UPSEE 2012 - Solved Paper

Physics

Question 1

The dimensions of capacitance are

Options:

A. $[ML^{-2}Q^{-2}T^{2}]]$

B. $[M^{-1}L^{2}T^{-2}Q^{-2}]$

C. $[M^{-1}L^{2}T^{-2}Q^{2}]$

D. $[M^{-1}L^{-2}T^{2}Q^{2}]$

Answer: D

Solution:

$$\text{Capacitance}=\frac{Charge}{Potential\ difference}$$

$$\text{And potential difference} = \frac{Work\ Charge}{Charge}$$

$$\text{Capacitance}=\frac{charge2}{Work\ done}$$

$$[C]=\frac{\left[Q^{2}\right]}{\left[ML^{2}T^{-2}\right]}$$

$$=[M^{-1}L^{-2}T^{2}Q^{2}]$$

Question 2

A scooter going to the east at 10 m/s turns right through an angle of 90^{0}. If the speed of the scooter remains unchanged in taking the turn, the change in the velocity of the scooter is

Options:

A. 20 m/s in south-west direction

B. zero

C. 10.0 m/s in south direction

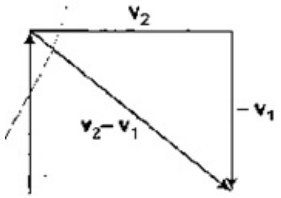
D. 14.14 m/s in south-western direction

Answer: D

Solution:

The velocity of the scooter when it is going in the east direction is, V_1 = 10 m/s

$|\Delta V| = |V_2 - V_1|$



Here, V_2 is perpendicular to V_1 and V_2 = V_1=10 m/s

$|\triangle V| = \sqrt{(V_2)^2 + (V_1)^2}$

$|\triangle V| = \sqrt{(10)^2 + (10)^2}$

$|\Delta V| = 10\sqrt{2} = 10 \text{x} 1.414$

$|\Delta V|$ =14.14 m/s

the change in the velocity of the scooter is 14.14 m/s in south-western direction.

Question 3

A man of height h walks in a straight path towards a lamp post of height H with uniform velocity u. Then, the velocity of the edge of the shadow on the ground will be

Options:

A. $\frac{hu}{H-h}$

B. $\frac{hu}{H+h}$

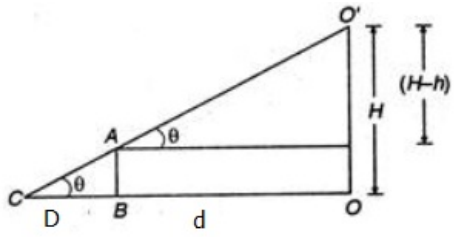
C. $\frac{H-h}{hu}$

D. $\frac{H+h}{hu}$

Answer: A

Solution:

Given that the velocity of the man is u.
Let the velocity of the edge of the shadow is v.
The distance covered by the man and edge of the shadow is respectively d and D in same time t.



From the above image

$\tan\theta = \frac{H-h}{d} = \frac{H}{d+D}$

$(H-h)(d+D) = Hd$

$Hd + HD - hd - hD = Hd$

$D(H-h) = hd$

$\frac{d}{D} = \frac{H-h}{h}$

Since d = ut and D = vt, So $\frac{d}{D} = \frac{u}{v}$

$\frac{u}{v} = \frac{H-h}{h}$

$\frac{v}{u} = \frac{h}{H-h}$

$v = \frac{hu}{H-h}$

Question 4

A ball is projected upward from the top of a tower with a velocity of 50 m/s making an angle of 30^0 with the horizontal. The height of the tower is 70m. After how many seconds from the instant of throwing will ball reach the ground ?

Options:

A. 2s

B. 5 s

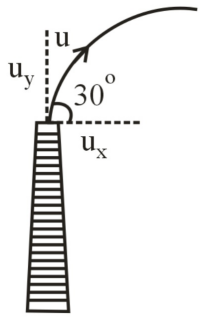
C. 7s

D. 9s

Answer: C

Solution:

The vertically upward component of the velocity of projection is $u_y = 50\sin 30^0$ m/s=25m/s.



Let at time t ball reach the ground then according to the equation of uniformly accelerated motion

$$s = u_y t + \frac{1}{2} g t^2$$

$$70 = -25t + \frac{1}{2} \times 10 \times t^2$$

$$70 = -25t + 5t^2$$

$$5t^2 - 25t - 70 = 0$$

$$t^2 - 5t - 14 = 0$$

$$t^2 - 7t + 2t - 14 = 0$$

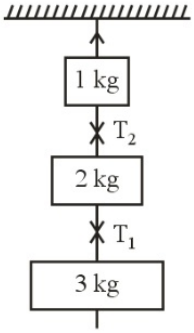
$$t(t-7) + 2(t-7) = 0$$

$$(t-7)(t+2) = 0$$

$t = -2s$ or t=7 sec

Question 5

In the system shown in the adjoining figure, the tension T_2 is



Options:

A. g

B. 2g

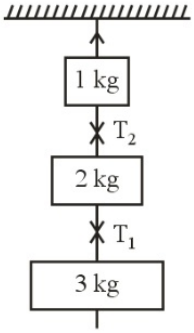
C. 5g

D. 6g

Answer: C

Solution:

Free body diagram of the system is



From the above figure

$T_1 = 3g$

$2g + T_1 = T_2$

$T_2 = 2g + 3g = 5g$

Question 6

A bullet when fired at a target with a velocity of 100 m/s penetrates one metre into it. If the bullet is fired at a similar target with a thickness 0.5. then, it will emerge from it with a velocity of

Options:

A. $50\sqrt{2}$ m/s

B. $\frac{50}{\sqrt{2}}$ m/s

C. 50 m/s

D. 10 m/s

Answer: A

Solution:

Let v be the velocity with which the bullet fired at a target of thickness 0.5.

According to the work-energy theorem

Change in kinetic energy = Work done

In first case,

$$\frac{1}{2}m(100)^2 - \frac{1}{2}m(0)^2 = F \times 1$$

$$\frac{1}{2}m(100)^2 = F \quad \text{.....(1)}$$

In second case,

$$\frac{1}{2}m(100)^2-\frac{1}{2}m(v)^2=F\times 0.5 \quad \text{.....(2)}$$

Dividing Eq. (2) by Eq. (1), we get

$$\frac{\frac{1}{2}m(100)^2-\frac{1}{2}m(v)^2}{\frac{1}{2}m(100)^2}=\frac{0.5F}{F}$$

$$\frac{(100)^2-v^2}{(100)^2}=\frac{0.5}{1}$$

$$1-\frac{v^2}{(100)^2}=\frac{1}{2}$$

$$\frac{v^2}{(100)^2}=\frac{1}{2}$$

$$v^2=\frac{1}{2}(100)^2$$

$$v=\frac{100}{\sqrt{2}}$$

$$v=50\sqrt{2}\ \text{m/s}$$

Question 7

A 500 kg boat is 9 m long and is floating without motion on still water. A man of mass 100 kg is at one end and if he runs to the other end of the boat and stops, the displacement of the boat is

Options:

A. 1.5 m in the direction of displacement of the man

B. 0.75 m in the direction of opposite to the displacement of the man

C. 1.5 m in the direction opposite to the displacement of the man

D. 0.75 m in the direction opposite to the displacement of the man

Answer: C

Solution:

Given that the system (Boat + man) is initially at rest, then the velocity of the centre of mass of the system is, $V_{CM}=0$

$$\therefore \frac{m_1v_1+m_2v_2}{m_1+m_2}=0$$

$$m_1v_1+m_2v_2=0$$

$$m_1\frac{\Delta r_1}{\Delta t}+m_2\frac{\Delta r_2}{\Delta t}$$

$$m_1\Delta r_1+m_2r_2=0$$

If the man moves towards right the boat will move towards left

$$m_1\Delta r_1\pm m_2\Delta r_2$$

If Δr_2 is the displacement of the boat relative to shore.

Then the displacement of man relative to shore would be (9 - Δr_2)

$$\Delta r_1 = 9 - \Delta r_2$$
$$m_1(9 - \Delta r_2) = m_2 \Delta r_2$$
$$100(9 - \Delta r_2) = 500 \Delta r_2$$
$$\Delta r_2 = \frac{100 \times 9}{600} = 1.5m$$

Hence the boat moves 1.5 m relative to shore in the direction opposite to the displacement of man.

Question 8

A man weighing 80 kg is standing on a trolley weighing 320 kg. The trolley is resting on frictionless horizontal rails. If the man starts walking on the trolley along the rails at speed 1 m/s. Then after 4 s, his displacement relative to ground will be

Options:

A. 4.5 m

B. 5 m

C. 8m

D. 3.2 m

Answer: D

Solution:

Let V_{mg} = velocity of the man with respect to ground
V_{tg} = velocity of the trolley with respect to ground
V_{mt} = velocity of man with respect to trolley = 1 m/s
Let M = mass of person = 80 kg
Let M' = the mass of trolley = 320kg
from relative motion velocity of the trolley with respect to ground is
$V_{tg} = V_{mg} - V_{mt}$ .....(1)

Since initially trolley and man were at rest hence the initial momentum = 0
According to the law of conservation of momentum

$V_{mg}M + V_{tg}M' = 0$

Substituting the value of V_{tg} from equation (1)

$V_{mg}M + (V_{mg} - V_{mt})M' = 0$
$V_{mg}(M + M') - V_{mt}M'$

$V_{mg}(320 + 80) = 1(320)$
$V_{mg} = 320/400 = 0.8$ m/s

Thus displacement of man with respect to observer = $V_{mg} \times$ time = $0.8 \times 4 = 3.2$ m

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Question 9

The magnitude of gravitational field at distances r_1 and r_2 from the centre of a uniform spheres of radius R and mass M is F_1 and F_2 respectively. Then

Options:

A. $\frac{F_1}{F_2}=\frac{r_1}{r_2}$ if $r_1<R$ and $r_2<R$

B. $\frac{F_1}{F_2}=\frac{r_2^2}{r_1^2}$ if $r_1>R$ and $r_2>R$

C. Both (A) and (B)

D. None of the above

Answer: C

Solution:

Consider the following two cases

Case I- when $r_1 > R$ and $r_2 > R$

The points are outside the sphere in this case, the gravitational field is

So, $F_1=\frac{GM}{r_1^2}$ and $F_2=\frac{GM}{r_2^2}$

$\frac{F_1}{F_2}=\frac{r_2^2}{r_1^2}$

Option b is correct

Case II- When $r_1<R$ and $r_2<R$

The points are inside the sphere within the sphere of radius

Now, $M_1=\frac{4}{3}\pi r^3\rho$

$F_1=G.\frac{4}{3}\pi r_1^3\rho/r_1^2$

$F_2=G.\frac{4}{3}\pi r_2^3\rho/r_2^2$

$\frac{F_1}{F_2}=\frac{r_1}{r_2}$

Option (a) is also correct.

Question 10

One end of a uniform bar of weight w_1 is suspended from the roof and a weight w_2 is suspended from the other end, the area of cross section is

A.What is the stress at the midpoint of the rod?

Options:

A. $\frac{w_1 + w_2}{A}$

B. $\frac{w_1 - w_2}{A}$

C. $\frac{(w_1/2) + w_2}{A}$

D. $\frac{W_2/2 + w_1}{A}$

Answer: C

Solution:

Stress-When a body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force.

Stress = $\frac{F}{A}$

Stress=$\frac{(Weight\ w2 + half\ of\ the\ weight\ of\ the\ rod}{area}$

Stress=$\frac{(w_1/2) + w_2}{A}$

Question 11

The potential energy of a molecules on the surface of a liquid compared to the one inside the liquid is

Options:

A. zero

B. lesser

C. equal

D. greater

Answer: D

Solution:

Surface tension- It is a property by virtue of which the free surface of liquid at rest behaves like stretched membrane tending to contract to possess minimum surface area.

Surface energy- It is defined as the amount of work done against the force of surface tension in increasing the given surface area of liquid surface at a constant temperature.

The potential energy of a molecule on the surface of a liquid compared to the one inside the liquid is greater as the surface tension is acting on the surface molecules; leading to more potential energy on the surface.

Question 12

The fraction of a floating object of a volume V_0 and density d_0 above the surface of a liquid of density d will be

Options:

A. $\frac{d_0}{d}$

B. $\frac{dd_0}{d+d_0}$

C. $\frac{d - d_o}{d}$

D. $\frac{dd_0}{d.d_0}$

Answer: C

Solution:

Weight if the object is

$d_o g V_{obj} = mg = d_o V_o g$; *where* $V_{object\ immersed} = V_{obj}$

Now, on solving, we get:

$\frac{V_{obj}}{V_o} = \frac{d_o}{d}$

$1 - \frac{V_{obj}}{V_o} = 1 - \frac{d_o}{d}$

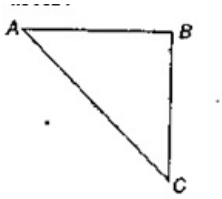
$\frac{V_o - V_{obj}}{V_o} = \frac{d - d_o}{d}$

$\frac{V_{floating}}{V_o} = \frac{d - d_o}{d} = Required\ fraction\ of\ floating\ portion\ of\ the$

object to the total volume of the object.

Question 13

In a metallic triangular sheet ABC, AB = BC = l . If M is mass of sheet, what is the moment of inertia about AC?



Options:

A. $\frac{(Ml)^2}{8}$

B. $\frac{Ml^2}{4}$

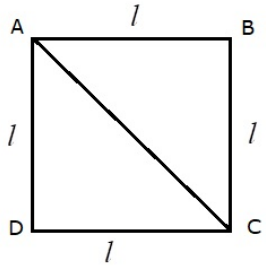
C. $\frac{Ml^2}{6}$

D. $\frac{(Ml)^2}{12}$

Answer: D

Solution:

Let us assume a square sheet ABCD of side l of the same material.



The mass of the square sheet is 2M.

The moment of Inertia of the square along the diagonal will be $\frac{(2Ml)^2}{12}$

Now, Moment of Inertia of the triangular sheet along AC will be $\frac{(Ml)^2}{12}$ which is half the moment of inertia of the sheet in square shape.

Question 14

A small ball describes a horizontal circle on the smooth inner surface of a conical funnel. If the height of the plane of the circle above the vertex be 10 cm, what is the speed of the particle?

Options:

A. 2 m/s

B. 4 m/s

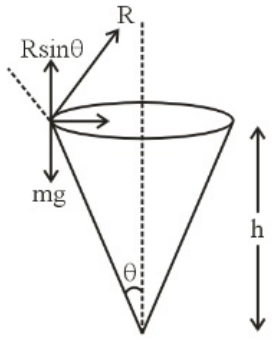
C. 16 m/s

D. 1m/s

Answer: D

Solution:

Let the ball move in a circle of radius R.



