CBSE Board
Class IX Mathematics
Sample Paper 6

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

1. Simplify the given expression: $(3+\sqrt{3})(2+\sqrt{2})$

OR

Rationalise the denominator of $\frac{1}{2-\sqrt{3}}$

2. Write the equation 7x = 3 in the Standard form?

3. If a line *l* intersects two concentric circles at P, Q, R and S, then state whether PQ = RS is true.

4. For a line whose equation is 2x + y = 5, does point (2, 1) lie on it?

OR

If x = 2 and y = 1 is a solution of the equation 2x + 3y = k, find the value of k.

5. Find the Mode of the given data 1, 1, 2, 2, 2, 2, 4, 4, 4, 4, 4, 3, 3, 3, 3, 1, 1, 2, 2, 2, 3, 3, 3.

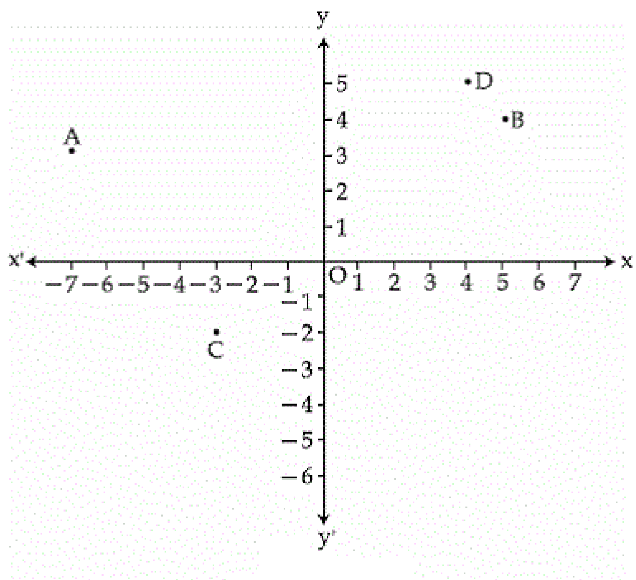
6. Three angles of a quadrilateral are 60°, 110° and 86°. What is the measure of the fourth angle of the quadrilateral?

Section B

(Questions 7 to 12 carry 2 marks each)

7. Express $0.\overline{975}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

8. See the given figure and answer the following:



(i) Coordinates of point A

(ii) Abscissa of point D

(iii) The point indentified by the coordinates (5, 4)

(iv) Coordinates of point C

9. Factorise: $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

OR

Evaluate : 997^2

10. What is the area of the triangle having sides of lengths 7 cm, 8 cm and 9 cm?

11. A rectangular sheet of card paper, 44 cm × 20 cm in size, is rolled along its length and a cylinder is formed. Find the volume of the cylinder.

12. Two angles are complementary. The larger angle is 3° less than twice the measure of the smaller angle. Find the measure of each angle.

OR

In a ΔABC, $\angle A + \angle B = 65^\circ$ and $\angle B + \angle C = 140^\circ$. Find the angles.

Section C

(Questions 13 to 22 carry 3 marks each)

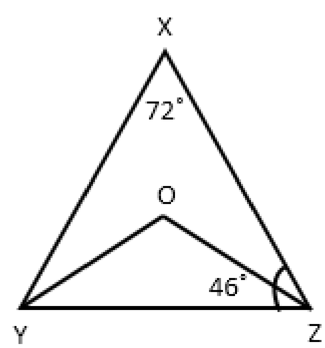
13. Express $5.3\overline{47}$ in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

OR

Simplify : $\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times \left[\left(\frac{25}{9}\right)^{\frac{-3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

14. $(x + 2)$ is one of the factors of the polynomial $x^3 + 13x^2 + 32x + 20$. Find its remaining factors.

15. In the figure, $\angle X = 72°$, $\angle XZY = 46°$. If YO and ZO are bisectors of $\angle XYZ$ and $\angle XZY$, respectively of ΔXYZ; find $\angle OYZ$ and $\angle YOZ$.



16. Find ab, if $a + b = \sqrt{11}$, $a^2 + b^2 = 5$

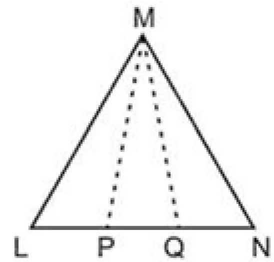
OR

Factorise : $a^4 + 4a^2 + 3$

17. In ΔABC, BE and CF are altitudes on the sides AC and AB, respectively, such that BE = CF. Using RHS congruency rule, prove that AB = AC.

