

Factorisation of Quadratic Polynomials

To express a given polynomial as the product of polynomials, each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorisation.

Examples : (i) $x^2 - 16 = (x - 4)(x + 4)$
(ii) $x^2 - 3x + 2 = (x - 2)(x - 1)$

Formulae for Factorisation

- (i) $(x + y)^2 = x^2 + y^2 + 2xy$
 - (ii) $(x - y)^2 = x^2 + y^2 - 2xy$
 - (iii) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
 - (iv) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
 - (v) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
 - (vi) $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$
- Thus, for factorisation, we have
- (i) $x^2 - y^2 = (x - y)(x + y)$
 - (ii) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
 - (iii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

METHODS OF FACTORISATION

Method 1 : When each term of an expression has a common factor, we divide each term by this factor and take it out as a multiple as shown below :

Example : $36a^3b - 60a^2bc = 12a^2b(3a - 5c)$

Example : $x(x - y)^3 + 3x^2y(x - y)$

$$= x(x - y)[(x - y)^2 + 3xy]$$

$$= x(x - y)(x^2 + y^2 - 2xy + 3xy)$$

$$= x(x - y)(x^2 + y^2 + xy)$$

Method 2 : Sometimes in a given expression it is not possible to take out a common factor directly. However the terms of the expression are grouped in such a manner that we may have a common factor. This can now be factorised as discussed above.

Example : Factorise : $6ab - b^2 + 12ac - 2bc$

$$\begin{aligned}\text{Sol. } 6ab - b^2 + 12ac - 2bc &= (6ab + 12ac) - (b^2 + 2bc) \\ &= 6a(b + 2c) - b(b + 2c) \\ &= (b + 2c)(6a - b)\end{aligned}$$

Example : Factorise : $x^2 + 18x + 81$

$$\begin{aligned}\text{Sol. } x^2 + 18x + 81 &= x^2 + 18x + 9^2 \\ &= x^2 + 2 \times 9 \times x + 9^2 \\ &= (x + 9)^2\end{aligned}$$

Example : Factorise : $64x^2 - 16x + 1$

$$\begin{aligned}\text{Sol. } 64x^2 - 16x + 1 &= (8x)^2 - 16x + 1^2 \\ &= (8x)^2 - 2 \cdot (8x) \cdot 1 + 1^2 = (8x - 1)^2\end{aligned}$$

Example : Factorise : $81 - 64x^2$

$$\begin{aligned}\text{Sol. } 81 - 64x^2 &= 9^2 - (8x)^2 \\ &= (9 + 8x)(9 - 8x)\end{aligned}$$

Method 3 : Factorisation of Quadratic Trinomials

Case I : Polynomial of the form $x^2 + bx + c$. We find integers p and q such that $p + q = b$ and $pq = c$. Then,

$$\begin{aligned}x^2 + bx + c &= x^2 + (p + q)x + pq \\ &= x^2 + px + qx + pq \\ &= x(x + p) + q(x + p) \\ &= (x + q)(x + p)\end{aligned}$$

Case II : Polynomial of the form $ax^2 + bx + c$. In this case, we find integers p and q such that $p + q = b$ and $pq = ac$.

$$\text{Then, } ax^2 + bx + c = ax^2 + (p + q)x + \frac{pq}{a}$$

$$\begin{aligned}&= a^2x^2 + apx + aqx + pq \\ &= ax(ax + p) + q(ax + p) \\ &= (ax + p)(ax + q)\end{aligned}$$

Example : Factorise : $x^2 + 9x + 18$

Sol. : We try to split 9 into two parts whose sum is 9 and product is 18.

$$\begin{aligned}\text{Clearly, } 6 + 3 &= 9 \text{ and } 6 \times 3 = 18 \\ \therefore x^2 + 9x + 18 &= x^2 + 6x + 3x + 18 \\ &= x(x + 6) + 3(x + 6) \\ &= (x + 3)(x + 6)\end{aligned}$$

Example : Factorise : $x^2 + 5x - 24$

Sol. : We try to split 5 into two parts whose sum is 5 and product is -24.

$$\begin{aligned}\text{Clearly, } 8 + (-3) &= 5 \text{ and } 8 \times (-3) = -24 \\ \therefore x^2 + 5x - 24 &= x^2 - 3x + 8x - 24 \\ &= (x^2 - 3x) + (8x - 24) \\ &= x(x - 3) + 8(x - 3) \\ &= (x - 3)(x + 8)\end{aligned}$$

Example : Factorise : $x^2 - 4x - 21$

Sol. : We try to split -4 into two parts whose sum is -4 and product is -21.

$$\begin{aligned}\text{Clearly, } (-7) + 3 &= -4 \text{ and } (-7) \times 3 = -21 \\ \therefore x^2 - 4x - 21 &= x^2 - 7x + 3x - 21 \\ &= x(x - 7) + 3(x - 7) \\ &= (x - 7)(x + 3)\end{aligned}$$

Method 4 : Factorisation of forms $x^3 - y^3$ and $x^3 + y^3$

Remember these formulae :

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Example : Factorise : $x^3 - 27y^3$

$$\begin{aligned}\text{Sol. : } x^3 - 27y^3 &= (x)^3 - (3y)^3 \\ &= (x - 3y) \{ (x)^2 + x \times 3y + (3y)^2 \} \\ &= (x - 3y) (x^2 + 3xy + 9y^2)\end{aligned}$$

Example : $8x^3 + 27$

$$\begin{aligned}\text{Sol. : } 8x^3 + 27 &= (2x)^3 + (3)^3 \\ &= (2x + 3) \{ (2x)^2 - 2x \times 3 + (3)^2 \} \\ &= (2x + 3) (4x^2 - 6x + 9)\end{aligned}$$

Method 5 : Factorisation of $x^3 + y^3 + z^3 - 3xyz$

Theorem : prove that

$$\begin{aligned}x^3 + y^3 + z^3 - 3xyz \\ &= (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)\end{aligned}$$

$$\text{Proof : } x^3 + y^3 + z^3 - 3xyz = (x^3 + y^3) + z^3 - 3xyz$$

$$= [(x + y)^3 - 3xy(x + y)] + z^3 - 3xyz$$

$$= u^3 - 3xyu + z^3 - 3xyz, \text{ जहाँ } (x + y) = u$$

$$= (u^3 + z^3) - 3xy(u + z)$$

$$= (u + z) (u^2 - uz + z^2) - 3xy(u + z)$$

$$= (u + z) (u^2 + z^2 - uz - 3xy)$$

$$= (x + y + z) [(x + y)^2 + z^2 - (x + y)z - 3xy]$$

$$= (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)$$

Note : $x^2 + y^2 + z^2 - xy - yz - zx$

$$= \frac{1}{2} (2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$$

$$= \frac{1}{2} [(x - y)^2 + (y - z)^2 + (z - x)^2]$$

Theorem : If $x + y + z = 0$ then prove that

$$x^3 + y^3 + z^3 = 3xyz$$

Sol : First Method :

$$\because x + y + z = 0$$

$$\therefore x + y = -z$$

Cubing both sides

$$(x + y)^3 = (-z)^3$$

$$\Rightarrow x^3 + y^3 + 3xy(x + y) = -z^3$$

$$\Rightarrow x^3 + y^3 + 3xy(-z) = -z^3$$

[Putting the value of $(x + y)$]

$$\Rightarrow x^3 + y^3 - 3xyz = -z^3$$

$$\therefore x^3 + y^3 + z^3 = 3xyz$$

Example : Factorise : $a^3 - 8b^3 + 64c^3 + 24abc$

$$\text{Sol. : } a^3 - 8b^3 + 64c^3 + 24abc$$

$$= a^3 + (-2b)^3 + (4c)^3 - 3a \times (-2b) \times (4c)$$

$$= x^3 + y^3 + z^3 - 3xyz$$

Where, $a = x$, $-2b = y$ and $4c = z$

$$= (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= [a + (-2b) + 4c] [a^2 + (-2b)^2 + (4c)^2 - a(-2b) - (-2b)(4c) - a(4c)]$$

$$= (a - 2b + 4c) (a^2 + 4b^2 + 16c^2 + 2ab + 8bc - 4ac)$$

SOLVED OBJECTIVE QUESTIONS

1. Factorise : $x^2 - \left(a + \frac{1}{a}\right)x + 1$

(1) $(x - a) \left(x - \frac{1}{a}\right)$ (2) $(x + a) \left(x + \frac{1}{a}\right)$

(3) $(x - a) \left(x + \frac{1}{a}\right)$ (4) None of these

2. Factorise : $x^2 - x \left(\frac{a^2 - 1}{a}\right) - 1$

(1) $(x - a) \left(x - \frac{1}{a}\right)$ (2) $(x - a) \left(x + \frac{1}{a}\right)$