$R\sin\theta = mg$

$R\cos\theta = \frac{mv^2}{r}$

$\therefore \tan\theta = \frac{rg}{v^2}$

Or $\frac{r}{h} = \frac{rg}{v^2}$

$v = \sqrt{gh}$

$v = \sqrt{10 \times \frac{10}{100}}$

$v = \sqrt{1}$

$v = 1 \text{ m/s}$

Question 15

A satellite of mass m is orbiting around the earth at a height equal to twice the radius of the earth R. Its potential energy is given by

Options:

A. -2 mgR

B. $-mg\frac{R}{2}$

C. $-2mg\frac{R}{3}$

D. $-mg\frac{R}{3}$

Answer: C

Solution:

Given that the mass of the satellite is m.
The height of the satellite from the center of the earth is, H = R + 2R = 3R
Then potential energy of the satellite is,

Potential energy $PE = \int_{R}^{3R} -\frac{GMm}{r^2} dr$

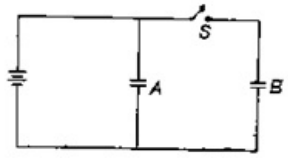
$$= -GMm\left[\frac{r^{-1}}{-1}\right]_{R}^{3R}$$

$$= GMm\left[\frac{1}{3R} - \frac{1}{R}\right] = \frac{2}{3}\frac{GMm}{R}$$

$$= \frac{2}{3}\frac{gR^2m}{R} = -\frac{2}{3}mgR$$

Question 16

Two identical air filled parallel plate capacitors are charged to the same potential in the manner shown by closing the switch S. If now the switch S is opened and the space between the plates is filled with a dielectric of relative permittivity ε_r, then



Options:

A. The potential difference as well as charge on each capacitor goes up by a factor ε_r

B. The potential difference as well as charge on each capacitor goes down by a factor ε_r

C. The potential difference across A remains constant and the charge on B remains unchanged.

D. the potential difference across B remains constant while the charge on A remains unchanged.

Answer: C

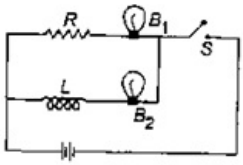
Solution:

When switch S is opened, capacitor A is still connected with battery. Therefore, potential difference across A remains constant. As B is disconnected from battery, charge on B remain fixed.

Question 17

Figure represents two bulbs B_1 and B_2, resistor R and an inductor L.

When the switch S in turned off



Options:

A. Both B_1 and B_2 die out promptly

B. both B_1 and B_2 die out with some delay

C. B_1 dies out promptly but B_2 with some delay

D. B_2 dies out promptly but B_1 with some delay

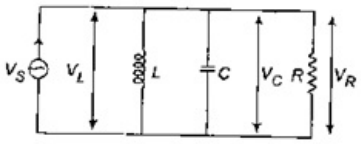
Answer: C

Solution:

Bulb B1 dies out promptly, but bulb B2 diet out with some delay. This is because of self-inductance emf across L, during delay.

Question 18

An AC source is connected in parallel with an L-C-R circuit as shown. Let I_S,I_L,I_C and I_R denote the currents through and V_S,V_L,V_C and V_R the voltages across the corresponding components. Then



Options:

A. $I_s=I_L+I_C+I_R$

B. $V_s=V_L+V_C+V_R$

C. $(I_L,I_C,I_R)<I_s$

D. I_L,I_C may be greater than I_s.

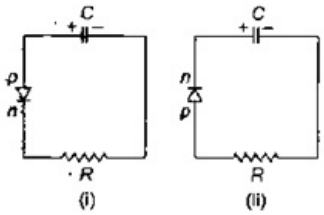
Answer: D

Solution:

In parallel resonant circuit, current through L and C may be greater that the source current i.e. I_L,I_c may be greater than I_S

Question 19

Two identical capacitors each of capacitance C are charged to the same potential V and are connected in two circuits (i) and (ii) at t=0 as shown. The charged on the capacitor at t=CR are



Options:

A. $\frac{CV}{e},\frac{CV}{e}$

B. CV,CV

C. $\frac{CV}{e}$,CV

D. $VC,\frac{CV}{e}$

Answer: C

Solution:

In Fig. (i) the p-n junction diode is forward biased and represents a very low resistance, the capacitor, therefore discharges itself through resistor R according to relation.

$q = q_0 e^{-t/CR}$

And $q_0 = CV$ at t=CR

$\therefore q = q_0 e^{-1} = \frac{CV}{e}$

In Fig.(ii), the p-n junction diode is reversed biased, the capacitor , therefore holds the charge intact.

$\therefore q = q_0 = CV$

Question 20

If a semiconductor has an intrinsic carrier concentration of 1.41 x $10^{16}/m^3$, when doped with $10^{21}/m^3$ phosphorous atoms, then the concentration of holes/m^3 at room temperature will be

Options:

A. $2x10^{21}$

B. $2x10^{11}$

C. $1.41x10^{10}$

D. $1.41x10^{16}$

Answer: D

Solution:

Given that the carrier concentration of semiconductor is, $1.41 \times 10^{16}/m^3$

Impurity of phosphorous is, $10^{21}/m^3$

Doping will increase the number of electrons only and not the holes.

So, number of holes will be equal to number of intrinsic carrier concentration $=1.41x10^{16}/m^3$

Question 21

A parallel beam of light is incident normally on a plane surface absorbing 40% of the light and reflecting the rest. If the incident beam carries 60W of power, the force exerted by it on the surface is

Options:

A. $3.2x10^{-18}N$

B. $3.2x10^{-7}N$

C. $5.12x10^{-7}N$

D. $5.12x10^{-8}N$

Answer: B

Solution:

Momentum of incident light per second

$$P_1 = \frac{E}{C} = \frac{60}{3\times10^8} = 2\times10^{-7}$$

Momentum of reflected light per second

$$P_2 = \frac{60}{100}\times\frac{E}{c}$$

$$= \frac{60}{100}\times\frac{60}{3\times10^8}$$

Force on the surface$=P_2-(-P_1)$

$F = P_2+P_1$

$F = (2+1.2)x10^{-7}$

$F = 3.2 \times 10^{-7}$ N

Question 22

A fraction f_1 of a radioactive sample decays in one mean life and a fraction f_2 is one half-life

Options:

A. $f_1 > f_2$

B. $f_1 < f_2$

C. $f_1 = f_2$

D. may be A., B. or C. depending of the values of the mean life and half-life

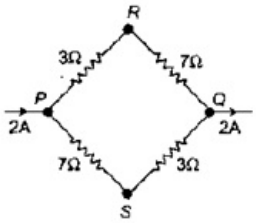
Answer: A

Solution:

Fraction decayed in one mean life must be bigger than the fraction decayed in one half life,
i.e., $f_1 > f_2$

Question 23

A current of 2 A flows in an electric circuit as shown in figure. The potential difference (V_R - V_S), in volts (V_R and V_S are potentials at R and S respectively is.



Options:

A. -4

B. +2

C. +4

D. -2

Answer: C

Solution:

Current through each arm PRQ and PSQ is = 1 Amp.
Potential across upper part and lower part is same and is equal to 10 V
Potential at R is V_R = 7V
Potential at S is V_S = 10 - 7 = 3V
V_R - V_S = 7V - 3V = 4V

Question 24

When a battery connected across a resistor of 16Ω, voltage across the resistor is 12V. When the same battery is connected across a resistor of 10Ω, voltage across it is 11V. The internal resistance of the battery (in ohms) is

Options:

A. $\frac{10}{7}$

B. $\frac{20}{7}$

C. $\frac{25}{7}$

D. $\frac{30}{7}$

Answer: B

Solution:

Given that the voltage (V) across resistance < emf (E) of the battery
emf of the battery is
$\therefore$ E = V + Ir
where r is the internal resistance of the battery.
Case I-
$E=12+\frac{12}{16}r$ .....(i)
Case II-
$E=11+\frac{11}{10}r$ .....(ii)
From eqn(i) and (ii)
$12+\frac{12}{16}r=11+\frac{11}{10}r$

$1 = \frac{11}{10}r - \frac{12}{16}r$

$1 = \frac{56}{160}r$

$r = \frac{160}{56}$

$r = \frac{20}{7}$

Question 25

One junction of a certain thermoelectric couple is at a fixed temperature T_r and the other junction is at temperature T. The thermo-electromotive force for this is expressed by $E = k(T - T_r)\left[T_0 - \frac{1}{2}(T + T_r)\right]$. At temperature $T = \frac{1}{2}T_0$, the thermoelectric power is

Options:

A. $\frac{1}{2}kT_0$

B. kT_0

C. $\frac{1}{2}kT_0^2$

D. $\frac{1}{2}k(T_0 - T_r)^2$

Answer: A

Solution:

Given that $E = k(T - T_r)\left[T_0 - \frac{1}{2}(T + T_r)\right]$

We know that thermoelectric power

$S = \frac{dE}{dT}$

$S = \frac{d}{dT}\left(K(T - T_r)\left[T_o - \frac{1}{2}(T + T_r)\right]\right)$

$S = K\left[T_o - \frac{1}{2}(T + T_r)\right] + K(T - T_r)\left(-\frac{1}{2}\right)$

$|S|_{T = \frac{T_o}{2}} = K\left[T_o - \frac{1}{2}\left(\frac{T_o}{2} + T_r\right)\right] + K\left(\frac{T_o}{2} - T_r\right)\left(-\frac{1}{2}\right)$

$|S| = KT_o - \frac{KT_o}{4} + KT_r - \frac{KT_o}{4} - KT_r$

$|S| = KT_o - \frac{KT_o}{2}$

$|S| = \frac{KT_o}{2}$

Question 26

Two concentric coils of 10 turns each are placed in the same plane. Their radii are 20cm and 40 cm and carry 0.2 A and 0.3 A current respectively in opposite directions. The magnetic induction (in Tesla) at the centre is

Options:

A. $\frac{3}{4}\mu_0$

B. $\frac{5}{4}\mu_0$

C. $\frac{7}{4}\mu_0$

D. $\frac{9}{4}\mu_0$

Answer: B

Solution:

Two coils carry currents in opposite directions, hence net magnetic field at centre will be difference of two fields.
i.e.,

$$B_{net} = \frac{\mu_0}{4\pi}.2\pi r\left[\frac{i_1}{r_1} - \frac{i_2}{r_2}\right]$$

$$= \frac{10\mu_0}{2}.\left[\frac{0.2}{0.2} - \frac{0.3}{0.4}\right]$$

$$= \frac{5}{4}\mu_0$$

Question 27

The number of turns in primary and secondary coils of a transformer is 50 and 200 respectively. IF the currents in the primary coil is 4 A, then the current in the secondary coil is

Options:

A. 1 A

B. 2 A

C. 4 A

D. 5 A

Answer: A

Solution:

In a transformer, Ratio of the number of turns in primary coil to the secondary coil is

$$\therefore \frac{N_P}{N_S} = \frac{I_S}{I_P}$$
$$\frac{50}{200} = \frac{I_S}{4}$$
$$\Rightarrow I_s = 1A$$

Question 28

X-rays of wavelength 0.140 nm are scattered from a block of carbon. What will be the wavelengths of X-rays scatterd at 90^0 ?

Options:

A. 0.140 nm

B. 0.142 nm

C. 0144 nm

D. 0.146 nm

Answer: B

Solution:

Given that the scattering angle, $\phi = 90^0, \cos\phi = 0$
The scattered wavelength of X-ray is

$$\lambda' = \lambda + \frac{h}{m_e c}$$
$$= 0.140\times10^{-9} + \frac{6.63\times10^{-34}}{(9.1\times10^{-31})(3\times10^8)}$$
$$= (0.140\times10^{-9} + 2.4\times10^{-12})m$$
$$= 0.142n - m$$

Question 29

In Fraunhoffer diffraction experiment, L is the distance between screen and the obstacle, b is the size of obstacle and λ is wavelength of incident light. The general condition for the applicability of Fraunhoffer diffraction is

Options:

A. $\frac{b^2}{L\lambda} >> 1$

B. $\frac{b^2}{L\lambda}=1$

C. $\frac{b^2}{L\lambda}<<1$

D. $\frac{b^2}{L\lambda}\neq 1$

Answer: C

Solution:

The general condition for Fraunhoffer differection is $\frac{b^2}{L\lambda}<<1$

Question 30

With a standard rectangular bar magnet the time period of a vibration magnetometer is 4 s. The bar magnet is cut parallel to its length into four equal pieces. The time period of vibration magnetometer when one piece is used (in second)(bar magnet breadth is small) is

Options:

A. 16

B. 8

C. 4

D. 2

Answer: C

Solution:

Time period of magnet, $T=2\pi\sqrt{\frac{I}{MB}}$
When magnet is cut parallel to its length into four equal pieces.
Then
Magnetic moment, M'=$\frac{M}{4}$
New moment of inertia I'=$\frac{I}{4}$
New time period T'=$2\pi\sqrt{\frac{I'}{M'B}}$
T'=4sec

Question 31

At a given place where acceleration due to gravity is g m/s2, a sphere of lead of density d kg/m3 is gently released in a column of liquid of density ρ kg/m3. If d> ρ, the sphere will

Options:

A. fall vertically with an acceleration g m/s2

B. fall vertically with no acceleration

C. fall vertically with an acceleration $g\left(\frac{d-\rho}{d}\right)$

D. fall vertically with an acceleration $g\left(\frac{\rho}{d}\right)$

Answer: C

Solution:

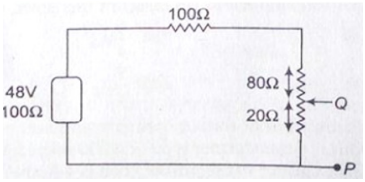
Apparent weight =actual weight - upthrust force

$$Vdg' = Vdg - V\rho g$$

$$g' = \left(\frac{d-\rho}{d}\right)g$$

Question 32

In the circuit, the potential difference across PQ will be nearest to



Options:

A. 9.6 V

B. 606 V

C. 4.8 V

D. 3.2 V

Answer: D

Solution:

Let the current i is flowing through the circuit. Since all the resistance are in series then current is

$$i = \frac{48}{100+100+80+20}$$

i = 0.16 Amp

Since the resistance across PQ is 20 ohm.
The potential difference across the resistance of 20Ω, V = i x 20
V =3.2 V

Question 33

A metal sphere of radius r and specific heat s is rotated about an axis passing through its centre at a speed of n rotations per second. It is suddenly stopped and 50% of its energy is used in increasing its temperature. Then the rise in temperature of the sphere is

Options:

A. $\frac{2\pi^2 n^2 r^2}{5s}$

B. $\frac{\pi^2 n^2}{10 r^2 s}$

C. $\frac{7}{8}\pi^2 r^2 n^2 s$

D. $\frac{5(\pi m)^2}{14s}$

Answer: A

Solution:

We know that the Angular frequency is $\omega = 2\pi n$ rad/s
Moment of inertia of a sphere is

$$I = \frac{2}{5}mr^2$$

$$KE = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}.\frac{2}{5}mr^2 \times (2\pi n)^2$$

$$= \frac{4}{5}m\pi^2 r^2 n^2$$

Since half of this energy is converted into heat then

$$dQ = \frac{4}{5}m\pi^2 n^2 r^2 \times \frac{1}{2}$$

Specific heat is

$$s = \frac{1}{m}\frac{dQ}{dT}$$

$$dT = \frac{dQ}{ms}$$

$$dT = \frac{2/5m\pi^2r^2n^2}{ms}$$

$$dT = \frac{2m\pi^2r^2n^2}{5s}$$

Question 34

Let n_p and n_e be the numbers of holes and conduction electrons in an extrinsic semiconductor, then

Options:

A. $n_p > n_e$

B. $n_p = n_e$

C. $n_p < n_e$

D. $n_p \neq n_e$

Answer: D

Solution:

S.No.	Option	Conclusion
1.	$n_p > n_e$	Given that the number of holes is greater then the number of electrons so this is the condition of p-type semiconductor.
2.	$n_p = n_e$	Given that the number of holes is equals to the number of electrons so this is the condition of an intrinsic semiconductor.
3.	$n_p < n_e$	Given that the number of holes is less then the number of electrons so this is the condition of the n-type semiconductor.
4.	$n_p \neq n_e$	Given that the number of electron is not equal to the number of holes which is completely satisfied with the condition of an extrinsic semiconductor.

