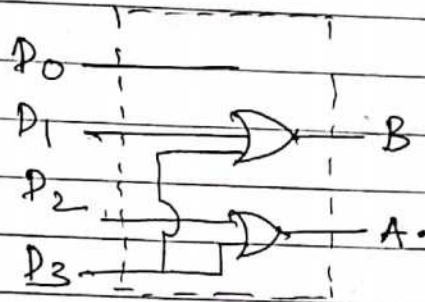
D_0	D_1	D_2	D_3	A	B
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1

\rightarrow Only one i/p has a value of '1' at any given time.

Logic expression =
 $A = D_2 + D_3$

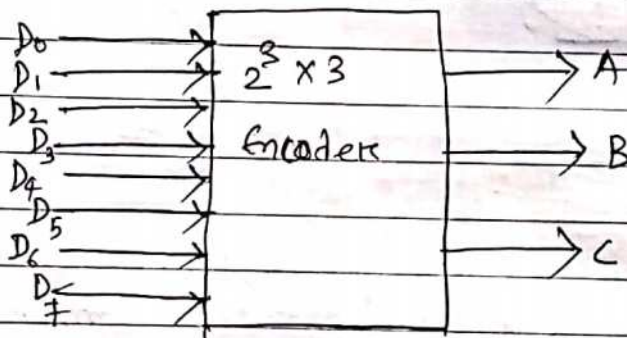
$B = D_1 + D_3$

Logic Circuit =



② 8x3 Encoders (Octal to binary Encoder)

$$8 = 2^3 = 3 \text{ O/P}$$



Truth table =

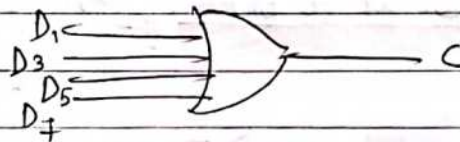
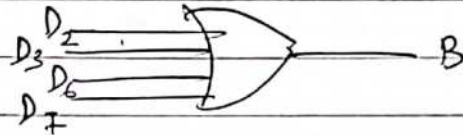
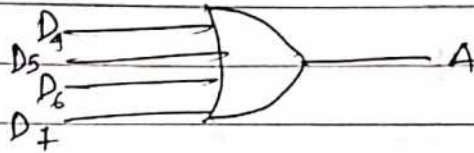
D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	A	B	C
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	0	0
0	0	0	0	0	0	0	1	1	1	1

Logic expression =

$$A = D_4 + D_5 + D_6 + D_7$$

$$B = D_2 + D_3 + D_6 + D_7$$

$$C = D_1 + D_3 + D_5 + D_7$$

Logic circuit —• Limitation —

(1) If two input are activate simultaneously the o/p of the encoder will be undefined combination.

ex- $D_2=1, D_4=1$
 010 100

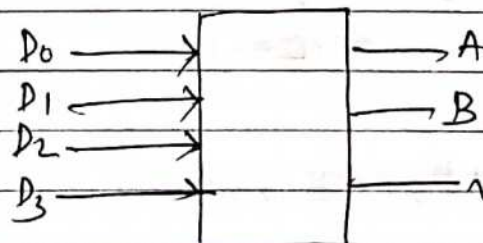
A B C
 (1 1 0) (D_6)

Solution \rightarrow establish a I/p priority:

(1) whe all inputs are '0'
 the o/p = $\overset{ABC}{000}$

$D_0=1 \rightarrow 000$

solution \rightarrow

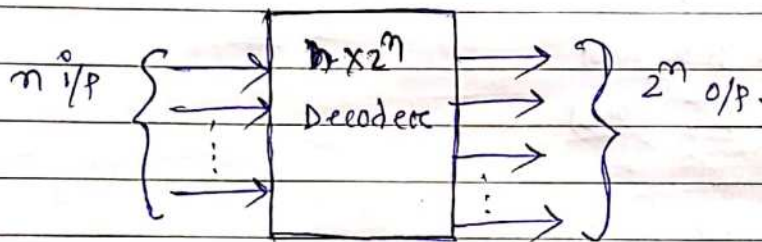
• 4 X 2 priority encoder —

(valid bit indicator = 1
 when one or more
 input are evaluate)

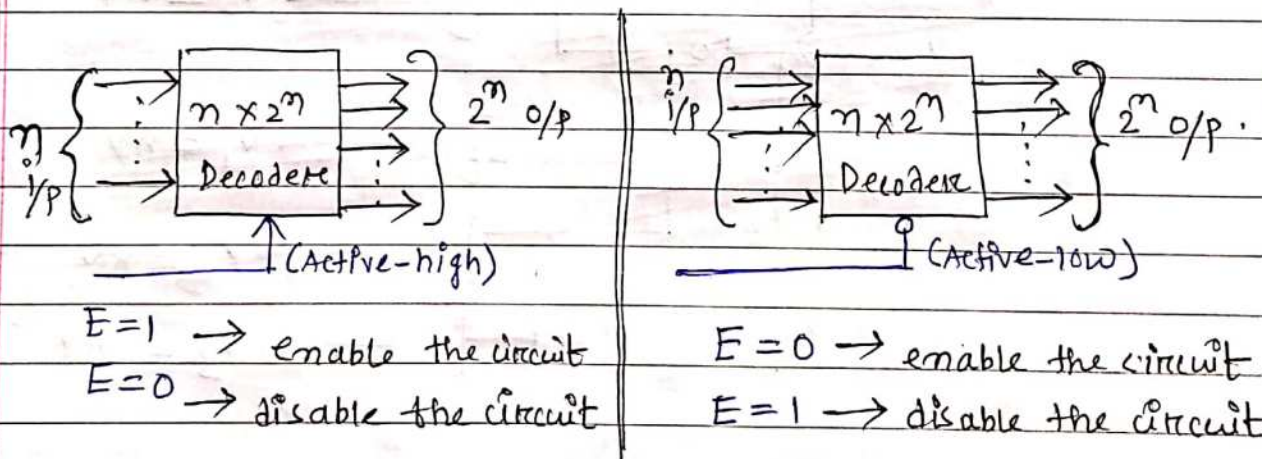
D ₀	D ₁	D ₂	D ₃	A	B	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

• Decoder :

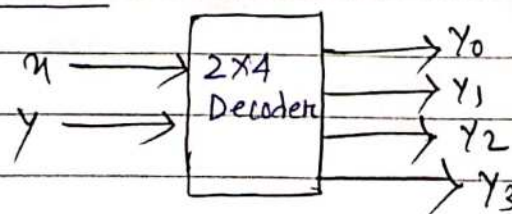
→ "A decoder is a combinational circuit that converts binary information from n input lines to a max 2^n unique output lines."



• Decoder with enable input —



• 2 x 4 Decoder —



truth table -

	x	y	y_0	y_1	y_2	y_3
(0)	0	0	①	0	0	0
(1)	0	1	0	①	0	0
(2)	1	0	0	0	①	0
(3)	1	1	0	0	0	①

logic expressions =

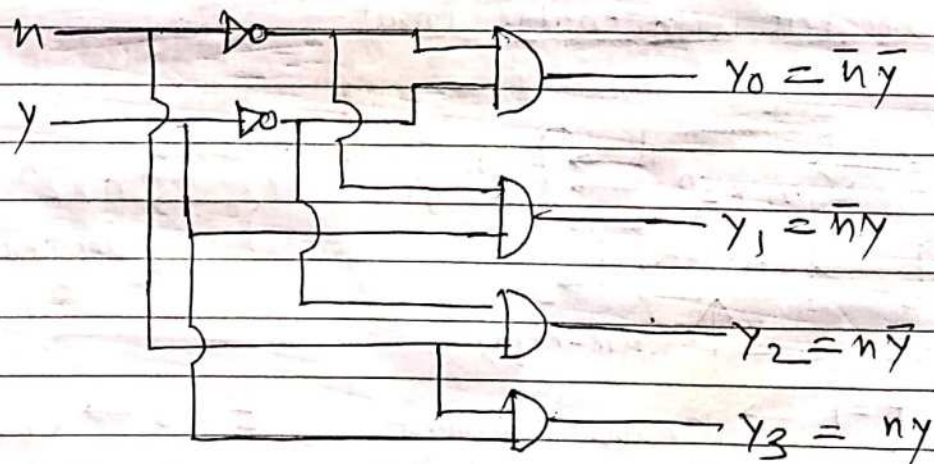
$$y_0 = \bar{x} \bar{y} \text{ (} m_0 \text{)}$$

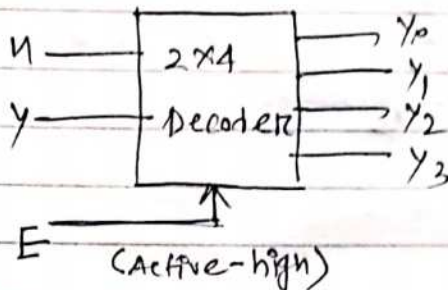
$$y_1 = \bar{x} y \text{ (} m_1 \text{)}$$

$$y_2 = x \bar{y} \text{ (} m_2 \text{)}$$

$$y_3 = x y \text{ (} m_3 \text{)}$$

logic diagram =



2x4 Decoder with enable i/p =table

E	n	y	y_0	y_1	y_2	y_3
0	X	X	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1

