

CHAPTER – 13
DIRECT AND INVERSE PROPORTIONS
EXERCISE – 13.1

Question – 1 Following are the car parking charges near a railway station upto

4 hours	Rs. 60
8 hours	Rs. 100
12 hours	Rs. 140
24 hours	Rs. 180

Check if the parking charges are in direct proportion to the parking time.

Answer:

For the data to be in direct proportion, the ratio of the charged money and the time should remain equal.

The ratio of parking charges to their respective number of hours may be calculated as

$$\frac{60}{4} = 15$$

$$\frac{100}{8} = \frac{25}{2} = 12.5$$

$$\frac{140}{12} = \frac{35}{3} = 11.67$$

$$\frac{180}{24} = \frac{15}{2} = 7.5$$

As, each ratio is not same therefore, the parking charges to their respective number of hours are not in direct proportion.

Question – 2 A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of Red Pigment	1	4	7	12	20
Parts of base	8				

Answer:

Parts of Red Pigment	1	4	7	12	20
Parts of base	8	X_1	X_2	X_3	X_4

A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base

x_1 = If 1 part of red pigment is mixed with 8 part of base, 4 parts of red pigment is mixed with 4×8 parts of base. Therefore,

$$X_1 = 4 \times 8 = 32$$

x_2 = If 1 part of red pigment is mixed with 8 part of base, 7 parts of red pigment is mixed with 7×8 parts of base. Therefore,

$$X_2 = 7 \times 8 = 56$$

x_3 = If 1 part of red pigment is mixed with 8 part of base, 12 parts of red pigment is mixed with 12×8 parts of base. Therefore,

$$X_3 = 12 \times 8 = 96$$

x_4 = If 1 part of red pigment is mixed with 8 part of base, 20 parts of red pigment is mixed with 20×8 parts of base. Therefore,

$$X_4 = 20 \times 8 = 160$$

Question – 3 In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base?

Answer:

The table of the given information is as follows:

Parts of Red Pigment	1	X
Parts of base	75 ml	1800 ml

The parts of the red pigment and parts of base are in direct proportion,

Therefore,

$$\frac{1}{75} = \frac{x}{1800}$$

$$\Rightarrow x = 24$$

Thus, 24 parts of the red pigment mixed with 1800 ml of base.

Method 2:

Given: 1 part of red pigment makes 75 ml
1 ml will be made from $\frac{1}{75}$ parts
Therefore, 1800 ml will be made from $\frac{1}{75} \times 1800 = 24$ parts

Question – 4 A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?

Answer:

The given information in the form of table is as follows:

Bottles	840	x
hours	6	5

Let x be the no of bottles filled in 5 hours.

The no of bottles and no of hours are in direct proportion as the larger the number of bottles the longer it takes to fill them and vice-versa.

Therefore,

$$\frac{840}{6} = \frac{x}{5}$$

$$6x = 5 \times 840$$

$$\rightarrow x = 700$$

Thus, 700 bottles will be filled in 5 hours.

Question – 5 A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the actual length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?

Answer:

Let the actual length of the bacteria be x cm and enlarged length be y cm when it is enlarged 20,000 times.

The given information in the form of table is as follows

Length of bacteria in cm	5	x	y
Number of times the photograph of bacteria was enlarged	50,000	1	20,000

The above information is in direct proportion.

Therefore,

$$\frac{x}{1} = \frac{5}{50000}$$

$$x = 10^{-4} \text{ cm}$$

Hence the actual length of bacteria is 10^{-4} cm

$$= \frac{5}{50000} = \frac{y}{20000}$$

$$y = 2 \text{ cm}$$

Hence, the enlarged length of bacteria when it is enlarged 20,000 times is 2cm

Question – 6 In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?

Answer:

Let the length of the mast of modal ship be x when the length of the actual ship be 28 cm

The given information in the form of table is as follows:

	Height of ship	Length of ship
Modal ship	9	x
Actual ship	12	28

The above information is in direct information:

$$9/12 = x/28$$

Cross-multiplying, we get,

$$9 \times 28 = 12 \times x$$

$$x = \frac{9 \times 28}{12} = 3 \times 7$$

$$\rightarrow x = 21 \text{ cm}$$

Question – 7 Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in

(i) 5 kg of sugar?

(ii) 1.2 kg of sugar?