(3) $(x + a) \left(x - \frac{1}{a}\right)$ (4) None of these

3. Factorise : $x^6 - 1$

(1) $(x - 1) (x^2 + x + 1) (x + 1) (x^2 - x + 1)$

(2) $(x - 1) (x^2 + x + 1)$

(3) $(x + 1) (x^2 - x + 1)$

(4) None of these

4. Factorise : $x^5 - x$

(1) $x(x^2 + 1)$

(2) $x(x^2 + 1)(x + 1)(x - 1)$

(3) $x(x^2 - 1)$

(4) None of these

5. Factorise : $x^2 - y^2 - 2y - 1$

(1) $(x + y + 1)(x + y - 1)$

(2) $(x + y + 1)^2$

(3) $(x + y + 1)(x - y - 1)$

(4) None of these

6. Factorise : $4x^2 - \frac{2x(a^2 - 1)}{a} - 1$

(1) $(2x - a)^2$ (2) $\left(2x + \frac{1}{a}\right)^2$

(3) $(2x + a) \left(2x - \frac{1}{a}\right)$ (4) $(2x - a) \left(2x + \frac{1}{a}\right)$

7. Factorise : $x^2 + ab - (a^2 - 3ab + 4b^2)$

(1) $(x + a - 2b)^2$

(2) $(x - a + 2b)^2$

(3) $(x + a - 2b)(x - a + 2b)$

(4) $(x - a + 2b)$

8. Factorise : $4x^2 + 12xy + 9y^2$

(1) $(2x + 5y)(2x + y)$

(2) $(2x + y)^2$

(3) $(2x + 3y)^2$

(4) None of these

9. Factorise : $4x^4 + 7x^2y^2 + 16y^4$

(1) $(2x^2 + 4y^2 + 3xy)^2$

(2) $(2x^2 + 4y^2 - 3xy)^2$

(3) $(2x^2 + 4y^2 + 3xy)(2x^2 + 4y^2 - 3xy)$

(4) None of these

FACTORISATION OF QUADRATIC POLYNOMIALS

10. Factorise : $x^4 + 4y^4$

- (1) $(x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$
 (2) $(x^2 + 2y^2 + 2xy)^2$
 (3) $(x^2 + 2y^2 - 2xy)^2$
 (4) None of these

11. Factorise : $x^2 + 2x - 899$

- (1) $(x + 31)(x - 29)$ (2) $(x + 31)^2$
 (3) $(x - 29)^2$ (4) $(x + 33)^2$

12. Factorise : $6x^3 - 31x^2 + 35x$

- (1) $x(2x - 7)(3x - 5)$ (2) $(2x - 7)(3x - 5)$
 (3) $(2x + 7)(3x - 5)^2$ (4) None of these

13. Factorise : $3 - 7x + 4x^2$

- (1) $(1 + x)(3 - 4x)$ (2) $(1 - x)(3 - 4x)$
 (3) $(1 - x)(3 + 4x)$ (4) None of these

14. Factorise : $9x^4 + 4y^4 - 13x^2y^2$

- (1) $(x^2 - y^2)(9x^2 - 4y^2)$
 (2) $(x^2 + y^2)(9x^2 - 4y^2)$
 (3) $(x^2 - y^2)(9x^2 + 4y^2)$
 (4) None of these

15. Factorise : $(2a + 3b)^2 - 14(2a + 3b)(3a - b) - 32(3a - b)^2$

- (1) $(19b - 46a)(8a + b)$
 (2) $(19b + 46a)(8a - b)$
 (3) $(8a + b)^2$
 (4) $(19b + 46b)^2$

16. Factorise : $(2x - 3y)^2 - 10x + 15y - 6$

- (1) $(2x - 3y - 6)^2$ (2) $(2x - 3y + 1)^2$
 (3) $(2x - 3y - 6)(2x - 3y + 1)$
 (4) None of these

17. Factorise : $7(a + b)^2 + 48(a + b)ab - 7a^2b^2$

- (1) $(a + b + 7ab)^2$ (2) $(7a + 7b - ab)^2$
 (3) $(a - b + 7ab)^2$
 (4) $(a + b + 7ab)(7a + 7b - ab)$

18. Factorise : $\left(\frac{a}{b} + \frac{b}{a}\right)^4 - 2\left(\frac{a^2}{b^2} - \frac{b^2}{a^2}\right)^2 + \left(\frac{a}{b} - \frac{b}{a}\right)^4 - c^2$

- (1) $(4 + c)(4 - c)$ (2) $(2 + c)(2 - c)$
 (3) $(3 + c)(3 - c)$ (4) None of these

19. Factorise : $27x^3 - 125y^3$

- (1) $(3x - 5y)^3$
 (2) $(9x^2 + 15xy + 25y^2)$
 (3) $(3x - 5y)(9x^2 + 15xy + 25y^2)$
 (4) None of these

20. Factorise : $64 - x^6$

- (1) $(2 + x)(4 - 2x + x^2)(2 - x)(4 + 2x + x^2)$
 (2) $(2 + x)^2(4 - 2x + x^2)^2$
 (3) $(2 - x)^2(4 + 2x + x^2)^2$
 (4) None of these

21. Factorise : $(x + y)^3 + (x - y)^3$

- (1) $2(x^2 + 3y^2)$ (2) $2x(x^2 + 3y^2)$
 (3) $2x(x^2 - 3y^2)$ (4) None of these

22. Factorise : $x^3 - 8y^3 - 2x^2y + 4y^2x$

- (1) $(x - 2y)(x^2 + 4y^2)$ (2) $(x + 2y)(x^2 - 4y^2)$
 (3) $(x - 2y)^2$ (4) None of these

23. Factorise : $(9a^2 - 16b^2)$

- (1) $(3a + 4b)(3a - 4b)$
 (2) $(3a - 4b)(2a + 5b)$
 (3) $(3a - 4b)(3a + 4b)$
 (4) $(3a - 4b)(3a - 4b)$

24. Factorise : $(4a^3 - 25ax^2)$

- (1) $a(2a + 5x)(2a + 5x)$
 (2) $a(3a + 2x)(5a + 2x)$
 (3) $a(2a - 5x)(2a - 5x)$
 (4) $a(2a + 5x)(2a - 5x)$

25. Factorise : $(36x^4 - 1)$

- (1) $(6x^2 - 1)(6x^2 - 1)$ (2) $(6x^2 + 1)(6x^2 - 1)$
 (3) $(9x^2 + 1)(4x^2 - 1)$ (4) $(6x^2 + 1)(6x^2 + 1)$

26. Factorise : $(16x^4 - 1)$

- (1) $(4x^2 - 1)(2x - 1)(2x + 1)$
 (2) $(4x^2 + 1)(2x + 1)(2x + 1)$
 (3) $(4x^2 - 1)(2x - 1)(2x - 1)$
 (4) $(4x^2 + 1)(2x + 1)(2x - 1)$

27. Factorise : $(16x^5 - 9x)$

- (1) $x(4x^2 - 3)(4x^2 - 3)$ (2) $x(4x^2 + 3)(4x^2 - 3)$
 (3) $x(4x^2 + 3)(4x^2 + 3)$ (4) $x(3x^2 + 4)(3x^2 - 4)$

28. Factorise : $(16x^5 - 81x)$

- (1) $x(4x^2 + 9)(2x + 3)(2x - 3)$
 (2) $x(3x^2 + 9)(2x - 3)(3x + 2)$
 (3) $x(4x^2 - 9)(2x - 3)(2x - 3)$
 (4) $x(4x^2 + 9)(2x + 3)(2x + 3)$

29. Factorise : $(1 - 16a^4)$

- (1) $(1 - 4a^2)(1 - 2a)(1 - 2a)$
 (2) $(1 + 4a^2)(1 + 2a)(1 + 2a)$
 (3) $(1 + 4a^2)(1 + 2a)(1 - 2a)$
 (4) $(1 - 4a^2)(1 + 2a)(1 + 2a)$

30. Factorise : $(x^2 - 81x^6)$

- (1) $x^2(1 + 9x^2)(1 + 3x)(1 + 3x)$
 (2) $x^2(1 - 9x^2)(1 - 3x)(1 - 3x)$
 (3) $x^2(1 + 9x^2)(1 + 3x)(1 + 3x)$
 (4) $x^2(1 + 9x^2)(1 + 3x)(1 - 3x)$

31. Factorise : $(36 - \frac{x^4}{a^2})$

- (1) $6\left(x - \frac{x^2}{a}\right)\left(6 + \frac{x^2}{a^2}\right)$ (2) $\left(6 + \frac{x^2}{a}\right)\left(6 - \frac{x^2}{a}\right)$
 (3) $\left(6 - \frac{x^2}{a}\right)\left(6 + \frac{x^2}{a}\right)$ (4) $\left(6 - \frac{x^2}{a}\right)\left(6 - \frac{x^2}{a}\right)$