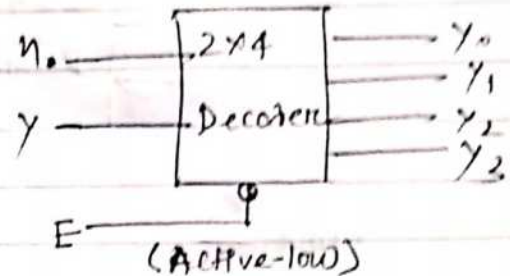
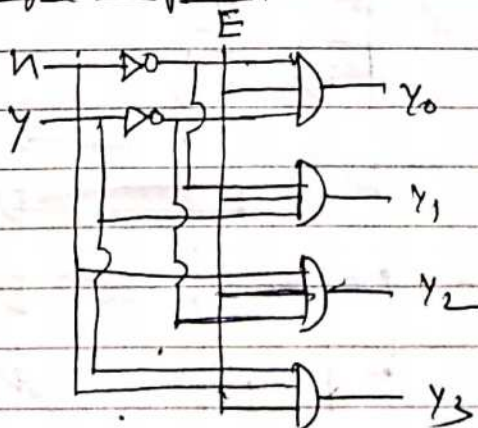
logic expression -

$$y_0 = \bar{n} \bar{y} \cdot E$$

$$y_1 = \bar{n} y \cdot E$$

$$y_2 = n \bar{y} \cdot E$$

$$y_3 = n y \cdot E$$

logic diagram -table

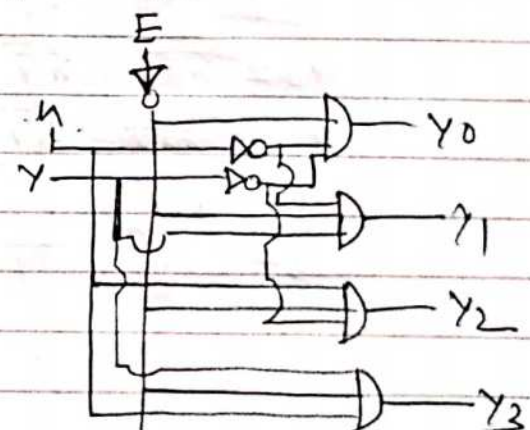
E	n	y	y_0	y_1	y_2	y_3
1	X	X	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	1	0	0
0	1	0	0	0	1	0
0	1	1	0	0	0	1

$$y_0 = \bar{n} \bar{y} \cdot \bar{E}$$

$$y_1 = \bar{n} y \cdot \bar{E}$$

$$y_2 = n \bar{y} \cdot \bar{E}$$

$$y_3 = n y \cdot \bar{E}$$

logic diagram -

• 2X4 Decoder with active low output =

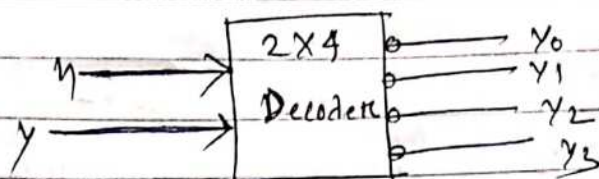


Table =

h	y	y ₀	y ₁	y ₂	y ₃
0	0	0	1	1	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

expression -

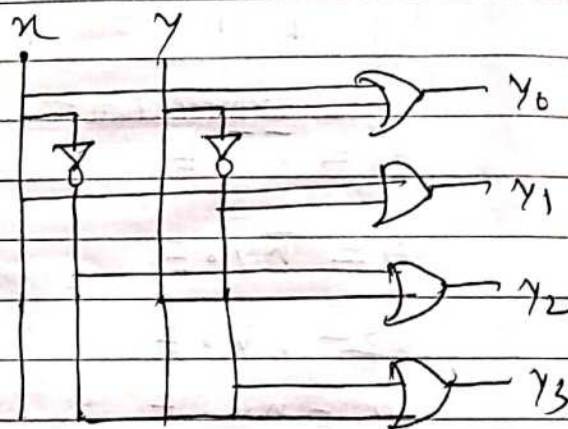
$$y_0 = h + y$$

$$y_1 = h + \bar{y}$$

$$y_2 = \bar{h} + y$$

$$y_3 = \bar{h} + \bar{y}$$

logic diagram -



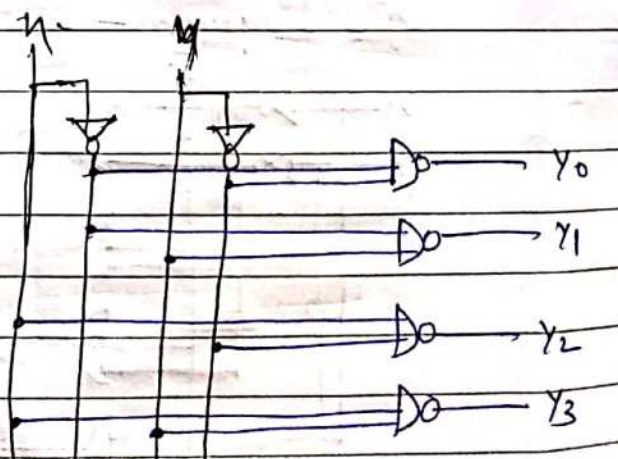
using NAND gates -

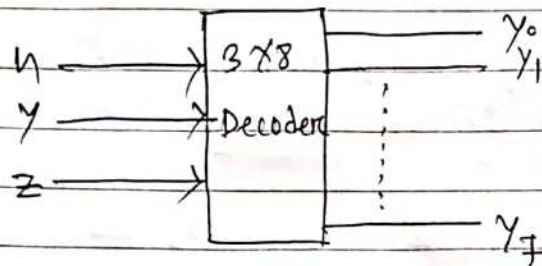
$$y_0 = \overline{h+y} = \overline{h \cdot \bar{y}}$$

$$y_1 = \overline{h+\bar{y}} = \overline{h \cdot y}$$

$$y_2 = \overline{\bar{h}+y} = \overline{\bar{h} \cdot \bar{y}}$$

$$y_3 = \overline{\bar{h}+\bar{y}} = \overline{\bar{h} \cdot \bar{y}}$$



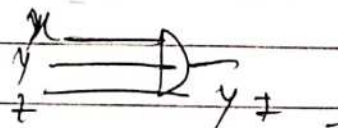
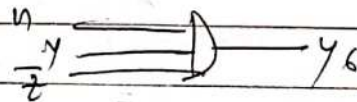
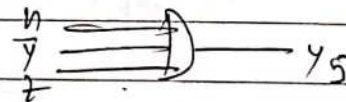
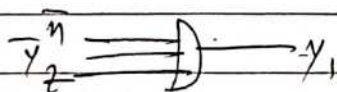
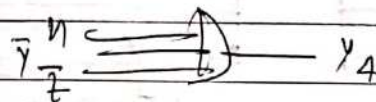
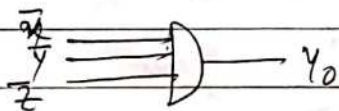
3 X 8 Decoder —Table —

	x	y	z	y ₀	y ₁	y ₂	y ₃	y ₄	y ₅	y ₆	y ₇
(0)	0	0	0	1	0	0	0	0	0	0	0
(1)	0	0	1	0	1	0	0	0	0	0	0
(2)	0	1	0	0	0	1	0	0	0	0	0
(3)	0	1	1	0	0	0	1	0	0	0	0
(4)	1	0	0	0	0	0	0	1	0	0	0
(5)	1	0	1	0	0	0	0	0	1	0	0
(6)	1	1	0	0	0	0	0	0	0	1	0
(7)	1	1	1	0	0	0	0	0	0	0	1

logic expression —

$$y_0 = \bar{x}\bar{y}\bar{z} \quad y_1 = \bar{x}\bar{y}z \quad y_2 = \bar{x}y\bar{z} \quad y_3 = \bar{x}yz \quad y_4 = x\bar{y}\bar{z}$$

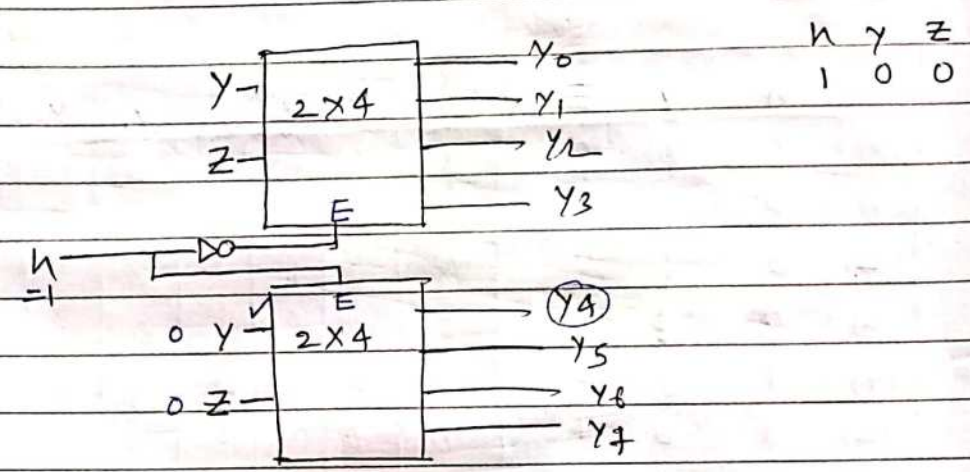
$$y_5 = x\bar{y}z \quad y_6 = xy\bar{z} \quad y_7 = xyz$$