OR

In the figure, it is given that LM = MN and LP = QN. Prove that $\Delta LMQ \cong \Delta NMP$



18. A survey was undertaken in 30 classes at a school to find the total number of left-handed students in each class. The table below shows the results:

No. of left-handed students	0	1	2	3	4	5
Frequency (no. of classes)	1	2	5	12	8	2

A class was selected at random.

(a) Find the probability that the class has 2 left-handed students.

(b) What is the probability that the class has at least 3 left-handed students?

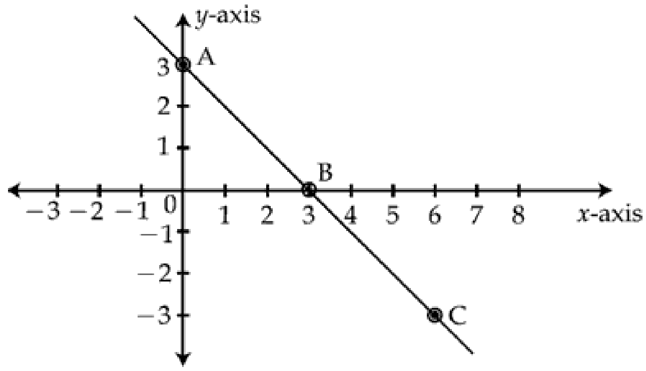
(c) Given that the total number of students in the 30 classes is 960, find the probability that a student randomly chosen from these 30 classes is left-handed.

19. Observe the graph and answer the following questions:

i. Write the co-ordinates of points B and C.

ii. Find one more solution of the line passing through A and B.

iii. Write equations of the x-axis and y-axis.



20. Solve: $x-\frac{2}{3}y=\frac{8}{3}, \frac{2x}{5}-y=\frac{7}{5}$.

21. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs. 210 per 100 m^2.

OR

The external diameter of a lead pipe is 2.4 cm and the thickness of the lead is 2 mm. Find the weight of a pipe of length 7 m, it being given that 1 cu cm of lead weighs 10 g.

22. The distance (in km) of 40 engineers from their residence to place of work were found as follows:

5	3	10	20	25	11	13	7	12	31
2	19	10	12	17	18	11	32	17	16
3	7	9	7	8	3	5	12	15	18
12	12	14	2	9	6	15	15	7	6

Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0 - 5 (5 not included). What main feature do you observe from this tabular representation?

Section D

(Questions 23 to 30 carry 4 marks each)

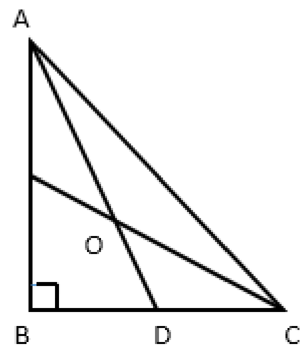
23. Prove that: $\left(\frac{1}{4}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 4^{0}+\left(\frac{9}{16}\right)^{-\frac{1}{2}}=\frac{16}{3}$

OR

If a and b are rational numbers and $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$, find the values of a and b.

24. In the figure, AD and CE are the bisectors of $\angle$A and $\angle$C, respectively.

If $\angle$ABC = 90°, then find $\angle$AOC.



25. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $(x + 1)$ leaves the remainder 19. Find the value of a. Also find the remainder when p(x) is divided by $x + 2$.

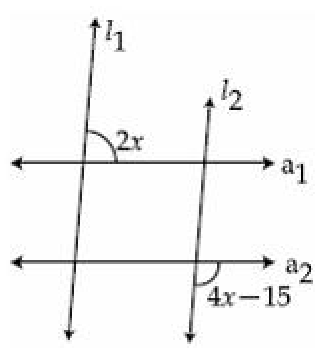
OR

Find the value of p if $2x^4 + 3x^3 + 2px^2 + 3x + 6$ is divisible by $x + 2$.

26. Prove that: If two lines intersect each other, then the vertically opposite angles are equal.

OR

In the figure, $l_1 \parallel l_2$ and $a_1 \parallel a_2$. Find the value of x.



27. Sonu and Monu had adjacent triangular fields with a common boundary of 25 m. The other two sides of Sonu's field were 52 m and 63 m, while Monu's were 114 m and

101 m. If the cost of fertilization is Rs 20 per sq m, then find the total cost of fertilization for both of Sonu and Monu together.