FACTORISATION OF QUADRATIC POLYNOMIALS

- 32. Factorise :** $(64a^4 - 49x^6)$
 (1) $(8a^2 + 7x^3)(8a^2 - 7x^3)$
 (2) $(8a^2 + 7x^3)(8a^2 + 7x^3)$
 (3) $(8a^2 - 7x^3)(8a^2 - 7x^3)$
 (4) $(7a^2 + 8x^3)(7a^2 - 8x^3)$
- 33. Factorise :** $(121 - m^6)$
 (1) $(10 - m^3)(10 + m^3)$ (2) $(11 + m^3)(11 + m^3)$
 (3) $(11 + m^3)(11 - m^3)$ (4) $(11 - m^3)(11 - m^3)$
- 34. Factorise :** $(49x^6a^{10} - 81)$
 (1) $(7x^3a^5 - 9)(7x^3a^5 - 9)$
 (2) None of these
 (3) $(7x^3a^5 + 9)(7x^3a^5 + 9)$
 (4) $(7x^3a^5 + 9)(7x^3a^5 - 9)$
- 35. Factorise :** $(a^2b^2 - 25c^2d^2)$
 (1) $(ab + 5cd)(ab - 5cd)$
 (2) $(ab + 5cd)(ab + 5cd)$
 (3) $(ab - 5cd)(ab - 5cd)$
 (4) $(5ab + cd)(5ab - cd)$
- 36. Factorise :** $(81x^{12} - 64a^{10})$
 (1) $(9x^6 + 8a^5)(9x^6 + 8a^5)$
 (2) $(9x^6 + 8a^5)(7x^6 - 3a^5)$
 (3) $(9x^6 - 8a^5)(9x^6 - 8a^5)$
 (4) $(9x^6 + 8a^5)(9x^6 - 8a^5)$
- 37. Factorise :** $(p^2q^4 - 100p^2)$
 (1) $p^2(q^2 - 10)(q^2 - 10)$
 (2) $p^2(q^2 + 10)(q^2 + 10)$
 (3) $p^2(q^2 + 10)(q^2 - 10)$
 (4) $p^2(q^2 + 10)(p^2 + 10)$
- 38. Factorise :** $(144x^7 - 25x^3a^4)$
 (1) $x^2(12x^2 - 5a^2)(12x^2 - 5a^2)$
 (2) $x^3(12x^2 + 5a^2)(12x^2 - 5a^2)$
 (3) $x^3(5x^2 + 12x^2)(5x^2 - 12x^2)$
 (4) $x^3(12x^2 + 5a^2)(12x^2 + 5a^2)$
- 39. Factorise :** $(192a^9 - 243a^5x^4)$
 (1) $3a^5(8a^2 - 9x^2)(8a^2 - 9x^2)$
 (2) $3a^5(8a^2 + 9x^2)(8a^2 - 9x^2)$
 (3) $3a^5(8a^2 - 9x^2)(8a^2 + 9x^2)$
 (4) None of these
- 40. Factorise :** $(98a^3x^5 - 128ax)$
 (1) $2ax(7ax^2 - 8)(7ax^2 - 8)$
 (2) $2ax(7ax^2 + 8)(7ax^2 - 8)$
 (3) $2ax(7ax^2 - 8)(7ax^2 + 8)$
 (4) None of these
- 41. Factorise :** $(324x^{17}a^9 - 484x^5a^3)$
 (1) $4x^5a^3(9x^{12}a^6 + 11)(9x^5a^3 + 11)$
 (2) $4x^5a^3(9x^{12}a^6 - 11)(9x^5a^3 - 11)$
 (3) $4x^5a^3(9x^{12}a^6 + 11)(9x^5a^3 - 11)$
 (4) None of these
- 42. Factorise :** $(245m^{23}n^{13} - 605m^{15}n^7)$
 (1) $5m^{15}n^7(7m^8n^6 + 11)(7m^8n^6 - 11)$
 (2) $5m^{12}n^7(7m^4n^3 - 11)(7m^4n^3 + 11)$
 (3) $5m^{16}n^7(7m^4n^3 + 11)(7m^4n^3 - 11)$
 (4) $5m^{18}n^7(7m^4n^3 - 11)(7m^4n^3 - 11)$
- 43. Factorise :** $[(a + 3b)^2 - 25c^2]$
 (1) $(a + 3b + 5c)^2$
 (2) $(a + 3b - 5c)^2$
 (3) $(a + 3b + 5c)(a + 3b - 5c)$
 (4) None of these
- 44. Factorise :** $[a^2 - (3b - 5c)^2]$
 (1) $(a + 3b + 5c)(a + 3b + 5c)$
 (2) $(a + 3b - 5c)(a - 3b + 5c)$
 (3) $(a - 3b - 5c)(a - 3b - 5c)$
 (4) $(a - 3b + 5c)(a + 3b - 5c)$
- 45. Factorise :** $(x + y)^2 - (x - y)^2$
 (1) $3xy$ (2) $2xy$
 (3) $4xy$ (4) $6xy$
- 46. Factorise :** $(3a + 2x)^2 - (2a + x)^2$
 (1) $(5a + 3x)(a + x)$ (2) $(5a - 3x)(a + x)$
 (3) $(5a - 3x)(a - x)$ (4) $(5a + 3x)(a - x)$
- 47. Factorise :** $4(a - b)^2 - 9(c - d)^2$
 (1) $(2a - 2b - 3c - 3d)(3a - 2b - 3c - 3d)$
 (2) $(2a + 2b + 3c + 3d)(3a + 2b + 3c + 3d)$
 (3) $(2a + 2b - 3c + 3d)(3a + 2b + 3c - 3d)$
 (4) $(2a - 2b + 3c - 3d)(3a - 2b - 3c + 3d)$
- 48. Factorise :** $(x^4 + x^2 + 1)$
 (1) $(x^2 - 1 + x)(x^2 - 1 - x)$
 (2) $(x^2 + 1 + x)(x^2 + 1 + x)$
 (3) $(x^2 + 1 - x)(x^2 + 1 + x)$
 (4) $(x^2 - 1 - x)^2$
- 49. Factorise :** $(x^8 + x^4 + 1)$
 (1) $(x^2 - x + 1)^2(x^4 - x^2 + 1)$
 (2) $(x^2 + 1 - x)(x^2 + x + 1)(x^4 + x^2 + 1)$
 (3) $(x^2 + x + x)^2(x^4 + x^2 + 1)$
 (4) $(x^2 + 1 + x)(x^2 - x + 1)(x^4 - x^2 + 1)$
- 50. Factorise :** $(a^4 + a^2x^2 + x^4)$
 (1) $(a^2 + x^2 + ax)^2$
 (2) $(a^2 + x^2 - ax)(a^2 - x^2 - ax)$
 (3) $(a^2 - x^2 - ax)^2$
 (4) $(a^2 + x^2 + ax)(a^2 + x^2 - ax)$
- 51. Factorise :** $(a^8 + a^4x^4 + x^8)$
 (1) $(a^2 + ax + x^2)(a^2 - ax + x^2)(a^4 - a^2x^2 + x^4)$
 (2) $(a^2 + ax - x^2)^2(a^4 + a^4x^4 + x^4)$
 (3) $(a^2 + ax + x^2)^2(a^4 - a^2x^2 + x^4)$
 (4) $(a^2 - ax + x^2)^2(a^4 - a^4x^4 - x^4)$
- 52. Factorise :** $(x^4 + 64)$
 (1) $(x^2 + 8 + 4x)(x^2 + 8 - 4x)$
 (2) $(x^2 - 4x - 8)^2$
 (3) $(x^2 + 4x + 8)^2$
 (4) $(x^2 - 8 - 4x)(x^2 + 8 + 4x)$
- 53. Factorise :** $(4x^4 + 81)$
 (1) $(2x^2 - 9 + 6x)(2x^2 + 6x + 9)$
 (2) $(2x^2 + 9 + 6x)^2$
 (3) $(2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$
 (4) $(2x^2 - 9 - 6x)^2$