logic diagram

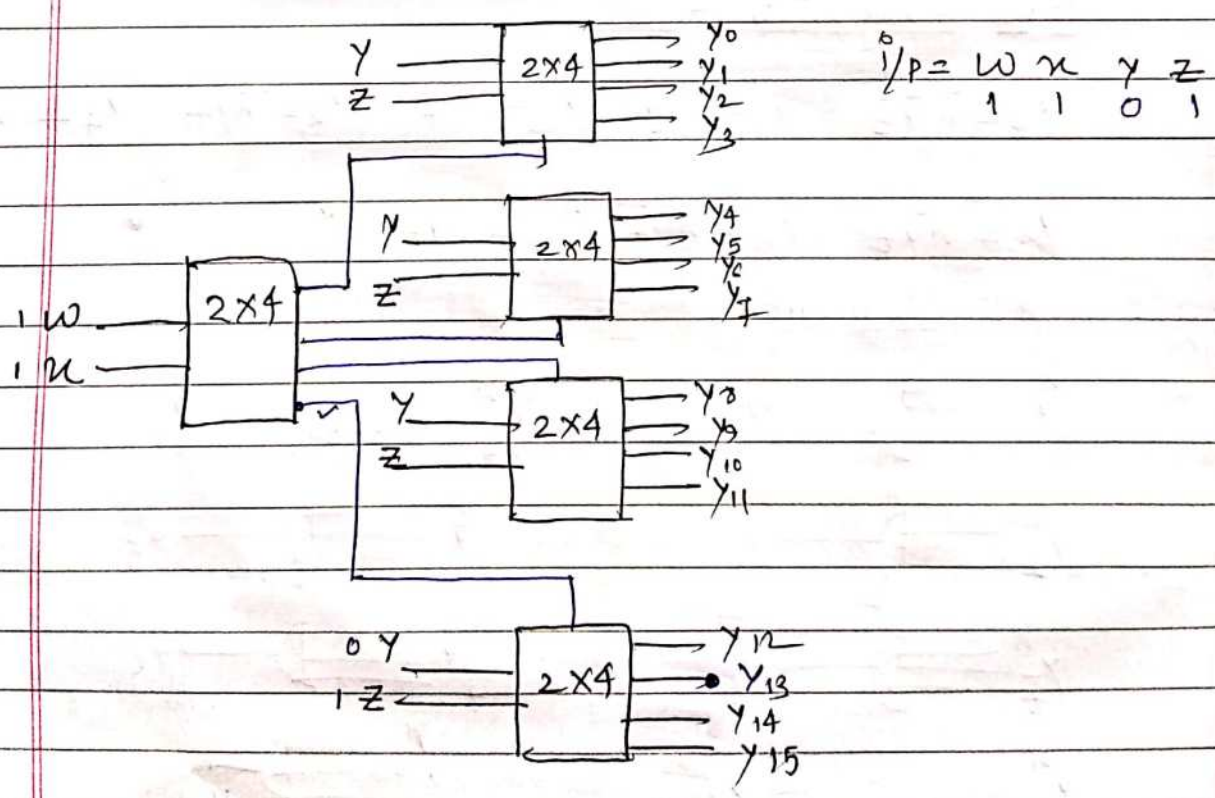
• Construction of larger Decoders using smaller decoders -

" Decoders with enable i/p can be connected together to form larger decoder circuits.

① 3x8 Decoder using 2x4 Decoders -

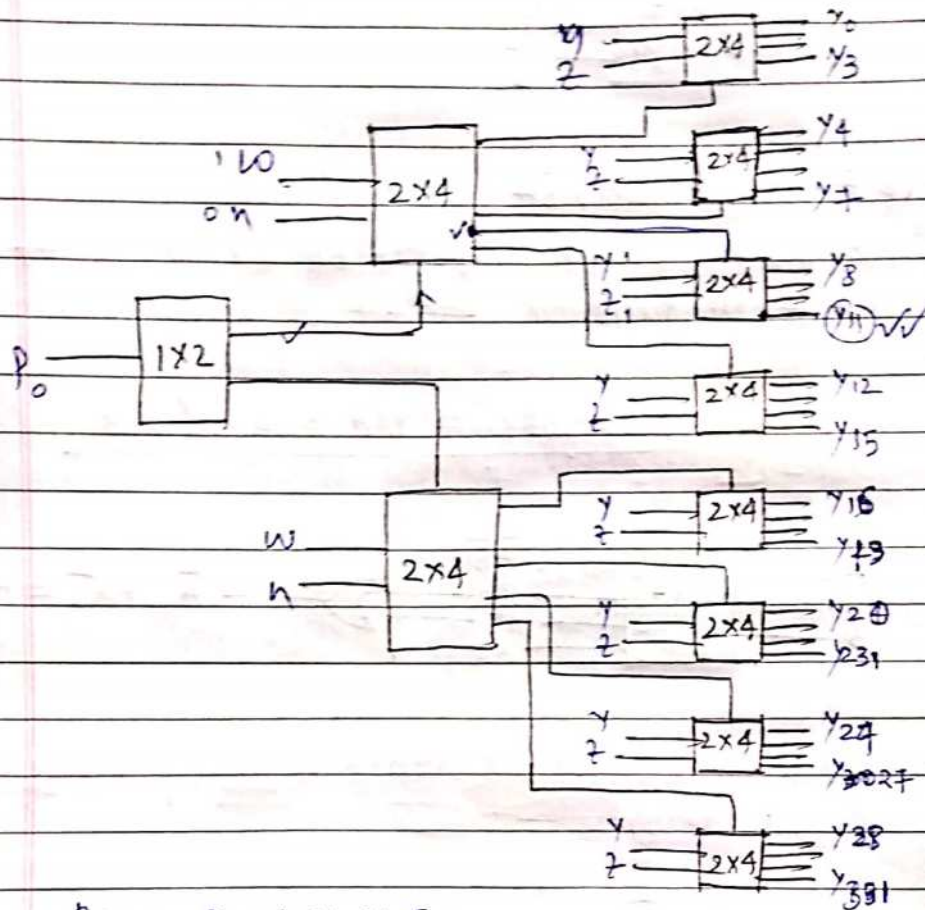


② 4x16 Decoder using 2x4 Decoders -



③ 5x32 Decoder using 2x4 Decoder —

$$\frac{32}{8}$$

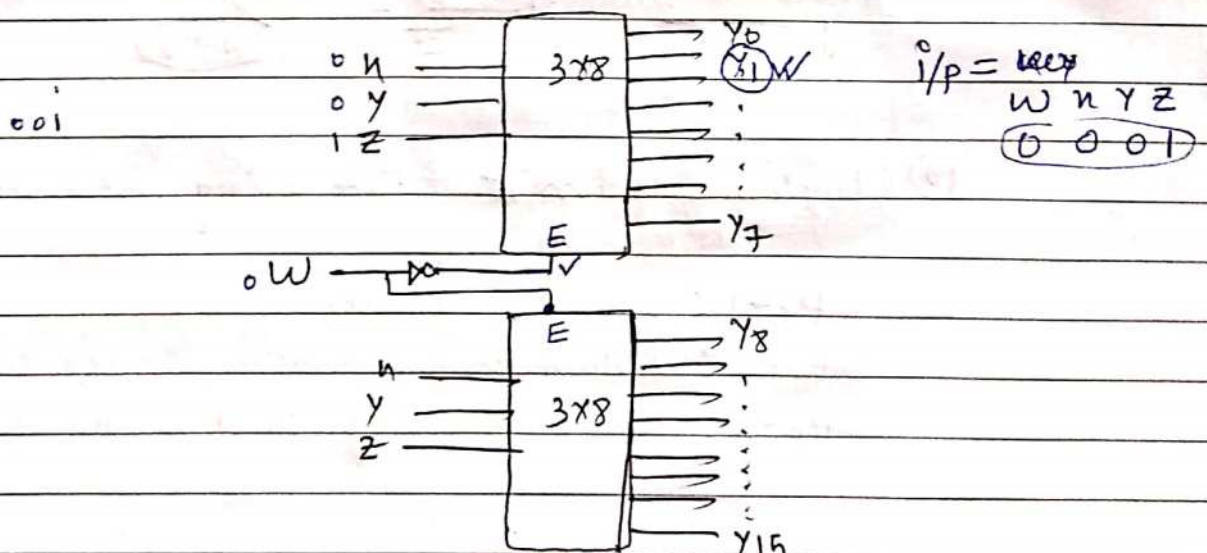


$$i/p = p \ w \ n \ y \ z$$

$$0 \ 1 \ 0 \ 1 \ 1$$

④ 4x16 Decoder using 3x8 Decoder —

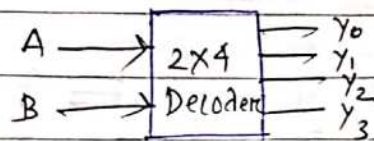
$$16/8 = 2$$



$$i/p = w \ n \ y \ z$$

$$0 \ 0 \ 0 \ 1$$

• Implementation of Boolean functions using Decoders —

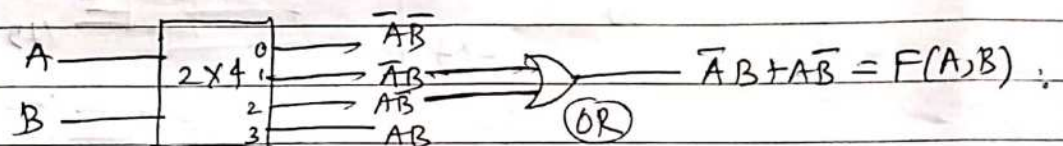


ex =

① ex $F(A, B) = \bar{A}B + A\bar{B}$

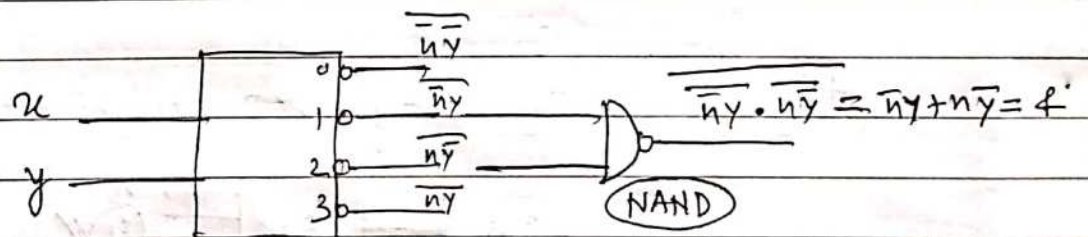
"Any Boolean function can be expressed by as sum of products or product of sum of minterms form."

Decoder + OR gate = $f(A, B)$ or $f(A, B, C)$



** ② Implement the EX-OR function using active low output decoder —

→ $f = x\bar{y} + \bar{x}y$ $f = \Sigma(1, 2)$



③ Implementation of F.A using 3x8 Decoder —

Step-1 : write truth table.

Step-2 : Obtain Boolean function in sop form.