28. Prove that the medians of an equilateral triangle are equal.

29. There is a triangular field PQR whose corner angles P, Q and R have been measured as $50°$, $60°$ and $70°$, respectively. Three friends Anuja, Nikita and Raghav daily go on morning walk and walk along AB, BC and AC, respectively. Who walk the maximum distance among these three? Who walks the least? What value is indicated from this action?

30. (i) Multiply $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$ by $3x - 5y - 4$ using suitable identities.

    (ii) Factorise: $a^2 + b^2 - 2(ab - ac + bc)$.

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Sample Paper 6 – Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. We have,

$$(3+\sqrt{3})(2+\sqrt{2}) = 3 \times 2 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{3}\times\sqrt{2}$$
$$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{3\times 2}$$
$$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$$

OR

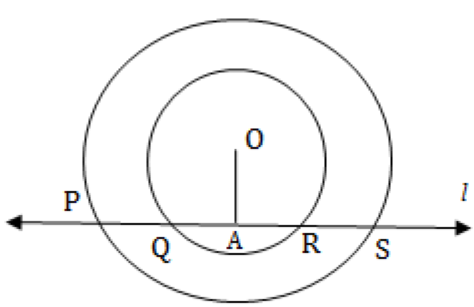
$$\frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$$
$$= \frac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$$
$$= \frac{2+\sqrt{3}}{4-3}$$
$$= 2+\sqrt{3}$$

2. Standard form of the Linear Equation in two variables is ax + by + c = 0.

Here, 7x = 3

$\therefore$ 7x + 0y – 3 = 0 is the Standard form.

3. Let OA $\perp$ l



Perpendicular from centre to the chord bisects the chord.

$\therefore$ PA = AS and AQ = AR ....(i)

$\therefore$ PA – QA = PQ and AS – AR = RS

(Since, A-Q-R and A-R-S)

$\therefore$ PQ = RS   (from i)

4. Substituting x = 2 and x = 1 in the equation 2x + y = 5, we get

L.H.S. = 2(2) + 1 = 5 = R.H.S.

$\therefore$ Point (2, 1) lie on the line 2x + y = 5.

OR

The given equation is 2x + 3y – k = 0

Since, x = 2 and y = 1 is a solution of the given linear equation,

2 × 2 + 3 – k = 0

7 – k = 0

k = 7

5. Number 1 occurs maximum number of times in the data.

$\therefore$ The Mode of the given data = 1

6. Let the fourth angle of a quadrilateral be x.

Since, the sum of all angles of a quadrilateral = 360°

$\therefore$ 60° + 110° + 86° + x = 360°

$\therefore$ 256° + x = 360° $\Rightarrow$ x = 104°

Section B

7. Let x = $0.\overline{975} = 0.975975975$ ....(1)

On multiplying both sides of equation (1) by 1000,

1000x = 975.975975   ....(2)

On subtracting equation (1) from equation (2),

999x = 975

$$\Rightarrow x = \frac{975}{999} = \frac{325}{333}$$

8. (i) Coordinates of A are (−7,3)

(ii) Abscissa of point D is 4.

(iii) Point is B.

(iv) Coordinates of C are (−3, −2)

9. $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

$= 7\sqrt{2}x^2 - 14x + 4x - 4\sqrt{2}$

$= 7\sqrt{2}\,(x - \sqrt{2}) + 4\,(x - \sqrt{2})$

$= (7\sqrt{2} + 4)(x - \sqrt{2})$

OR

$997^2 = (1000 - 3)^2$

$= 1000^2 + 3^2 - 2 \times 1000 \times 3$ $\quad\because (a - b)^2 = a^2 - 2ab + b^2$

$= 1000000 + 9 - 6000$

$= 994009$

10. Let a = 7 cm, b = 8 cm and c = 9 cm.

$\therefore$ Semi-perimeter = $s = \dfrac{a+b+c}{2} = \dfrac{7 \text{ cm} + 8 \text{ cm} + 9 \text{ cm}}{2} = 12 \text{ cm}$

Using Heron's formula,

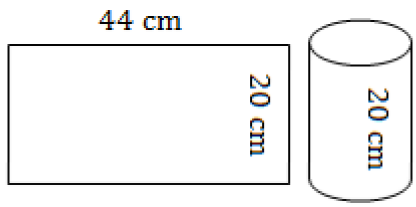
Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12 \times (12-7) \times (12-8) \times (12-9)} \text{ cm}^2$

$= \sqrt{12 \times 5 \times 4 \times 3} \text{ cm}^2$

$= 12\sqrt{5} \text{ cm}^2$

11. Height of the cylinder (h) = 20 cm



Let, the radius of the cylinder be r cm.