FACTORISATION OF QUADRATIC POLYNOMIALS

54. Factorise : $(9x^4 + 36)$
 (1) $9(x^2 + x + 3)^2$
 (2) $9(x^2 + 2 + 2x)(x^2 - 2x + 2)$
 (3) $9(x^2 - x - 3)^2$
 (4) $8(x^2 - x + 3)(x^2 - x - 3)$
55. Factorise : $(a^4 + 2a^2 + 9)$
 (1) $(a^2 + 2a + 3)^2$
 (2) $(a - 2a - 3)^2$
 (3) $(a^2 + 2a + 3)(a^2 - 2a + 3)$
 (4) $(a - 2a - 3)(a^2 + 2a - 3)$
56. Factorise : $(x^4 - 7x^2 + 9)$
 (1) $(x^2 - x - 3)^2$
 (2) $(x^2 + x + 3)(x^2 - x - 3)$
 (3) $(x^2 + x - 3)^2$
 (4) $(x^2 - x + 3)(x^2 + x + 3)$
57. Factorise : $(4x^4 + 8x^2 + 9)$
 (1) $(2x^2 + 2x + 3)(2x^2 - 2x + 3)$
 (2) $(2x^2 - 2x - 3)^2$
 (3) $(2x^2 + 2x + 3)^2$
 (4) $(2x^2 + 2x - 3)(2x^2 + 2x + 3)$
58. Factorise : $(4x^4 - 16x^2 + 9)$
 (1) $(2x^2 - 2x - 3)^2$
 (2) $(2x^2 + 2x + 3)^2$
 (3) $(2x^2 - 2x + 3)(2x^2 + 2x - 3)$
 (4) $(2x^2 + 2x - 3)(2x^2 - 2x - 3)$
59. Factorise : $(4x^4 + 3x^2 + 9)$
 (1) $(2x^2 + 3x + 3)(2x^2 - 3x + 3)$
 (2) $(2x^2 - 3x - 3)^2$
 (3) $(2x^2 + 3x + 3)^2$
 (4) $(2x^2 - 3x - 3)(2x^2 + 3x - 3)$
60. Factorise : $(4a^4 - 37a^2 + 9)$
 (1) $(2a^2 - 5a + 3)(2a^2 + 5a - 3)$
 (2) $(2a^2 - 5a - 3)^2$
 (3) $(2a^2 + 5a + 3)^2$
 (4) $(2a^2 + 5a - 3)(2a^2 - 5a - 3)$
61. Factorise : $(4a^4 + 625)$
 (1) $(2a^2 + 10a - 25)(2a^2 - 25 - 10a)$
 (2) $(2a^2 + 25a + 10a)^2$
 (3) $(2a^2 + 10a + 25)(2a^2 + 25 - 10a)$
 (4) $(2a^2 - 25a + 10a)^2$
62. Factorise : $(9x^4 + 23x^2 + 16)$
 (1) $(3x^2 - 4 - x)^2$
 (2) $(3x^2 + 4 + x)(3x^2 + 4 - x)$
 (3) $(3x^2 + 4 + x)^2$
 (4) $(3x^2 - 4 + x)(3x^2 + 4 - x)$
63. Factorise : $(9a^4 - 25a^2 + 16)$
 (1) $(3a^2 - 4 - a)^2$
 (2) $(3a^2 + 4 + a)^2$
 (3) $(3a^2 - 4 + a)(3a^2 - a - 4)$
 (4) $(3a^2 + 4 - a)(3a^2 - a - 4)$
64. Factorise : $(9x^4 - 33x^2 + 16)$
 (1) $(3x^2 - 3x + 4)(3x^2 + 3x - 4)$
 (2) $(3x^2 + 3x - 4)(3x^2 - 3x - 4)$
 (3) $(3x^2 + 3x + 4)^2$
 (4) $(3x^2 - 3x + 4)^2$
65. Factorise : $(9a^4 - a^2 + 16)$
 (1) $(3a^2 + 4 + 5a)(3a^2 + 4 - 5a)$
 (2) $(3a^2 + 4 + 5a)^2$
 (3) $(3a^2 - 4 + 5a)(3a^2 + 4 - 5a)$
 (4) $(3a^2 - 4 - 5a)^2$
66. Factorise : $(16x^4 + 4x^2a^2 + 25a^4)$
 (1) $(4x^2 - 6ax - 5a^2)^2$
 (2) $(4x^2 + 6ax + 5a^2)(4x^2 - 6ax + 5a^2)$
 (3) $(4x^2 + 6ax - 5a^2)(4x^2 - 6ax - 5a^2)$
 (4) $(4x^2 + 6ax + 5a^2)^2$
67. Factorise : $(9a^4 - 19a^2x^2 + 25x^4)$
 (1) $(3a^2 - 7ax - 5x^2)^2$
 (2) $(3a^2 + 7ax + 5x^2)(3a^2 + 5x^2 - 7ax)$
 (3) $(3a^2 + 7ax + 5x^2)(3a^2 + 5x^2 - 7ax)$
 (4) $(3a^2 + 7ax + 5x^2)^2$
68. Factorise : $(x^4 + 8x^2 + 144)$
 (1) $(x^2 + 4x + 12)(x^2 - 4x + 12)$
 (2) $(x^2 - 4x - 12)^2$
 (3) $(x^2 + 4x + 12)^2$
 (4) $(x^2 - 4x + 12)(x^2 - 4x - 12)$
69. Factorise : $(a^4 - 35a^2b^2 + 25b^4)$
 (1) $(a^2 - 5ab + 5b^2)(a^2 + 5ab + 5b^2)$
 (2) $(a^2 - 5ab - 5b^2)^2$
 (3) $(a^2 - 5ab - 5b^2)^2$
 (4) $(a^2 + 5ab - 5b^2)(a^2 - 5ab - 5b^2)$
70. Factorise : $(36a^4 - 16a^2b^2 + b^4)$
 (1) $(6a^2 + b^2 + 2ab)^2$
 (2) $(6a^2 - b^2 + 2ab)(6a^2 + b^2 - 2ab)$
 (3) $(6a^2 - b^2 + 2ab)(6a^2 - b^2 - 2ab)$
 (4) $(6a^2 - b^2 - 2ab)^2$
71. Factorise : $(49m^4 + 16n^2 - 60m^2n^2)$
 (1) $(7m^2 - 2mn + 4n^2)^2$
 (2) $(7m^2 + 2mn - 4n^2)(7m^2 - 2mn - 4n^2)$
 (3) $(m^2 + 2mn + 4n^2)^2$
 (4) $(7m^2 + 2mn - 4n^2)(7m^2 - 2mn + 4n^2)$
72. Factorise : $(64a^4 + 81x^4)$
 (1) $(8a^2 + 12ax + 9x^2)^2$
 (2) $(8a^2 - 12ax - 9x^2)^2$
 (3) $(8a^2 - 12ax + 9x^2)(8a^2 - 12ax - 9x^2)$
 (4) $(8a^2 + 12ax + 9x^2)(8a^2 - 12ax + 9x^2)$
73. Factorise : $(a^3 - 8b^3)$
 (1) $(a - 2b)(a^2 - 2ab - 4a^2)$
 (2) $(a - 2b)(a^2 + 2ab + 4b^2)$
 (3) $(a + 2b)(a^2 + 2ab + 4a^2)$
 (4) $(a - 2b)(a^2 + 2ab - 4a^2)$
74. Factorise : $(a^4 - 27ax^3)$
 (1) $a(a + 3x)(a^2 - 3ax - 9x^2)$
 (2) $a(a - 3x)(a^2 - 3ax + 9x^2)$
 (3) $a(a - 3x)(a^2 + 3ax + 9x^2)$
 (4) $a(a + 3x)(a^2 + 3ax - 9x^2)$

