# Wave Optics



#### 1. Wave Nature of Light: Huygen's Theory

There are some phenomena like interference, diffraction and polarisation which could not be explained by Newton's corpuscular theory. These were explained by wave theory first proposed by Huygen.

The assumptions of Huygen's wave theory are: (i) A source sends waves in all possible directions. The locus of particles of a medium vibrating in the same phase is called a **wavefront**. For a point source, the wavefront is spherical; while for a line source the wavefront is cylindrical. A distant wavefront is plane. (ii) Each point of a wavefront acts as a source of secondary wavelets. The envelope of all wavelets at a given instant gives the position of a new wavefront.

#### 2. Wavefront

A wavefront is defined as the locus of all the particles which are vibrating in the same phase. The perpendicular line drawn at any point on the wavefront represents the direction of propagation of the wave at that point and is called the '**ray**'.

**Types of Wavefronts:** The wavefronts can be of different shapes. In general, we experience three types of wavefronts.

(*i*) **Spherical Wavefront:** If the waves in a medium are originating from a point source, then they propagate in all directions. If we draw a spherical surface centred at point-source, then all the particles of the medium lying on that spherical surface will be in the same phase, because the disturbance starting from the source will reach all these points simultaneously. Hence in this case, the wavefront will be spherical and the rays will be the radial lines [Fig (*a*)].



(*ii*) **Cylindrical Wavefront:** If the waves in a medium are originating from a line source, then they too propagate in all directions. In this case the locus of particles vibrating in the same phase will be a cylindrical surface. Hence in this case the wavefront will be cylindrical. [Fig. (*b*)]



(*iii*) **Plane Wavefront:** At large distance from the source, the radii of spherical or cylindrical wavefront will be too large and a small part of the wavefront will appear to be plane. At infinite distance from the source, the wavefronts are always plane and the rays are parallel straight lines.

The equation  $y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$ 

represents the plane wave propagating along positive direction of X-axis.

#### 3. Coherent and Incoherent Sources of Light

The sources of light emitting waves of same frequency having zero or constant initial phase difference are called coherent sources.

The sources of light emitting waves with a random phase difference are called incoherent sources. For interference phenomenon, the sources must be coherent.

**Methods of Producing Coherent Sources:** Two independent sources can never be coherent sources. There are two broad ways of producing coherent sources for the same source.

- (*i*) By division of wavefront: In this method the wavefront (which is the locus of points of same phase) is divided into two parts. The examples are Young's double slit and Fresnel's biprism.
- *(ii)* By division of amplitude: In this method the amplitude of a wave is divided into two parts by successive reflections, *e.g.*, Lloyd's single mirror method.

#### 4. Interference of Light

Interference is the phenomenon of superposition of two light waves of same frequency and constant phase different travelling in same direction. The positions of maximum intensity are called *maxima*, while those of minimum intensity are called *minima*.

**Conditions of Maxima and Minima:** If  $a_1$  and  $a_2$  are amplitudes of interfering waves and  $\phi$  is the phase difference at a point under consideration, then

Resultant intensity at a point in the region of superposition

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$
  
=  $I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$ 

where

$$I_1 = a_1^2 =$$
 intensity of one wave

 $I_2 = a_2^2 =$  intensity of other wave

#### **Condition of Maxima:**

Phase difference,  $\phi = 2n\pi$ 

or path difference,  $\Delta = n\lambda$ , *n* being integer

Maximum amplitude,  $A_{\text{max}} = a_1 + a_2$ 

Maximum intensity,  $I_{\text{max}} = A^2_{\text{max}} = (a_1 + a_2)^2$ 

$$=a_1^2+a_2^2+2a_1a_2=I_1+I_2+2\sqrt{I_1I_2}$$

**Condition of Minima:** Phase difference,  $\phi = (2n - 1) \pi$ Path difference,  $\Delta = (2n - 1) \frac{\lambda}{2}$ , n = 1, 2, 3, ...Minimum amplitude,  $A_{\min} = (a_1 - a_2)$ Minimum intensity,  $I_{\min} = (a_1 - a_2)^2 = a_1^2 + a_2^2 - 2a_1a_2$  $= I_1 + I_2 - 2\sqrt{I_1I_2}$ 