Question 35

Two particles P and Q describe SHM of same amplitude a frequency v along the same straight line. The maximum distance between two particle is $\sqrt{2a}$. The initial phase difference between the particles is

Options:

A. zero

B. $\pi/2$

C. $\pi/6$

D. $\pi/3$

Answer: B

Solution:

Let φ is the phase difference between the both particle SHM motion. Then equation of motion

$$y_1 = a\sin\omega t$$

$$y_2 = a\sin(\omega t+\phi)$$

$$y_2 - y_1 = a\sqrt{2} = a\sin(\omega t+\phi) - a\sin\omega t$$

$$\sqrt{2}a = 2a\cos\left\{\frac{\omega t+\phi+\omega t}{2}\right\}\sin\left(\frac{\phi+\omega t-\omega t}{2}\right)$$

$$\sqrt{2}a = 2a\cos\left(\omega t+\frac{\phi}{2}\right)\sin\frac{\phi}{2}$$

For maximum value, $\cos\left(\omega t+\frac{\phi}{2}\right)=1$

$$2\sin\frac{\phi}{2} = \sqrt{2}$$

$$\sin\frac{\phi}{2} = \frac{1}{\sqrt{2}}$$

$$\frac{\phi}{2} = \frac{\pi}{4}$$

$$\phi = \frac{\pi}{2}$$

Question 36

A wire is stretched as to change its diameter by 0.25%. The percentage change in resistance is

Options:

A. 4.0%

B. 2.0%

C. 1.0%

D. 0.5%

Answer: C

Solution:

On stretching the wire, volume (V) of the wire remains same.

So, V=Al

Or l=$\frac{V}{A}$

The resistance of the wire is,

$$R = \rho.\frac{l}{A}$$

$$= \rho.\frac{V}{A^2} = \frac{\rho V}{\pi^2 D^4/16}$$

$$= \frac{16\rho V}{\pi^2 D^4}$$

Taking logarithm of both sides and differentiating, we get

$$\frac{\Delta R}{R} = -4\frac{\Delta D}{D}$$

$$\frac{\Delta R}{R} = -4 \times 0.25 = 1.0\%$$

Question 37

A coil of inductance 8.4 mH and resistance 6Ω is connected to 12V battery. The current in the coil is 1 A at approximately the time

Options:

A. 500s

B. 20s

C. 35 ms

D. 1 ms

Answer: D

Solution:

The current-time (i-t) equation in L-R circuit is given by (growth current in L-R circuit)

$$i = i_0(1 - e^{-t/\tau_L}) \qquad \text{.....(i)}$$

Where, $i_0 = \frac{V}{R} = \frac{12}{6} = 2A$

$$\tau_L = \frac{L}{R}$$

$$\tau_L = \frac{8.4 \times 10^{-3}}{6}$$

$$\tau_L = 1.4 \times 10^{-3}\ s$$

And $i = 1A(given)$

Putting the above values in Eq.(i), we get

$$1 = 2\left(1 - e^{-\frac{t}{1.4\times10^{-3}}}\right)$$

$$\left(1 - e^{-\frac{t}{1.4\times10^{-3}}}\right) = 0.5$$

$$e^{-\frac{t}{1.4\times10^{-3}}} = 0.5$$

Taking log on both side

$$-\frac{t}{1.4\times10^{-3}} = \ln(0.5)$$

$$-\frac{t}{1.4\times10^{-3}} = -0.69$$

$$t = 0.97\times10^{-3}$$

$$t = 1\ ms$$

Question 38

In a stationary wave,

Options:

A. strain is maximum at antinodes

B. strain is maximum at nodes

C. strain is minimum at nodes

D. amplitude is zero at all points

Answer: B

Solution:

Question 39

A string of density 7.5 g cm^{-3} and area of cross-section 0.2 mm^2 is stretched under a tension of 20N. When it is plucked at the mid point, the speed of the transverse waves on the wire

Options:

A. 116 m/s

B. 40 m/s

C. 200 m/s

D. 80 m/s

Answer: A

Solution:

T=20N, A=0.2mm^2=0.2x10^{-6}m^2

And ρ=7.5g cm^{-2}=7.5x10^3 kgm^{-3}

Mass per unit length of the wire $\mu = A\rho$

=0.2x10^{-6}x7.5x10^3

=1.5x10^{-3} kgm^{-1}

When a string is plucked at the mid-point, it vibrates with fundamental frequency.

Fundamental frequency

$$v = \frac{1}{2l}\sqrt{\frac{T}{M}}$$

$$= \frac{1}{2l}\sqrt{\frac{20}{1.5\times10^{-3}}}$$

Speed of transverse wave $v = vx$

$$v = \frac{1}{2l}\sqrt{\frac{20}{1.5\times10^{-3}}}\times x$$

$$= 116m/s$$

Question 40

Amplitude of a wave is represented by $A = \frac{c}{a+b-c}$ Then, resonance will occur when

Options:

A. b=-c/2

B. b=0 and a=c

C. b=1a/2

D. None of these

Answer: B

Solution:

$A = \frac{c}{a+b-c}$, when b=0, a=c

Amplitude $A \to \infty$, this corresponds to resonance

Question 41

Capacitance of a capacitor made by a thin metal foil is 2μF. If the foil is

folded with paper of thickness 0.15 mm, dielectric constant of paper is 2.5 and width of paper is 400 mm. the length of the foil will be

Options:

A. 0.34 m

B. 1.33 m

C. 13.4 m

D. 33.9 m

Answer: D

Solution:

Given that the capacitance of the capacitor is, C = $2\mu F$
The dielectric constant of the paper is 2.5
If the length of the foil is l, then

$$C = \frac{K\varepsilon_0(l \times b)}{d}$$

$$2\times10^{-6} = \frac{2.5\times8.85\times10^{-12}(l\times400\times10^{-3})}{0.15\times10^{-3}}$$

l=33.9 cm

Question 42

A rod of a certain metal is 1.0 m long and 0.6 cm in diameter. Its resistance is $3.0\text{x}10^{-3}\,\Omega$. Another disc made of the same metal is 2.0 cm in diameter and 1.0 m thick. What is the resistance between the round faces of the disc ?

Options:

A. $1.35\text{x}10^{-8}\,\Omega$

B. $2.70\text{x}10^{-7}\,\Omega$

C. $4.05\text{x}10^{-6}\,\Omega$

D. $8.10\text{x}10^{-5}\,\Omega$

Answer: B

Solution:

The resistivity of the material of the rod

$$\rho = \frac{RA}{l} = \frac{3\times10^{-3}\times\pi(0.3\times10^{-2})^2}{1}$$

$$= 27\times10^{-9}\pi\Omega m$$

The resistivity of the disc,

$R=\frac{Resistivity\ of\ rod \times thickness}{Area\ of\ cross-section}$

$=27\times10^{-9}\pi\times\frac{10^{-3}}{\pi\times(1\times10^{-2})^2}$

$=2.7\times10^{-7}\ \Omega$

Question 43

The cyclotron frequency of an electron grating in a magnetic field of 1 T is approximately

Options:

A. 28 MHz

B. 280 MHz

C. 2.8 MHz

D. 28 GHz

Answer: D

Solution:

The frequency of the cyclotron is

$v=\frac{Bq}{2\pi m}\ \Rightarrow v=\frac{1\times1.6\times10^{-19}}{2\times3.14\times9.1\times10^{-31}}$

v = 2.79 x 10^{10} Hz

v = 28 GHz

Question 44

An electron is accelerated through potential difference of 45.5 volt. The velocity acquired by it is (in m/s)

Options:

A. 4x10^{6}

B. 4x10^{-6}

C. 10^{6}

D. zero

Answer: A

Solution:

Velocity acquired by the accelerated charged particle is

$$v = \sqrt{\frac{2eV}{m}}$$

where m is the mass of the charged particle, e is the charge, and V is the potential difference

$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 45.5}{9.1 \times 10^{-31}}}$$

$$v = 4 \times 10^{6}\, m/s$$

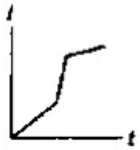
Question 45

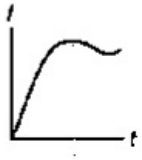
When a battery is connected across a series combination of self-inductance L and resistance R, the variation in the current I with time t is the best represented by

Options:

A. 

B. 

C. 

D. 

Answer: B

Solution:

Current equation for RL circuit is,

$$i = i_0(1 - e^{-R/Lt})$$

At t = 0, i = 0

When $t >> \tau$ where $\tau = \frac{L}{R}$

$$i = i_o$$

So current will exponentially increased and become constant.

Question 46

The magnetic moment produced in a substance of 1 g is $6x10^{-7}$ A-m^2. If its density is 5g/km^2, then the intensity of magnetization in A/m will be

Options:

A. $8.3x10^{6}$

B. 3.0

C. $1.2x10^{-7}$

D. $3x10^{-6}$

Answer: B

Solution:

The intensity of magnetization is

$$I = \frac{M}{V}$$

$$= \frac{M}{mass / density}$$

Given that the mass = 1gm = 10^{-3}kg

And density=5g/cm^3

$$= \frac{5\times10^{-3}}{(10^{-2})^5}$$

$$= 5\times10^{3} kg / m3$$

$$I = \frac{6\times10^{-7}\times5\times10^{3}}{10^{-3}}$$

Question 47

A moving coil galvanometer has 48 turns and area of coil is $4x10^{-2}$ m^2. If the magnetic field is 0.2T, then to increase the current sensitivity by 25% without changing area (A) and field (B) the number of turns should become

Options:

A. 24

B. 36

C. 60

D. 54

Answer: C

Solution:

Current sensitivity $S_1 = \frac{NBA}{C}$

So, $S_1 \propto N$

$\therefore \frac{S_1}{S_2} = \frac{N_1}{N_2} \Rightarrow \frac{100}{125} = \frac{48}{N_2}$

$N_2 = 60$

Question 48

A coil carrying electric current is placed in uniform magnetic field, then

Options:

A. Torque is formed

B. Emf is induced

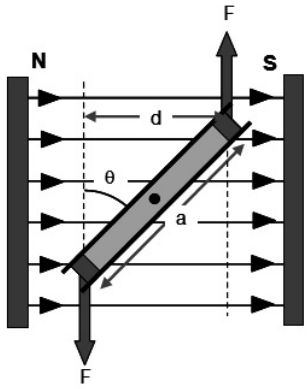
C. Both A. and B. are correct

D. None of the above

Answer: A

Solution:

If a current carrying coil is placed in a magnetic field then the coil will experience a turning and twisting force on it. Which causes torque on that coil.



Question 49

Two copper balls each weighing 10 g are kept in air 10 cm apart. If one electron from every 10^6 atoms is transferred from one ball to the other, the coulomb force berween them is (atomic weight of copper is 63.5)

Options:

A. $2.0x10^{10}$ N

B. 2.0×10^{4} N

C. 2.0×10^{8} N

D. 2.0×10^{6} N

Answer: C

Solution:

Number of atoms in given mass is

$N = \frac{10}{63.5} \times 6.02 \times 10^{23}$

$N = 9.48x10^{22}$

Transfer of electron between balls

$= \frac{9.48 \times 10^{22}}{10^{6}}$

$= 9.48 \times 10^{16}$

Hence the magnitude of charge gained by each ball

$Q = 9.48x10^{16}x1.6x10^{-19}$ C

Q = 0.015 C

Force of attraction between the balls

$F = 9 \times 10^{9} \times \frac{(0.015)^2}{(0.1)^2} = 2 \times 10^{8} N$

Question 50

A simple telescope consisting of an objective of focal length 60 cm and a single eye lens of focal length 5 cm is focused on a distant object is such a way that parallel rays come out from the eye lens. If the object subtends an angle 2 ° at the objective, the angular width of the image

Options:

A. 10°

B. 24°

C. 50°

D. (1/6)°

Answer: B

Solution:

Magnification of telescope is

$m = \frac{\text{Angle subtended by the image}}{\text{Angle subtended by the object}}$

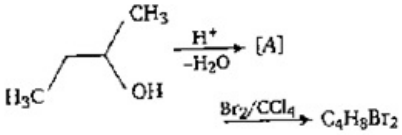
$$\frac{f_0}{f} = \frac{\alpha}{\beta}$$

$$\Rightarrow \alpha = \frac{f_0 \times \beta}{f} = \frac{60 \times 2}{5} = 24^0$$

Chemistry

Question 51

In the following reaction, number of possible structure is



Options:

A. 1

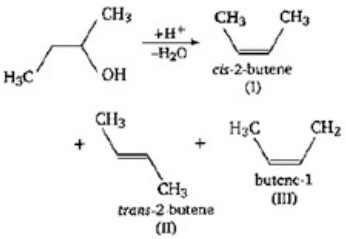
B. 2

C. 5

D. 6

Answer: C

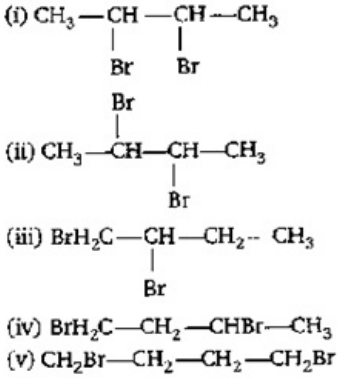
Solution:



Therefore in X, order of stability of alkaline is as follows

II>I>III

All of these alkenes, with Br_2/CCl_4, produce additive product having molecular formula $C_4H_8Br_2$. The possible products are

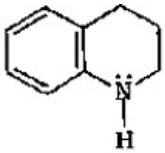


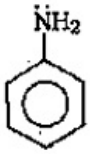
Question 52

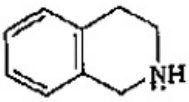
Which of the following is the most basic

Options:

A. 

B. 

C. 

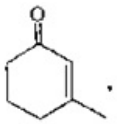
D. 

Answer: D

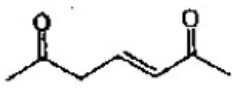
Solution:

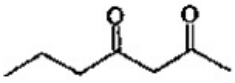
Lone pair of electrons present at N in (a) is in sp^2-hybrid orbital (less available).
Lone pair of electrons present at N in (b) is in hyperconjugation with benzene ring therefore, these are also less available.
Aniline is less basic in nature.
Lone pair of electrons of N in (d) are available freely. So, this compound is the most basic.

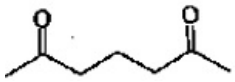
Question 53

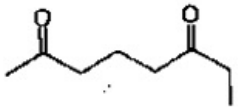
Which of the following will yield  **when reacted with base ?**

Options:

A. 

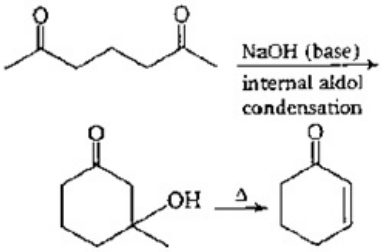
B. 

C. 

D. 

Answer: C

Solution:



Question 54

Which of the following is formed on reduction of oxalic acid with magnesium mercury amalgam and H_2SO_4 ?

Options:

A. Glyoxalic acid

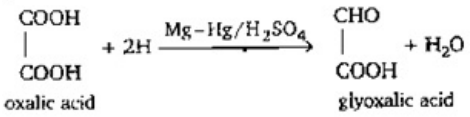
B. Formic acid

C. Glycolic acid

D. CO_2 and H_2O

Answer: A

Solution:



Question 55

On mixing urea with nitrous acid, the composition of the products is

Options:

A. NH_2-NH_2+CO_2

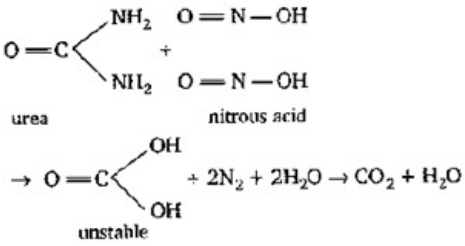
B. N_2+C+H_2O

C. NH_3+CO_2+H_2O

D. N_2+CO_2+H_2O

Answer: D

Solution:



Question 56

Mark out the correct decreasing order of ease of addition of halogen.