FACTORISATION OF QUADRATIC POLYNOMIALS

- 75. Factorise : $(512x^9 + 1)$**
 (1) $(2x + 1)(4x^2 - 2x + 1)(64x^6 - 8x^3 + 1)$
 (2) $(2x + 1)(4x^2 - 2x + 1)(64x^6 + 8x^3 - 1)$
 (3) $(2x - 1)(4x^2 + 2x + 1)(64x^6 - 8x^3 + 1)$
 (4) $(2x - 1)(4x^2 + 2x - 1)(64x^6 - 8x^3 - 1)$
- 76. Factorise : $(a^9 - 512b^9)$**
 (1) $(a + 2b)(a^2 - 2ab - 4b^2)(a^6 - 8a^3b^3 - 64b^6)$
 (2) $(a + 2b)(a^2 + 2ab - 4b^2)(a^6 + 8a^3b^3)$
 (3) $(a - 2b)(a^2 + 2ab + 4b^2)(a^6 + 8a^3b^3 + 64b^6)$
 (4) $(a - 2b)(a^2 + 2ab + 4b^2)(a^6 - 8a^3b^3)$
- 77. Factorise : $(27a^6 + 125x^6)$**
 (1) $(3a - 5x^2)(9a^4 + 15a^2x^2 + 25x^4)$
 (2) $(3a^2 + 5x^2)(9a^4 + 15a^2x^2 + 25x^4)$
 (3) $(3a^2 + 5x^2)(9a^4 - 15a^2x^2 + 25x^4)$
 (4) $(3a - 5x^2)(9a^4 - 15a^2x^2 - 25x^4)$
- 78. Factorise : $(m^6 - n^6)$**
 (1) $(m - n)(m - n)(m^4 + mn + n^4)$
 (2) $(m + n)(m - n)(m^4 + mn + n^4)$
 (3) $(m + n)(m^4 - mn - n^4)$
 (4) $(m - n)(m^4 - mn - n^4)$
- 79. Factorise : $(343x^3 + 512y^3)$**
 (1) $(7x - 8y)(49x^2 - 56xy - 64y^2)$
 (2) $(7x - 8y)(49x^2 + 56xy + 64y^2)$
 (3) $(7x + 8y)(49x^2 + 56xy + 64y^2)$
 (4) $(7x + 8y)(49x^2 - 56xy + 64y^2)$
- 80. Factorise : $(64x^{12} - 1)$**
 (1) $(2x^2 + 1)(2x^2 - 1)(16x^8 + 4x^4 + 1)$
 (2) $(2x^2 + 1)^2(16x^8 - 4x^4 - 1)$
 (3) $(2x^2 + 1)^2(16x^8 + 4x^4 + 1)$
 (4) $(2x^2 - 1)^2(16x^8 - 4x^4 - 1)$
- 81. Factorise : $(a^6 - 64x^{12})$**
 (1) $(a + 2x^2)^2(a^4 + 4a^2x^2 + 16x^8)$
 (2) $(a^2 + 2x^2)(a^4 - 4a^2x^2 - 16x^8)$
 (3) $(a - 2x^2)^2(a^4 - 4a^2x^2 + 16x^8)$
 (4) $(a - 2x^2)(a + 2x^2)(a^4 + 4a^2x^2 + 16x^8)$
- 82. Factorise : $(125x^3 - 216a^3)$**
 (1) $(5x^3 - 6a^3)(25x^6 + 30x^3a^3 - 36a^6)$
 (2) $(5x^3 - 6a^3)(25x^6 + 30x^3a^3 + 36a^6)$
 (3) $(5x^3 + 6a^3)(25x^6 - 30x^3a^3 + 36a^6)$
 (4) None of these
- 83. Factorise : $(64a^{13}b + 343ab^{13})$**
 (1) $ab(4a^4 - 7a^4)(16a^8 + 28a^4b^4 + 49b^8)$
 (2) $ab(4a^4 + 7a^4)(16a^8 + 28a^4b^4 + 49b^8)$
 (3) $ab(4a^4 - 7a^4)(16a^8 - 28a^4b^4 - 49b^8)$
 (4) $ab(4a^4 + 7b^4)(16a^8 - 28a^4b^4 + 49b^8)$
- 84. Factorise : $(729x^{20}y^2 - 64x^2y^{20})$**
 (1) $(x^2 + y^2)(81x^{12} + 36x^6y^6 + 6y^{12})$
 (2) $x^2y^2(3x^3 - 2y^3)(3x^3 + 2y^3)(81x^{12} + 36x^6y^6 + 6y^{12})$
 (3) $x^2y^2(3x^3 + 2y^3)(81x^{12} - 36x^6y^6)$
 (4) None of these
- 85. Factorise : $(a^2 + b^2)^3 + 8a^3b^3$**
 (1) $(a - b)^2(a^4 - b^4 + 6a^2b^2 + 2a^3b + 2ab^3)$
 (2) $(a + b)^2(a^4 + b^4 + 6a^2b^2 - 2a^3b - 2ab^3)$
 (3) $(a + b)^2(a^4 - b^4 - 6a^2b^2 - 2a^3b + 2ab^3)$
 (4) $(a - b)^2(a^4 + b^4 + 6a^2b^2 + 2a^3b + 2ab^3)$
- 86. Factorise : $(2x^2 - 3y^2)^3 + y^6$**
 (1) $(2x^2 - 2y^2)(4x^4 - 14x^2y^2 + 13y^4)$
 (2) $(2x^2 - 4y^2)(4x^4 - 14x^2y^2 - 13y^4)$
 (3) $(2x^2 + 4y^2)(4x^4 + 14x^2y^2 + 13y^4)$
 (4) $(2x^2 + 4y^2)(4x^4 - 14x^2y^2 + 13y^4)$
- 87. Factorise : $(2a^3 - b^3)^3 - b^9$**
 (1) $2(a - b)(a^2 - ab - b^2)(4a^6 - b^6 - 2a^3b^3)$
 (2) $2(a - b)(a^2 + ab + b^2)(4a^6 + b^6 - 2a^3b^3)$
 (3) $2(a + b)(a^2 - ab + b^2)(4a^6 + b^6 + 2a^3b^3)$
 (4) None of these
- 88. Factorise : $(a^6 + \frac{b^6}{27})$**
 (1) $\left(a^2 + \frac{b^2}{3}\right)\left(a^4 + \frac{a^2b^2}{3} + \frac{a^2b^7}{9}\right)$
 (2) $\left(a^2 + \frac{b^2}{3}\right)\left(a^4 - \frac{a^2b^2}{3} + \frac{b^4}{9}\right)$
 (3) $\left(a^2 - \frac{b^2}{3}\right)\left(a^4 + \frac{a^2b^2}{3} + \frac{a^2b^7}{9}\right)$
 (4) $\left(a^2 - \frac{b^2}{3}\right)\left(a^4 - \frac{a^2b^2}{3} + \frac{a^2b^7}{9}\right)$
- 89. Factorise : $(a^3 - \frac{8}{b^3})$**
 (1) $\left(a - \frac{2}{b}\right)\left(a^2 + \frac{2a}{b} + \frac{4}{b^2}\right)$
 (2) $\left(a + \frac{2}{b}\right)\left(a^2 - \frac{2a}{b} - \frac{4}{b^2}\right)$
 (3) $\left(a + \frac{2}{b}\right)\left(a^2 + \frac{2a}{b} + \frac{4}{b^2}\right)$
 (4) $\left(a - \frac{2}{b}\right)\left(a^2 - \frac{2a}{b} + \frac{4}{b^2}\right)$
- 90. Factorise : $(\frac{1}{8x^3} + \frac{8}{y^3})$**
 (1) $\left(\frac{1}{2x} - \frac{2}{y}\right)\left(\frac{1}{4x^2} + \frac{1}{xy} + \frac{4}{y^2}\right)$
 (2) $\left(\frac{1}{2x} + \frac{2}{y}\right)\left(\frac{1}{4x^2} + \frac{1}{xy} + \frac{4}{y^2}\right)$
 (3) $\left(\frac{1}{3x} - \frac{2}{y}\right)\left(\frac{1}{4x^2} + \frac{1}{xy} + \frac{4}{y^2}\right)$
 (4) $\left(\frac{1}{2x} + \frac{2}{y}\right)\left(\frac{1}{4x^2} - \frac{1}{xy} + \frac{4}{y^2}\right)$

ANSWERS

1. (1)	2. (2)	3. (1)	4. (2)	5. (3)
6. (4)	7. (3)	8. (1)	9. (3)	10. (1)
11. (1)	12. (1)	13. (2)	14. (1)	15. (1)
16. (3)	17. (4)	18. (1)	19. (3)	20. (1)
21. (2)	22. (1)	23. (1)	24. (4)	25. (2)
26. (4)	27. (2)	28. (1)	29. (3)	30. (4)
31. (2)	32. (1)	33. (3)	34. (4)	35. (1)
36. (4)	37. (3)	38. (2)	39. (2)	40. (2)
41. (3)	42. (1)	43. (3)	44. (2)	45. (3)
46. (1)	47. (4)	48. (3)	49. (4)	50. (4)
51. (1)	52. (1)	53. (3)	54. (2)	55. (3)
56. (2)	57. (1)	58. (4)	59. (1)	60. (4)
61. (3)	62. (2)	63. (3)	64. (2)	65. (1)
66. (2)	67. (3)	68. (1)	69. (4)	70. (3)
71. (2)	72. (4)	73. (2)	74. (3)	75. (1)
76. (3)	77. (3)	78. (2)	79. (4)	80. (1)
81. (4)	82. (2)	83. (4)	84. (2)	85. (2)
86. (1)	87. (2)	88. (2)	89. (1)	90. (4)

EXPLANATIONS

1. (1) $x^2 - \left(a + \frac{1}{a}\right)x + 1$

$$= x^2 - ax - \frac{x}{a} + 1 = x(x-a) - \frac{1}{a}(x-a)$$

$$= (x-a) \left(x - \frac{1}{a}\right)$$

2. (2) $x^2 - x\left(a - \frac{1}{a}\right) - 1$

$$= x^2 - x\left(a - \frac{1}{a}\right) - 1 = x^2 - xa + x \cdot \frac{1}{a} - 1$$