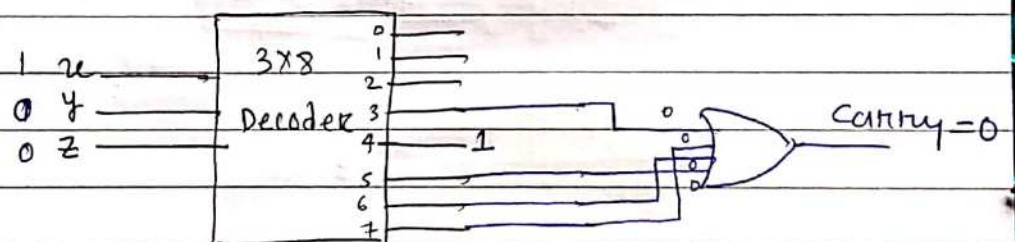
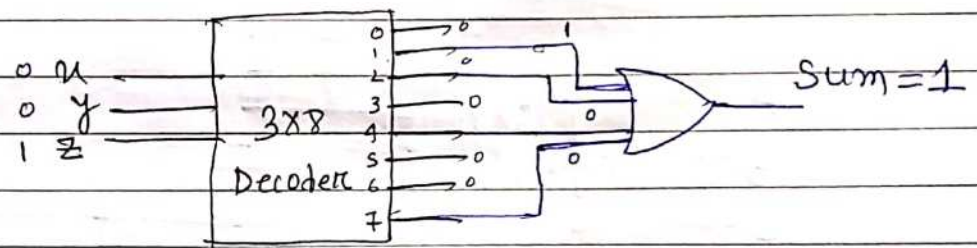
Step-3 : Implement using decoder and OR gate.

table

	x	y	z	sum	carry
(0)	0	0	0	0	0
(1)	0	0	1	①	0
(2)	0	1	0	①	0
(3)	0	1	1	0	①
(4)	1	0	0	①	0
(5)	1	0	1	0	①
(6)	1	1	0	0	①
(7)	1	1	1	①	①

$$\text{sum} = \Sigma(1, 2, 4, 7)$$

$$\text{carry} = \Sigma(3, 5, 6, 7)$$



(Worked)

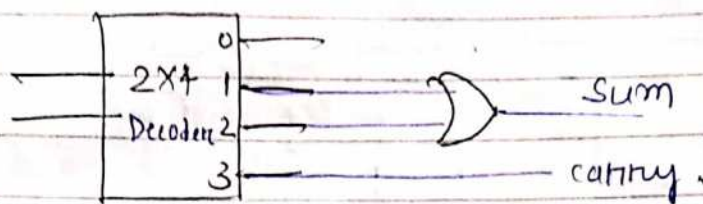
④ Implementation of H.A using Decoder —

table

	A	B	S	C
(0)	0	0	0	0
(1)	0	1	①	0
(2)	1	0	①	0
(3)	1	1	0	①

$$\text{sum} = \Sigma(1, 2)$$

$$\text{carry} = \Sigma(3)$$



⑤ Implement the following function using Decoder —

$$f(A, B, C) = A + BC$$

$$= A(B + \bar{B})(C + \bar{C}) + BC(A + \bar{A})$$

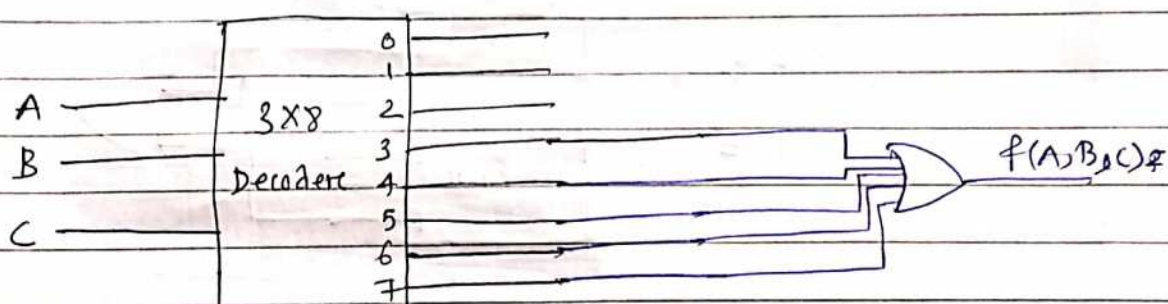
$$= \underset{\substack{\uparrow \uparrow \uparrow \\ 111}}{ABC} + \underset{\substack{\uparrow \uparrow \uparrow \\ 110}}{ABC} + \underset{\substack{\uparrow \uparrow \uparrow \\ 101}}{ABC} + \underset{\substack{\uparrow \uparrow \uparrow \\ 100}}{ABC} + \underset{\substack{\uparrow \uparrow \uparrow \\ 111}}{ABC} + \underset{\substack{\uparrow \uparrow \uparrow \\ 011}}{ABC}$$

A 2 1

$$= ABC + ABC + ABC + ABC + ABC + ABC$$

$$(m_7) \quad (m_6) \quad (m_5) \quad (m_4) \quad (m_3)$$

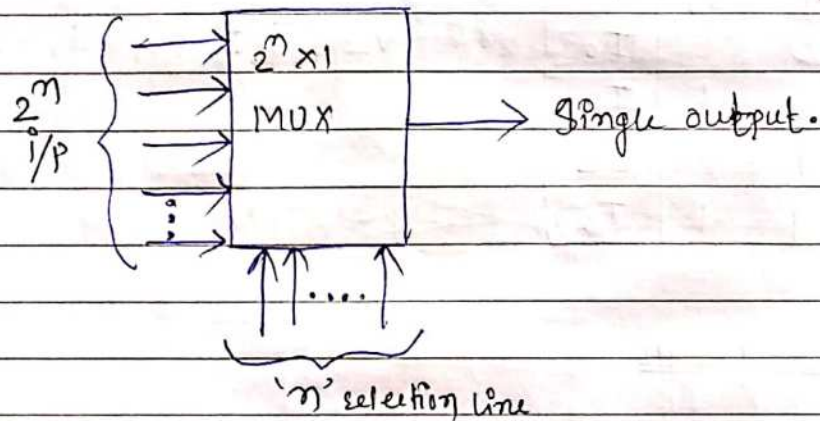
$$= \Sigma(3, 4, 5, 6, 7)$$



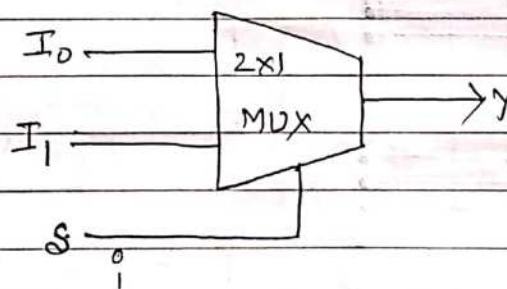
** • Multiplexer : (MUX) or (Data Selector)

"A Multiplexer is a combinational circuit that select binary information from one of many i/p lines and directs it to a single o/p line".

The selection of a particular i/p controlled by a set of selection lines".



• 2x1 Multiplexer (MUX) —

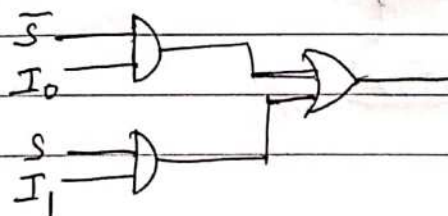


Truth Table:

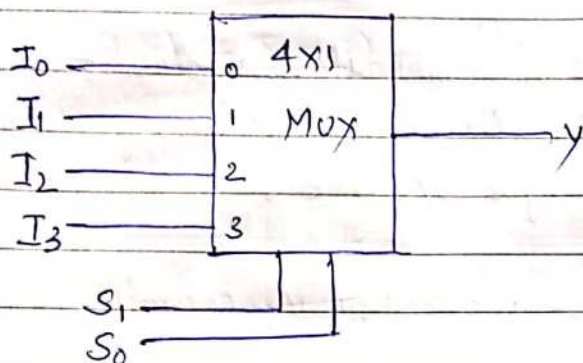
S	Y
0	I_0
1	I_1

$$Y = \bar{S}I_0 + SI_1$$

Logic Diagram:



• 4x1 MUX:



