CHAPTER 2

Polynomials

IMPORTANT POINTS

Constant : A symbol having a fixed numerical value is called a constant.

Examples : 8, – 7, $\frac{5}{9}$, π etc. are all constants.

Variables : A symbol which may be assigned different numerical vlaues is known as a variable.

Example : We know that area of circle is given by the formula $A = \pi r^2$ where r is the radius of the circle. Here, π is constant while A and r are variables.

Algebraic Expressions : A combination of constants and variables, connected by some or all of the operations +, –, × and ÷, is known as an algebraic expression.

Terms of An Algebraic Expression

The several parts of an algebraic expression separated by + or – operations are called the terms of the expression.

Examples : (i) $5 + 9x - 7x^2y + \frac{3}{7}xy$

(ii) $x^3 + 3x^2y + 3xy^2 + y^3 + 7$

Polynomials : An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

General Form : $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ is a polynomial in variable x, where $a_0, a_1, a_2, a_3 ... a_n$ are real numbers and n is non-negative integer.

Examples : (i) $6x^3 - 4x^2 + 7x - 3$ is a polynomial in one variable x.

(ii) $9y^5 + 6y^4 + 7y^3 + 10y^2 - 8y + \frac{2}{7}$ is a polynomial in one variable y.

(iii) $3 + 2x^2 - 6x^2y + 5xy^2$ is a polynomial in two variables x and y.

(iv) $5 + 8x^{5/2} + 7x^3$ is an expression but not a polynomial since it contains a term containing $x^{5/2}$ where $\frac{5}{2}$ is not a non-negative integer.

Coefficients : In the polynomial $6x^3 - 5x^2 + 5x - 7$ we say that coefficients of x^3, x^2 and x are 6, –5 and 5 respectively and we also say that –7 is the constant term in it.

Degree of A Polynomial in One Variable : In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.

Examples :

(i) $3x + 5$ is a polynomial in x of degree 1.

(ii) $4y^2 - \frac{7}{2}y + 5$ is a polynomial in y of degree 2.

Degree of A Polynomial in Two Or More Variables :

In case of polynomials in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of polynomial.

Examples :

(i) $7x^2 - 5x^2y^2 + 3xy + 7y + 9$ is a polynomial in x and y of degree 4.

(ii) $4x^3y^3 - 5xy^2 + 2x^4 - 7$ is a polynomial in x and y of degree 6.

Polynomials of Various Degrees

(1) Linear Polynomial : A polynomial of degree 1 is called a linear polynomial.

Examples :

(i) $2x + 7$ is a linear polynomial in x.

(ii) $2x + y + 7$ is a linear polynomial in x and y.

(2) Quadratic Polynomial : A polynomial of degree 2 is called a quadratic polynomial.

Examples :

(i) $x^2 + 2x + 3$ is a quadratic polynomial in x.

(ii) $xx + yz + zx$ is a quadratic polynomial in x, y and z.

(3) Cubic Polynomial : A polynomial of degree 3 is called a cubic polynomial.

Examples :

(i) $5x^3 - 3x^2 + 7x + 7$ is a cubic polynomial in x and y.

(ii) $4x^2y + 7xy^2 + 7$ is a cubic polynomial in x and y.

(4) Biquadratic Polynomial : A polynomial of degree 4 is called a biquadratic polynomial.

Examples :

(i) $x^4 - 7x^3 + 15x^2 + 7x - 9$ is a biquadratic polynomial in x.

(ii) $x^2y^2 + xy^3 + y^4 - 8xy + 2y^2 + 9$ is a biquadratic polynomial in x and y.

Number of Terms in A Polynomial

(i) Monomial : A polynomial containing one non-zero term is called a monomial.

Example : 5, 3x, $\frac{7xy}{4}$ are all monomials.

(ii) Binomial : A polynomial containing two non-zero terms is called a binomial.

Examples : $(7 + 5x)$, $(x - 7y)$, $(5x^2y + 3yz)$ are all binomials.

(iii) Trinomial : A polynomial containing three nonzero terms is called a trinomial.

Examples :

$(8 + 5x + x^2)$, $3x - 5xy + 7y^2$ are all trinomials.

Constant Polynomial

A polynomial containing one term only, consisting of a constant is called a constant polynomial.

Examples :

7, – 5, $\frac{7}{9}$ etc. are all constant polynomials.

Clearly, the degree of a non-zero constant polynomial is zero.