$\Rightarrow 2\pi r = 44$

$\Rightarrow r = \frac{44}{2\pi} = \frac{44}{2} \times \frac{7}{22} = 7$ cm

$\therefore$ Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 20 = 3080\ cm^3$

12. Let the measure of the smaller angle be x and that of the larger angle be y.

The larger angle is 3° less than twice the measure of the smaller angle.

$\Rightarrow y = 2x - 3°$ ....(1)

Now, x + y =180° (Two angles are complementary)

$\Rightarrow x + (2x - 3°) = 180°$

$\Rightarrow 3x = 183°$

$\Rightarrow x = 61°$

Substitute value of x in equation (1)

$y = 2(61) - 3°$

$\Rightarrow y = 119°$

So, the measures of the two angles are 61° and 119°.

OR

$\angle A + \angle B = 65°$ and $\angle B + \angle C = 140°$

$\angle A + \angle B + \angle B + \angle C = 65° + 140°$

$\angle A + \angle B + \angle C + \angle B = 205°$

$180° + \angle B = 205°$

$\angle B = 25°$

$\angle C = 140° - 25° = 115°$

$\angle A = 65° - 25° = 40°$

$\angle A = 40°, \angle B = 25°$ and $\angle C = 115°$

Section C

13. Let $x = 5.3\overline{47} = 5.34747.....$

Multiplying by 10, we get

$10x = 53.4747......$ (i)

Multiplying by 100, we get

$1000x = 5347.47......$ (ii)

Subtracting (i) from (ii), we get

$1000x - 10x = 5347.47...... - 53.47.....$

$\Rightarrow 990x = 5294$

$\Rightarrow x = \frac{5294}{990} = \frac{2647}{495}$

$\therefore\ 5.3\overline{47} = \frac{2647}{495}$

OR

$$\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times \left[\left(\frac{25}{9}\right)^{\frac{-3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right] = \left(\frac{3^4}{2^4}\right)^{\frac{-3}{4}} \times \left[\left(\frac{5^2}{3^2}\right)^{\frac{-3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{4\times\frac{-3}{4}} \times \left[\left(\frac{5}{3}\right)^{2\times\frac{-3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left[\left(\frac{5}{3}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{2}{3}\right)^{3} \times \left[\left(\frac{3}{5}\right)^{3} \div \left(\frac{2}{5}\right)^{3}\right]$$

$$= \left(\frac{2^{3}}{3^3}\right) \times \left[\left(\frac{3^3}{5^3}\right) \div \left(\frac{2^3}{5^3}\right)\right]$$

$$= \left(\frac{2^{3}}{3^3}\right) \times \left[\left(\frac{3^3}{5^3}\right) \times \left(\frac{5^3}{2^3}\right)\right]$$

$= 1$

14. $x^3 + 13x^2 + 32x + 20$ is a cubic polynimial, so it has three factors, of which one is $(x + 2)$.

$$
\begin{array}{r|l}
 & x^2 + 11x + 10 \\
x + 2 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + 2x^2 \\
 & (-)\ (-) \\
\hline
 & 11x^2 + 32x \\
 & 11x^2 + 22x \\
 & (-)\quad (-) \\
\hline
 & 10x + 20 \\
 & 10x + 20 \\
 & (-)\quad (-) \\
\hline
 & 0 \\
\hline
\end{array}
$$

Now, $x^2 + 11x + 10 = x^2 + 10x + x + 10$

$= (x + 1)(x + 10)$

$\Rightarrow x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$

15. In ΔXYZ,

$\angle X + \angle Y + \angle Z = 180°$

$72° + \angle Y + 46° = 180°$

$\angle Y = 62°$

$\angle OYZ = \frac{1}{2}\angle Y$ [since OY is bisector of $\angle Y$]

$\therefore\ \angle OYZ = \frac{1}{2} \times 62^o = 31^o$

$\angle OZY = \frac{1}{2}\angle Z$ [since OZ is bisector of $\angle Z$]

$= \frac{1}{2} \times 46^o = 23^o$

In ΔOYZ

$\angle OYZ + \angle YOZ + \angle OZY = 180^o$ [Angle sum property]

$31^o + \angle YOZ + 23^o = 180^o$

$\angle YOZ = 126^o$

OR

LM = MN (Given)

$\Rightarrow \angle$ MLN = $\angle$MNL (angles opposite equal sides are equal)

$\Rightarrow \angle$ MLQ = $\angle$MNP

LP = QN (Given)

$\Rightarrow$ LP + PQ = PQ + QN (adding PQ on both sides)

$\Rightarrow$ LQ = PN

In ΔLMQ and Δ NMP

LM = MN

$\angle$MLQ = $\angle$MNP

LQ = PN

Δ LMQ $\cong$ ΔNMP (SAS congruence rule)

16.