Options:

A. $CH_2=CH_2 > CH_3-CH=CH-CH_3 > CH_3-C(CH_3)=CH_2 > (CH_3)_2C=C(CH_3)_2$

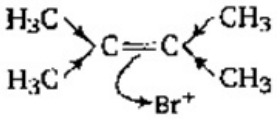
B. $(CH_3)_2C=C(CH_3)_2 > CH_3-C(CH_3)=CH_2 > CH_3-CH=CH-CH_3 > CH_2=H_2$

C. $(CH_3)_2C=C(CH_3)_2 > CH_3-CH=CHCH_3 > CH_3-C(CH_3)=CH_2 > CH_2=CH_2$

D. $CH_2=CH_2 > CH_3-C(CH_3)=CH_2 > CH_3-CH=CH-CH_3 > (CH_3)_2C=C(CH_3)_2$

Answer: B

Solution:



Maximum +I effect, high electron density, higher rate of E^+ addition. As substituent ($-CH_3$) decrease tendency decreases

---

Question 57

IUPAC name of the compound is

Options:

A. 2-methyl-6-ethyloct-1,5-dienal

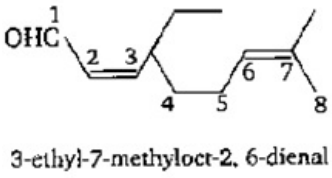
B. 3-ethyl-7-methyloct-2, 6-dienal

C. 7-methyl-3-ethyloct-2, 6-dienal

D. None of the above

Answer: B

Solution:



3-ethyl-7-methyloct-2, 6-dienal

Question 58

Which of the following nitro compounds are not soluble in NaOH ?

Options:

A. $(CH_3)_3C\text{-}NO_2$

B. $(CH_3)_2CH\text{-}NO_2$

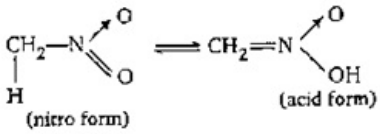
C. $CH_3CH_2\text{-}NO_2$

D. $Ph\text{-}CH_2\text{-}NO_2$

Answer: A

Solution:

Nitro compounds having minimum one α-H, show tautomerism. Due to which their nitro group convert into acid group. In this acidic form, they dissolve in base.



But nitro compounds having no α-H, do not show tautomerism. So, they cannot convert into acid form and thus, cannot dissolve in base.

Question 59

Transport number of Cl- is least in

Options:

A. HCl

B. NaCl

C. KCl

D. CsCl

Answer: A

Solution:

Transport number(T)

$=\frac{Current\ carried\ by\ ion}{total\ current}$

Transport number$\propto$speed of ion

$$T_{Cl^-}(HCl)=\frac{v_{Cl^-}}{v_{Cl^-}+v_{H^+}}$$

$$T_{Cl^-}(NaCl)=\frac{v_{Cl^-}}{v_{Cl^-}+v_{Na^+}}$$

As $v_{H^+}>v_{Na^+}$ or v_{K^+} or v_{Cs^+}

Question 60

Iron pipes, lying in acidic soil, are often attached to the blocks of magnesium for their protection from rusting, because magnesium

Options:

A. is lighter than iron

B. is readily converted into positive ion

C. Forms a corrosion-resistant alloy with iron

D. Prevents air from reaching the surface of iron

Answer: B

Solution:

Magnesium is readily converted into positive ions when they come in contact of acidic soil. They form layer of oxide over acidic soil and help in prevention of rusting of iron pipes.

Question 61

Decomposition of H_2O_2 is a first order reaction. Initial Volume of solution of H_2O_2 having half-life time 15 min is 16 volume then time after which solution becomes 1 volume-

Options:

A. 4

B. 15 min

C. 30 min

D. 60 min

Answer: D

Solution:

$$N_t = N_0 \times \left(\frac{1}{2}\right)^n$$

Where, N_t=after n half lives times volume concentration of H_2O_2=1 volume.

N_0=initial concentration of H_2O_2=16 volume

$$\therefore 1 = 16 \times \left(\frac{1}{2}\right)^n$$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$$

$$N = 4$$

Therefore, after 4 half lives times, volume of H_2O_2 solution becomes 1 volume. Time used in this procedure=4x15 min

=60 min

Question 62

What is the name of the element with atomic number 105 ?

Options:

A. Dubnium

B. Holmium

C. Kurchatovium

D. Nobelium

Answer: A

Solution:

The element with atomic number 105 is Dubnium (Db). Its IUPAC name is Un-nil-pentium.

Question 63

Consider the ground state of Cr atom (Z=24). The numbers of electrons with the azimuthal quantum numbers, l=1 and 2 are respectively

Options:

A. 12 and 4

B. 12 and 5

C. 16 and 4

D. 16 and 5

Answer: B

Solution:

The electronic configuration of Cr(24)=$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1 3d^5$

EC of Cr (Z = 24)	principal quantum number 'n	azimuthal quantum number 'l l=0;s orbital l=1;p orbital l=2;d orbital l=3;f orbital	No. of electrons
$1s^2$	1	0	2
$2s^2$	2	0	2
$2p^6$	2	1	6

$3s^2$	3	0	2
$3p^6$	3	1	6
$4s^1$	4	0	1
$3d^5$	3	2	5

Thus, the number of electrons with l=1 is (6+6)=12 and with l=2 is 5.
Hence option(2) is correct.

Question 64

Sodium bicarbonate on heating decomposes to form sodium carbonate, CO_2 and water. If 0.2 moles of sodium bicarbonate is completely decomposed, how many moles of sodium carbonate is formed

Options:

A. 0.1

B. 0.2

C. 0.05

D. 0.025

Answer: A

Solution:

$\underset{2mol}{2NaHCO_3} \xrightarrow{\Delta} \underset{1mol}{Na_2CO_3} + H_2O + CO_2$ $\because$2 mol $NaHCO_3$ on decomposition gives =1 mol Na_2CO_3

$\therefore$0.2 mil $NaHCO_3$ on decomposition will give $=\frac{1}{2}\times 0.2$
$=0.1$ mol Na_2CO_3

Question 65

Intermolecular hydrogen bonding is strongest in

Options:

A. HCHO

B. CH_3OH

C. C_6H_5OH

D. CH_3NH_2

Answer: B

Solution:

Hydrogen bond is found in compound in which H-bond is directly attached to highly electronegative elements as F, O, N. HCHO show very less H-bonding as H-atoms is not attached to O atom. CH_3OH has stronger H-bond as compared to C_6H_5OH dut to steric hinderence of bulky $-C_6H_5$ group in phenol.

Question 66

What is the two third life of a first order reaction having k=5.48x10^{-14} s^{-1} ?

Options:

A. 1x10^{23} s

B. 2x10^{13}s

C. 8x10^{13}s

D. 5x10^{14}s

Answer: B

Solution:

$t_n = \frac{2.303}{k}\log\frac{[A_0]}{A}$

Let [A]0=a

$\therefore$[A]= $a - \frac{2}{3}a = \frac{a}{3}$

Therefore, $t_{2/3} = \frac{2.303}{5.48\times10^{-24}}\log\frac{a}{\left(\frac{a}{3}\right)}$

$= \frac{2.303}{5.48\times10^{-14}}\log 3$

$= \frac{2.303\times0.4771}{5.48\times10^{-14}}$

$= 2.01\times10^{23}s$

Question 67

In a cell that utilizes the reaction

$Zn(s)+2H^+(aq)\rightarrow Zn^{2+}(aq)+H_2(g)$ **addition of H_2SO_4 to cathode compartment will**

Options:

A. lower the E and shift equilibrium to the left

B. lower the E and shift the equilibrium to the right

C. increase the E and shift the equilibrium to the right

D. Increase the E and shift the equilibrium to the left

Answer: C

Solution:

$$Zn(s)+2H^+ \rightarrow Zn^{2+}(aq)+H_2(g)$$

Reaction quotient Q= $\frac{[Zn^{2+}]}{[H^+]^2}$

Corresponding cell is

$$Zn\left|Zn^{2+}_{anode}(C_1)\right\|H^+(aq)\left|Pt(H_2)\right._{cathode}$$

And $E_{cell}=E^0{}_{cell}-\frac{0.0591}{2}\log K$

$$=E^0{}_{cell}-\frac{0.0591}{2}\log\frac{[Zn^{2+}]}{[H^+]^2}$$

$$=E^0{}_{cell}+\frac{0.0591}{2}\log\frac{[H^+]^2}{[Zn^{2+}]}$$

If H_2SO_4 is added to cathodic compartment, (towards reactant side), then Q decreases (due to increase in H^+).
Hence equilibrium is displaced towards right and E_{cell} increases.

Question 68

Which of the following solvents are aprotic ?
I. NH_3 II. SO_2
III. CH_3CN IV. CH_3CO_2H

Options:

A. I,II,III

B. I,III,IV

C. II,II

D. I,III

Answer: A

Solution:

A solvent molecule lacking a polar X-H bond is called aprotic solvent. NH_3, SO_2 and CH_3CN are aprotic solvents while CH_3COOH is protic solvent.

Question 69

The most efficient agent for the absorption of SO_3 is

Options:

A. 80% H2SO4

B. 98%H_2SO_4

C. 50%H_2SO_4

D. 20%$H_2S_2O_7$

Answer: B

Solution:

$\because$98% H_2SO_4 is used for absorbing dense fog of acid which is formed by dissolving SO_3 in water.
$\therefore$98% H_2SO_4 is the most efficient reagent for absorption of SO_3

Question 70

Which of the following reaction is used to make a fuel cell ?

Options:

A. $Cd(s)+2Ni(OH)_3(s)\rightarrow CdO(s)+2Ni(OH)_2(s)+H_2O(l)$

B. $2Fe(s)+O_2(g)+4H^+(aq)\rightarrow 2Fe^{2+}(aq)+2H_2O(l)$

C. $Pb(s)+PbO_2(s)+2H_2O(l)\rightarrow 2PbSO_4(s)+2H_2O(l)$

D. $2H_2(g)+O_2(g)\rightarrow 2H_2O(l)$

Answer: D

Solution:

Cell which convert chemical energy of a fuel directly into electrical energy is called fuel cell. H_2-O_2 cell is a fuel cell. It is also known as Becon cell.

$2H_2(g)+O_2(g)\rightarrow 2H_2O(l)$

Question 71

If 20 g of a solute was dissolve in 500 ml of water and osmotic pressure of the solution was found to be 600 mm of Hg at 15^0C, then molecular weight of the solute is

Options:

A. 1000

B. 1200

C. 1400

D. 1800

Answer: B

Solution:

Osmotic pressure is a colligative property.

$\pi V = nRT$

Or $\pi = \frac{n}{V}RT = \frac{mRT}{MV}$

Where,

π=osmotic pressure=600 mm of Hg=600/760 atm

m=mass of solute=20 g

M=molecular mass of solute=?

R=0.821 L atm $mol^{-1}K^{-1}$

T=150C=15+273=288K

V=500mL=0.5L

$\frac{600}{760} = \frac{20\times 0.0821\times 288}{M\times 0.5}$

Or, M=1200

$\therefore$Molecular mass of solute is 1200.

Question 72

If standard enthalpies of formation of CaCl (s) (hypothetical) and that of $CaCl_2$(s) are -188 $Jmol^{-1}$and -795 $kJmol^{-1}$respectively. Calculate the value of standard heat of reaction for the following disproportionation reaction

$2CaCl(s)\rightarrow CaCl_2(s)+Ca(s)$

Options:

A. -607 $kJmol^{-1}$

B. +607 $kJmol^{-1}$

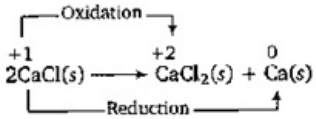
C. -419 $kJmol^{-1}$

D. +419 $kJmol^{-1}$

Answer: C

Solution:

In disproportionation reaction same species (here Ca^{2+}) is oxidised as well as reduced.



Standard heat of reaction,

$$\Delta H^0 = \Delta H_P{}^0 - \Delta H^0{}_R$$
$$= \Delta H^0{}_F(CaCl_2) + \Delta H^0{}_F(Ca) - 2\Delta H^0{}_F(CaCl)$$
$$= (-795 + 0) - (-2 \times 188) \qquad = -419\ \text{kJ mol}^{-1}$$

Question 73

Which of the following is not a polymer ?

Options:

A. Cholesterol

B. Oxytocin

C. Amylose

D. RNA

Answer: A

Solution:

(a) cholesterol-It is a steroid (not polymer)
(b) Oxytocin-It is a polyamide
(c) Amylose- It is a polysaccharide
(d) RNA- It is a polymer of nucleotides

Question 74

Aluminium Chloride exists as dimer, Al_2Cl_6 in solid state as well as solution of non-polar solvents such as benzene. When dissolved in water, it gives

Options:

A. $Al^{3+}+3Cl^-$

B. $[Al(H_2O)_6)]^{3+} + Cl^-$

C. $[Al(HO)_6]^{3-} + 3HCl$

D. $Al_2O_3 + 6HCl$

Answer: B

Solution:

$AlCl_3$ is covalent but in water it become ionic due to large hydration energy of Al^{3+}

$$AlCl_3 + 6H_2O \rightleftharpoons [Al(H_2O)_6]^{3+} + 3Cl^-$$

Question 75

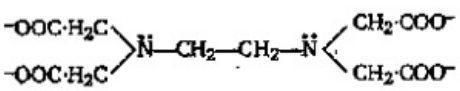
Which ligand is useful for removal of the toxic effect of lead metal in body in chelate therapy treatment ?

Options:

A. AsO_4^{3-}

B. CH_3COO^-

C. 

D. 

Answer: D

Solution:

EDTA (ethylene di ammine tetra acetate ion) is used for removal of the toxic effect of lead metal in body in chelate therapy treatment because it forms complex with lead.

Question 76

The number of sodium atoms in 2 moles of sodium ferrocyanite is

Options:

A. $12x12^{23}$

B. $26x10^{23}$

C. $34x10^{23}$

D. $48x10^{23}$

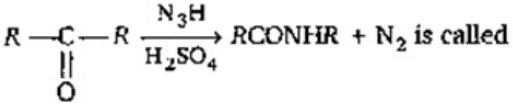
Answer: D

Solution:

Formula of sodium ferrocyanide is: $Na_4[Fe(CN)_6]$ .The formula suggests that it has four sodium(Na) atoms.
since,number of Na atoms
=(Number of moles of the complex) x (number of atoms per molecule of the complex) x (Avogadro's number)
Given,
number of moles of complex=2
number of atoms per molecule of the complex=4
Avogadro's number=$6.023x10^{23}$
Hence,
number of sodium atoms =$2 \times 4 \times 6.023 \times 10^{23}$
= 48×10^{23}
So option(4) is correct.

Question 77

The reaction,



Options:

A. Claisen-Schmidt reaction

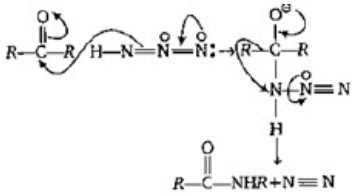
B. Kolbe-Schmidt reaction

C. Schmidt reaction

D. Kolbe’s reaction

Answer: C

Solution:



This reaction is called Schmidt reaction

Question 78

The ions N^{3-}, O^{2-}, F^{-}, Na^{+}, Mg^{2+} are isoelectronic. Their ionic radii show

Options:

A. An increase from N^{3-} to F^{-} and then decrease from N^{3-} to Mg^{2+}

B. A decrease from N^{3-} to F^{-} and then increase from Na^{+} to Mg^{2+}

C. a significant increase from $N^{3-}$to Mg^{2+}

D. a significant decrease from N^{3-} to Mg^{2+}

Answer: D

Solution:

Ionic radii

$$\propto \frac{1}{effective\ nuclear\ charge\ (Zeff)}$$

$$\propto \text{negative charge}$$

$$\propto \frac{1}{positive\ charge}$$

Hence, as the negative charge increases, ionic radii increases and with increase in positive charge ionic radii decrease

Hence, ionic radii decreases from N^{3-} to Mg^{2+}

Question 79

Which of the following is not true

Options:

A. In a first order reaction, the half life is independent to the initial concentration of reactant.

B. A given piece of charcoal shows increase in its surface area in its powdered form

C. In valence bond of H2, both the electrons spend their time around its own nucleus

D. In valence bond of H2, both the electrons spend their time around both the nucleus.

Answer: C

Solution:

(i) For first order reaction, $t_{1/2} = \frac{0.693}{k}$
Where, $t_{1/2}$ =half life period
K=rate constant
So, the half-life period of first order reaction is independent of the initial concentration of reaction.
(ii) A given piece of charcoal shows increase in its surface area in its powdered form.
(iii)Two electrons, one belonging to each atom originally, spend maximum time in the region of overlap and thus, are attracted simultaneously by the two nuclei.