Zero Polynomial

A polynomial consisting of one term, namely zero only, is called a zero polynomial. The degree of a zero polynomial is not defined.

Zeros of A Polynomial

Let $p(x)$ be a polynomial. If $p(a) = 0$, then we say that a is a zero of the polynomial $p(x)$. Finding the zeros of a polynomial $p(x)$ means solving the equation $p(x) = 0$.

Example : If $f(t) = 3t^2 - 10t + 6$, find $f(0)$.

Solution : $f(t) = 3t^2 - 10t + 6$

$\Rightarrow f(0) = 3 \times 0^2 - 10 \times 0 + 6 = 6$

Example : If $p(x) = 2x^2 - 5x + 4$, find $p(2)$

Solution : $p(x) = 2x^2 - 5x + 4$

$\Rightarrow p(2) = (2 \times 2^2 - 5 \times 2 + 4) = 8 - 10 + 4 = 2$

Example : Find a zero of polynomial

$p(x) = x - 7$

Solution : $p(x) = x - 7$

Now, $p(x) = 0 \Rightarrow x - 7 = 0 \Rightarrow x = 7$

$\therefore$ 7 is a zero of polynomial $p(x)$

Addition and Difference of Two Polynomials

Addition of two polynomials is determined by arranging terms of same degrees with signs and adding the co-efficients. The operation of subtraction is similar to the operation of addition. Only difference is that the signs of the polynomial to be subtracted are changed and then operation of addition is performed.

Example : If $p(x)$

$= x^4 - 5x^3 + 3x + 9$ and $q(x) = 2x^4 - 3x^3 + 5x - 4$

Then, $p(x) + q(x)$

$= (x^4 - 5x^3 + 3x + 9) + (2x^4 - 3x^3 + 5x - 4)$

$= (x^4 + 2x^4) + (-5x^3 - 3x^3) + (3x + 5x) + (9 - 4)$

$= 3x^4 - 8x^3 + 8x + 5$

For the sake of convenience, the above operation can be written in the following form :

$p(x) = x^4 - 5x^3 + 3x + 9$

$q(x) = 2x^4 - 3x^3 + 5x - 4$

$\therefore p(x) + q(x) = 3x^4 - 8x^3 + 8x + 5$

and, $p(x) - q(x)$

$= (x^4 - 5x^3 + 3x + 9) - (2x^4 - 3x^3 + 5x - 4)$

$= (x^4 - 5x^3 + 3x + 9) + (-2x^4 + 3x^3 - 5x + 4)$

$= (x^4 - 2x^4) + (-5x^3 + 3x^3) + (3x - 5x) + (9 + 4)$

$= -x^4 - 2x^3 - 2x + 13$

For the sake of convenience,

$\Rightarrow \quad p(x) = x^4 - 5x^3 + 3x + 9$

$\quad\quad q(x) = 2x^4 - 3x^3 + 5x - 4$

$\quad\quad\quad - \quad - \quad + \quad - \quad +$

$\therefore p(x) - q(x) = -x^4 - 2x^3 - 2x + 13$

Multiplication of Two Polynomials

To determine the product of two polynomials, the distributive law of multiplication is used first and then grouping is made of terms of same degrees for addition and subtraction.

$x^3 - 6x^2 + x + 1$

$x^2 - 3x + 2$

$x^5 - 6x^4 + x^3 + x^2$

$\quad - 3x^4 + 18x^3 - 3x^2 - 3x$

$\quad\quad + 2x^3 - 12x^2 + 2x + 2$

$x^5 - 9x^4 + 21x^3 - 14x^2 - x + 2$

Remember : $(-x) \times (-x) = +x^2$

$(+x) \times (-x) = -x^2$

$(x) \times (x) = x^2$ etc.

or, $(-) \times (-) = +$

$(-) \times (+) = -$

$(+) \times (-) = -$

$(+) \times (+) = +$

Division of Polynomial by Another Polynomial

Let $p(x)$ and $q(x)$ be two polynomials and $q(x) \neq 0$. If we find two polynomials $g(x)$ and $r(x)$ such that

$p(x) = g(x)\, q(x) + r(x)$

i.e. Dividend = Divisor × Quotient + Remainder

Where degree of $r(x)$ < degree of $g(x)$, then we say that on dividing $p(x)$ by $q(x)$, the quotient is $g(x)$ and remainder is $r(x)$. If remainder $r(x)$ = zero, we say that $q(x)$ is a factor of $p(x)$.

Let's take a few examples to illustrate the method of division of a polynomial by a polynomial of lesser degree.