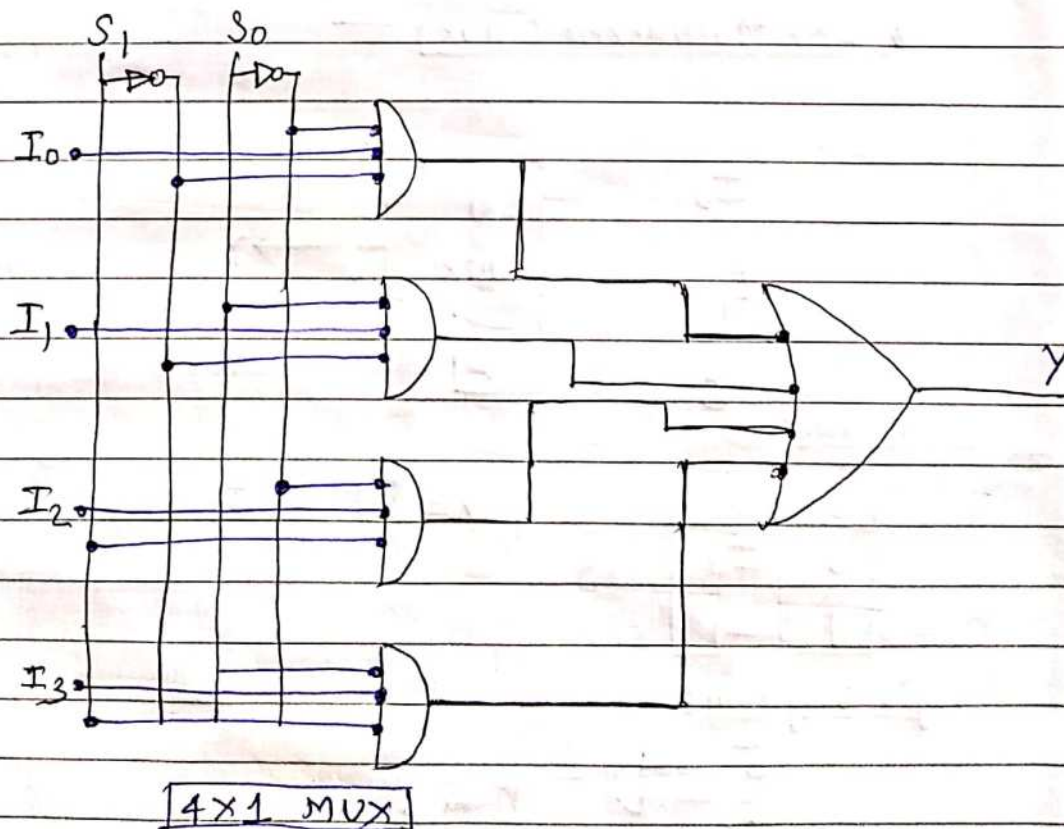
Table

	S_1	S_0	y
(0)	0	0	I_0
(1)	0	1	I_1
(2)	1	0	I_2
(3)	1	1	I_3

expression -

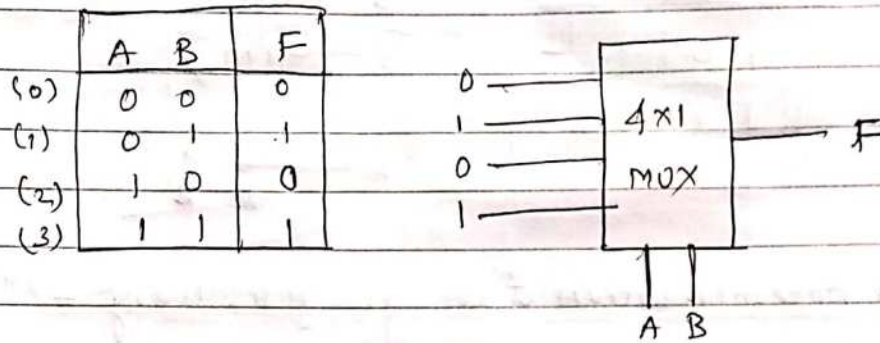
$$y = \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1 \bar{S}_0 I_2 + S_1 S_0 I_3$$

logic diagram

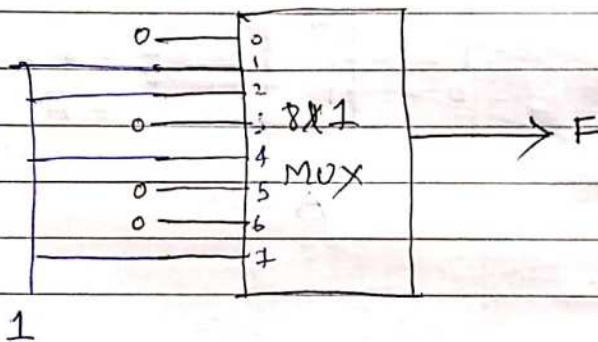


• Implementation of Boolean function using MUX:

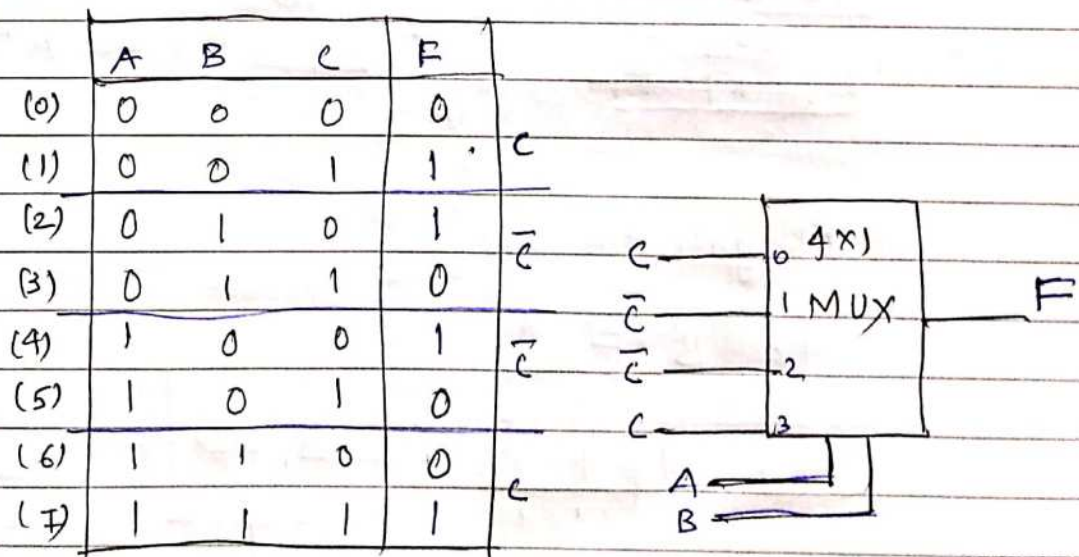
① $F(A,B) = \Sigma(1,3)$ using 4×1 MUX.



② $F(A,B,C) = \Sigma(1,2,4,7)$ using 8×1 MUX -



③ $F(A,B,C) = \Sigma(1,2,4,7)$ using 4×1 MUX -

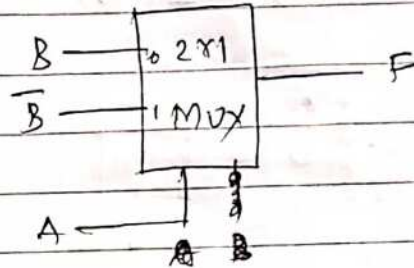


A B C'
0 0 0

④

using 2x1 MUX -

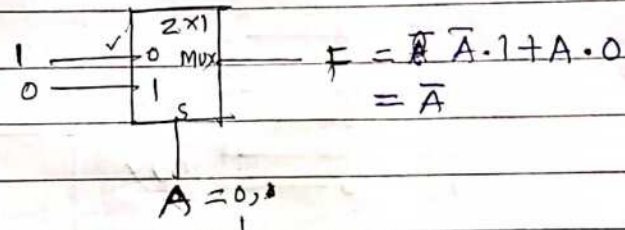
A	B	F
0	0	0
0	1	1
1	0	1
1	1	0



• Implementation of all logic gates using 2x1 MUX -

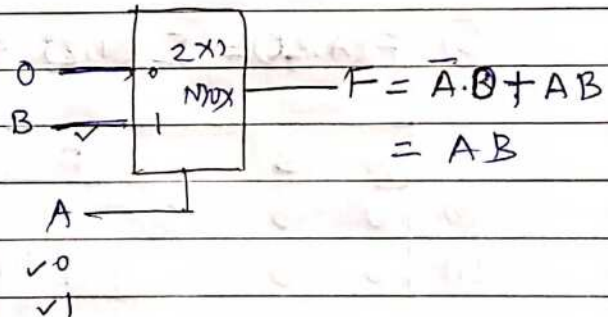
(I) NOT gate -

A	F
0	1
1	0



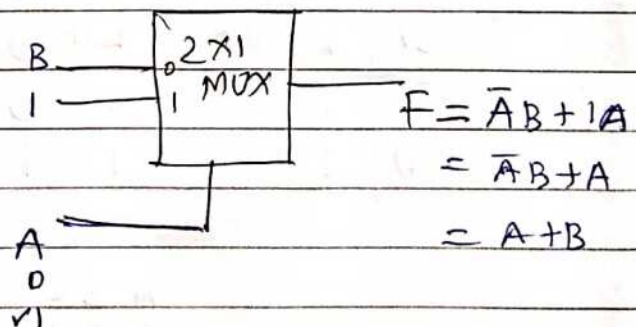
(II) AND gate -

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1



(III) OR gate -

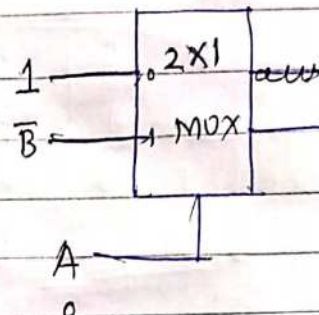
A	B	F
0	0	0
0	1	1
1	0	1
1	1	1



(IV) NAND gate -

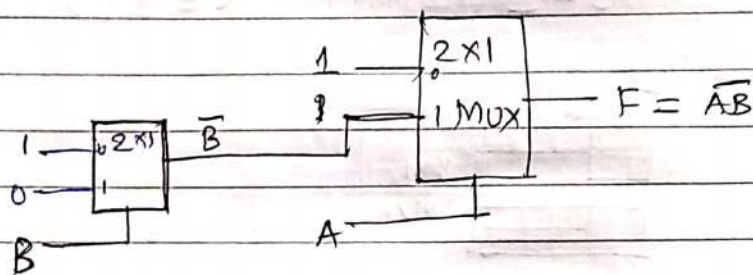
A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

B 1

 \bar{B} 

$$\begin{aligned}
 F &= \bar{A} \cdot 1 + A \cdot \bar{B} \\
 &= \bar{A} + A \cdot \bar{B} \\
 &= \bar{A} + \bar{B} \\
 &= \overline{A \cdot B}
 \end{aligned}$$

* using no complement i/p - 1



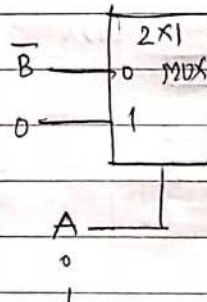
$$F = \overline{A \cdot B}$$

(V) NOR gate -

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

 \bar{B}

0

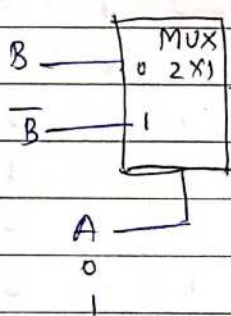


$$\begin{aligned}
 F &= \bar{A} \cdot \bar{B} + 0 \cdot A \\
 &= (\bar{A} + \bar{B})
 \end{aligned}$$