$a+b=\sqrt{11}$

$\Rightarrow (a+b)^2=\left(\sqrt{11}\right)^2$

$\Rightarrow a^2+b^2+2ab=11$

$\Rightarrow 5+2ab=11$

$\Rightarrow 2ab=6$

$\Rightarrow ab=3$

OR

$a^4 + 4a^2 + 3$

Consider, $a^2 = y$

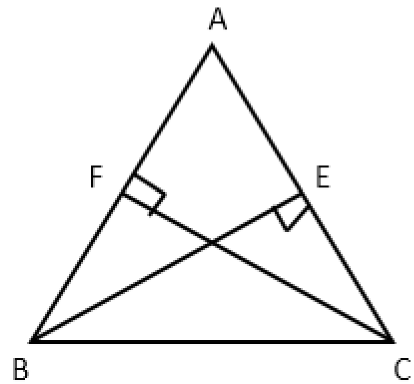
$y^2 + 4y + 3 = y^2 + 3y + y + 3$

$= y(y + 3) + (y + 3)$

$= (y + 3)(y + 1)$

But $y = a^2$ hence,

$a^4 + 4a^2 + 3 = (a^2 + 3)(a^2 + 1)$



In right angled triangles, BEC and CFB

$\angle BEC = \angle CFB$ (90° each)

BE = CF (given)

BC = BC (Common)

$\therefore \Delta BEC \cong \Delta CFB$ (RHS congruence)

$\angle BCE = \angle CBF$ (CPCT)

$\Rightarrow \angle BCA = \angle CBA$ (same angle)

In ΔABC, we have

$\angle BCA = \angle CBA$ (Proved above)

AB = AC [sides opposite equal angles are equal]

OR

LM = MN &. Given

$\Rightarrow \angle MLN = \angle MNL$ (angles opposite equal sides are equal)

$\Rightarrow \angle MLQ = \angle MNP$

LP = QN (Given)

$\Rightarrow LP + PQ = PQ + QN$ (adding PQ on both sides)

$\Rightarrow LQ = PN$

In ΔLMQ and ΔNMP

LM = MN

$\angle MLQ = \angle MNP$

LQ = PN

$\Delta LMQ \cong \Delta NMP$ (SAS congruence rule)

18. Let S be the sample space.

Thus, n(S) = 30

(a) Let A be the event of a class having 2 left-handed students.

$\therefore$ n(A) = 5

$\therefore P(A) = \frac{5}{30} = \frac{1}{6}$

(b) Let B be the event of a class having at least 3 left-handed students.

$\therefore$ n(B) = 12 + 8 + 2 = 22

$\therefore P(B) = \frac{22}{30} = \frac{11}{15}$

(c) First find the total number of left-handed students:

No. of left-handed students, x	0	1	2	3	4	5
Frequency, f (no. of classes)	1	2	5	12	8	2
fx	0	2	10	36	32	10

Total number of left-handed students = 2 + 10 + 36 + 32 + 10 = 90

Here, the sample space is the total number of students in the 30 classes, which was given as 960.

Let T be the sample space and C be the event that a student is left-handed.

n(T) = 960

n(C) = 90

$\therefore P(C) = \frac{90}{960} = \frac{3}{32}$

19.

i. Co-ordinates of points B and C are (3, 0) and (6, −3) respectively.

ii. (1, 2) is a solution of line passing through A and B.

iii. Equation of the x-axis is y = 0 and the y-axis is x = 0.