Example : Divide $p(x) = x^3 + 3x^2 - 12x + 4$ by $g(x) = x - 2$.

Solution :

$$\begin{array}{r|l} & x^2+5x-2 \\ \hline x-2 & x^3+3x^2-12x+4 \\ & x^3-2x^2 \\ & -\quad + \\ \hline & \quad 5x^2-12x+4 \\ & \quad 5x^2-10x \\ & \quad -\quad + \\ \hline & \qquad -2x+4 \\ & \qquad -2x+4 \\ & \qquad +\quad - \\ \hline & \qquad\qquad 0 \end{array}$$

Note : It is to be noted that the degree of q(x) is less than that of $p(x)$ and polynomial of higher degree is always divided by a polynomial of lower degree. The operation of division ends when the remainder is either zero or the degree of remainder is less than that of divisor.

In the above example, the quotient is $x^2 + 5x - 2$ and remainder is zero. As the remainder is zero, $(x - 2)$ is a factor of $x^3 + 3x^2 - 12x + 4$.

Example : Divide $p(x) = x^3 - 14x^2 + 37x - 60$ by $g(x) = x - 2$.

Solution :

$$\begin{array}{r|l} & x^2-12x+13 \\ \hline x-2 & x^3-14x^2+37x-60 \\ & x^3-2x^2 \\ & -\quad + \\ \hline & \quad -12x^2+37x-60 \\ & \quad -12x^2+24x \\ & \quad +\quad - \\ \hline & \qquad 13x-60 \\ & \qquad 13x-26 \\ & \qquad -\quad + \\ \hline & \qquad\qquad -34 \end{array}$$

Here, quotient = $x^2 - 12x + 13$ and remainder = -34

Since remainder $\neq 0$, then $(x - 2)$ is not a factor of $x^3 - 14x^2 + 37x - 60$.

Remainder Theorem

Let $f(x)$ be a polynomial of degree $n \geq 1$, and let a be any real number. When $f(x)$ is divided by $(x - a)$, then the remainder is $f(a)$.

Proof : Suppose that when $f(x)$ is divided by $(x - a)$, the quotient is $g(x)$ and the remainder is $r(x)$.

Then, degree $r(x) <$ degree $(x - a)$

$\Rightarrow$ degree $r(x) < 1$

$\Rightarrow$ degree $r(x) = 0$ [$\because$ degree of $(x - a) = 1$]

$\Rightarrow r(x)$ is constant, equal to r (say).

Thus, when $f(x)$ is divided by $(x - a)$, then the quotient is $g(x)$ and the remainder is r.

$\therefore f(x) = (x - a) \cdot g(x) + r$ ... (i)

Putting $x = a$ in (i), we get $r = f(a)$.

Thus, when $f(x)$ is divided by $(x - a)$, then the remainder is $f(a)$.

Remarks

(i) If a polynomial $p(x)$ is divided by $(x + a)$, the remainder is the value of $p(x)$ at $x = -a$ i.e. $p(-a)$

[$\because x + a = 0 \Rightarrow x = -a$]

(ii) If a polynomial $p(x)$ is divided by $(ax - b)$, the remainder is the value of $p(x)$ at $x = \frac{b}{a}$ i.e. $p\left(\frac{b}{a}\right)$.

[$\because ax - b = 0 \Rightarrow x = \frac{b}{a}$]

(iii) If a polynomial $p(x)$ is divided by $(ax + b)$, then remainder is the value of $p(x)$ at $x = -\frac{b}{a}$ i.e. $p\left(-\frac{b}{a}\right)$

[$\because ax + b = 0 \Rightarrow x = -\frac{b}{a}$]

(iv) If a polynomial $p(x)$ is divided by $b - ax$, the remainder is the value of $p(x)$ at $x = \frac{b}{a}$ i.e. $p\left(\frac{b}{a}\right)$

[$\because b - ax = 0 \Rightarrow x = \frac{b}{a}$]

Example : Let $p(x) = x^4 - 3x^2 + 2x + 5$. Find remainder when $p(x)$ is divided by $(x - 1)$.

Solution :

$$\begin{array}{r|l} & x^3+x^2-2x \\ \hline x-1 & x^4+0x^3-3x^2+2x+5 \\ & x^4-x^3 \\ & -\quad + \\ \hline & \quad x^3-3x^2+2x+5 \\ & \quad x^3-x^2 \\ & \quad -\quad + \\ \hline & \qquad -2x^2+2x+5 \\ & \qquad -2x^2+2x \\ & \qquad +\quad - \\ \hline & \qquad\qquad 5 \end{array}$$

Here, remainder = 5

Find the value of p(1) from the above example.