(VI) Ex-OR gate -

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

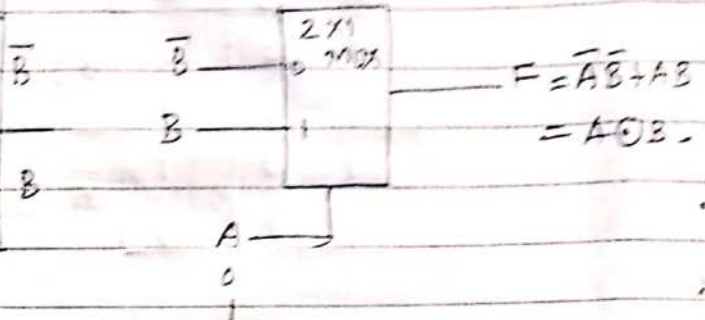
B

 \bar{B} 

$$\begin{aligned}
 F &= \bar{A} \cdot B + A \cdot \bar{B} \\
 &= A \oplus B
 \end{aligned}$$

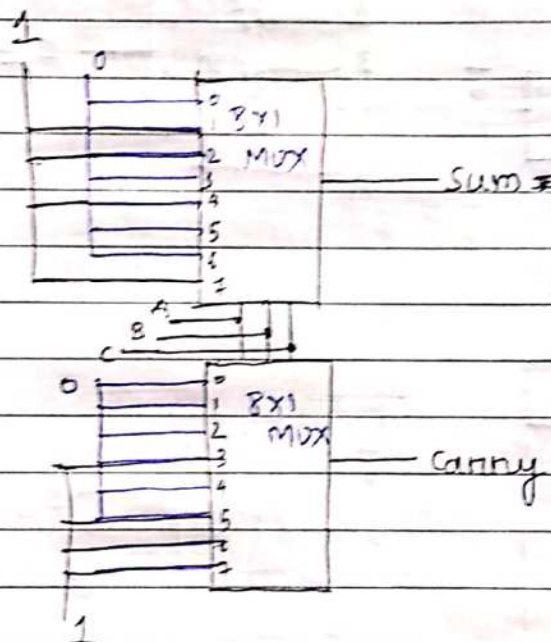
(vii) EX-NOR gate —

A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

(viii) Full-Adder using 2x1 MUX —

$$S = \Sigma (1, 2, 4, 7)$$

$$C = \Sigma (3, 5, 6, 7)$$



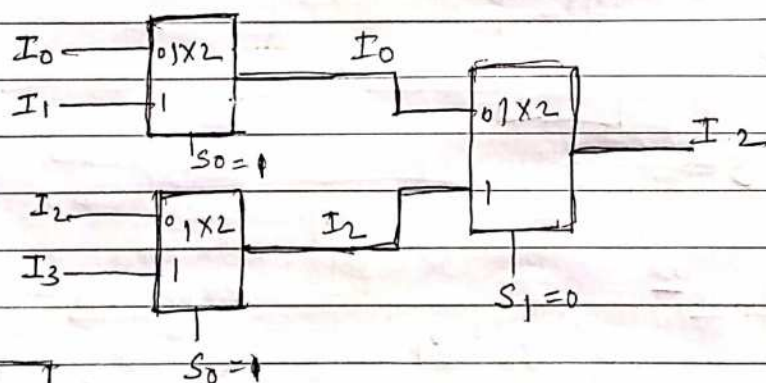
A	B	C	S	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

• Design of Large size multiplexer using Other MUX:

Given MUX	To be implemented	Required no. of MUX
2x1	4x1	3
2x1	8x1	7
2x1	16x1	15
2x1	64x1	63
2x1	256x1	255
2x1	$2^n \times 1$	$(2^n - 1)$
4x1	16x1	$4 + 1 = 5$
4x1	64x1	$16 + 4 + 1 = 21$
8x1	64x1	$8 + 1 = 9$
8x1	256x1	$32 + 8 + 1 = 37$

$\frac{64}{4} = 16$
 $\frac{64}{8} = 8$
 $\frac{256}{8} = 32$
 $\frac{256}{32} = 8$

① Implement 4x1 MUX using 2x1 MUX -



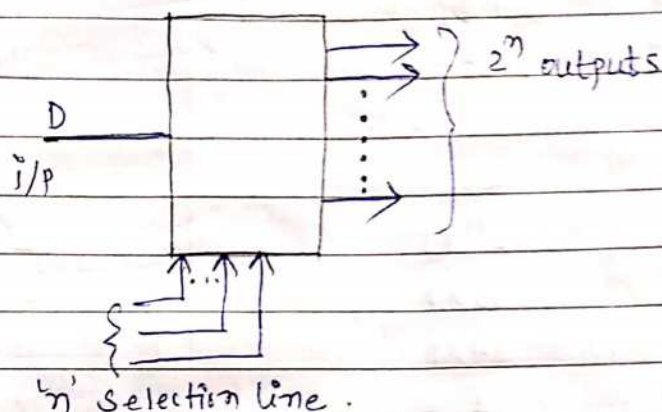
S_1	S_0	F
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

$\checkmark S_1 S_0$
 $010 = I_2$

3 - (2:1) MUX required.

Demultiplexers (De-MUX) (Data Distributor).

- It performs the inverse of Multiplexer.
- It takes single i/p and distributes several outputs.



① 1x2 Demux or 1 line to 2 line De-mux -

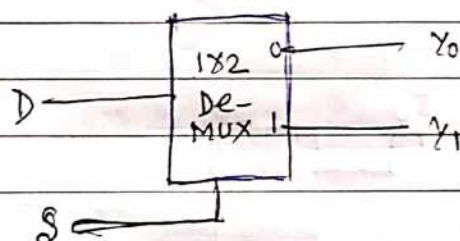


table -

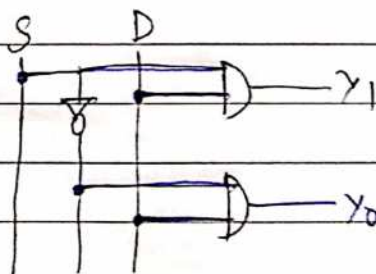
S	Y ₀	Y ₁
0	D	0
1	0	D

expression -

$$Y_0 = \bar{S} D$$

$$Y_1 = S D$$

logic diagram -



② 1x4 De-Mux with Enable i/p -

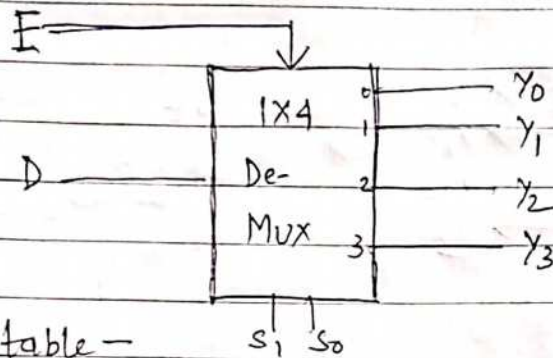


table -

E	S ₁	S ₀	Y ₀	Y ₁	Y ₂	Y ₃
0	X	X	0	0	0	0
1	0	0	D	0	0	0
1	0	1	0	D	0	0
1	1	0	0	0	D	0
1	1	1	0	0	0	D

Y₀ expression -

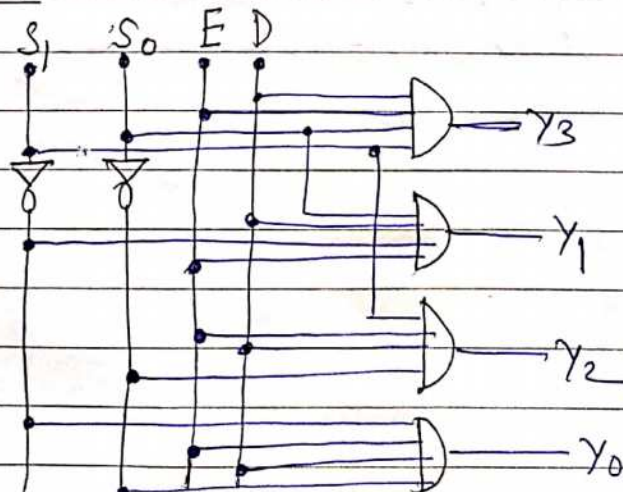
$$Y_0 = E \cdot \bar{S}_1 \cdot \bar{S}_0 \cdot D$$

$$Y_1 = E \cdot \bar{S}_1 \cdot S_0 \cdot D$$

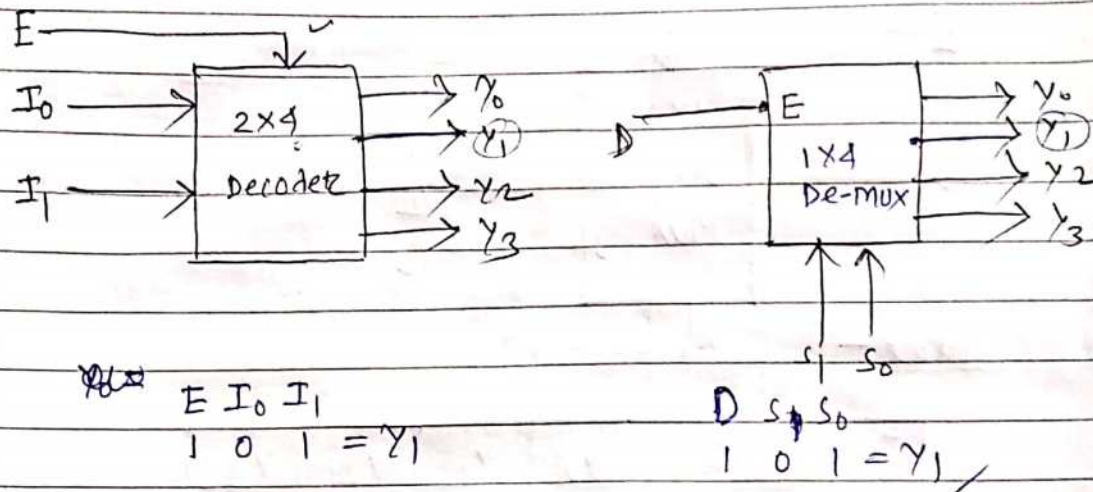
$$Y_2 = E \cdot S_1 \cdot \bar{S}_0 \cdot D$$