Question 80

When glucose reacts with acetone in acidic medium, the main product is

Options:

A. laevulic acid

B. glycosazone

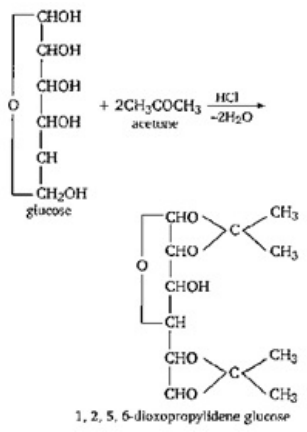
C. dioxopropylidene

D. None of the above

Answer: C

Solution:

In the presence of HCl glucose condenses with acetone to yield. 1, 2, 5, 6-dioxopropylidene glucose.



Question 81

Which of the following is not true for antibiotics ?

Options:

A. *Tetracycline* is one of the broad spectrum antibiotics which are effective against a large number of harmful microorganism

B. Streptomycin is highly effective against microorganism which cause tuberculosis

C. Penicillin has a narrow spectrum and certain persons are sensitive to it

D. Penicillin may be administered without testing the patients foe sensitivity to it

Answer: D

Solution:

Penicillin has a narrow spectrum and is effective only against disease caused by various cocii and some gram positive bacteria. Some person have allergy to penicillin. Hence, it is very essential to test the sensitivity of the patient by giving a test dose before giving the full dose.

Question 82

Match the Column I and Column II and choose the correct code given below

S.No.	Column I		Column II
A.	Peroxyacetyl nitrile	(i)	Waste incineration
B.	Indigo	(ii)	Vat dye
C.	IR active molecules	(iii)	Global warming
D.	Dioxins	(iv)	Photochemical smog

Options:

A. A-(iv), B-(ii), C-(iii), D-(i)

B. A-(i), B-(ii), C-(iV), D-(iii)

C. A-(ii), B-(i), C-(iv), D-(iii)

D. A-(iv), B-(i), C-(iii), D-(ii)

Answer: A

Solution:

(A) Peroxyacetyl nitrate (PAN) is used in photochemical smog.
(B) Indigo is a vat dye. Dyeing with indigation was carried out in wooden vats in the form of water soluble indigotin-white
(C) IR active molecules are used in global warming.
(D) Dioxins are used to burn the waste into ashes.

Question 83

250 ml of a Na_2CO_3 solution contains 2.65 g of Na_2CO_3. 10 ml of this solution is added to x mL of water to obtain 0.001 M Na_2CO_3 solution. The value of x is (molecular weight of Na_2CO_3 =106)

Options:

A. 1000 mL

B. 990 mL

C. 9990 mL

D. 90 mL

Answer: B

Question 84

The densities of graphite and diamond at 298 K are 2.25 and 3.31 gcm^{-3} respectively. If the standard free energy difference is 1895 J mol^{-1}, the pressure at which graphite will be transformed into diamond is

Options:

A. 9.92x10^8 Pa

B. 9.92x10^7 Pa

C. 9.92x10^6 Pa

D. None of these

Answer: D

Solution:

$$\Delta G = W_{compress} = p\Delta V$$

$$1895 = p\left[\left(\frac{12}{2.25} - \frac{12}{3.31}\right)\times 10^{-6} m^3\right]$$

$$1895 = p(5.34 - 3.62)\times 10^{-6}$$

$$p = \frac{1895 \times 10^6}{1.73} Pa$$
$$= 11.02 \times 10^8 Pa$$

Question 85

Which of the following statement is false ?

Options:

A. Raolt’s law states that the vapour pressure of a component over a solution is proportional to its mole fraction.

B. The osmotic pressure (π) of a solution is given by the equation $\pi = MRT$, where M is the molarity of the solution

C. The correct order of osmotic pressure for 0.01 M aqueous solution of each compound is $BaCl_2$>KCl>CH_3COOH>Sucrose

D. Two sucrose solution of same molality prepared in different solvents will have the same freezing point depression.

Answer: D

Solution:

(a) $p_A = X_A p^0{}_A$

Thus (a) is true

(b) $\pi = iMRT = MRT$

Thus, (b) is true

(if van’t Hoff factor, i=1)

(c) $i = [1 + (y-1)x]$

Where,

Y=number of ions

X=degree of ionisation

i=3, for $BaCl_2$ x=1(strong electrolyte)

i=2, for KCl x=1 (strong electrolyte)

i=(i+x), for CH_3COOH>sucrose

Thus, (c) is also true

(d) $\Delta T_f = K_f m$

K_f is dependent on solvent.

Thus, freezing points[=T (solvent)-ΔT_f] are different.

Thus, (d) is false.

Question 86

In reduction of $KMnO_4$ by warm acidified oxalic acid, the oxidation number of Mn changes from

Options:

A. +4 to +2

B. +6 to +4

C. +7 to +2

D. +7 to +4

Answer: C

Solution:

$$[MnO^{4-} + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O] \times 2$$

$$\frac{[C_2O_4{}^{2-} \rightarrow 2CO_2 + 2e^-] \times 5}{2Mn^{2+}O_4^- + 5C_2O_4{}^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O}$$

So, oxidation number of Mn changes in the above reaction from +7 to +2.

Question 87

How much ethyl alcohol C_2H_5OH, must be added to 1.00L of water so that the solution will not freeze at -4^0F ?

Options:

A. 211 g

B. 495 g

C. 85 g

D. 46 g

Answer: B

Solution:

$$-4^0F = \frac{5}{9}(-4-32)^0C = -20^0C$$

$$\Delta T_f = 20^0C = Kf_m = (1.86^0C/m)(m)$$

$$m = \frac{20^0C}{1.86^0C/m} = 10.70m$$

$$= (10.7mol)(46.0g/\text{mol}) = 495\,\text{g}$$

Question 88

The reagent (s) used in the preparation of aspirin from salicylic acid

Options:

A. $SOCl_2$, pyridine

B. $(CH_3CH)_2O$, H^+

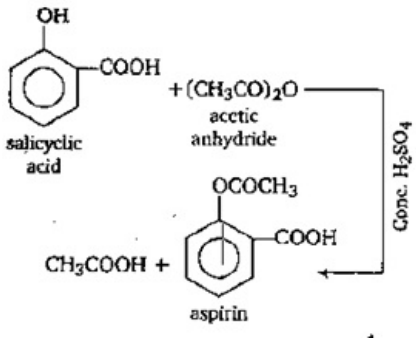
C. CH_3CO_2H, HCl

D. CH_3Cl, $AlCl_3$

Answer: B

Solution:

Aspirin (2-acetoxy benzoic acid) is an analgesic. It is obtained by acetylating salicyclic acid with acetic anhydride and conc. H_2SO_4.



Question 89

Chlorobenzene is 0, p-directing in electrophilic substitution reaction. The directing influence is explained by

Options:

A. +M of Ph

B. +I of Cl

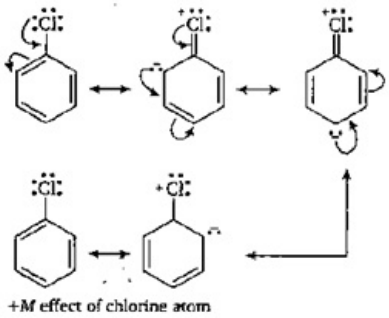
C. +M of Cl

D. -I of Ph

Answer: C

Solution:

Chlorobenzene is 0,p-directing in electrophilic substitution reaction. The directing influence is expalined by +M of Cl atom.



Question 90

The correct order of decreasing dipole moment of (I) toluene, (II) m-dichlorobenzene (III) o-dichlorobenzene and (IV) p-dichlorobenzene is

Options:

A. IV<II<I<III

B. IV<I<II<III

C. I<IV<II<II

D. IV<I<III<II

Answer: B

Solution:

Dipole moment of p-dichlorobenzene is zero, while that of o-dichlorobenzene is 2.54D and for m-dichlorobenzene is 1.48D. This shows that benzene ring is planar and in p-$C_6H_4Cl_2$, two C-Cl bonds are directed in the plane of the ring, thus on adding, they becomes zero. In this, two C-Cl bonds contain an angle of 60^0 and in m-$C_6H_4Cl_2$, the C-Cl bonds contain an angle of 120^0 and their dipole moments on vector addition give a higher value of dipole moment. Therefore, sequence of dipole moment is as follows p-dichlorobenzene<toluene<m-dichlorobenzene<o-dichlorobenzene

---

Question 91

Which of the following is most stable compound

Options:

A. $Ph_3\overset{+}{C}$

B. $Ph_2\overset{+}{C}H$

C. $Ph_2\overset{+}{C}H_2$

D. $Ph\overset{+}{C}H_2$

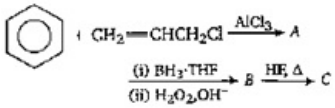
Answer: A

Solution:

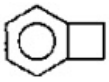
Resonance stabilises the compound.
Therefore, Ph_3C^+ is most stable due to presence of three benzene rings which are resonance stabilised.

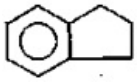
Question 92

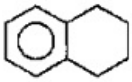
The end product 'C' in the following reaction

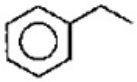


Options:

A. 

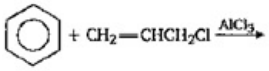
B. 

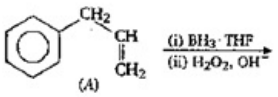
C. 

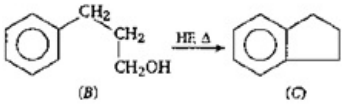
D. 

Answer: B

Solution:







Question 93

Moth repellents are

Options:

A. 

B. perchlorethane

C. 

D. All of these

Answer: D

Solution:

p-dichlorobenzene, perchloroethane and naphthalene are used as moth repellents.

Question 94

Alkaline earth’s metals are denser than alkali metals because metallic bonding in alkaline earth\s metal is

Options:

A. stronger

B. Weaker

C. volatile

D. not present

Answer: A

Solution:

Alkaline earth metals (ns^2) are denser than alkali metals (ns^1) because metallic bonding in alkaline earth metals is stronger than alkali metals due to presence of two electrons in valence shell as compared to one electron in alkali metals.

Question 95

Transition metals have the electronic configuration $(n-1)^{1-10}ns^{1-2}$. The d-orbitals are degenerate. Colour of transition metal ions is due to absorption of some wavelength. This results is

Options:

A. d-s transition

B. s-d transition

C. s-s transition

D. d-d transition

Answer: D

Solution:

When a transition metal compound is formed, the degenracy of d-orbitals is resolved giving two sets of orbitals slightly differing in energy. Absorption of light causes the promotion of electrons fromlower energy set of orbitals to higher energy set within the same d-subshell. This is called d-d transition.

Question 96

Which one of the following complexes is an outer orbital complex ? (Atomic number of Mn=25, Fe=26, C0=27, Ni=28)

Options:

A. $[Fe(CN)^6]^{4-}$

B. $[Mn(CN)_6]^{4-}$

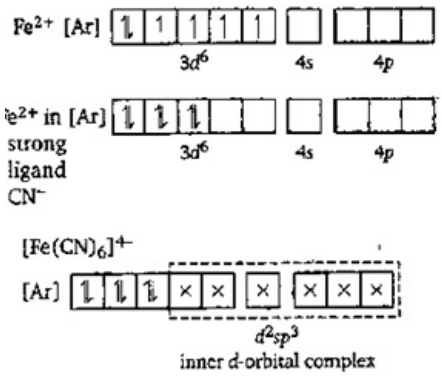
C. $[Co(NH_3)_6]^{3+}$

D. $[Ni(NH_3)_6]^{2+}$

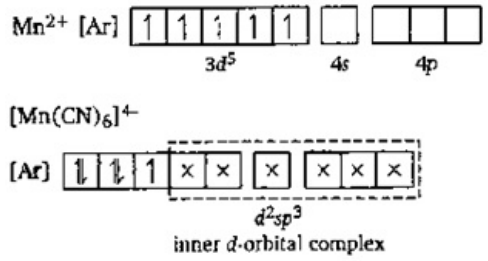
Answer: D

Solution:

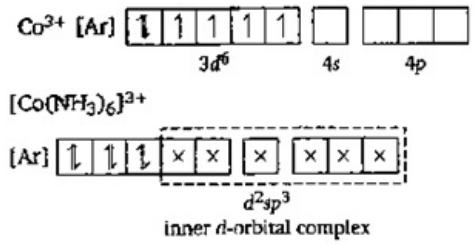
(a)



(b)

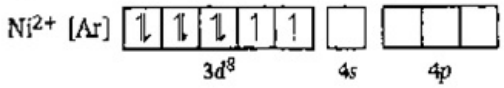


(c)

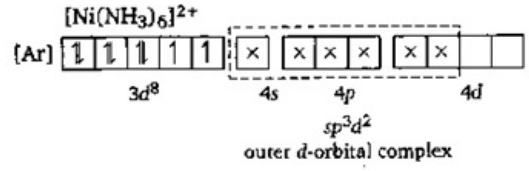


(d)

NH_3 is strong ligand.



In this case, NH3 is a strong ligand but electrons remain unpaired since only one orbital is left vacent in 3d. Thus,



Question 97

Which one of the following statements regarding helium is incorrect

Options:

A. It is used to fill gas balloons instead of hydrogen because it is lighter and non-inflammable.

B. It is used as a cryogenic agent foe carrying out experiments at low temperatures

C. It is used to produce and sustain powerful superconducting magnets

D. It is used in gas cooled nuclear reactors.

Answer: C

Solution:

Helium is not used to produce and sustain powerful superconducting magnets. All others are the uses of helium.

Question 98

Wolframite ore is separated from tin stone ore by the process of

Options:

A. roasting

B. electromagnetic

C. smelting

D. calcination

Answer: B

Solution:

Wolframite ore (FeWO4) is present in tin stone in the form of impurities and same mass per unit as tin stone. Wolframite is magnetic in nature and tin stone is non-magnetic in nature. They are seperated by magnetic seperation,
Wolframite is attracted by magnet and falls just below magnet while tin stone falls away from it. Roasting and calcination are method for conversation of ore into oxide, Smelting is used for iron.

Question 99

For a gas in equilibrium with a liquid, the ratio of the concentration of the gas in the solution phase to that in the gaseous phase is constant at constant temperature, only if molecules undergo

Options:

A. ionisation

B. dissociation

C. isolation

D. reaction with solvent

Answer: C

Solution:

That's Henry's law

$$\frac{(Concentration\ of\ gas\ in\ solution)}{(Concentration\ of\ gas\ in\ the\ gaseous\ phase)}$$

=Constant

Only when there is simple solvation (isolation, then stabilisation). Not possible in the case of ionisation or dissociation.