#### Young's Double Slit Experiment

Let  $S_1$  and  $S_2$  be coherent sources at separation d and D be the distance of screen from sources, then path difference between waves reaching at P can be shown as

$$\Delta = \frac{y_n d}{D}$$

For maxima  $\Delta = n\lambda$ 

$$\therefore$$
 Position of *n*th maxima  $y_n = \frac{nD\lambda}{d}$ 

 $\therefore$  Position of *n*th minima  $y_n = \left(n - \frac{1}{2}\right) \frac{D\lambda}{d}$ 

**Fringe width:** Fringe width is defined as the separation between two consecutive maxima or minima.

Linear fringe width, 
$$\beta = y_{-n+1} - y_n = \frac{D\lambda}{d}$$
  
Angular fringe width,  $\beta_{\theta} = \frac{\beta}{D} = \frac{\lambda}{d}$ .

**Use of white light:** When white light is used to illuminate the slit, we obtain an interference pattern consisting of a central white fringe having few coloured fringes on two sides and uniform illumination.

**Remark:** If waves are of same intensity,

$$I_1 = I_2 = I_0 \text{ (say) then}$$
$$I = 2I_0 + 2I_0 \cos \phi$$
$$= 2I_0(1 + \cos \phi)$$
$$= 4I_0 \cos^2 \frac{\phi}{2}$$

#### 5. Diffraction of Light

The bending of light from the corner of small obstacles or apertures is called diffraction of light. **Diffraction due to a Single Slit:** 

When a parallel beam of light is incident normally on a single slit, the beam is diffracted from the slit and the diffraction pattern consists of a very intense central maximum, and secondary maxima and minima on either side alternately.

If *a* is width of slit and  $\theta$  the angle of diffraction, then the directions of maxima are given by

$$a\sin\theta = \left(n + \frac{1}{2}\right)\lambda$$
  $n = 1, 2, 3, ...$ 

The position of *n*th minima are given by

$$a\sin\theta = n\lambda$$
,

where  $n = \pm 1, \pm 2, \pm 3, ...$  for various minima on either side of principal maxima.

#### Width of Central Maximum:

The width of central maximum is the separation between the first minima on either side. The condition of minima is

 $a \sin \theta = \pm n\lambda \ (n = 1, 2, 3,...).$ 

The angular position of the first minimum (n=1) on either side of central maximum is given by

$$a \sin \theta = \pm \lambda$$
$$\Rightarrow \qquad \theta = \pm \sin^{-1} \left(\frac{\lambda}{a}\right)$$





- $\therefore$  Half-width of central maximum,  $\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$
- $\therefore$  Total width of central maximum,  $\beta = 2\theta = 2 \sin^{-1} \left(\frac{\lambda}{a}\right)$



Diffraction due to a single slit by a monochromatic light

**Linear Width:** If *D* is the distance of the screen from slit and *y* is the distance of *n*th minima from the centre of the principal maxima, then

$$\sin\theta \simeq \tan\theta \simeq \theta = \frac{y}{D}$$

Now,

$$\theta = \frac{\lambda n}{a} = \frac{y_n}{D}$$

 $n\lambda = a\sin\theta \simeq a\theta$ 

$$\Rightarrow \qquad y_n = \frac{n \lambda D}{a}$$

Linear half-width of central maximum,  $y = \frac{\lambda D}{a}$ 

Total linear width of central maximum,  $\beta = 2y = \frac{2\lambda D}{a}$ 

#### 6. Resolving Power

The resolving power of an optical instrument is its ability to form distinct images of two neighbouring objects. It is measured by the smallest angular separation between the two neighbouring objects whose images are just seen distinctly formed by the optical instrument. This smallest distance is called the *limit of resolution*.