$p(1) = 1 - 3 \times 1 + 2 \times 1 + 5 = 5$

Thus, remainder obtained on dividing $p(x)$ by $(x - 1)$ is same as $p(1)$.

Factor Theorem

Let $p(x)$ be a polynomial of degree greater than or equal to 1 and a be a real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$

Proof : First, let $p(x)$ be a polynomial of degree greater than or equal to one and a be a real number such that $p(a) = 0$, then we have to show that $(x - a)$ is a factor of $p(x)$.

Let $q(x)$ be the quotient, when $p(x)$ is divided by $(x-a)$.

By remainder theorem,

$p(x)$ when divided by $(x-a)$ gives remainder equal to $p(a)$.

$\therefore p(x) = (x-a)\, q(x) + p(a)$

$\Rightarrow p(x) = (x-a)\, q(x)$ $\quad [\because p(a) = 0]$

$\Rightarrow (x-a)$ is a factor of $p(x)$

Conversely, Let $(x-a)$ is a factor of $p(x)$. Then, we have to prove that $p(a) = 0$.

Now $(x-a)$ is a factor of $p(x)$

$\Rightarrow p(x)$, when divided by $(x-a)$ gives remainder zero. But by Remainder theorem,

$p(x)$ when divided by $(x-a)$ gives the remainder equal to $p(a)$.

$\therefore p(a) = 0$

Remarks

(i) $(x+a)$ is a factor of a polynomial iff (if and only if) $p(-a) = 0$

(ii) $(ax-b)$ is a factor of a polynomial iff $p\left(\frac{b}{a}\right) = 0$

(iii) $(ax+b)$ is a factor of a polynomial $p(x)$ iff $p\left(-\frac{b}{a}\right) = 0$

(iv) $(x-a)(x-b)$ are factors of a polynomial $p(x)$ iff $p(a) = 0$ and $p(b) = 0$

G.C.D & L.C.M OF POLYNOMIALS

Greatest Common Divisor/Highest Common Factor (GCD/HCF) : The GCD of two polynomials $p(x)$ and $q(x)$ is that common divisor which has the highet degree among all common divisors and the coefficient of the highest degree term be positive.

Example : What is the HCF of $(x+4)^2 (x-3)^3$ and $(x-1)(x+4)(x-3)^2$?

Sol. $p(x) = (x+4)^2 (x-3)^2$

$q(x) = (x-1)(x+4)(x-3)^2$

We see that $(x+4)(x-3)^2$ is such a polynomial that is a common divisor and whose degree is highest among all common divisors.

$\therefore$ HCF $= (x+4)(x-3)^2$

Lowest Common Multiple (LCM) : The LCM of two polynomials $p(x)$ and $q(x)$ is a polynomial of lowest degree of which $p(x)$ and $q(x)$ both are multiples.

Example : Find the LCM of $(x-1)(x+2)^2$ and $(x-1)^3(x+2)$.

Sol. $p(x) = (x-1)(x+2)^2$

$q(x) = (x-1)^3 (x+2)$

We make a polynomial by taking each factor of $p(x)$ and $q(x)$. If a factor is common in both, then we take that factor which has highest degree in $p(x)$ and $q(x)$.

$\therefore$ LCM $= (x-1)^3 (x+2)^2$

Relation Between Two Polynomials And Their HCF And LCM

$p(x) = 8(x^3 - 3x + 2)$ and

$q(x) = 14(x^2 + x - 2)$

Now, $p(x) = 8(x^2 - 3x + 2)$

$= 2 \times 2 \times 2 \times (x-2)(x-1)$

$q(x) = 14(x^2 + x - 2)$

$= 2 \times 7 \times (x+2)(x-1)$

HCF $= 2(x-1)$

LCM $= 2 \times 2 \times 2 \times 7 \times (x-1)(x-2)(x+2)$

$= 56(x-1)(x-2)(x+2)$

$\therefore$ HCF $\times$ LCM $= 112(x-1)^2(x-2)(x+2)$

Also, $p(x) \times q(x)$

$= 112(x-1)^2(x-2)(x+2)$

$\therefore$ HCF $\times$ LCM $= p(x) \times q(x)$

$\therefore$ HCF of polynomials $\times$ LCM of same polynomials = Product of same polynomials.