$$= x(x-a) + \frac{1}{a}(x-a) = (x-a) \left(x + \frac{1}{a}\right)$$

3. (1) $x^3 - 1$

$$= (x^3)^2 - (1)^2$$

$$= (x^3 - 1)(x^3 + 1)$$

$$= (x-1)(x^2+x+1)(x+1)(x^2-x+1)$$

4. (2) $x^5 - x$

$$= x(x^4 - 1) = x[(x^2)^2 - 1] = x(x^2 + 1)(x^2 - 1)$$

$$= x(x^2 + 1)(x+1)(x-1)$$

5. (3) $x^2 - y^2 - 2y - 1 = x^2 - (y^2 + 2y + 1)$

$$= x^2 - (y+1)^2 = (x+y+1)(x-y-1)$$

6. (4) $4x^2 - 2x\left(\frac{a^2-1}{a}\right) - 1$

$$= 4x^2 - 2xa + \frac{2x}{a} - 1 = 2x(2x-a) + \frac{1}{a}(2x-a)$$

$$= (2x-a) \left(2x + \frac{1}{a}\right)$$

7. (3) $x^2 + ab - a^2 + 3ab - 4b^2$

$$= x^2 - a^2 + 4ab - 4b^2$$

$$= x^2 - (a^2 - 4ab + 4b^2)$$

$$= x^2 - (a-2b)^2$$

$$= (x+a-2b)(x-a+2b)$$

8. (1) $4x^2 + 12xy + 9y^2 - 4y^2$

$$= (2x)^2 + 2 \times 2x \times 3y + (3y)^2 - 4y^2$$

$$= (2x+3y)^2 - (2y)^2$$

$$= (2x+3y+2y)(2x+3y-2y)$$

$$= (2x+5y)(2x+y)$$

9. (3) $4x^4 + 7x^2y^2 + 16y^4$

$$= 4x^4 + 16x^2y^2 + 16y^4 - 9x^2y^2$$

$$= (2x^2 + 4y^2)^2 - (3xy)^2$$

$$= (2x^2 + 4y^2 + 3xy)(2x^2 + 4y^2 - 3xy)$$

10. (1) $y^4 + 4x^2y^2 + 4y^4 - 4x^2y^2$

$$= (x^2 + 2y^2)^2 - (2xy)^2$$

$$= (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$$

11. (1) $x^2 + 2x - 899$

$$= x^2 + 31x - 29x - 899$$

$$= x(x+31) - 29(x+31)$$

$$= (x+31)(x-29)$$

12. (1) $6x^3 - 31x^2 + 35x$

$$= 6x^3 - 21x^2 - 10x^2 + 35x$$

$$= 3x^2(2x-7) - 5x(2x-7)$$

$$= (2x-7)(3x^2-5x)$$

$$= (2x-7)x(3x-5)$$

$$= x(2x-7)(3x-5)$$

13. (2) $3 - 7x + 4x^2 = 3 - 3x - 4x + 4x^2$

$$= 3(1-x) - 4x(1-x)$$

$$= (1-x)(3-4x)$$

14. (1) $9x^4 + 4y^4 - 13x^2y^2$

$$= 9x^4 - 13x^2y^2 + 4y^4$$

$$= 9x^4 - 9x^2y^2 - 4x^2y^2 + 4y^4$$

$$= 9x^2(x^2 - y^2) - 4y^2(x^2 - y^2)$$

$$= (x^2 - y^2)(9x^2 - 4y^2)$$

$$= (x-y)(x+y)(3x-2y)(3x+2y)$$

15. (1) Let, $2a + 3b = x$ और $3a - b = y$

$$\therefore \text{Expression} = x^2 - 14xy - 32y^2$$

$$= x^2 - 16xy + 2xy - 32y^2$$

$$= x(x-16y) + 2y(x-16y)$$

$$= (x-16y)(x+2y)$$

$$= [(2a+3b) - 16(3a-b)][2a+3b] + 2(3a-b)$$

$$= (19b-46a)(8a+b)$$

$$16. (3) (2x - 3y)^2 - 5(2x - 3y) - 6$$

$$\text{Let } 2x - 3y = a$$

$$= a^2 - 5a - 6$$

$$= a^2 - 6a + a - 6$$

$$= a(a - 6) + 1(a - 6)$$

$$= (a - 6)(a + 1)$$

$$= (2x - 3y - 6)(2x - 3y + 1) \text{ (Putting the value of } a)$$

$$17. (4) 7(a + b)^2 + 48(a + b)ab - 7a^2b^2$$

$$\text{Let } a + b = x$$

$$\therefore \text{Expression} = 7x^2 + 48xab - 7a^2b^2$$

$$= 7x^2 + 49xab - xab - 7a^2b^2$$

$$= 7x(x + 7ab) - ab(x + 7ab)$$

$$= (x + 7ab)(7x - ab)$$

$$\text{Putting the value of } x$$

$$= (a + b + 7ab)[7(a + b) - ab]$$

$$= (a + b + 7ab)(7a + 7b - ab)$$

$$18. (1) \left(\frac{a}{b} + \frac{b}{a}\right)^4 - 2\left(\frac{a^2}{b^2} - \frac{b^2}{a^2}\right)^2 + \left(\frac{a}{b} - \frac{b}{a}\right)^4 - c^2$$

$$= \left(\frac{a}{b} + \frac{b}{a}\right)^4 - 2\left(\frac{a}{b} + \frac{b}{a}\right)^2\left(\frac{a}{b} - \frac{b}{a}\right)^2 + \left(\frac{a}{b} - \frac{b}{a}\right)^4 - c^2$$

$$\text{Let } \frac{a}{b} + \frac{b}{a} = x \text{ and } \frac{a}{b} - \frac{b}{a} = y$$

$$= x^4 - 2x^2y^2 + y^4 - c^2$$

$$= (x^2 - y^2)^2 - c^2$$

$$= \left[\left(\frac{a}{b} + \frac{b}{a}\right)^2 - \left(\frac{a}{b} - \frac{b}{a}\right)^2\right]^2 - c^2$$

(Putting the values of x, y)

$$= \left[\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\frac{a}{b} \times \frac{b}{a} - \frac{a^2}{b^2} - \frac{b^2}{a^2} + 2\frac{a}{b} \times \frac{b}{a}\right]^2 - c^2$$

$$= (4)^2 - c^2 = (4 + c)(4 - c)$$

$$19. (3) 27x^3 - 125y^3$$

$$= (3x)^3 - (5y)^3$$

$$= (3x - 5y)[(3x)^2 + 3x \times 5y + (5y)^2]$$

$$= (3x - 5y)(9x^2 + 15xy + 25y^2)$$

$$20. (1) 64 - x^6$$

$$= (8)^2 - (x^3)^2$$

$$= (8 + x^3)(8 - x^3)$$

$$= (2^3 + x^3)(2^3 - x^3)$$

$$= (2 + x)[(2)^2 - 2 \times x + x^2](2 - x)[2^2 + 2 \times x + x^2]$$

$$= (2 + x)(4 - 2x + x^2)(2 - x)(4 + 2x + x^2)$$

$$21. (2) \text{ Let } x + y = a, x - y = b$$

$$\therefore \text{Expression}$$

$$= a^3 + b^3$$

$$= (a + b)(a^2 - ab + b^2)$$

$$= (x + y + x - y)[(x + y)^2 - (x + y)(x - y) + (x - y)^2]$$

(Putting the value of a, b)

$$= (2x)[x^2 + y^2 + 2xy - (x^2 - y^2) + x^2 + y^2 - 2xy]$$

$$\therefore 2x(x^2 + y^2 + 2xy - x^2 + y^2 + x^2 + y^2 - 2xy)$$

$$= 2x(x^2 + 3y^2)$$

$$22. (1) x^3 - 8y^3 - 2x^2y + 4y^2x$$

$$= x^3 - (2y)^3 - 2xy(x - 2y)$$

$$= (x - 2y)[x^2 + 2xy + (2y)^2] - 2xy(x - 2y)$$

$$= (x - 2y)(x^2 + 2xy + 4y^2) - 2xy(x - 2y)$$

$$= (x - 2y)(x^2 + 2xy + 4y^2 - 2xy)$$

$$= (x - 2y)(x^2 + 4y^2)$$

$$23. (1) 9a^2 - 16b^2$$

$$= (3a)^2 - (4b)^2$$

$$= [(3a + 4b)(3a - 4b)] \quad [\because a^2 - b^2 = (a + b)(a - b)]$$

$$24. (4) 4a^3 - 25ax^2 = a[4a^2 - 25x^2]$$

$$= a[(2a)^2 - (5x)^2] = a[(2a + 5x)(2a - 5x)]$$

$$25. (2) 36x^4 - 1$$

$$= (6x^2)^2 - (1)^2 = (6x^2 + 1)(6x^2 - 1)$$

$$26. (4) 16x^4 - 1$$

$$= (4x^2)^2 - (1)^2$$

$$= (4x^2 + 1)(4x^2 - 1)$$

$$= (4x^2 + 1)[(2x)^2 - (1)^2]$$

$$= (4x^2 + 1)(2x + 1)(2x - 1)$$

$$27. (2) 16x^5 - 9x = x[16x^4 - 9]$$

$$= x[(4x^2)^2 - (3)^2] = x(4x^2 + 3)(4x^2 - 3)$$

$$28. (1) 16x^5 - 81x$$

$$= x[16x^4 - 81]$$

$$= x[(4x^2)^2 - (9)^2]$$

$$= x[(4x^2 + 9)(4x^2 - 9)]$$

$$= x(4x^2 + 9)[(2x)^2 - (3)^2]$$

$$= x(4x^2 + 9)(2x + 3)(2x - 3)$$

$$29. (3) 1 - 16a^4$$

$$= (1)^2 - (4a^2)^2$$

$$= (1 + 4a^2)(1 - 4a^2)$$

$$= (1 + 4a^2)[(1)^2 - (2a)^2]$$

$$= (1 + 4a^2)(1 + 2a)(1 - 2a)$$

$$30. (4) x^2 - 81x^6 = x^2[1 - 81x^4]$$

$$= x^2[(1)^2 - (9x^2)^2]$$

$$= x^2(1 + 9x^2)(1 - 9x^2)$$

$$= x^2[(1 + 9x^2)(1)^2 - (3x)^2]$$

$$= x^2(1 + 9x^2)(1 + 3x)(1 - 3x)$$

$$31. (2) 36 - \frac{x^4}{a^2} = (6)^2 - \left(\frac{x^2}{a}\right)^2 = \left(6 + \frac{x^2}{a}\right)\left(6 - \frac{x^2}{a}\right)$$