Smaller the limit of resolution, greater is the resolving power.

The angular limit of resolution of eye is 1' or  $\left(\frac{1}{60}\right)^{\circ}$ . It means that if two objects are so close that angle subtended by them on eye is less than 1' or  $\left(\frac{1}{60}\right)^{\circ}$ , they will not be seen as separate.

The best criterion of limit of resolution was given by Lord Rayleigh. He thought that each object forms its diffraction pattern. For just resolution, the central maximum of one falls on the first minimum of the other [Fig. (a)]. When the central maxima of two objects are closer, then these objects appear overlapped and are not resolved [Fig. (b)]; but if the separation between them is more than this, they are said to be well resolved.





**Telescope:** If *a* is the aperture of telescope and  $\lambda$  the wavelength, then resolving limit of telescope  $d\theta \propto \frac{\lambda}{a}$ 

For spherical aperture,  $d\theta = \frac{1.22\lambda}{a}$ 

**Microscope:** In the case of a microscope,  $\theta$  is the well resolved semi-angle of

cone of light rays entering the telescope, then limit of resolution  $= \frac{\lambda}{2n \sin \theta}$ where *n* sin  $\theta$  is called numerical aperture.

#### 7. Polarisation

The phenomenon of restriction of vibrations of a wave to just one direction is called *polarisation*. It takes place only for transverse waves such as heat waves, light waves etc.

**Unpolarised Light:** The light having vibrations of electric field vector in all possible directions perpendicular to the direction of wave propagation is called the ordinary (or unpolarised) light.

**Plane (or Linearly) Polarised Light:** The light having vibrations of electric field vector in only one direction perpendicular to the direction of propagation of light is called plane (or linearly) polarised light.

The unpolarised and polarised light is represented as:

- (a) Unpolarised light
- (b) Polarised light
- (c) Partially polarised light

**Polarisation by Reflection:** *Brewster's Law:* If unpolarised light falls on a transparent surface of refractive index n at a certain angle  $i_B$  called polarising angle, then reflected light is plane polarised.

**Brewster's law:** The polarising angle  $(i_B)$  is given by  $n = \tan i_B$ 

This is called Brewster's law.



Under this condition, the reflected and refracted rays are mutually perpendicular, *i.e.*,

$$i_B + r = 90$$

where r is angle of refraction into the plane.

**Malus Law:** It states that if completely plane polarised light is passed through an analyser, the intensity of light transmitted  $\propto \cos^2 \theta$ , where  $\theta$  is angle between planes of transmission of polariser and analyser *i.e.*,



θ

 $I = I_0 \cos^2 \theta$  (Malus Law)

If incident light is unpolarised, then  $I = \frac{I_0}{2}$ ,

since  $(\cos^2 \theta)_{\text{average}}$  for all directions =  $\frac{1}{2}$ .

Polaroid: Polaroid is a device to produce and detect plane polarised light.

#### Some uses of polaroid are:

- (i) Sun glasses filled with polaroid sheets protect our eyes from glare.
- (ii) Polaroids reduce head light glare of motor car being driven at night.

(iii) Polaroids are used in three-dimensional pictures *i.e.*, in holography.

**Analysis of a given light beam:** For this, given light beam is made incident on a polaroid (or Nicol) and the polaroid/Nicol is gradually rotated:

- (i) If light beam shows no variation in intensity, then the given beam is unpolarised.
- (i) If light beam shows variation in intensity but the minimum intensity is non-zero, then the given beam is partially polarised.
- (*iii*) If light beam shows variation in intensity and intensity becomes zero twice in a rotation, then the given beam of light is plane polarised.

# Selected NCERT Textbook Questions

- Q. 1. Monochromatic light of wavelength 589 nm is incident from air on a water surface. What is the wavelength, frequency and speed of (*a*) reflected and (*b*) refracted light? Refractive index of water is 1.33.
- **Ans.** Given =  $589 \text{ nm} = 589 \times 10^{-9} \text{ m}$

Speed of light in air,  $c = 3 \times 10^8$  m/s

Refractive index of water  $n_w = 1.33$ 

(a) The reflected and incident rays are in the same medium, so all physical quantities (wavelength, frequency and speed) remain unchanged.