Example : Find the HCF of following pair of polynomials :

$p(x) = (x^2 - 9)(x-3)$

$q(x) = x^2 + 6x + 9$

Sol. $p(x) = (x^2 - 9)(x-3)$

$= (x+3)(x-3)(x-3)$

$= (x+3)(x-3)^2$

$q(x) = x^2 + 6x + 9$

$= x^2 + 3x + 3x + 9$

$= x(x+3) + 3(x+3) = (x+3)(x+3)$

$= (x+3)^2$

HCF of $p(x)$ and $q(x) = x + 3$

Example : Find the LCM of $p(x) = (x+3)(x-2)^2$ and $q(x) = (x-2)(x-6)$.

Sol. $p(x) = (x+3)(x-2)^2$

$q(x) = (x-2)(x-6)$

HCF of $p(x)$ and $q(x) = (x-2)$

LCM of $p(x)$ and $q(x) = \frac{p(x) \times q(x)}{\text{HCF}}$

$= \frac{(x+3)(x-2)^2 \times (x-2)(x-6)}{(x-2)}$

$= (x+3)(x-2)^2(x-6)$

SOLVED OBJECTIVE QUESTIONS

1. When the polynomial $f(x) = x^4 + 2x^3 - 3x^2 + x - 1$ is divided by $(x - 2)$ what will be the remainder?
(1) 21 (2) 22
(3) 23 (4) 29

2. When $f(x) = x^3 - 3x^2 + 4x + 50$ is divided by $(x + 3)$, the remainder is
(1) 16 (2) − 16
(3) 15 (4) − 15

3. When polynomial $f(x) = 4x^3 - 12x^2 + 14x - 3$ is divided by $2x - 1$, the remainder is :
(1) $\frac{2}{3}$ (2) $\frac{1}{3}$
(3) $\frac{3}{2}$ (4) $\frac{2}{7}$

4. If polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 2x + a$ are divided by $(x - 2)$, the same remainders are obtained. Find the value of a.
(1) −3 (2) 3
(3) −4 (4) −9

5. If the polynomial $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $(x - 1)$ and $(x + 1)$, the remainders are respectively 5 and 19. The values of a and b are :
(1) $a = 8, b = 7$ (2) $a = 5, b = 8$
(3) $a = 8, b = 5$ (4) $a = 6, b = 8$

6. The factor of polynomial $f(x) = x^4 + x^2 - 17x + 15$ is
(1) $x - 3$ (2) $x - 5$
(3) $x + 3$ (4) $x + 1$

7. For what value of a, $(x - a)$ is a factor of $f(x) = x^5 - a^2x^3 + 2x + a - 3$?
(1) − 1 (2) 1
(3) − 2 (4) 2

8. For what value of a, $(x + a)$ is a factor of polynomial $f(x) = x^3 + ax^2 - 2x + a + 6$?
(1) 2 (2) 3
(3) − 2 (4) − 3

9. For what value of k, $(2x^4 + 3x^3 + 2kx^2 + 3x + 6)$ is exactly divisible by $(x + 2)$?
(1) 1 (2) 2
(3) − 2 (4) − 1

10. For what values of a and b, $(x^3 - 10x^2 + ax + b)$ is exactly divisible by $(x - 1)$ and $(x - 2)$?
(1) $a = 23, b = -14$ (2) $a = -23, b = 14$
(3) $a = 21, b = -14$ (4) $a = -21, b = 15$

11. The LCM of $x^3 - 1$, $x^4 + x^2 + 1$ and $x^4 - 5x^2 + 4$ is :
(1) $(x - 1)(x + 1)(x - 2)$
(2) $(x - 1)(x + 1)(x + 2)$
(3) $(x^2 - 1)(x^2 - 4)$
(4) $(x^2 - 1)(x^2 - 4)(x^2 + x + 1)(x^2 + 1 - x)$

12. What is the LCM of $x^2 - 1$, $x^2 + 4x + 3$ and $x^2 - 3x + 2$?
(1) $(x + 3)(x + 1)$
(2) $(x^2 - 1)(x - 2)(x + 3)$
(3) $(x - 1)(x - 2)(x - 3)$
(4) $(x^2 - 1)(x + 2)(x + 3)$

13. What is the HCF of $x^2 - x - 12$, $x^2 - 7x + 12$ and $2x^2 - 11x + 15$?
(1) 1 (2) $(x - 3)$
(3) $2x - 5$ (4) $x - 4$