$$32. (1) 64a^4 - 49x^6$$

$$= (8a^2)^2 - (7x^3)^2 = (8a^2 + 7x^3)(8a^2 - 7x^3)$$

$$33. (3) 121 - m^6$$

$$= (11)^2 - (m^3)^2 = (11 + m^3)(11 - m^3)$$

$$34. (4) 49x^6a^{10} - 81$$

$$= (7x^3a^5)^2 - (9)^2 = (7x^3a^5 + 9)(7x^3a^5 - 9)$$

$$35. (1) a^2b^2 - 25c^2d^2$$

$$= (ab)^2 - (5cd)^2 = (ab + 5cd)(ab - 5cd)$$

$$36. (4) 81x^{12} - 64a^{10}$$

$$= (9x^6)^2 - (8a^5)^2$$

$$= (9x^6 + 8a^5)(9x^6 - 8a^5)$$

37. (3) $p^2q^4 - 100p^2 = p^2(q^4 - 100)$
 $= p^2[(q^2)^2 - (10)^2] = p^2(q^2 + 10)(q^2 - 10)$
38. (2) $144x^2 - 25x^3a^4 = x^2[144x^4 - 25a^4]$
 $= x^2[(12x^2)^2 - (5a^2)^2] = x^2[(12x^2 + 5a^2)(12x^2 - 5a^2)]$
39. (2) $192a^3 - 243a^5x^4 = 3a^3[64a^4 - 81x^4]$
 $= 3a^3[(8a^2)^2 - (9x^2)^2] = 3a^3(8a^2 + 9x^2)(8a^2 - 9x^2)$
40. (2) $98a^3x^5 - 128ax$
 $= 2ax[49a^2x^4 - 64]$
 $= 2ax[(7ax^2)^2 - (8)^2]$
 $= 2ax(7ax^2 + 8)(7ax^2 - 8)$
41. (3) $324x^{17}a^9 - 484x^6a^3$
 $= x^6a^3[324x^{12}a^6 - 484]$
 $= x^6a^3[(18x^6a^3)^2 - (22)^2]$
 $= x^6a^3(18x^6a^3 + 22)(18x^6a^3 - 22)$
 $= 4x^6a^3(9x^6a^3 + 11)(9x^6a^3 - 11)$
42. (1) $245m^{23}n^{13} - 605m^{15}n^7$
 $= 5m^{15}n^7[49m^8n^6 - 121]$
 $= 5m^{15}n^7[(7m^4n^3)^2 - (11)^2]$
 $= 5m^{15}n^7(7m^4n^3 + 11)(7m^4n^3 - 11)$
43. (3) $(a + 3b)^2 - 25c^2$
 $= (a + 3b)^2 - (5c)^2 = (a + 3b + 5c)(a + 3b - 5c)$
44. (2) $(a)^2 - (3b - 5c)^2 = (a + 3b - 5c)(a - 3b + 5c)$
45. (3) $(x + y)^2 - (x - y)^2$
 $= (x + y + x - y)(x + y - x + y)$
 $= (2x)(2y) = 4xy$
46. (1) $(3a + 2x)^2 - (2a + x)^2$
 $= (3a + 2x + 2a + x)(3a + 2x - 2a - x)$
 $= (5a + 3x)(a + x)$
47. (4) $4(a - b)^2 - 9(c - d)^2$
 $= [2(a - b)]^2 - [3(c - d)]^2$
 $= (2a - 2b + 3c - 3d)(2a - 2b - 3c + 3d)$
48. (3) $x^4 + x^2 + 1 = (x^4 + 2x^2 + 1) - x^2$
 $= (x^2 + 1)^2 - (x)^2 = (x^2 + 1 - x)(x^2 + 1 + x)$
49. (4) $x^6 + x^4 + 1$
 $= x^6 + 2x^4 + 1 - x^4$
 $= (x^4 + 1)^2 - (x^2)^2$
 $= (x^4 + 1 - x^2)(x^4 + 1 + x^2)$
 $= (x^2 + 1 - x)(x^2 + 1 + x)(x^4 - x^2 + 1)$
50. (4) $a^4 + a^2x^2 + x^4$
 $= a^4 + 2a^2x^2 + x^4 - a^2x^2$
 $= (a^2 + x^2)^2 - (ax)^2$
 $= (a^2 + x^2 + ax)(a^2 + x^2 - ax)$
51. (1) $a^6 + a^4x^4 + x^6$
 $= a^6 + 2a^4x^4 + x^6 - a^4x^4$
 $= (a^4 + x^4)^2 - (a^2x^2)^2$
 $= (a^4 + x^4 + a^2x^2)(a^4 + x^4 - a^2x^2)$
 $= (a^2 + ax + x^2)(a^2 - ax + x^2)(a^4 - a^2x^2 + x^4)$
52. (1) $x^4 + 64 = x^4 + 16x^2 + 64 - 16x^2$
 $= (x^2 + 8)^2 - (4x)^2 = (x^2 + 8 + 4x)(x^2 + 8 - 4x)$
53. (3) $4x^4 + 81$
 $= 4x^4 + 81 + 36x^2 - 36x^2$
 $= (2x^2 + 9)^2 - (6x)^2$
 $= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$
54. (2) $9x^4 + 36$
 $= 9(x^4 + 4)$
 $= 9[(x^2 + 4x^2 + 4) - 4x^2]$
 $= 9[(x^2 + 2)^2 - (2x)^2]$
 $= 9(x^2 + 2 + 2x)(x^2 - 2x + 2)$
55. (3) $a^4 + 2a^2 + 9$
 $= a^4 + 6a^2 + 9 - 4a^2$
 $= (a^2 + 3)^2 - (2a)^2$
 $= (a^2 + 2a + 3)(a^2 - 2a + 3)$
56. (2) $x^4 - 7x^2 + 9$
 $= x^4 - 7x^2 + 9 + x^2 - x^2$
 $= x^4 - 6x^2 + 9 - x^2$
 $= (x^2 - 3)^2 - (x)^2$
 $= (x^2 + x + 3)(x^2 - x - 3)$
57. (1) $4x^4 + 8x^2 + 9$
 $= 4x^4 + 12x^2 + 9 - 4x^2$
 $= (2x^2 + 3)^2 - (2x)^2$
 $= (2x^2 + 2x + 3)(2x^2 - 2x + 3)$
58. (4) $4x^4 - 16x^2 + 9$
 $= 4x^4 - 16x^2 + 9 + 4x^2 - 4x^2$
 $= 4x^4 - 12x^2 + 9 - 4x^2$
 $= (2x^2 - 3)^2 - (2x)^2$
 $= (2x^2 + 2x - 3)(2x^2 - 2x - 3)$
59. (1) $4x^4 + 3x^2 + 9 = 4x^4 + 12x^2 + 9 - 9x^2$
 $= (2x^2 + 3)^2 - (3x)^2 = (2x^2 + 3x + 3)(2x^2 - 3x + 3)$
60. (4) $4a^4 - 37a^2 + 9$
 $= 4a^4 - 37a^2 + 9 + 25a^2 - 25a^2$
 $= 4a^4 - 12a^2 + 9 - 25a^2$
 $= (2a^2 - 3)^2 - (5a)^2$
 $= (2a^2 - 3 + 5a)(2a^2 - 3 - 5a)$
61. (3) $4a^4 + 625$
 $= 4a^4 + 100a^2 + 625 - 100a^2$
 $= (2a^2 + 25)^2 - (10a)^2$
 $= (2a^2 + 10a + 25)(2a^2 + 25 - 10a)$
62. (2) $9x^4 + 23x^2 + 16$
 $= 9x^4 + 24x^2 + 16 - x^2$
 $= (3x^2 + 4)^2 - (x)^2$
 $= (3x^2 + 4 + x)(3x^2 + 4 - x)$
63. (3) $9a^4 - 25a^2 + 16$
 $= 9a^4 - 25a^2 + 16 + a^2 - a^2$
 $= 9a^4 - 24a^2 + 16 - a^2$
 $= (3a^2 - 4)^2 - (a)^2$
 $= (3a^2 - 4 + a)(3a^2 - 4 - a)$
64. (2) $9x^4 - 33x^2 + 16$
 $= 9x^4 - 33x^2 + 16 + 9x^2 - 9x^2$
 $= 9x^4 - 24x^2 + 16 - 9x^2$
 $= (3x^2 + 4)^2 - (3x)^2$
 $= (3x^2 - 4 + 3x)(3x^2 - 4 - 3x)$
65. (1) $9a^4 - a^2 + 16$
 $= 9a^4 - a^2 + 16 + 25a^2 - 25a^2$
 $= 9a^4 + 24a^2 + 16 - 25a^2$
 $= (3a^2 + 4)^2 - (5a)^2$
 $= (3a^2 + 4 + 5a)(3a^2 + 4 - 5a)$