$$Y_3 = E \cdot S_1 \cdot S_0 \cdot D$$

Logic diagram -

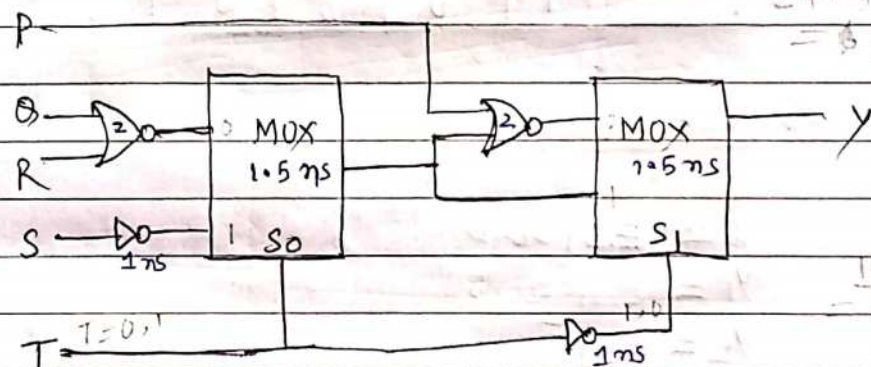


③ Decoder with enable pin can act as a De-MUX -

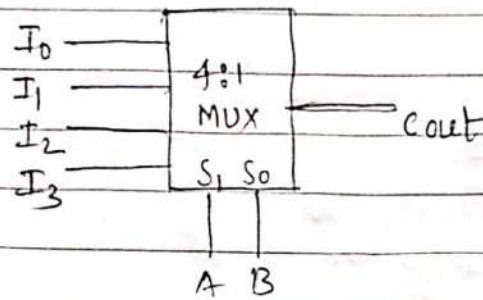


• GATE Questions on Combinational circuit -

2016
Q-①



The delay of NOR gate, MUX and inverters are 2 ns, 1.5 ns, and 1 ns respectively. If all the i/p P, Q, R, S applied at same time, the max propagation delay of the circuit is 6 ns.

2016-Gate
Q-2

Which one of the following statements correctly describes the choice of input signals to be connected to the i/p's I_0, I_1, I_2 & I_3 so that the o/p is C_{out} ?

- ☒ (A) $I_0 = 0, I_1 = C_{in}, I_2 = C_{in}, I_3 = 1$,
☐ (B) $I_0 = 1, I_1 = C_{in}, I_2 = C_{in}, I_3 = 1$,
☐ (C) $I_0 = C_{in}, I_1 = 0, I_2 = 1, I_3 = C_{in}$,
☐ (D) $I_0 = 0, I_1 = C_{in}, I_2 = 1, I_3 = C_{in}$

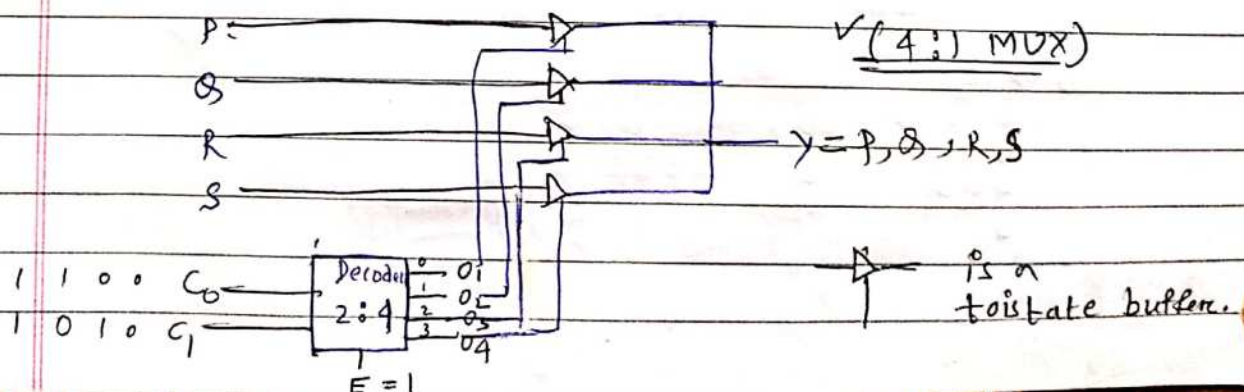
⇒

A	B	C_{in}	$C_{out} (carry)$	
0	0	0	0	0 (I_0)
0	0	1	0	
0	1	0	0	C_{in} (I_1)
0	1	1	1	
1	0	0	0	C_{in} (I_2)
1	0	1	1	
1	1	0	1	1 (I_3)
1	1	1	1	

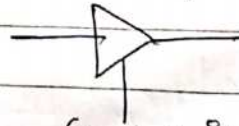
Q-16

Q-3

The functionality implemented by the ckt below is



Three state gate -



(control i/p)

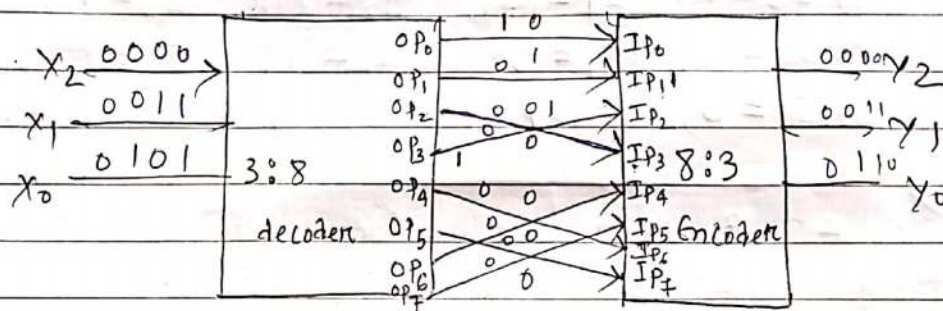
if $c=0$, $y = \text{high impedance}$

$c=1$, $y = A$

2016

Q-4

Identify the ckt below -

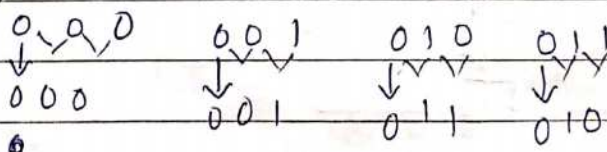


4 2 1
0 1 0

- ✓ (a) Binary to Gray code converter.
(b) Binary to XS3 converter.
(c) Gray to binary converter.

X_2	X_1	X_0	Y_2	Y_1	Y_0
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
:	:	:	:	:	:

B → G



2019

Q-5

In a Half Subtractor CKT with x & y as I/p, the Borrow (M) & Difference ($N = x - y$) are give by -

(a) $M = x \oplus y$, $N = xy$ ☐

(b) $M = xy$, $N = x \oplus y$ ☐

(c) $M = \bar{x}y$, $N = x \oplus y$ ☒

(d) $M = \bar{x}y$, $N = \overline{x \oplus y}$ ☐

\Rightarrow

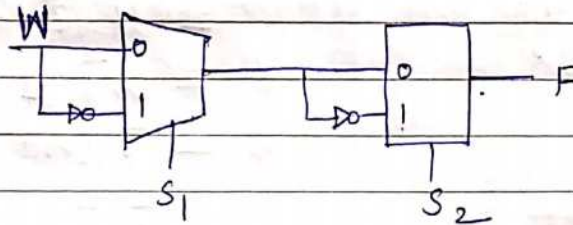
x	y	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$B = \bar{x}y$$

$$D = \bar{x}y + x\bar{y} = x \oplus y$$

10 A

Q-6 consider the Max 'based ckt shown in fig -



Which of the following Boolean function is realized by the ckt is ?

(a) $F = W\bar{S}_1\bar{S}_2$

(b) $F = WS_1 + WS_2 + S_1S_2$

(c) $F = \bar{W} + S_1 + S_2$

(d) $F = W \oplus S_1 \oplus S_2$ ☒

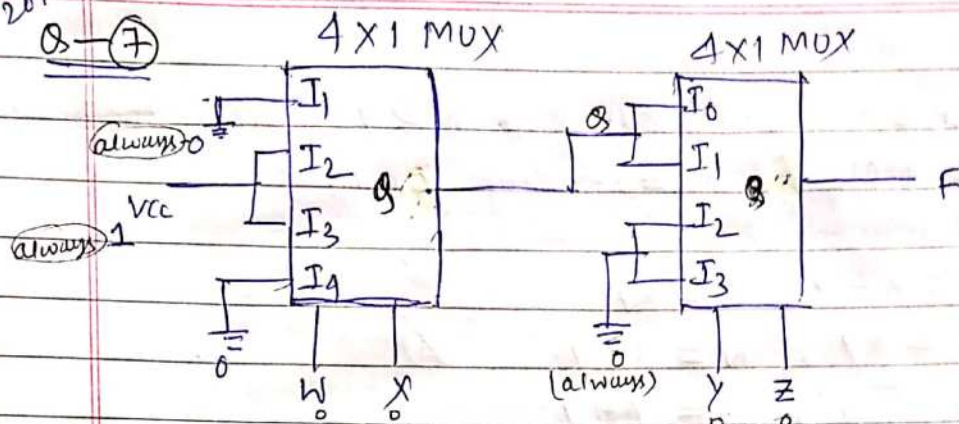
\Rightarrow

S_1	S_2	F
0	0	$W\bar{S}_1\bar{S}_2$
0	1	$\bar{W}S_1S_2$
1	0	$\bar{W}S_1\bar{S}_2$
1	1	WS_1S_2

$$\begin{aligned}
 F &= W\bar{S}_1\bar{S}_2 + \bar{W}S_1S_2 + \bar{W}S_1\bar{S}_2 + WS_1S_2 \\
 &= W(\bar{S}_1\bar{S}_2 + S_1S_2) + \bar{W}(S_1\bar{S}_2 + S_1S_2) \\
 &= W(S_1 \oplus S_2) + \bar{W}(S_1 \oplus S_2) \\
 &= W \oplus S_1 \oplus S_2
 \end{aligned}$$