Answer:

(i) Let x be the sugar crystals in 5kg of sugar

The above information in the form of table is as follows:

Sugar Weight	2 kg	5 kg
No. of crystals	9×10^6	x

The above information are in direct proportion:

Number of Crystals \propto Weight of Sugar (The more the weight, the more the number of crystals and vice versa)

$$\frac{\text{Weight}}{\text{Number of Crystals}} = \text{Constant}$$

$$\frac{2}{9 \times 10^6} = \frac{5}{x}$$

Cross Multiplying, we get,

$$2x = 5 \times 9 \times 10^6$$

$$x = \frac{45 \times 10^6}{2}$$

$$x = 22.5 \times 10^6$$

(ii) Let x be the sugar crystals in 1.2kg of sugar

The above information in the form of table is as follows:

Sugar Weight	2 kg	1.2 kg
No. of crystals	9×10^6	x

The above information is in direct proportion:

Therefore,

$$2/(9 \times 10^6) = 1.2/x$$

$$\rightarrow x = 5.4 \times 10^6 \text{ crystals}$$

Question – 8 Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

Answer:

Let the distance measured on map be x cm

The above information in the form of table is as follows:

Distance covered on map in cm	1	x
Distance covered on road in km	18	72

The above information is in direct proportion because distance covered in cm will increase or decrease with distance covered in km.

Therefore,

$$\frac{18}{72} = \frac{1}{x}$$

$$1 \times 18 = x \times 72$$

$$x = 4 \text{ cm}$$

Method 2: 1 cm on map = 18 km on road
1 km on road = $1/18$ cm on map
72 km on road = $1/18 \times 72$ cm on map = 4 cm on map.

Question – 9 A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time

- (i) the length of the shadow cast by another pole 10 m 50 cm high
- (ii) the height of a pole which casts a shadow 5m long.

Answer:

(i) let the length of the shadow of other pole be x m

The given information in the form of table is as follows:

Length of shadow in m	3.20	x
Length of pole in m	5.60	10.50

The above information is in direct proportion.

$$\frac{3.2}{5.6} = \frac{x}{10.5}$$

$$5.6x = 3.2 \times 10.5$$

$$5.6x = 33.6 \quad x = 33.6/5.6$$

$$\rightarrow x = 6\text{m}$$

(ii) let the length of other pole be x m

The given information in the form of table is as follows:

Length of shadow in m	3.20	5
Length of pole in m	5.60	x

The above information is in direct proportion.

$$\frac{3.2}{5.6} = \frac{5}{x}$$

$$3.2x = 5 \times 5.6$$

$$3.2x = 28$$

$$x = \frac{28}{3.2}$$

$$\rightarrow x = 8.75 \text{ m}$$

Question – 10 A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

Answer:

Let the distance covered by truck in 5 hours be x km

The given information in the form of table is as follows:

Distance in km	14	x
Time in minutes	25	$5 \times 60 = 300$

Given: In 25 minutes, the truck travels 14km

Now, in 5 hours the truck will travel more distance. Therefore, above information is in direct proportion.

Therefore,

$$14/25 = x/300$$

$$25x = 14 \times 300$$

$$x = \frac{14 \times 300}{25} = 11 \times 12$$

$$\rightarrow x = 168 \text{ km}$$

EXERCISE – 13.2

Question – 1 Which of the following are in inverse proportion?

- (i) The number of workers on a job and the time to complete the job.
- (ii) The time taken for a journey and the distance travelled in a uniform speed.
- (iii) Area of cultivated land and the crop harvested.
- (iv) The time taken for a fixed journey and the speed of the vehicle.
- (v) The population of a country and the area of land per person.

Answer:

- (i) Yes, these are in inverse proportion because if there are more workers, then it will take lesser time to complete that job.
- (ii) No, these are not in inverse proportion because in more time, we may cover more distance with a uniform speed. It is a direct Proportion.
- (iii) No, these are not in inverse proportion because in more area, more quantity of crop may be harvested. It is a direct Proportion.
- (iv) Yes, these are in inverse proportion because with more speed, we may complete a certain distance in a lesser time.
- (v) Yes, these are in inverse proportion because if the population is increasing, then area of the land per person will be decreasing accordingly.

Question – 2 In a Television game show, the prize money of Rs 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners?

Name of winners	1	2	4	5	8	10	20
Prize of each winner (in Rs.)	1,00,000	50,000

Answer:

Name of winners	1	2	4	5	8	10	20
Prize of each winner (in Rs.)	1,00,000	50,000	X_1	X_2	X_3	X_4	X_5

From the table, we obtain:

$$1 \times 100,000 = 2 \times 50,000 = 100,000$$

Thus, the number of winners and amount given to an individual winner is inversely proportional.

Therefore,

$$1 \times 100,000 = x_1 \times 4$$

$$\rightarrow x_1 = 25,000$$

$$1 \times 100,000 = x_2 \times 5$$

$$\rightarrow x_2 = 20,000$$

$$1 \times 100,000 = x_3 \times 8$$

$$\rightarrow x_3 = 12500$$

$$1 \times 100,000 = x_4 \times 10$$

$$\rightarrow x_4 = 10,000$$

$$1 \times 100,000 = x_5 \times 20$$

$$\rightarrow x_5 = 5,000$$

Question – 3 Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following

table.



Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°

- (i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?
- (ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.
- (iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40° ?

Answer:

- (i) The table of the required information is as follows:

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	X_1	X_2	X_3

$$4 \times 90^\circ = 6 \times 60^\circ = 360^\circ$$

Therefore, the number of spokes and the angles formed between the pairs of consecutive spokes is in inverse proportion.

$$x_1 = \frac{360^\circ}{8} = 45^\circ$$

$$x_2 = \frac{360^\circ}{10} = 36^\circ$$

$$x_3 = \frac{360^\circ}{12} = 30^\circ$$

- (ii) $4 \times 90^\circ = 15 \times \text{angle}$

$$\text{Angle} = 24^\circ$$

the angle between a pair of consecutive spokes on a wheel with 15 spokes is 24°

$$(iii) 4 \times 90^\circ = 40^\circ \times \text{spokes}$$

$$\rightarrow \text{spokes} = 9$$

9 spokes would be needed, if the angle between a pair of consecutive spokes is 40°

Question – 4 If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

Answer:

According to the question:

a box of sweets is divided among 24 children, they will get 5 sweets each

$$\text{total no of sweets} = 24 \times 5 = 120$$

the number of the children is reduced by 4.

$$\text{Now, total no of children} = 24 - 4 = 20$$

$$\text{Sweets get by each children} = \frac{120}{20} = 6$$

Question – 5

A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

Answer:

A farmer has enough food to feed 20 animals in his cattle for 6 days.

More the number of animals, the lesser will be the number of days food lasts.

Therefore, the number of animals and number of days are inversely proportional to each other.

Hence,

$$20 \times 6 = 30 \times x$$

$$x = \frac{20 \times 6}{30} = 2 \times 2$$

$$\rightarrow x = 4 \text{ days}$$

Now, food will last in 4 days.

Question – 6 A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

Answer:

If the number of persons be more, it will take less time to complete the job

Therefore,

The number of persons and time required to complete the work are inversely proportional to each other.

Hence,

$$3 \times 4 = 4 \times x \text{ \{let } x \text{ be the number of days required to complete the work by 4 persons\}}$$

$$\rightarrow x = 3 \text{ days}$$

Question – 7 A batch of bottles were packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?



Answer:

A batch of bottles were packed in 25 boxes with 12 bottles in each box.

1 box contains 12 bottles, so 25 boxes will contain = 25×12 bottles

Total no of bottles = $25 \times 12 = 300$

If the same batch is packed using 20 bottles in each box

Now the total number of bottles = 300

And each box contains 20 bottles.

Then, no of boxes = $300/20 = 15$

Question – 8 A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Answer:

The more the no of machines, the less days will be required to produce a given no of articles.

Therefore, the no of machines and no of days are in inverse proportion.

Hence,

$42 \times 63 = 54 \times x$ {x be the no of machines required to produce a given no of articles in 54 days}

$$\rightarrow x = 49$$

Question – 9 A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h?

Answer:

Distance = speed \times time

Therefore, speed and time are inversely proportional

This means that,

Speed \times Time = Constant $2 \times 60 = 80 \times t$ { t be the time required to cover a given distance at 80 km/hr}

$$t = \frac{2 \times 60}{80} = \frac{3}{2} = 1.5 \text{ hr}$$

$$\rightarrow t = 1.5 \text{ hr}$$

Question – 10 Two persons could fit new windows in a house in 3 days.

(i) One of the persons fell ill before the work started. How long would the job take now?

(ii) How many persons would be needed to fit the windows in one day?

Answer:

(i) If the number of persons be more, it will take less time to complete the job. Now, one of the persons fell ill before the work started.

$$\text{Persons left} = 2 - 1 = 1$$

Therefore,

The number of persons and time required to complete the work are inversely proportional to each other.

Therefore,

$$2 \times 3 = 1 \times t \text{ (t be the time required to fit the windows by 1 person)}$$

$$\rightarrow t = 6 \text{ days}$$

(ii) If the number of persons be more, it will take less time to complete the job. Now, one of the persons fell ill before the work started.

Therefore,

The number of persons and time required to complete the work are inversely proportional to each other

Let n be the numbers of persons required to complete the work in 1 day.

$$2 \times 3 = 1 \times n$$

$$\rightarrow n = 6 \text{ persons}$$

Question – 11 A school has 8 periods a day each of 45 minutes' duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours tube the same?

Answer:

A school has 8 periods a day each of 45 minutes duration.

$$\text{Total time} = 8 \times 45 \text{ minutes}$$

if the school has 9 periods a day.

$$\text{Then duration of period} = (8 \times 45) / 9$$

$$= 40 \text{ minutes}$$