Question 100

The homogeneous catalysis is shown by

Options:

A. Haber's process, $N_2 + 3H_2 \xrightarrow{Fe} 2NH_3$

B. Ostwald process $4NH_3 + 5O_2 \xrightarrow{Pt} 4NO + 6H_2O$

C. Contact process $2SO_2 + O_2 \xrightarrow{Pt} 2SO_3$

D. $2C_2H_5OH \xrightarrow{H_2SO_4} C_2H_5 - O - C_2H_5 + H_2O$

Answer: D

Solution:

If the catalyst is present in the same phase as the reactants, it is called a homogeneous catalyst and this type of catalyst is called homogeneous catalysis. For example,

$2C_2H_5OH(aq) \xrightarrow{H_2SO_4(aq)}$

$C_2H_5 - O - C_2H_5(aq) + H_2O(l)$

Here, both reactants and catalyst are present in the liquid phase.

Maths

Question 101

Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is

Options:

A. 128

B. 216

C. 240

D. 160

Answer: D

Solution:

Given, n(U)=800, n(C)=224, n(H)=240, n(B)=336
n(H∩B)=64, n(B∩C)=80, n(H∩C)=40 , n(C∩H∩B)=24
∴ $n(C^c \cap H^c \cap B^c)$
= n[(C∪H∪B)] = n(U) - n(C∪H∪B)
= 800 - [n(C) + n(H) + n(B) - n(H∩C) - n(H∩B) - n(C∩B)+n(C∩H∩B)
=800-[224+240+336-40-64-80+24]
=800-640
=160

Question 102

An integer m is said to be related to another integer n, if m is a multiple of n. then the relation is

Options:

A. Reflexive and symmetric

B. Reflexive and transitive

C. symmetric and transitive

D. an equivalence relation

Answer: B

Solution:

For any integern, we have n/n$\Rightarrow$nRn
So, nRn for all $n \in Z \Rightarrow R$ is reflexive.
Now, 2/6 but 6+2$\Rightarrow (2,6) \in R \notin$ R.
So, R is not symmetric.
Let $(m,n) \in R$ and $(n,p) \in R$
then $\left.\begin{matrix}(m,n) \in R \Rightarrow m/n \\ (n,p) \in R \Rightarrow n/p\end{matrix}\right\} \Rightarrow m/p \Rightarrow (m,p) \in R$ So, R is transitive.
Hence, R is reflexive and transitive but it is not symmetric.

Question 103

The real part of $(1-i)^{-i}$ is

Options:

A. $-e^{-\pi/4}\sin\left(\frac{1}{2}\log 2\right)$

B. $e^{\pi/4}\cos\left(\frac{1}{2}\log 2\right)$

C. $e^{-\pi/4}\sin\left(\frac{1}{2}\log 2\right)$

Answer: B

Solution:

let $z=(1-i)^{-i}$

On taking log on both sides, we get

$\Rightarrow \log z=-i\log(1-i)$

$=-i\log\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$

$=-i\log(\sqrt{2}.e^{-i\pi/4})$

$=-i\left[\frac{1}{2}\log 2+\log e^{-i\pi/4}\right]$

$=-i\left[\frac{1}{2}\log 2-\frac{i\pi}{4}\right]=\frac{i}{2}\log 2-\frac{\pi}{4}$

$\Rightarrow z=e^{-\pi/4}.e^{-i/2}\log 2$

On taking real part only,

$\Rightarrow \mathrm{Re}(z)=e^{-\pi/4}.\cos\left(\frac{1}{2}\log 2\right)$

Question 104

If $x_n=\cos\left(\frac{\pi}{4^n}\right)+i\sin\left(\frac{\pi}{4^n}\right)$, then $x_1.x_2.x_3....^{\infty}$ is

Options:

A. $\frac{1+i\sqrt{3}}{2}$

B. $\frac{-1+i\sqrt{3}}{2}$

C. $\frac{1-i\sqrt{3}}{2}$

D. $\frac{-1-i\sqrt{3}}{2}$

Answer: A

Solution:

$x_1.x_2.x_3...\infty$

$$\left[\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right]\left[\cos\left(\frac{\pi}{4^2}\right)+i\sin\left(\frac{\pi}{4^2}\right)\right]\left[\cos\left(\frac{\pi}{4^3}\right)+i\sin\left(\frac{\pi}{4^3}\right)\right]...\infty$$

$$\left[\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right]\left[\cos\left(\frac{\pi}{4^2}\right)+i\sin\left(\frac{\pi}{4^2}\right)\right]=\left[\cos\left(\frac{\pi}{4^3}\right)+i\sin\left(\frac{\pi}{4^3}\right)\right]...\infty$$

$$=\cos\left(\frac{\pi}{4}+\frac{\pi}{4^2}+\frac{\pi}{4^3}+...\infty\right)$$

$$+i\sin\left(\frac{\pi}{4}+\frac{\pi}{4^2}+\frac{\pi}{4^3}+...\infty\right)$$

$$=\cos\left(\frac{\pi/4}{1-1/4}\right)+i\sin\left(\frac{\pi/4}{1-1/4}\right)$$

$$=\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)=\frac{1+\sqrt{3}i}{2}$$

Question 105

The number of roots of the $|x|^2-7|x|+12=0$ is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: D

Solution:

$$|x|^2-7|x|+12=0$$
$$|x|^2-4|x|-3|x|+12=0$$
$$(|x|-4)(|x|-3)=0$$
$$|x|=4,3$$
$$x=\pm 4,\pm 3$$

Question 106

If the roots of the equation $ax^2+bx+c=0$ are real and of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then the value of $(a+b+c)^2$ is

Options:

A. b^2-4ac

B. b^2-2ac

C. $2b^2-ac$

D. None of these

Answer: A

Solution:

Here, $\frac{\alpha+1}{\alpha}+\frac{\alpha}{\alpha-1}=\frac{-b}{a}$

And $\frac{\alpha+1}{\alpha-1}=\frac{c}{a}$

$\therefore \alpha=\frac{c+a}{c-a}$ and $\frac{2\alpha^2-1}{\alpha(\alpha-1)}=\frac{-b}{a}$

On substituting α, we get

$(a+b+c)^2=b^2-4ac$

Question 107

If $a^2+b^2+c^2=-2$ and

$$f(x)=\begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Then f(x) is a polynomial of degree

Options:

A. 3

B. 2

C. 1

D. 0

Answer: B

Solution:

Applying $C_1 \to C_1 + C_2 + C_3$ we get

$$f(x) = \begin{vmatrix} 1+a^2x+\left(1+b^2\right)x+(1+c^2)x & \left(1+b^2\right)x & (1+c^2)x \\ (1+a^2)x+1+b^2x+(1+c^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x+\left(1+b^2\right)x+1+c^2x & \left(1+b^2\right)x & 1+c^2x \end{vmatrix}$$

$$f(x) = \begin{vmatrix} 1+\left(a^2+b^2+c^2\right)x+2x & \left(1+b^2\right)x & (1+c^2)x \\ 1+\left(a^2+b^2+c^2\right)x+2x & 1+b^2x & (1+c^2)x \\ 1+\left(a^2+b^2+c^2\right)x+2x & \left(1+b^2\right)x & 1+c^2x \end{vmatrix}$$

Since $a^2+b^2+c^2=-2$

$$f(x) = \begin{vmatrix} 1-2x+2x & \left(1+b^2\right)x & (1+c^2)x \\ 1-2x+2x & 1+b^2x & (1+c^2)x \\ 1-2x+2x & \left(1+b^2\right)x & 1+c^2x \end{vmatrix}$$

$$f(x) = \begin{vmatrix} 1 & \left(1+b^2\right)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & \left(1+b^2\right)x & 1+c^2x \end{vmatrix}$$

Again, applying

$R_2 \to R_2 - R_1$,

$R_3 \to R_3 - R_1$,

we get

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = (1-x)^2$$

Hence, the degree of the function is 2.

Question 108

If $A = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix}$ **and B=(adj A), and C=5A, then** $\frac{|adjB|}{|C|}$ **is**

Options:

A. 5

B. 25

C. -1

D. 1

Answer: D

Solution:

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix} = 1(3)+1(6)+1(-4) = 5 \quad B = adj(A) = \begin{vmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{vmatrix}$$

$$adj(B) = \begin{vmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{vmatrix} = 5A$$

and C=5A

$$C = adj(B); |C| = |adj(B)|; \therefore \frac{|adj(B)|}{|C|} = 1$$

Question 109

If matrix $A = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{vmatrix}$ **and its inverse is denoted by** $A^{-1} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$**, then the value of a_{23} is**

Options:

A. $\frac{21}{20}$

B. $\frac{1}{5}$

C. $-\frac{2}{5}$

D. $\frac{2}{5}$

Answer: D

Solution:

The adjoint matrix is the transpose of the co-factor matrix
Thus the element a_{ij} of the adjoint matrix= b_{ji} of the cofactor matrix
b_{23} of the adjoint matrix= b_{32} of the cofactor matrix
The element b_{32} of the cofactor matrix is

$$= (-1)^{3+2} \begin{vmatrix} 1 & -1 \\ 3 & 5 \end{vmatrix}$$

= - (1x5 - (-1)x3)
=- (5+3)=-8

The inverse of a matrix is defined as

A^{-1} = adj A / |A|

|A|=1(4x7 - 6x5) -0+(-1)(3x6-0)
=28-30-18
=-20

Thus,
$a_{23} = b_{32}$ / |A|
= -8/-20
=2/5

Question 110

The number of solutions of the system of equations 2x + y – z = 7, x - 3y + 2z = 1 and x + 4y - 3z = 5

Options:

A. 3

B. 2

C. 1

D. 0

Answer: D

Solution:

According to Cramer's rule if the determinant of the coefficient matrix is zero, then either the system of equations has no solution or infinite solutions. Check the value of the coefficient determinant.

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & -3 \end{vmatrix}$$

=2(9-8)-1(-3-2)-1(4+3)
=7-7=0

Question 111

The number of triangles that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line, is

Options:

A. 185

B. 175

C. 115

D. 105

Answer: A

Solution:

The minimum number of points needed to draw a triangle.
No triangle can be formed if three points are collinear.
Since seven points are collinear, then no triangle is formed from the combination of these seven points.

The required number of ways = Total number of 3 point combinations - Total number of collinear point combinations
$= {}^{12}C_3 - {}^{7}C_3$
$=[12x11x10 / 3x2x1] - [7x6x5 / 3x2x1]$
$=220-35$
$=185$

Question 112

In the expansion of $(1+x+x^3+x^4)^{10}$, the coefficient of x^4 is

Options:

A. ${}^{40}C_4$

B. ${}^{10}C_4$

C. 210

D. 310

Answer: D

Solution:

$(1+x+x^3+x^4)^{10}$
$= (1+x+x^3(1+x))^{10}$
$=((1+x)(1+x^3))^{10}$

$=(1+x)^{10}\ (1+x^3)^{10}$

$=$[Coefficient of x^0 in the expansion of $(1+x)^{10}$][Coefficient of x^4 in the expansion of $(1+x^3)^{10}$]
+[Coefficient of x in the expansion of $(1+x)^{10}$][Coefficient of x^3 in the expansion of $(1+x^3)^{10}$]
+[Coefficient of x^2 in the expansion of $(1+x)^{10}$][Coefficient of x^2 in the expansion of $(1+x^3)^{10}$]
+[Coefficient of x^3 in the expansion of $(1+x)^{10}$][Coefficient of x in the expansion of $(1+x^3)^{10}$]
+[Coefficient of x^4 in the expansion of $(1+x)^{10}$][Coefficient of x^0 in the expansion of $(1+x^3)^{10}$]

$=[^{10}C_0\ (1)^{10}\ x^0\ ][0]$
$+[^{10}C_1\ (1)^9\ x\ ][^{10}C_1\ (1)^9\ (x^3)\ ]$
$+[^{10}C_2\ (1)^8\ x^2\ ][0\ ]$
$+[^{10}C_3\ (1)^7\ x^3\ ][0\ ]$
$+[^{10}C_4\ (1)^6\ x^4\ ][^{10}C_0\ (1)^{10}\ (x^3)^0\ ]$

$=[^{10}C_1\ (1)^9\ x\ ][^{10}C_1\ (1)^9\ (x^3)\ ]+[^{10}C_4\ (1)^6\ x^4\ ][^{10}C_0\ (1)^{10}\ (x^3)^0\ ]$
$=10x10+(10x9x8x7/4x3x2x1)(1)$
$=100+210$
$=310$

Question 113

Let P(n) denote the statement that n^2+n is odd. It is seen that P(n)⇒P(n+1), P(n) is true for all

Options:

A. n>1

B. n

C. n>2

D. None of these

Answer: D

Solution:

P(n): [n^2+n] is odd.
P(n+1): [$(n+1)^2$ + (n+1)] is odd
or
P(n+1): [n^2+1+2n+n+1] is odd
or
P(n+1): [(n^2+n) + 2n+2] is odd

The sum of an odd number with an even number is always an odd number while the sum of an even number with an even number is an even number.

P(n)⇒P(n+1) means
If P(n) is true then, P(n+1) is true.
If P(n) is false then, P(n+1) is false.

Rewrite statement P(n)
P(n): [n^2+n] is odd.
or
P(n): [n(n+1)] is odd

The product of two consecutive numbers is always an even number because it is obvious that any one of the numbers is even. Thus, P(n) is false, and so P(n+1) is also false.

Hence, P(n)⇒P(n+1) is true

Question 114

If the coefficient of $(2r-3)^{th}$ and $(r-3)^{th}$ terms is the expansion of $(1+x)^{18}$ are equal, then r is

Options:

A. 12

B. 10

C. 8

D. 6

Answer: D

Solution:

$^{18}C_{2r-3} = {}^{18}C_{r-3}$

$$\Rightarrow \frac{18!}{(2r+3)!(18-2r-3)!} = \frac{18!}{(r-3)!(18-r+3)!}$$

$$\Rightarrow \frac{18!}{(2r+3)!(15-2r)!} = \frac{18!}{(r-3)!(21-r)!}$$

$$\Rightarrow \frac{(21-r)!}{(15-2r)!} = \frac{(2r+3)!}{(r-3)!}$$

r>7 is not possible because that will make 15-2r<0. Thus, options with r=12,10,8 are neglected
r=6 satisfies the above equation. Hence, the answer is r=6.

Question 115

$\binom{n}{0}+2\binom{n}{1}+2^2\binom{n}{2}+...+2^n\binom{n}{n}$ **is equal to**

Options:

A. 2^n

B. 0

C. 3^n

D. None of these

Answer: C

Solution:

$$(1+x)^n = {}^nC_0 + x.{}^nC_1 + x^2.{}^nC_2 + ... + x^n.{}^nC_n$$

Put x=2

$$\Rightarrow 3^n = {}^nC_0 + 2.{}^nC_1 + 2^2.{}^nC_2 + 2^3.{}^nC_3 + ... + 2^n.{}^nC_3$$

Question 116

Let the sequence a_1 , a_2 , a_3 ,..., a_{2n} form an AP. Then a_1^2 - a_2^2 - a_3^2

+ a_4^2 - a_5^2 + ……+a^2_{2n-1} - a^2_{2n} is

Options:

A. $\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$

B. $\frac{2n}{n-1}(a_{2n}^2 - a_1^2)$

C. $\frac{n}{n+1}(a_1^2 + a_{2n}^2)$

D. None of the above

Answer: A

Solution:

Since, $a_1,a_2,a_3…,a_{2n}$ form an AP
Therefore,
$a_2-a_1=a_4-a_3=…=a_{2n}-a_{2n-1}=d$
Here, $a_1^2-a_2^2+a_3^2-a_4^2+…+a_{2n-1}^2-a_{2n}^2$
$=(a_1-a_2)(a_1+a_2)+(a_3-a_4)(a_3+a_4)+…+(a_{2n-1}-a_{2n})(a_{2n-1}+a_{2n})$
$=-d(2a_1+2a_2+…+2a_{2n})$
$=-d(2(n/2)(a_1+a_{2n}))$
$=-d(n(a_1+a_{2n}))$

Also, we know
$a_{2n}=a_1+(2n-1)d$
$-d=(a_1-a_{2n})/(2n-1)$

Therefore, the sum is
$=[(a_1-a_{2n})/(2n-1)]((n/2)(a_1+a_{2n}))$
$=[n\{a_{2n}^2-a_1^2\}/(2n-1)]$

Question 117

The value of $\frac{1}{(1+a)(2+a)}+\frac{1}{(2+a)(3+a)}+\frac{1}{(3+a)(4+a)}+…+\infty$ is, (where, a is a constant)

Options:

A. $\frac{1}{1+a}$

B. $\frac{2}{1+a}$

C. ∞

D. None of the above

Answer: A

Solution:

$$\frac{1}{(1+a)(2+a)}+\frac{1}{(2+a)(3+a)}$$

$$+\frac{1}{(3+a)(3+a)}+\ldots+\infty$$

nth term of series

$$T_n=\frac{1}{(n+a)(n+1+a)}=\frac{1}{n+a}-\frac{1}{(n+1+a)}$$

$$T_1=\frac{1}{1+a}-\frac{1}{2+a}$$

$$T_2=\frac{1}{2+a}-\frac{1}{3+a}$$

$$T_3=\frac{1}{3+a}-\frac{1}{4+a}$$

.....