20. We have,

$$x - \frac{2}{3}\ y = \frac{8}{3}, \frac{2x}{5} - y = \frac{7}{5}$$

$$x - \frac{2}{3}y = \frac{8}{3} \quad ....(1)$$

$$\frac{2x}{5} - y = \frac{7}{5} \quad ....(2)$$

From (1)

$$x = \frac{8}{3} + \frac{2}{3}y = \frac{8+2y}{3} \quad ......(3)$$

Substituting the value of x in (2),

$$\frac{2}{5}\left(\frac{8+2y}{3}\right) - y = \frac{7}{5}$$

$$\Rightarrow \frac{16+4y}{15} - y = \frac{7}{5}$$

$$\Rightarrow 16 + 4y - 15y = 21$$

$$\Rightarrow -11y = 5$$

$$y = \frac{-5}{11}$$

Substituting the value of y in (3),

$$x = \frac{8 + 2\left(\frac{-5}{11}\right)}{3} = \frac{8 - \frac{10}{11}}{3}$$

$$= \frac{88-10}{11 \times 3} = \frac{78}{11 \times 3}$$

$$x = \frac{26}{11}$$

$$x = \frac{26}{11}, y = \frac{-5}{11}$$

21. Slant height (l) of conical tomb = 25 m

Base radius (r) of tomb = $\frac{14}{2}$ m = 7 m

C.S.A. of conical tomb = $\pi rl = \left(\frac{22}{7} \times 7 \times 25\right) m^2 = 550\ m^2$

Cost of white-washing 100 m^2 = Rs. 210

Cost of white-washing 550 m^2 = Rs. $\left(\frac{210 \times 550}{100}\right)$ = Rs. 1155

Thus, the cost of white washing the conical tomb is Rs. 1155.

OR

Length of the pipe, h = 7 × 100 = 700 cm

External radius of the pipe, R = 1.2 cm

Internal radius of the pipe, r = 1.2 – 0.2 = 1 cm

External volume of the pipe = $\pi R^2 h$ = $\frac{22}{7} \times 1.2 \times 1.2 \times 700 = 3168 \text{ cm}^3$

Internal volume of the pipe = $\pi r^2 h$ = $\frac{22}{7} \times 1 \times 1 \times 700 = 2200 \text{ cm}^3$

Volume of lead = external volume – internal volume = 3168 – 2200 = 968 cm^3

Weight of the pipe = $\frac{968 \times 10}{1000} = 9.68 \text{ kg}$

22. Given that we have to construct a grouped frequency distribution table of class size 5. So, the class intervals will be as 0 – 5, 5 – 10, 10 – 15, 15 –20, and so on.

Required grouped frequency distribution table is as follows:

Distance (in km)	**Tally marks**	**Number of engineers**
0 – 5	~~IIII~~	5
5 – 10	~~IIII~~ ~~IIII~~ I	11
10 –15	~~IIII~~ ~~IIII~~ I	11
15 – 20	~~IIII~~ IIII	9
20 – 25	I	1
25 – 30	I	1
30 – 35	II	2
Total		40

Only 4 engineers have homes at a distance of more than or equal to 20 km from their work place.

Most of the engineers have their workplace at a distance of upto 15 km from their homes.

Section D

23. We have

$$\left(\frac{1}{4}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 4^{0}+\left(\frac{9}{16}\right)^{-\frac{1}{2}}$$

$$=\left(\frac{1}{2^{2}}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 1+\left(\frac{3^{2}}{4^{2}}\right)^{\frac{1}{2}}$$

$$=\left(2^{-2}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 1+\left(\frac{3^{2\times\frac{-1}{2}}}{4^{2\times\frac{-1}{2}}}\right)$$

$$=2^{(-2)\times(-2)}-3\times 8^{\frac{2}{3}}+\left(\frac{3^{-1}}{4^{-1}}\right)$$

$$=2^{4}-3\times 2^{3\times\frac{2}{3}}+\frac{4}{3}$$

$$=2^{4}-3\times 2^{2}+\frac{4}{3}$$

$$=2^{4}-3\times 4+\frac{4}{3}$$

$$=16-12+\frac{4}{3}$$

$$=4+\frac{4}{3}=\frac{12+4}{3}$$

$$=\frac{16}{3}$$

$$\Rightarrow \left(\frac{1}{4}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 4^{0}+\left(\frac{9}{16}\right)^{-\frac{1}{2}}=16$$

OR

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}=\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}\times\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}}$$

$$=\frac{\left(\sqrt{11}-\sqrt{7}\right)^{2}}{\left(\sqrt{11}\right)^{2}-\left(\sqrt{7}\right)^{2}}$$

$$=\frac{11+7-2\sqrt{77}}{11-7}$$

$$= \frac{18 - 2\sqrt{77}}{4}$$

$$= \frac{18}{4} - \frac{2}{4}\sqrt{77} = a - b\sqrt{77}$$

The values of a and b are $a = \frac{18}{4} = \frac{9}{2}$, $b = \frac{2}{4} = \frac{1}{2}$

24. In fig., AD and CE are the bisectors of $\angle A$ and $\angle C$ respectively. If $\angle ABC = 90°$ then find $\angle AOC$.