2014

Q-7



The O/p is given by -

(a) $F = W\bar{X} + \bar{W}X + \bar{Y}Z$

(b) $F = W\bar{X} + \bar{W}X + \bar{Y}Z$

(c) $F = W\bar{X}\bar{Y} + \bar{W}X\bar{Y}$

✓ (d) $F = (\bar{W} + \bar{X}) \bar{Y} Z$

$$\Rightarrow Q = \bar{W}X \cdot 1 + W\bar{X} \cdot 1$$

$$= \bar{W}X + W\bar{X}$$

$$= W \oplus X$$

$$F = \bar{Y}ZQ + \bar{Y}ZQ$$

$$= (\bar{Y}Z + YZ)Q$$

$$= Q\bar{Y}(Z + \bar{Z})$$

$$= \bar{Y}(W \oplus X)$$

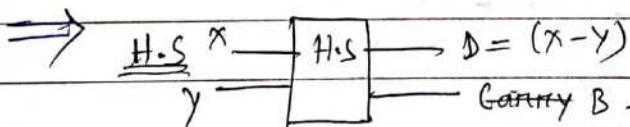
$$= \bar{Y}[W\bar{X} + \bar{W}X]$$

$$F = \bar{W}\bar{X}\bar{Y} + \bar{W}X\bar{Y}$$

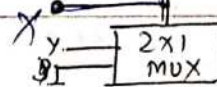
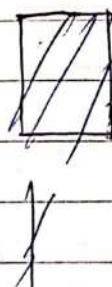
2014

Q-8

if x and y are i/p's and Difference (D) or Δ the Borrow (B) are the o/p's. Implement it using 2×1 MUX

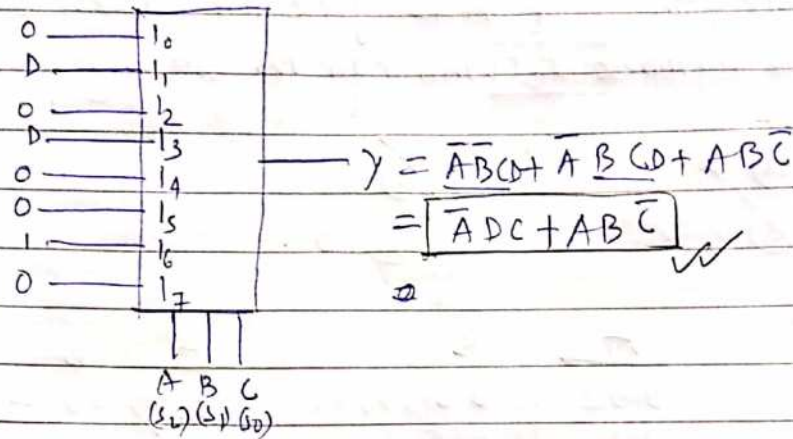


x	y	D	C
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



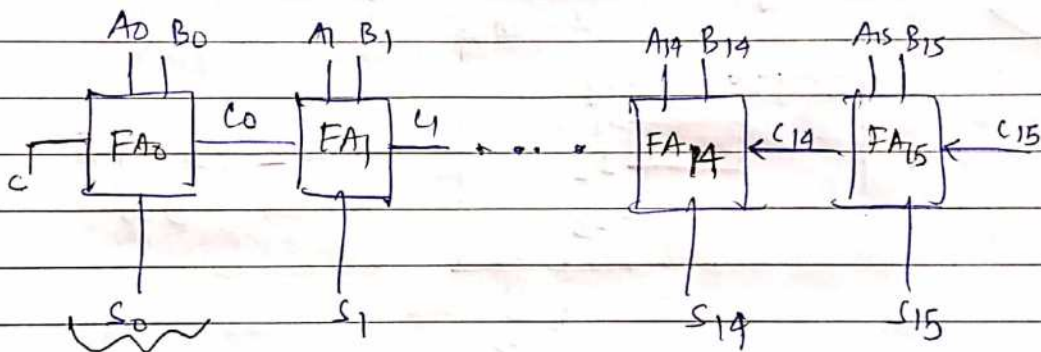
2014

Q-9 An 8-to-1 MUX is used to implement a logic y as shown in fig. The O/p.



Q-10

16 bit RCA is realized using 16 identical Full adders as shown in fig. The carry propagation delay of each FA is 12 ns and the sum propagation delay is 15 ns. The worst case delay (in ns) the 16 bit adder will be -



\Rightarrow carry propagation delay ≈ 12 ns
sum ≈ 15 ns.

Total propagation delay ≈ 15 ns $+ 15 \times 12$ ns
 $= 15$ ns $+ 180$ ns
 $= 195$ ns

Q-12
Q-11

The o/p 'Y' of a 2-bit comparator is logic '1' whenever the 2-bit i/p A is greater than 2-bit i/p B.
The number of combinations for which the o/p is logic 1 is -

- (a) 4 (b) 6
(c) 8 (d) 10

⇒

A	B
00	00
01	01
10	10
11	11

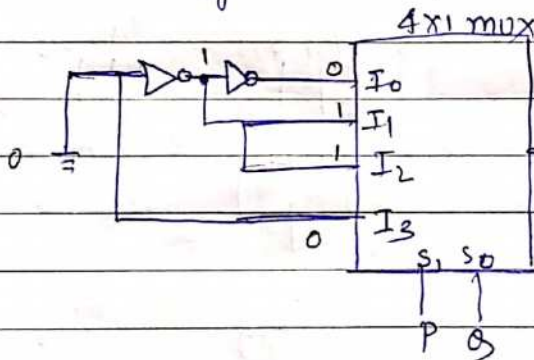
A	B
01	00 - 1
10	00 - 1
	01 - 1
11	00 - 1
	01 - 1
	10 - 1

6

Q-11

Q-12

The logic function implemented by the ckt below is (ground implies a logic '0')



$$F = \bar{P}Q + PQ$$

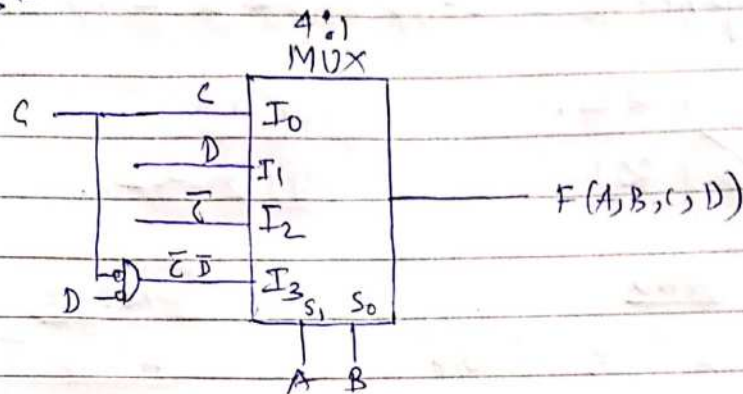
$$= P \oplus Q$$

- (a) $F = \text{AND}(P, Q)$ (b) $F = \text{OR}(P, Q)$
(c) $F = \text{XNOR}(P, Q)$ (d) $F = \text{XOR}(P, Q)$

Q-10

Q-13

The Boolean function realized by the logic circuit shown is -



(A) $F = \sum m(0, 1, 3, 5, 9, 10, 14)$

(B) $F = \sum m(2, 3, 5, 7, 8, 12, 13)$

(C) $F = \sum m(1, 2, 4, 5, 11, 14, 15)$

(D) $F = \sum m(2, 3, 5, 7, 8, 9, 12)$

\Rightarrow	A	B	C	D	F
00	0	0	0	0	0 (0)
	0	0	0	1	0 (1)
	0	0	1	0	1 (2) ✓
	0	0	1	1	1 (3) ✓
01	0	1	0	0	0 (4)
	0	1	0	1	1 (5) ✓
	0	1	1	0	0 (6)
	0	1	1	1	1 (7) ✓
10	1	0	0	0	1 (8) ✓
	1	0	0	1	1 (9) ✓
	1	0	1	0	0 (10)
	1	0	1	1	0 (11)
11	1	1	0	0	1 (12) ✓
	1	1	0	1	0 (13)
	1	1	1	0	0 (14)
	1	1	1	1	0 (15)

cr-2

Q-14) What are the min numbers of 2-to-1 MUX required to generate a 2-i/p AND & a 2-i/p EX-OR gate?

(a) 1 & 2.

(c) 1 & 3.

✓ (b) 1 & 1.

(d) 2 & 2.

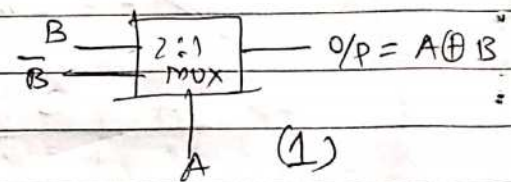
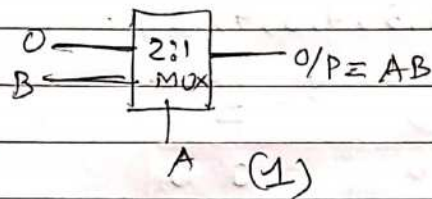
⇒

AND

	A	B	O/P
0	0	0	0
	0	1	0
1	1	0	0
	1	1	1

EXd-OR

	A	B	O/P
	0	0	0
	0	1	1
	1	0	1
	1	1	0



cr-3

Q-15) without any additional circuitry, an 8:1 MUX can be used to obtain -

(a) some but not all Boolean function of 3-variables.

(b) all functions of 3-variables but none of 4-variables.

✓ (c) " " " " and some but not all of 4-variables.

(d) all functions of 4 variables.

7/18