.....

.....

$$T_{n-1}=\frac{1}{n-1+a}-\frac{1}{n+a}$$

$$T_n=\frac{1}{n+a}-\frac{1}{n+1+a}$$

$$\therefore S_n=T_1+T_2+T_3+\ldots+T_n$$

$$=\frac{1}{1+a}-\frac{1}{n+1+a}$$

$$=\frac{n}{(1+a)(n+1+a)}$$

$$S_n=\frac{1}{(1+a)\left(1+\frac{1}{n}+\frac{a}{n}\right)}$$

$$S_\infty=S_n$$

$$n\to\infty$$

$$\therefore S_\infty=\frac{1}{1+a}$$

Question 118

The harmonic mean of two numbers is 4 and the arithmetic and geometric mean satisfies the relation $2A+G^2=27$, the numbers are

Options:

A. 6,3

B. 5,4

C. 5,-2.5

D. -3,1

Answer: A

Solution:

Let numbers be x and y.

Then, $A=\frac{1}{2}(x+y), \sqrt{xy}=G$

Or $G^2=xy$

The harmonic mean of x and y

$$H=\frac{2}{\frac{1}{x}+\frac{1}{y}}$$

$$H=\frac{2xy}{x+y}$$

$$4=\frac{G^2}{A}$$

$$\Rightarrow G^2=4A$$

Also, $2A+G^2=2A+4A=6A=27$

$\Rightarrow A=27/6=9/2$

So, $G^2=4(9/2)=18$

Thus,

x+y=9

and

xy=18

$\Rightarrow(9-y)y=18$

$\Rightarrow 9y - y^2 =18$

$\Rightarrow y^2-9y+18=0$

$\Rightarrow y^2 -6y-3y+18=0$

$\Rightarrow y(y-6) - 3(y-6)=0$

⇒(y-3)(y-6)=0
⇒y=3,6
So, x=6,3

Hence, numbers are 6 and 3.

Question 119

If f be the greater integer function and g be the modulus function, then $(gof)\left(-\frac{5}{3}\right)-(fog)\left(-\frac{5}{3}\right)$ is

Options:

A. 1

B. -1

C. 2

D. 4

Answer: A

Solution:

Given:
f(x)=[x]
g(x)=|x|

fog(x)=f(|x|)=[|x|]
gof(x)=g([x])=|[x]|

gof(-5/3)=|[-5/3]|=|[-2+1/3]|=|-2|=2
fog(-5/3)=[|-5/3|]=[5/3]=[1+2/3]=1

So,
gof(-5/3) - fog(-5/3)
=2-1=1

Question 120

$\lim_{x\to 0}\frac{x}{|x|+x^2}$ **is equal to**

Options:

A. 1

B. -1

C. 0

D. does not exist

Answer: A

Solution:

LHL

$$\lim_{x\to 0-h}\frac{x}{|x|+x^2}$$

$$=\lim_{h\to 0}\frac{-h}{-h+h^2}$$

$0/0$ form, apply L'Hospital's Rule

$$=\lim_{h\to 0}\frac{-1}{-1+2h}$$

$$=1$$

RHL

$$\lim_{x\to 0+h}\frac{x}{|x|+x^2}$$

$$=\lim_{h\to 0}\frac{h}{h+h^2}$$

$0/0$ form, apply L'Hospital's Rule

$$=\lim_{h\to 0}\frac{1}{1+2h}$$

$$=1$$

Hence, the answer is 1.

Question 121

If a_1=1 and $a_{n+1}=\frac{4+3a_n}{3+2a_n}\quad n\geq 1$ **and a_n>0 if,**

$\lim_{n\to\infty} a_n = a$ **then the value of a is**

Options:

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. 2

D. None of these

Answer: A

Solution:

We have,

$$a_{n+1}=\frac{4+3a_n}{3+2a_n}$$

So,

$$a_n=\frac{4+3a_{n-1}}{3+2a_{n-1}}$$

$$\Rightarrow \lim_{n\to\infty} a_{n+1}=\lim_{n\to\infty}\frac{4+3a_n}{3+2a_n}$$

$$\Rightarrow \lim_{n\to\infty} a_n=\frac{4+3\lim_{n\to\infty} a_{n-1}}{3+2\lim_{n\to\infty} a_{n-1}}$$

$$\Rightarrow \lim_{n\to\infty} a_n=\frac{4+3\lim_{n\to\infty} a_n}{3+2\lim_{n\to\infty} a_n}$$

$$\Rightarrow a=\frac{4+3a}{3+2a}$$

$\Rightarrow a(3+2a)=4+3a$

$\Rightarrow 2a^2+3a=4+3a$

$\Rightarrow 2a^2=4$

$\Rightarrow a^2=2$

$\Rightarrow a=\sqrt{2}$

And $a\neq-\sqrt{2}$ because of each $a_n>0$

Question 122

$\lim_{x\to0}\frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to

Options:

A. $-\pi$

B. π

C. $\frac{\pi}{2}$

D. 1

Answer: B

Solution:

$$\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}\left(\frac{0}{0}\,form\right)$$

$$=\lim_{x\to 0}\frac{\{\cos(\pi\cos^2 x).\pi.2\cos x(-\sin x)\}}{2x}\text{ (using L'Hospital's rule)}$$

$$=\lim_{x\to 0}\pi cos(\pi\cos^2 x).\cos x\left(\frac{-\sin x}{x}\right)$$

$$=(\pi).(-1).1.(-1)=\pi$$

Question 123

$$f(x)=\begin{cases}\dfrac{1-\cos 4x}{x2}, when, x<0\\ a, when, x=0\\ \dfrac{\sqrt{x}}{\sqrt{(16+\sqrt{x}-4}}, \text{when, x} > 0\end{cases}$$

If
Continuous at x=0, then the value of a will be

Options:

A. 8

B. -8

C. 4

D. None of these

Answer: A

Solution:

$$\lim_{x\to 0-} f(x)=\lim_{x\to 0-}\frac{1-\cos 4x}{x^2}$$

$$=\lim_{x\to 0-}\left(\frac{2\sin^2 2x}{(2x)^2}\right).4=8$$

$$\lim_{x\to 0+} f(x)=\lim_{x\to 0+}\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}-4}}$$

$$=\lim_{x\to 0+}\sqrt{16+\sqrt{x}}+4=8$$

Hence, a=8

Question 124

If f is a real-valued differentiable function satisfying $|f(x)-f(y)|\le(x-y)^2,\ x,y\in R$ and f(0)=0, then f(1) Is equal to

Options:

A. 2

B. 1

C. -1

D. 0

Answer: D

Solution:

$$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right|\le\lim_{x\to y}|x-y|$$

Or $|f'(x)|\le 0$

$\Rightarrow f'(x)=0\Rightarrow f(x)$ is constant

As $f(0)=0$

$\therefore f(1)=0$

Question 125

$\int\frac{dx}{\sin x-\cos x+\sqrt{2}}$ is equal to

Options:

A. $-\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)+C$

B. $\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)+C$

C. $\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+C$

D. $-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+C$

Answer: D

Solution:

$$I=\int\frac{dx}{\sin x-\cos x+\sqrt{2}}$$

$$=\int\frac{dx}{\sqrt{2}\left(\sin x.\sin\frac{\pi}{4}-\cos x.\cos\frac{\pi}{4}+1\right)}$$

$$=\frac{1}{\sqrt{2}}\int\frac{dx}{1-\cos\left(x+\frac{\pi}{4}\right)}$$

$$=\frac{1}{\sqrt{2}}\int\frac{dx}{1-\cos 2\left(\frac{x}{2}+\frac{\pi}{8}\right)}$$

$$=\frac{1}{\sqrt{2}}\int\frac{dx}{2\sin^2\left(\frac{x}{2}+\frac{\pi}{8}\right)}$$

$$=\frac{1}{2\sqrt{2}}\int\cos ec^2\left(\frac{x}{2}+\frac{\pi}{8}\right)dx$$

$$=\frac{1}{2\sqrt{2}}\frac{-\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)}{\frac{1}{2}}+C$$

$$=-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+C$$

Question 126

$\int ex/2\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$ **is equal to**

Options:

A. $e^{x/2}\cos\frac{x}{2}+C$

B. $\sqrt{2}e^{x/2}\cos\frac{x}{2}+C$

C. $e^{x/2}\sin\frac{x}{2}+C$

D. $\sqrt{2}e^{x/2}\sin\frac{x}{2}+C$

Answer: D

Solution:

Let $I=\int e^{x/2}.\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$

$$=2\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)e^{x/2}-\int\cos\left(\frac{x}{2}+\frac{\pi}{4}\right).\frac{1}{2}.2e^{x/2}dx+C$$

$$=2\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)e^{x/2}--2e^{x/2}.\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$-\int \sin\left(\frac{x}{2}+\frac{\pi}{4}\right).\frac{1}{2}.2e^{x/2}dx$$

$$2I = 2e^{x/2}\left\{\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)-\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\right\}$$

$$I = e^{x/2}\left\{\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)-\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\right\}$$

$$= \sqrt{2}e^{x/2}\left(\sin\frac{x}{2}\right) = \sqrt{2}.e^{x/2}.\sin\frac{x}{2}+C$$

Trick: By inspection

$$\frac{d}{dx}\left[\sqrt{2}e^{x/2}.\sin\frac{x}{2}+C\right]$$

$$= \sqrt{2}\left[\frac{1}{2}\cos\frac{x}{2}+\frac{1}{\sqrt{2}}\sin\frac{x}{2}\right]$$

$$= e^{x/2}.\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

Question 127

The part of straight line y=x+1 between x=2 and x=3 is revolved about x-axis, then the curved surface of the solid thus generated is

Options:

A. $\frac{37\pi}{3}$

B. $7\pi\sqrt{2}$

C. 37π

D. $\frac{7\pi}{\sqrt{2}}$

Answer: B

Solution:

Curved surface=

$$= \int_a^b 2\pi y\sqrt{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}dx$$

Given that, a=2, b=3 and y=x+1

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 1+0$ or $\frac{dy}{dx} = 1$

Therefore, curved surface

$$= \int_2^3 2\pi(x+1)\sqrt{\left[1+(1)^2\right]}dx$$

$$= \int_2^3 2\pi(x+1)\sqrt{2}dx$$

$$= 2\sqrt{2}\int_2^3 (x+1)dx = 2\sqrt{2}\pi\left[\frac{(x+1)^2}{2}\right]_2^3$$

$$= \frac{2\sqrt{2}}{2}\pi[(3+1)^2-(2+1)^2]$$

$$= \sqrt{2}\pi(16-9)$$

$$= 7\sqrt{2}\pi = 7\pi\sqrt{2}$$

Question 128

The part of circle $x^2 + y^2 = 9$ in between y=0 and y=2 is revolved about y-axis. The volume of generating solid will be

Options:

A. $\frac{46\pi}{3}$

B. 12π

C. 16π

D. 28π

Answer: A

Solution:

The part of circle $x^2+y^2=9$ in between $y=0$ and $y=2$ is revolved about y-axis. Then, a frustum of sphere will be formed.
The volume of this frustum

$$= \pi\int_0^2 x^2dy = \pi\int_0^2 (9-y^2)dy$$

$$= \pi\left[9y-\frac{1}{3}y3\right]_0^3$$

$$= \pi\left[9\times 2-\frac{1}{3}(2)^3-\left(9.0-\frac{1}{3}.0\right)\right]$$

$$= \pi\left(18-\frac{8}{3}\right) = \frac{46}{3}\pi$$

Question 129

If $\int_0^{t^2} xf(x)dx = \frac{2}{5}t^5, t>0$, then $f\left(\frac{4}{25}\right)$ is

Options:

A. $\frac{2}{5}$

B. $\frac{5}{2}$

C. $-\frac{5}{2}$

D. None of the above

Answer: A

Solution:

$\int_0^{t^2} xf(x)dx = \frac{2}{5}t^5, t>0$

On differentiating both sides w.r.t. t, we het,

$t^2 f(t^2)2t = 2t^4$ (Leibnitz rule)

$\Rightarrow f(t^2) = t$

Put, $t = \frac{2}{5}$, we get $f\left(\frac{4}{25}\right) = \frac{2}{5}$

Question 130

The solution of the equation $\sin^{-1}\left(\frac{dy}{dx}\right) = x+y$ is

Options:

A. $\tan(x+y)+\sec(x+y) = x+c$

B. $\tan(x+y)-\sec(x+y) = x+c$

C. $\tan(x+y)+\sec(x+y)+x+c = 0$

D. None of the above

Answer: B

Solution:

Here, $\frac{dy}{dx} = \sin(x+y)$

Now, put x+y=v

And $\frac{dy}{dx}=\frac{dv}{dx}-1$

Therefore, $\frac{dy}{dx}=\sin(x+y)$ reduces to $\frac{dv}{1+\sin v}=dx$

Now, on integraticg both sides, we get $\tan v-\sec v=x+C$

Or $\tan(x+y)-\sec(x+y)=x+C$

Question 131

If the coordinates of the vertices of a triangle be (1,a),(2,b) and (c^2,3) then the centroid of the triangle

Options:

A. lies at the origin

B. cannot lie on x-axis

C. cannot lie on y-axis

D. none of the above

Answer: C

Solution:

Centroid is $\left(\frac{1+2+c^2}{3},\frac{a+b+3}{3}\right)$

Now, on y-axis x=0

$\Rightarrow \frac{3+c^2}{3}=0$

$\Rightarrow c^2=-3$

Which is impossible. Hence it cannot lie on y-axis.

Question 132

The area of a parallelogram formed by the lines $ax\pm by\pm c=0$, is

Options:

A. $\frac{c^2}{ab}$

B. $\frac{2c^2}{ab}$

C. $\frac{c^2}{2ab}$

D. None of these

Answer: B

Solution:

$ax \pm by \pm c = 0 \Rightarrow \frac{x}{\pm c/a} + \frac{y}{\pm c/b} = 1$

Which meets on axes at $A\left(\frac{c}{a}, 0\right), C\left(\frac{-c}{a}, 0\right)$

$B\left(0, \frac{c}{b}\right)$ and $D\left(0, \frac{-c}{b}\right)$

Therefore, the diagonals AC and BD of quadrilateral ABCD are perpendicular, hence it is a rhombus whose area is given by

$= \frac{1}{2} AC \times BD = \frac{1}{2} \times \frac{2c}{a} \times 2\frac{c}{b} = \frac{2c^2}{ab}$

Question 133

The equation of the circle with origin as centre passing the vertices of an equilateral triangle whose median is of length 3a is

Options:

A. $x^2+y^2=9a^2$

B. $x^2+y^2=16a^2$

C. $x^2+y^2=a^2$

D. None of the above

Answer: D

Solution:

Centre (0,0), radius= $3a \times \frac{2}{3} = 2a$

Hence, circle $x^2 + y^2 = 4a^2$ as centroid divides median in the ratio of 2:1.