$\angle DAC = \frac{1}{2}\angle A$ [$\because$ AD is bisector of $\angle A$]

$\Rightarrow \angle OAC = \frac{1}{2}\angle A$ --- (i) [$\because \angle OAC = \angle DAC$]

$\angle ECA = \frac{1}{2}\angle C$ [$\because$ CE is angle bisector of $\angle C$]

$\Rightarrow \angle OCA = \frac{1}{2}\angle C$ ---(ii) [$\because \angle OCA = \angle ECA$]

In Δ ABC

$\angle A + \angle B + \angle C = 180^o$ (Angle sum property)

$\angle A + \angle C + 90^o = 180^o$

$\angle A + \angle C = 90^o$

$\frac{1}{2}\angle A + \frac{1}{2}\angle C = 45^0$ --- (iii)

$\Rightarrow \angle OAC + \angle OCA = 45^0$ --- (iv)

In ΔOAC,

$\angle AOC + \angle OAC + \angle OCA = 180^o$ (Angle sum property)

$\angle AOC + 45^o = 180^o$

$\angle AOC = 135^o$

25. p(x) = $x^4 - 2x^3 + 3x^2 - ax + 3a - 7$

Now, p(−1) = 19

$\Rightarrow (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) - 7 = 19$

$\Rightarrow 1 + 2 + 3 + a - 7 = 19$

$\Rightarrow a - 1 = 19$

$\Rightarrow a = 18$

$\therefore$ The given polynomial is $x^4 - 2x^3 + 3x^2 - 18x + 3 \times 18 - 7$,

i.e. $p(x) = x^4 - 2x^3 + 3x^2 - 18x + 47$

The remainder when p(x) is divided by $(x + 2) = p(-2)$

$\therefore p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 18(-2) + 47$

$= 16 + 16 + 12 + 36 + 47$

$= 127$

OR

Consider,

p(x) = $2x^4 + 3x^3 + 2px^2 + 3x + 6$

According to the question,

p(-2) = $2 \times (-2)^4 + 3 \times (-2)^3 + 2p(-2)^2 + 3(-2) + 6$

p(-2) = 32 – 24 + 8p – 6 + 6

p(-2) = 8 + 8p

By factor theorem,

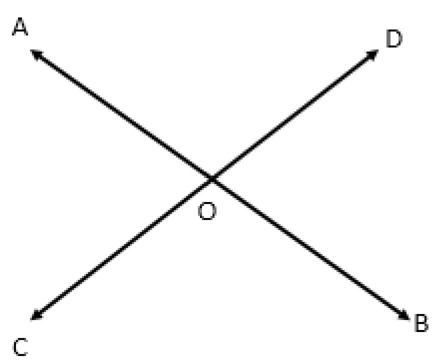
p(-2) = 0

8 + 8p = 0

p = -1

26. Given: Two lines AB and CD which intersect each other at O.

To prove: ∠AOC = ∠BOD and ∠AOD = ∠BOC



Proof: Since AB is line and ray OD stands on it

∴ ∠AOD + ∠BOD = 180° ....(1)(Linear pair axiom)

Since CD is a line and ray OA stands on it

$\therefore$ ∠AOC + ∠AOD = 180°   ....(2)(Linear pair axiom)

From (i) and (ii)

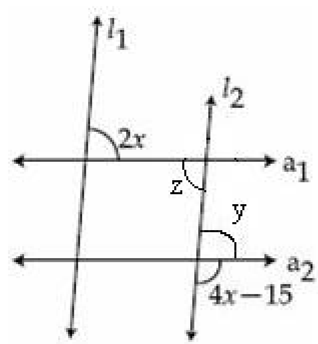
∠AOC + ∠AOD = ∠AOD + ∠BOD

$\Rightarrow$ ∠ AOC = ∠BOD

Similarly, we can prove that

∠ BOC = ∠AOD

OR



2x = z (Alternate angles, as $l_1 \parallel l_2$)

y = z (Alternate angles, as $a_1 \parallel a_2$)