Wavelength of reflected wave, $\lambda$ Speed of reflected wave,c

Frequency of reflected wave,

$$\lambda = 589 \text{ nm}$$
  
 $c = 3 \times 10^8 \text{ m/s}$   
 $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{589 \times 10^{-9}} = 5.1 \times 10^{14} \text{ Hz}$ 

(b) Refracted wave is in second medium; its frequency remains unchanged; the speed becomes  $\frac{c}{n}$  times and wavelength changes accordingly.

Frequency of refracted wave,	$v = 5.1 \times 10^{14} \mathrm{Hz}$
Speed of refracted wave,	$v = \frac{c}{n} = \frac{3 \times 10^8}{1.33} = 2.26 \times 10^8 \text{ m/s}$
Wavelength of refracted wave,	$\dot{\lambda} = \frac{v}{v} = \frac{2.26 \times 10^8}{5.1 \times 10^{14}}$
	$= 443 \times 10^{-9} \text{m} = 443 \text{ nm}$

- Q. 2. (a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is  $3.0 \times 10^8 \text{ ms}^{-1}$ ).
  - (b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours, red and violet, travels slower in the glass prism?
- **Ans.** (*a*) Speed of light in glass,

$$v = \frac{c}{n_g} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$$



(b) No, the speed of light in glass depends on the colour of light.

$$v \propto \frac{1}{n}$$
 As  $n_V > n_R$ ,  $\therefore v_V < v_R$ 

That is, violet colour travels slower in glass prism.

Q. 3. In Young's double slit experiment the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth fringe is measured to be 1.2 cm. Determine the wavelength of light used in this experiment.

**Ans.** Given 
$$d = 0.28 \text{ mm} = 0.28 \times 10^{-3} \text{ m}$$
,  $D = 1.4 \text{ m}$ 

Position of *n*th bright fringe from central fringe is  $y_n = \frac{nD\lambda}{d}$ 

Here 
$$n = 4$$
,  $y_4 = 1.2$  cm  $= 1.2 \times 10^{-2}$  m

$$\Rightarrow \text{Wavelength } \lambda = \frac{y_4 \cdot d}{4D} = \frac{(1.2 \times 10^{-2} \text{ m}) \times (0.28 \times 10^{-3} \text{ m})}{4 \times 1.4 \text{ m}} = 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

- Q. 4. In Young's double slit experiment using monochromatic light of wavelength  $\lambda$  the intensity at a point on the screen where path difference is  $\lambda$  is K units. What is the intensity of light at a point where path difference is  $\frac{\lambda}{3}$ ?
- **Ans.** Resultant intensity at any point having a phase difference  $\phi$  is given by  $I = 4I_0 \cos^2 \frac{\phi}{2}$ When path difference is  $\lambda$ , phase difference is  $2\pi$

$$\therefore \qquad I = 4I_0 \cos^2 \pi = 4I_0 = K \qquad (given) \qquad \dots (i)$$

When path difference,  $\Delta = \frac{\lambda}{3}$ , the phase difference

*.*..

$$\phi' = \frac{2\pi}{\lambda} \Delta = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

$$I' = 4I_0 \cos^2 \frac{2\pi}{2 \times 3} \qquad (\text{since } K = 4I_0)$$

$$K \cos^2 \frac{\pi}{3} = K \times \left(\frac{1}{2}\right)^2 = \frac{1}{4}K$$

- Q. 5. A beam of light consisting of two wavelength 650 nm and 520 nm, is used to obtain interference fringes in a Young's double slit experiment on a screen 1.2 m away. The separation between the slits is 2 mm.
  - (a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.
  - (b) What is the least distance from the central maximum when the bright fringes due to both the wavelength coincide?