Question 134

The axis of the parabola $9y^2$ - 16x - 12y – 57 = 0 is

Options:

A. 3y=2

B. x+3y=3

C. 2x=3

D. y=3

Answer: A

Solution:

Since, $9y^2-16x-12y-57=0$

$\Rightarrow\left(y-\frac{2}{3}\right)^2=\frac{16}{9}\left(x+\frac{61}{16}\right)$

Put $y-\frac{2}{3}=y$ and $x+\frac{61}{16}=x$

$\Rightarrow y^2=4\left(\frac{4}{9}\right)x$

Axis of this parabola is y=0

$\Rightarrow y-\frac{2}{3}=0\Rightarrow y=\frac{2}{3}$

$\Rightarrow 3y=2$

Question 135

The angle between the pair of tangents drawn to the ellipse $3x^2+2y^2=5$ from the point (1,2) is

Options:

A. $\tan^{-1}\left(\frac{12}{5}\right)$

B. $\tan^{-1}(6\sqrt{5})$

C. $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$

D. $\tan^{-1}12\sqrt{5}$

Answer: C

Solution:

Here, $S=3x^2+2y^2-5=0$

$S_1=3(1)^2+2(2)^2-5=3+8-5=66$

[$\because$Point(1,2)]

$T = 3x.x_1 + 2y.y_1 - 5$

$= 3x(1) + 2y(2) - 5$

$= 3x + 4y - 5$

Now use, $SS_1 = T^2$

$(3x^2 + 2y^2 - 5).6 = (3x + 4y - 5)^2$

$18x^2 + 12y^2 - 30 = 9x^2 + 16y^2$

$+24xy + 25 - 30x - 40y$

$\Rightarrow 9x^2 - 4y^2 - 24xy + 30x + 40y - 55 = 0$ Here, a=9, b=-4 and h=-12

$\therefore \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\sqrt{144+36}}{9-4} = \frac{1\sqrt{180}}{5}$

$= \frac{6\sqrt{20}}{5} = \frac{12}{\sqrt{5}}$

Question 136

The angle between two diagonals of a cube will be

Options:

A. $\sin^{-1}\left(\frac{1}{3}\right)$

B. $\cos^{-1}\left(\frac{1}{3}\right)$

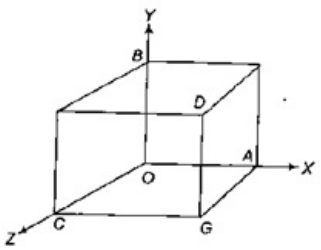
C. variable

D. None of these

Answer: B

Solution:

Let the cube of the side a.
(0,0,0), D(a,a,a),B(0,a,0), G(a,0,a). then equation of OD and BG are $\frac{x}{a} = \frac{y}{a} = \frac{z}{a}$ and $\frac{x}{a} = \frac{y-a}{-a} = \frac{z}{a}$, respectively.



Hence, angle between OD and BG is

$\cos^{-1}\left(\frac{a^2 - a^2 + a^2}{\sqrt{3a^2}.\sqrt{3a^2}}\right) = \cos^{-1}\left(\frac{1}{3}\right)$

Note: *Students should remember this question as a fact.*

Question 137

The lines
$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta}$ **and**
$\frac{x-b+c}{\beta-r}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+r}$
are coplanar and the equation to the plane in which they lie is

Options:

A. x+y+z=0

B. x -y+z=-0

C. x-2y+z=0

D. x+y-2z=0

Answer: C

Solution:

The lines will be coplanar, if

$$\begin{vmatrix} a-d-b+c & a-b & a+d-b-c \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-r & \beta & \beta+r \end{vmatrix}=0$$

Add 3rd column to first and it becomes twice the second
and hence the determinant is zero, as
the two columns are identical.
Again, the equation of the plane in which they lie is

$$\begin{vmatrix} x+a+d & y-a & z-a-d \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-r & \beta & \beta+r \end{vmatrix}=0$$

On adding 1st and 3rd columns and subtracting twice the 2nd, we get,

$$\begin{vmatrix} x+z-2y & y-a & z-a-d \\ 0 & \alpha & \alpha+\delta \\ 0 & \beta & \beta+r \end{vmatrix}=0$$

$$\Rightarrow [\alpha(\beta+r)-\beta(\alpha+\delta](x+z-2y)=0$$
$$\Rightarrow (x+z-2y)=0$$

Question 138

Three forces of magnitudes 1, 2 and 3 dynes meet in a point and act along diagonals of three adjacent faces of a cube. The resultant force is

Options:

A. 114 dynes

B. 6 dynes

C. 5 dynes

D. None of the above

Answer: C

Solution:

Three vectors meeting at a point are i+j,j+k and k+i. Forces of 1, 2 and 3dyenes are acting along these directions respectively, Therefore resultant force

$$= \frac{i+j}{\sqrt{2}} + \frac{2(j+k)}{\sqrt{2}} + \frac{3(k+i)}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(4i+3j+5k)$$

$$\therefore Magnitude = \frac{5\sqrt{2}}{\sqrt{2}} = 5dyenes$$

Question 139

The vectors $AB = 3i+5j+4k$ and AC=5i-5J+2k are side of a ΔABC. The length of the median through A is

Options:

A. $\sqrt{13}$ units

B. $2\sqrt{5}$ units

C. 5 units

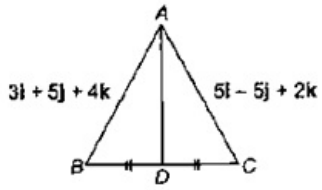
D. 10 units

Answer: C

Solution:

Position vector of AD

$$= \frac{(3+5)i+(5-5)j+(4+2)k}{2}$$



$$AD = \frac{8i+6k}{2} = 4i+3k$$

$\therefore$ Length of median $= |AD|$

$= \sqrt{16+9} = 5$ units.

Question 140

Let a=2i+j+k, b=i+2j-k, and a unit vector c be coplanar, If c is perpendicular to a, then c is

Options:

A. $\frac{1}{\sqrt{2}}(-j+k)$

B. $\frac{1}{\sqrt{3}}(-i-j-k)$

C. $\frac{1}{\sqrt{5}}(i-2j)$

D. $\frac{1}{\sqrt{3}}(i-j-k)$

Answer: A

Solution:

c is coplanar with **a** and **b**.

$\therefore c = xa + yb$

$\Rightarrow c = x(2i+j+k) + y(i+2j-k)$

$\Rightarrow c = (2x+y)i + (x+2y)j + (x-y)k$

$a.c = 0$

$\therefore 2(2x+y) + x + 2y + x - y = 0$

$\Rightarrow y = -2x$

$c = -3xj + 3xk = 3x(-j+k)$

$|c| = 1$

$\therefore 9x^2 + 9x^2 = 1$

$\Rightarrow x = \pm\frac{1}{3\sqrt{2}}$

$\Rightarrow c = \frac{1}{\sqrt{2}}(-j+k)$

Question 141

Odds 8 to 5 against a person who is 40 yr old living till he is 70 and 4 to 3 against another person now 50 till he will be living 80. Probability that one of them will be alive next 30 yr.

Options:

A. $\frac{59}{91}$

B. $\frac{44}{91}$

C. $\frac{51}{91}$

D. $\frac{32}{91}$

Answer: B

Solution:

Probability [Person A will die in 30 years]=$\frac{8}{8+5}$

$P(A)=\frac{8}{13}\Rightarrow P(\bar{A})=\frac{5}{13}$

Similarly, $P(B)=\frac{4}{7}\Rightarrow P(\bar{B})=\frac{3}{7}$

There are two ways in which one person is alive after 30 yr. $\bar{A}B$ and $A\bar{B}$ and event are independent.

So, required probability

$=P(\bar{A}).P(B)+P(A).P(\bar{B})$

$=\frac{5}{13}.\frac{4}{7}+\frac{8}{13}.\frac{3}{7}=\frac{44}{91}$

Question 142

A purse contains 4 copper coins and 3 silver coins, the second purse contains 6 copper coins and 2 silver coins. If a coin is drawn out of any purse, then the probability that it is a copper coin is

Options:

A. $\frac{4}{7}$

B. $\frac{3}{4}$

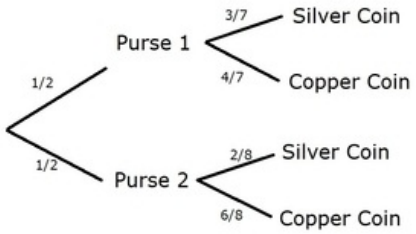
C. $\frac{37}{56}$

D. None of these

Answer: C

Solution:

The tree diagram for the problem is



$$=\frac{1}{2}.\frac{4}{7}+\frac{1}{2}.\frac{6}{8}=\frac{37}{56}$$

Question 143

The mean of the values 0,1,2,...,n having corresponding weight ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ respectively is

Options:

A. $\frac{2^n}{(n+1)}$

B. $\frac{2^{n+1}}{\mathrm{n}(n+1)}$

C. $\frac{n+1}{2}$

D. $\frac{n}{2}$

Answer: D

Solution:

The required mean is

$$\bar{x}=\frac{0.1+1.{}^nC_1+2.{}^nC_2+3.{}^nC_3+\ldots+{}^nC_n}{1+{}^nC_1+{}^nC_2+\ldots+{}^nC_n}=\frac{\sum_{r=0}^{n} r.{}^nC_r}{\sum_{r=0}^{n} {}^nC_r}=\frac{\sum_{r=1}^{n} r.\frac{n}{r}.{}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r}$$

$$=\frac{n\sum_{r=1}^{n} {}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r}$$

$$=\frac{n.2^{n-1}}{2^n}=\frac{n}{2}$$

Question 144

Mean of 100 observations is 45. It was later found that two observations 19 and 31 were in correctly recorded as 91 and 13. The correct mean is

Options:

A. 44.0

B. 44.46

C. 45.00

D. 45.54

Answer: B

Solution:

Sum of 100 items=45x100=4500
Sum of items added=19+31=50
Sum of items replaced=91+13=104
New Sum=4500-104+50=4446
So, New mean= $\frac{4446}{100}=44.46$

Question 145

The value of $\sin\theta+\cos\theta$ will be greatest, when

Options:

A. $\theta=30^0$

B. $\theta=45^0$

C. $\theta=60^0$

D. $\theta = 90^0$

Answer: B

Solution:

$$f(x) = \sin\theta + \cos\theta = \sqrt{2}.\sin\left(\theta + \frac{\pi}{4}\right)$$

let $But -1 \le \sin\left(\theta + \frac{\pi}{4}\right) \le 1$ Hence, the maximum value of $(\sin\theta + \cos\theta)$

$$\Rightarrow -\sqrt{2} \le \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) \le \sqrt{2}$$

$$i.e., \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) = \sqrt{2}$$

$$\therefore \sin\left(\theta + \frac{\pi}{4}\right) = 1 \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \sin\frac{\pi}{2}$$

$$\Rightarrow \theta + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} = 45^0$$

Question 146

A house subtends a right at the window of the opposite house and the angle of elevation of the window from the bottom of the first house is 60. If the distance between the two houses be 6 m, then the height of the first house is

Options:

A. $6\sqrt{3}$ m

B. $8\sqrt{3}$ m

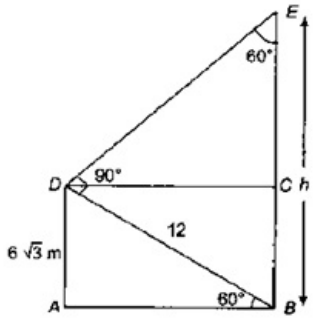
C. $4\sqrt{3}$ m

D. None of these

Answer: B

Solution:

$\frac{12}{h} = \sin 60^0$ (in ΔABD)



$\Rightarrow \quad h = 12 \times \frac{2}{\sqrt{3}} = 8\sqrt{3}\ \text{m}$

Question 147

$\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]$ **is equal to**

Options:

A. $\frac{2a}{b}$

B. $\frac{2b}{a}$

C. $\frac{a}{b}$

D. $\frac{b}{a}$

Answer: B

Solution:

$\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]$ Let $\frac{1}{2}\cos^{-1}\frac{a}{b}=\theta$

$\Rightarrow \cos 2\theta = \frac{a}{b}$

Thus, $\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)$

$=\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}$

$=\frac{(1+\tan\theta)^2+(1-tan\theta)^2}{(1-\tan^2\theta)}$

$=\frac{1+\tan^2\theta+2\tan\theta+1+\tan^2\theta-2\tan\theta}{(1-\tan^2\theta)}$

$=\frac{2(1+\tan^2\theta)}{(1-\tan^2\theta)}=2\sec 2\theta=\frac{2}{\cos 2\theta}$

$=\frac{2}{a/b}=\frac{2b}{a}$

Question 148

The proposition (p⇒~p)^(~p⇒p) is a

Options:

A. Tautology and contradiction

B. Neither tautology nor contradiction

C. Contradiction

D. Tautology

Answer: C

Solution:

p	~p	(p⇒~p)	(~p⇒p)	(p⇒~p)^(~p⇒p)
F	T	T	F	F
T	F	F	T	F

(p⇒~p)^(~p⇒p) is always false. Hence, the (p⇒~p)^(~p⇒p) is a contradiction.

A contradiction is a statement which is always false.

Question 149

The solution of the differential equation $xdy - ydx = (\sqrt{x^2 + y^2})dx$ is

Options:

A. $y - \sqrt{x^2 + y^2} = Cx^2$

B. $y + \sqrt{x^2 + y^2} = Cx^2$

C. $y + \sqrt{x^2 + y^2} + Cx^2 = 0$

D. None of the above

Answer: B

Solution:

$$xdy - ydx = \sqrt{x^2 - y^2}\,dx$$

$$dy = \frac{y}{x}dx = \sqrt{1 + \left(\frac{y}{x}\right)^2}\,dx$$

$$\frac{dy}{dx} = \left(\sqrt{1 + \left(\frac{y}{x}\right)^2} + \frac{y}{x}\right)$$

$$Let\ \frac{y}{x} = v$$

$$y = xv$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \sqrt{1 + v^2}$$

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

$$\ln|v + \sqrt{1 + v^2}| = \ln|x| + \ln C$$

$$\ln|v + \sqrt{1 + v^2}| = \ln|Cx|$$

$$y + \sqrt{x^2 + y^2} = Cx^2$$

Question 150

The solution of $\frac{dy}{dx} = \cos x(2 - y\ \text{cosec}\ x),$ where y = 2, when x=π/2 is

Options:

A. y = sin x + cosec x

B. $y = \tan\frac{x}{2} + \cot\frac{x}{2}$

C. $y = \frac{1}{\sqrt{2}}\sec\frac{x}{2} + \sqrt{2}\cos\frac{x}{2}$

D. None of the above

Answer: A

Solution:

$$\frac{dy}{dx} = \cos x(2 - y\,\mathrm{cosec}\,x)$$

$$\frac{dy}{dx} + y\cot x = 2\cos x$$

$$P(x) = \cot x$$

$$IF = e^{\int \cot x\,dx}$$

$$= e^{\ln \sin x}$$

$$= \sin x$$

$$\sin x \times y = \int \sin x \times 2\cos x\,dx$$

$$= \sin^2 x + c$$

$$y = \sin x + \frac{c}{\sin x}$$

Since, $y = 2$

$$at\ x = \frac{\pi}{2}$$

$$2 = 1 + c$$

$$c = 1$$

$$y = \sin x + \frac{1}{\sin x}$$

$$y = \sin x + \mathrm{cosec}\,x$$