14. If the LCM and HCF of two quadratic polynomials are $x^3 - 7x + 6$ and $(x - 1)$ respectively, find the polynomials.
(1) $(x^2 - 3x + 2)$, $(x^2 + 2x + 3)$
(2) $(x^2 + 3x - 2)$, $(x^2 - 2x + 3)$
(3) $(x^2 - 3x + 2)$, $(x^2 + 2x - 3)$
(4) $(x^2 + 3x + 2)$, $(x^2 + 2x + 3)$

15. The HCF of two expressions is $x - 7$ and their LCM is $x^3 - 10x^2 + 11x + 70$. If one of them is $x^2 - 5x - 14$, find the other.
(1) $x^2 - 12x + 35$ (2) $x^2 + 12x - 35$
(3) $x^2 - 14x + 35$ (4) $x^2 + 14x - 35$

16. The HCF and LCM of two expressions are respectively $(x + 3)$ and $(x^3 - 7x + 6)$. If one of the polynomials is $x^2 + 2x - 3$, the other polynomial is
(1) $x^2 - x + 6$ (2) $x^2 - 2x + 6$
(3) $x^2 - 2x + 8$ (4) $x^2 + x - 6$

17. If HCF and LCM of two terms x and y are a and b respectively and $x + y = a + b$, then $x^2 + y^2 = ?$
(1) $a^2 - b^2$ (2) $2a^2 + b^2$
(3) $a^2 + b^2$ (4) $a^2 + 2b^2$

18. If common factor of $x^2 + bx + c$ and $x^2 + mx + n$ is $(x + a)$, then the value of a is :
(1) $\frac{c-n}{b-m}$ (2) $\frac{c-n}{b+m}$
(3) $\frac{c+1}{b-m}$ (4) $\frac{c-n}{m-b}$

19. If $(x + 1)$ and $(x - 2)$ be the factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, then the values of a and b will be
(1) 2 and 8 (2) 1 and 7
(3) 5 and 3 (4) 3 and 7

[SSC Graduate Level Tier-I Exam, 2012]

20. The value of λ for which the expression $x^3 + x^2 - 5x + \lambda$ will be divisible by $(x - 2)$ is :
(1) 2 (2) −2
(3) −3 (4) 4

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (IInd Sitting)]

ANSWERS

1. (1)	**2.** (2)	**3.** (2)	**4.** (1)	**5.** (2)
6. (1)	**7.** (2)	**8.** (3)	**9.** (4)	**10.** (1)
11. (4)	**12.** (2)	**13.** (2)	**14.** (3)	**15.** (1)
16. (4)	**17.** (3)	**18.** (1)	**19.** (2)	**20.** (2)

EXPLANATIONS

1. (1) Here, $x-2=0 \Rightarrow x=2$

By Remainder Theorem, when polynomial $f(x)$ is divided by $(x-2)$, the remainder is $f(2)$.

$\therefore\ f(2)=2^4+2\times2^3-3\times2^2+2-1$

[**Remember :** x has been replaced by 2]

$=16+16-12+2-1=21$

$\therefore$ Remainder = 21

2. (2) Divisor $=x+3 \quad \therefore\ x+3=0 \Rightarrow x=-3$

By Remainder Theorem,

Remainder $=f(-3)=[(-3)^3-3(-3)^2+4(-3)+50]$

$=(-27-27-12+50)=-16$

3. (2) Here, $2x-1=0 \Rightarrow x=\frac{1}{2}$

By Remainder Theorem,

Remainder

$$=f\left(\frac{1}{2}\right)=\left[4\times\left(\frac{1}{2}\right)^3-12\left(\frac{1}{2}\right)^2+14\left(\frac{1}{2}\right)-3\right]$$

$$=\left(\frac{1}{2}-3+7-3\right)=\frac{3}{2}$$

4. (1) $f(x)=2x^3+ax^2+3x-5$

$g(x)=x^3+x^2-2x+a$

By Remainder Theorem,

$f(2)=(2\times2^3+a\times2^2+3\times2-5)=17+4a$

Again, $g(2)=(2^3+2^2-2\times2+a)=8+a$

$\therefore 17+4a=8+a$

$\Rightarrow 3a=-9 \qquad \Rightarrow a=-3$

5. (2) $f(x)=x^4-2x^3+3x^2-ax+b$

$f(1)=1-2+3-a+b=2-a+b$

$[x-1=0 \Rightarrow x=1]$

$f(-1)=1+2+3+a+b$

$=6+a+b$

$[x+1=0 \Rightarrow x=-1]$

$\therefore\ 2-a+b=5 \Rightarrow b-a=3$ ... (i)

and, $6+a+b=19 \Rightarrow a+b=13$ ... (ii)

By adding equations (i) and (ii),

$2b=16 \Rightarrow b=8$

From equation (ii),

$a+b=13 \Rightarrow a=13-8=5$

6. (1) By Factor Theorem,

If $f(a)=0$, then $(x-a)$ is a factor of $f(x)$.