$$\begin{aligned}
66. (2) & 16x^4 + 4x^2a^2 + 25a^4 \\
&= 16x^4 + 40x^2a^2 + 25a^4 - 36a^2x^2 \\
&= (4x^2 + 5a^2)^2 - (6xa)^2 \\
&= (4x^2 + 6ax + 5a^2)(4x^2 - 6ax + 5a^2) \\
67. (3) & 9a^4 - 19a^2x^2 + 25x^4 \\
&= 9a^4 - 19a^2x^2 + 25x^4 + 49a^2x^2 - 49a^2x^2 \\
&= 9a^4 + 30a^2x^2 + 25x^4 - 49a^2x^2 \\
&= (3a^2 + 5x^2)^2 - (7ax)^2 \\
&= (3a^2 + 7ax + 5x^2)(3a^2 + 5x^2 - 7ax) \\
68. (1) & x^4 + 8x^2 + 144 \\
&= x^4 + 24x^2 + 144 - 16x^2 \\
&= (x^2 + 12)^2 - (4x)^2 \\
&= (x^2 + 4x + 12)(x^2 - 4x + 12) \\
69. (4) & a^4 - 35a^2b^2 + 25b^4 \\
&= a^4 - 35a^2b^2 + 25b^4 + 25a^2b^2 - 25a^2b^2 \\
&= a^4 - 10a^2b^2 + 25b^4 - 25a^2b^2 \\
&= (a^2 - 5b^2)^2 - (5ab)^2 \\
&= (a^2 + 5ab - 5b^2)(a^2 - 5ab - 5b^2) \\
70. (3) & 36a^4 - 16a^2b^2 + b^4 \\
&= 36a^4 - 16a^2b^2 + b^4 + 4a^2b^2 - 4a^2b^2 \\
&= 36a^4 - 12a^2b^2 + b^4 - 4a^2b^2 \\
&= (6a^2 - b^2)^2 - (2ab)^2 \\
&= (6a^2 - b^2 + 2ab)(6a^2 - b^2 - 2ab) \\
71. (2) & 49m^4 + 16n^4 - 60m^2n^2 \\
&= 49m^4 - 56m^2n^2 + 16n^4 - 4m^2n^2 \\
&= (7m^2 - 4n^2)^2 - (2mn)^2 \\
&= (7m^2 + 2mn - 4n^2)(7m^2 - 2mn - 4n^2) \\
72. (4) & 64a^4 + 81x^4 \\
&= 64a^4 + 144a^2x^2 + 81x^4 - 144a^2x^2 \\
&= (8a^2 + 9x^2)^2 - (12ax)^2 \\
&= (8a^2 + 12ax + 9x^2)(8a^2 - 12ax + 9x^2) \\
73. (2) & a^3 - 8b^3 \\
&= (a)^3 - (2b)^3 = (a - 2b)(a^2 + 2ab + 4b^2) \\
74. (3) & a^4 - 27ax^3 = a(a^3 - 27x^3) \\
&= a[(a)^3 - (3x)^3] = a(a - 3x)(a^2 + 3ax + 9x^2) \\
75. (1) & 512x^9 + 1 \\
&= (8x^3)^3 + (1)^3 \\
&= (8x^3 + 1)(64x^6 - 8x^3 + 1) \\
&= [(2x)^3 + (1)^3][(64x^6 - 8x^3 + 1)] \\
&= (2x + 1)(4x^2 - 2x + 1)(64x^6 - 8x^3 + 1) \\
76. (3) & a^9 - 512b^9 \\
&= (a^3)^3 - (8b^3)^3 \\
&= (a^3 - 8b^3)(a^6 + 8a^3b^3 + 64b^6) \\
&= [(a)^3 - (2b)^3][(a^6 + 8a^3b^3 + 64b^6)] \\
&= (a - 2b)(a^2 + 2ab + 4b^2)(a^6 + 8a^3b^3 + 64b^6) \\
77. (3) & 27a^6 + 125x^6 \\
&= (3a^2)^3 + (5x^2)^3 \\
&= (3a^2 + 5x^2)(9a^4 - 15a^2x^2 + 25x^4) \\
78. (2) & m^6 - n^6 \\
&= (m^2)^3 - (n^2)^3 \\
&= (m^2 - n^2)(m^4 + mn^2 + n^4) \\
&= (m + n)(m - n)(m^4 + mn^2 + n^4) \\
79. (4) & 343x^3 + 512y^3 \\
&= (7x)^3 + (8y)^3 \\
&= (7x + 8y)(49x^2 - 56xy + 64y^2)
\end{aligned}$$

$$\begin{aligned}
80. (1) & 64x^{12} - 1 \\
&= (4x^4)^3 - (1)^3 \\
&= (4x^4 - 1)(16x^8 + 4x^4 + 1) \\
&= [(2x^2)^2 - (1)^2](16x^8 + 4x^4 + 1) \\
&= (2x^2 + 1)(2x^2 - 1)(16x^8 + 4x^4 + 1) \\
81. (4) & a^6 - 64x^{12} \\
&= (a^2)^3 - (4x^4)^3 \\
&= (a^2 - 4x^4)(a^4 + 4a^2x^4 + 16x^8) \\
&= [(a)^2 - (2x^2)^2][(a^4 + 4a^2x^4 + 16x^8)] \\
&= (a - 2x^2)(a + 2x^2)(a^4 + 4a^2x^4 + 16x^8) \\
82. (2) & 125x^9 - 216a^9 \\
&= (5x^3)^3 - (6a^3)^3 \\
&= (5x^3 - 6a^3)(25x^6 + 30x^3a^3 + 36a^6) \\
83. (4) & 64a^{13}b + 343ab^{13} \\
&= ab(64a^{12} + 343b^{12}) \\
&= ab[(4a^4)^3 + (7b^4)^3] \\
&= ab(4a^4 + 7b^4)(16a^8 - 28a^4b^4 + 49b^8) \\
84. (2) & 729x^{20}y^2 - 64x^2y^{18} \\
&= x^2y^2[729x^{18} - 64y^{16}] \\
&= x^2y^2[(9x^6)^3 - (4y^4)^3] \\
&= x^2y^2(9x^6 - 4y^4)(81x^{12} + 36x^6y^4 + 16y^{12}) \\
&= x^2y^2[(3x^2)^2 - (2y^2)^2][(81x^{12} + 36x^6y^4 + 16y^{12})] \\
&= (x^2y^2)(3x^2 - 2y^2)(3x^2 + 2y^2)(81x^{12} + 36x^6y^4 + 16y^{12}) \\
85. (2) & (a^2 + b^2)^3 + 8a^3b^3 \\
&= (a^2 + b^2)^3 + (2ab)^3 \\
&= (a^2 + b^2 + 2ab)(a^4 + b^4 + 2a^2b^2 - 2a^3b - 2ab^3 + 4a^2b^2) \\
&= (a + b)^2(a^4 + b^4 + 6a^2b^2 - 2a^3b - 2ab^3) \\
86. (1) & (2x^2 - 3y^2)^3 + y^6 \\
&= (2x^2 - 3y^2)^3 + (y^2)^3 \\
&= (2x^2 - 3y^2 + y^2)(4x^4 + 9y^4 - 12x^2y^2 - 2x^2y^2 + 3y^4 + y^4) \\
&= 2(x + y)(x - y)(4x^4 - 14x^2y^2 + 13y^4) \\
87. (2) & (2a^3 - b^3)^3 + (b^3)^3 \\
&= (2a^3 - b^3 + b^3)(4a^6 + b^6 - 2a^3b^3) \\
&= 2(a - b)(a^2 + ab + b^2)(4a^6 + b^6 - 2a^3b^3) \\
88. (2) & a^6 + \frac{b^6}{27} = (a^2)^3 + \left(\frac{b^2}{3}\right)^3 \\
&= \left(a^2 + \frac{b^2}{3}\right)\left(a^4 - \frac{a^2b^2}{3} + \frac{b^4}{9}\right) \\
89. (1) & a^3 - \frac{8}{b^3} = (a)^3 - \left(\frac{2}{b}\right)^3 \\
&= \left(a - \frac{2}{b}\right)\left(a^2 + \frac{2a}{b} + \frac{4}{b^2}\right) \\
90. (4) & \frac{1}{8x^3} + \frac{8}{y^3} = \left(\frac{1}{2x}\right)^3 + \left(\frac{2}{y}\right)^3 \\
&= \left(\frac{1}{2x} + \frac{2}{y}\right)\left(\frac{1}{4x^2} - \frac{2}{2xy} + \frac{4}{y^2}\right) \\
&= \left(\frac{1}{2x} + \frac{2}{y}\right)\left(\frac{1}{4x^2} - \frac{1}{xy} + \frac{4}{y^2}\right)
\end{aligned}$$