So, 2x = y

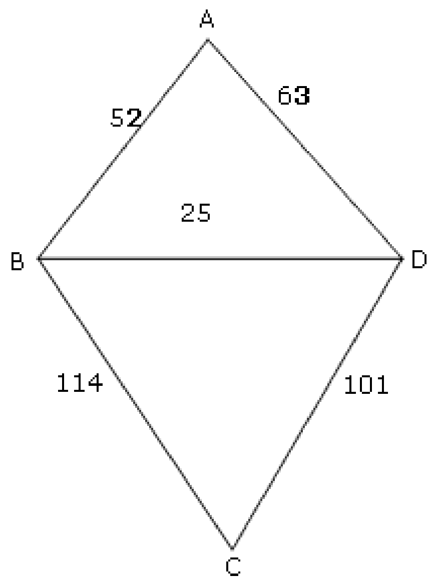
Now, y + 4x – 15 = 180° (linear pair)

2x + 4x – 15 = 180°

6x = 195° $\Rightarrow$ x = 32.5

27.

Sonu and Monu's field together form a quadrilateral ABCD.



Sonu's field is ΔABD,

$$s = \frac{a+b+c}{2} = \frac{52+25+63}{2} = 70$$

$s - a = 70 - 52 = 18$, $s - b = 70 - 25 = 45$ and $s - c = 70 - 63 = 7$

Area of ΔABD =

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{70.18.45.7} = 630 \text{ sq m}$$

Monu's field is ΔBCD,

$$s = \frac{a+b+c}{2} = \frac{114+25+101}{2} = 120$$

$s - a = 120 - 114 = 6$, $s - b = 120 - 25 = 95$ and $s - c = 120 - 101 = 19$

Area of ΔBCD =

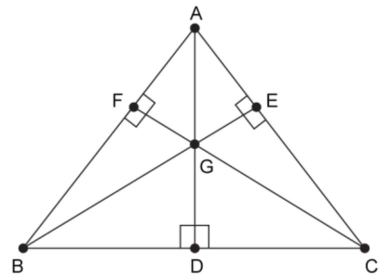
$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{120.6.95.19} = 1140 \text{ sq m}$$

Total area is = 630 + 1140 = 1770 sq m

The cost of fertilization is Rs 20 per sq m.

Therefore the total cost is = 1770 x 20 = Rs 35,400.

28.



Given: ABC is an equilateral triangle. AD, BE and CF are medians of triangle ABC.

To prove: AD = BE = CF

Proof:

In ΔCBF and ΔBCE

$\angle B = \angle C = 60^\circ$ [Angles of an equilateral triangle]

$BC = BC$ [common]

$BF = EC$ $\left[\because AB = AC \therefore \frac{1}{2}AB = \frac{1}{2}AC\right]$

By SAS, $\Delta CBF \cong \Delta BCE$

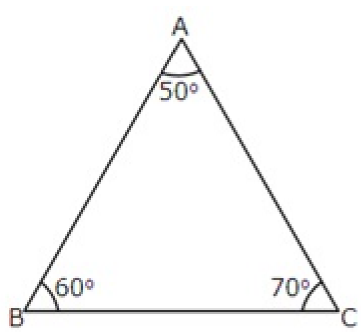
$\therefore BE = CF$ [$c.p.c.t$]

Similarly AD = BE

$\therefore AD = BE = CF$.

Hence proved.

29. Given: Three friends Anuja, Nikita and Raghav daily go on morning walks and walk along PQ, QR and PR of triangular field PQR, respectively.



Here, 50° < 60° < 70°

i.e. ∠ A < ∠B < ∠C

We know that side opposite to greater angle is longer.

Hence, BC < CA < AB

So, distance AB is the maximum. Thus, Anuja walks the maximum distance. Further, distance QR is the least. Thus, Nikita walks the least distance.

Value: Awareness for a daily walk for better health without gender and religion bias.

30.

(i) $(3x - 5y - 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16)$

$= (3x + (-5y) + (-4))\ [(3x)^2 + (-5y)^2 + (-4)^2 - (3x)(-5y) - (-5y)(-4) - (3x)(-4)]$

$= (3x)^3 + (-5y)^3 + (-4)^3 - 3(3x)(-5y)(-4)$

$[(a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$

$= 27x^3 - 125y^3 - 64 - 180xy$

(ii) $a^2 + b^2 - 2\ (ab - ac + bc)$

$= a^2 + b^2 - 2ab + 2ac - 2bc$

$= (a^2 + b^2 - 2ab) + 2c(a - b)$

$= (a - b)^2 + 2c(a - b)$

$= (a - b)(a - b + 2c)$