$\therefore\ f(3)=3^3+3^2-17\times3+15$

$=27+9-51+15=0$

$\therefore\ (x-3)$, is a factor of $f(x)$.

Remember : $f(5)\neq0;\ f(-3)\neq0;\ f(-1)\neq0$

7. (2) $(x-a)$, is a factor of polynomial.

$x^5-a^2x^3+2x+a-3 \ \therefore\ f(a)=0$

$\Rightarrow a^5-a^5+2a+a-3=0$

$\Rightarrow 3a=3 \Rightarrow a=1$

8. (3) Here, $x+a=0 \Rightarrow x=-a \therefore f(-a)=0$

$\Rightarrow (-a)^3+a(-a)^2-2(-a)+a+6=0$

$\Rightarrow 3a=-6 \Rightarrow a=-2$

9. (4) Here, $x+2=0 \Rightarrow x=-2$

By Factor Theorem,

$f(-2)=0$

$\Rightarrow 2(-2)^4+3(-2)^3+2k(-2)^2+3(-2)+6=0$

$\Rightarrow 32-24+8k-6+6=8k+8=0$

$\Rightarrow 8k=-8 \Rightarrow k=-1$

10. (1) $f(x)=x^3-10x^2+ax+b$

By Factor Theorem,

$f(1)=1-10+a+b=a+b-9$

$[\because\ x-1=0 \Rightarrow x=1]$

$\therefore\ f(1)=0 \Rightarrow a+b=9$ ...(i)

$f(2)=8-40+2a+b=2a+b-32$

$\therefore\ f(2)=0 \Rightarrow 2a+b=32$ ...(ii)

From equation (ii) – equiation (i), $a=23$

From equation (i),

$b=9-23=-14$

11. (4) (i) $x^3-1=(x-1)(x^2+x+1)$

(ii) $x^4+x^2+1=x^2+2x^2+1-x^2$

$=(x^2+1)^2-x^2=(x^2+1-x)(x^2+1+x)$

(iii) $x^4-5x^2+4=x^4-4x^2-x^2+4$

$=x^2(x^2-4)-1(x^2-4)=(x^2-4)(x^2-1)$

$=(x-2)(x+2)(x-1)(x+1)$

LCM

$=(x-1)(x+1)(x-2)(x+2)(x^2+x+1)(x^2+1-x)$

12. (2) (i) $x^2-1=x^2-(1)^2=(x-1)(x+1)$

(ii) $x^2+4x+3=x^2+3x+x+3$

$=x(x+3)+1(x+3)=(x+3)(x+1)$

(iii) $x^2-3x+2=x^2-2x-x+2$

$=x(x-2)-1(x-2)=(x-2)(x-1)$

LCM $=(x-1)(x+1)(x-2)(x+3)$

13. (2) (i) $x^2-x-12=x^2-4x+3x-12$

$=x(x-4)+3(x-4)$

$=(x-4)(x+3)$

(ii) $x^2 - 7x + 12 = x^2 - 3x - 4x + 12$
$= x(x-3) - 4(x-3)$
$= (x-3)(x-4)$

(iii) $2x^2 - 11x + 15 = 2x^2 - 6x - 5x + 15$
$= 2x(x-3) - 5(x+3)$
$= (x-3)(2x-5)$

HCF = 1 as no term is common.

14. (3) Putting $x - 1 = 0$ i.e. $x = 1$ in $(x-1)$ respectively.

Remainder $= (+1)^3 - 7(1) + 6 = 1 - 7 + 6 = 0$

$\therefore (x-1)$ is a factor of expression $x^3 - 7x + 6$

Now, $x^3 - 7x + 6 = x^2(x-1) + x(x-1) - 6(x-1)$
$= (x-1)(x^2 + x - 6)$
$= (x-1)[x^2 + 3x - 2x - 6]$
$= (x-1)[x(x+3) - 2(x+3)]$
$= (x-1)(x-2)(x+3)$

LCM $= x^3 - 7x + 6 = (x-1)(x-2)(x+3)$ and their HCF $= (x-1)$

$\because (x-1)$ is common in both.

$\therefore$ First expression $= (x-1)(x-2) = x^2 - 3x + 2$

and second expression
$= (x-1)(x+3) = x^2 + 2x - 3$

15. (1) $p(x) \times q(x) =$ HCF $\times$ LCM

$(x^2 - 5x - 14) \times q(x)$
$= (x-7)(x^3 - 10x^2 + 11x + 70)$

$$\therefore \ q(x) = \frac{(x-7)(x^3 - 10x^2 + 11x + 70)}{(x^2 - 5x - 14)}$$

Putting $x = 5$ in $x^3 - 10x^2 + 11x + 70$

Remainder $= (5)^3 - 10(5)^2 + 11 \times 5 + 70$
$= 125 - 250 + 55 + 70 = 0$

$\therefore (x-5)$ is a factor.

Now, $x^3 - 10x^2 + 11x + 70$
$= x^2(x-5) - 5x(x-5) - 14(x-5)$
$= (x-5)(x^2 - 5x - 14)$

$\therefore$ Second expression

$$= \frac{(x-7)(x-5)(x^2 - 5x - 14)}{(x^2 - 5x - 14)}$$

$= (x-7)(x-5) = x^2 - 12x + 35$

16. (4) $p(x) \times q(x) =$ HCF $\times$ LCM

$\Rightarrow (x^2 + 2x - 3) \times q(x) = (x+3) \times (x^3 - 7x + 6)$

$$\therefore \ \text{Second expression} = \frac{(x+3)(x^3 - 7x + 6)}{x^2 + 2x - 3}$$

$$= \frac{(x+3)(x-1)(x-2)(x+3)}{(x-1)(x+3)}$$

$= (x+3)(x-2) = x^2 + x - 6$

17. (3) $\because p(x) \times q(x) =$ LCM $\times$ HCF

$x \times y = b \times a$

$\therefore xy = ab$

and, $x + y = a + b$ (Given)

Squaring both sides,

$\therefore (x+y)^2 = (a+b)^2$

or, $x^2 + y^2 + 2xy = a^2 + b^2 + 2ab$

or, $x^2 + y^2 + 2ab = a^2 + b^2 + 2ab$ ($\because xy = ab$)

$\therefore x^2 + y^2 = a^2 + b^2$

18. (1) Since $(x + a)$ is common factor of $x^2 + bx + c$ and $x^2 + mx + n$ therefore, remainder = 0 when both polynomials are divided by $(x + a)$

$\because x + a = 0$

or, $x = -a$

For first polynomial, remainder $= (-a)^2 + b(-a) + c$

$0 = a^2 - ab + c$

or, $a^2 = ab - c$ ....(i)

For second polynomial, remainder
$= (-a)^2 + m(-a) + n$

$0 = a^2 - ma + n$

or, $a^2 = ma - n$ ....(ii)

Comparing values of a^2 from equations (i) and (ii),

$ab - c = ma - n$

$\Rightarrow ab - ma = c - n$

$\Rightarrow a(b - m) = c - n$

$$\Rightarrow a = \frac{c-n}{b-m}$$

19. (2) If $(x + a)$, is a factor of polynomial function $f(x)$, then

$f(-a) = 0$

$\therefore$ On putting $x = -1$ in

$x^3 + (a+1)x^2 - (b-2)x - 6$

$\Rightarrow -1 + a + 1 + b - 2 - 6 = 0$

$\Rightarrow a + b = 8$ .....(i)

Similarly,

$23 + (a+1) \times 2^2 - (b-2) \times 2 - 6 = 0$

$\Rightarrow 8 + 4a + 4 - 2b + 4 - 6 = 0 \Rightarrow 4a - 2b = -10$

$\Rightarrow 2a - b = -5$ ....(ii)

By equations (i) + (ii),

$3a = 3 \Rightarrow a = 1$

From equation (i),

$b = 8 - 1 = 7$

20. (2) $(x-2)$ is a factor of polynomial $P(x)$
$= x^3 + x^2 - 5x + \lambda$.

$\therefore$ P(2) = 0 (i.e., on putting $x = 2$)

$\Rightarrow 2^3 + 2^2 - 5 \times 2 + \lambda = 0$

$\Rightarrow 8 + 4 - 10 + \lambda = 0$

$\Rightarrow \lambda + 2 = 0$

$\Rightarrow \lambda = -2$