**Ans.** Given  $\lambda_1 = 650 \text{ nm} = 650 \times 10^{-9} \text{ m}, \lambda_2 = 520 \text{ nm} = 520 \times 10^{-9} \text{ m}$ 

(a) 
$$y_n = \frac{nD\lambda_1}{d}$$
  
 $\Rightarrow y_3 = \frac{3 \times 1.2 \times 650 \times 10^{-9}}{2 \times 10^{-3}} = 1.17 \times 10^{-3} \text{ m} = 1.17 \text{ mm}$ 

(b) For least distance of coincidence of fringes, there must be a difference of 1 in order of  $\lambda_1$  and  $\lambda_2$ .

$$\begin{split} n_1 \beta_1 &= n_2 \beta_2, \quad \frac{n_1 D \lambda_1}{d} = \frac{n_2 D \lambda_2}{d} \quad \Rightarrow \quad n_1 \lambda_1 = n_2 \lambda_2 \\ \text{As} \quad \lambda_1 > \lambda_2 \text{ , } n_1 < n_2 \end{split}$$

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If bright fringe will coincide at a least distance y,  $n_1 = n, n_2 = n + 1$ 

$$\therefore \quad (y_n)\lambda_1 = (y_{n+1})\lambda_2$$

$$\Rightarrow \quad \frac{nD\lambda_1}{d} = \frac{(n+1)D\lambda_2}{d}$$

$$\Rightarrow \quad n\lambda_1 = (n+1)\lambda_2$$

$$\Rightarrow \qquad n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{520 \text{ nm}}{(650 - 520) \text{ nm}}$$
or
$$n = \frac{520}{130} = 4$$

$$\therefore \text{ Least distance} \quad y_{\min} = \frac{nD\lambda_1}{d} = \frac{4 \times 1.2 \times 650 \times 10^{-9}}{2 \times 10^{-3}}$$

$$= 1.56 \times 10^{-3} \text{ m}$$

$$= 1.56 \text{ mm}$$

Q. 6. In Young's double slit experiment, the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water as  $\frac{4}{3}$ .

**Ans.** Angular fringe width 
$$\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$
 ...(*i*)

...(*ii*)

If apparatus is dipped in water,  $\lambda$  changes to  $\lambda_w = \frac{\lambda}{n_w} = \frac{\lambda}{4/3} = \frac{3\lambda}{4}$ 

$$\therefore$$
 New angular fringe width  $\theta_w = \frac{\lambda_w}{d}$ 

*:*.

$$\frac{\theta_w}{\theta} = \frac{\lambda_w}{\lambda} = \frac{(3\lambda/4)}{\lambda} = \frac{3}{4}$$
$$\theta_w = \frac{3}{4}\theta = \frac{3}{4} \times 0.2^\circ = 0.15^\circ$$

- Q. 7. What is Brewster angle for air to glass transition? Refractive index of glass = 1.5.
- Ans. From Brewster's law,

$$n = \tan i_B$$

Given n = 1.5

Brewster's angle,  $i_B = \tan^{-1} n = \tan^{-1} (1.5) = 56.3^{\circ}$ 

Q. 8. Light of wavelength 5000 Å falls on a plane reflecting surface. What is the wavelength and frequency of the reflected light?

For what angle of incidence is the reflected ray normal to the incident ray?

Ans. Given  $\lambda = 5000 \text{ Å} = 5 \times 10^{-7} \text{ m}$ 

Speed of light,  $c = 3 \times 10^8$  m/s

The frequency, wavelength and speed of light of reflected wave are same as of incident ray. Wavelength of reflected ray,

$$\lambda_{reflected} = \lambda_{incident} = 5000 \text{ \AA}$$



Frequency of reflected ray,

$$v_r = \frac{c}{\lambda_{reflected}} = \frac{3 \times 10^8}{5 \times 10^{-7}} = 6 \times 10^{14} \,\mathrm{Hz}$$

By law of reflection, i = r

Given, 
$$i + r = 90^\circ \implies 2i = 90^\circ \text{ or } i = 45^\circ$$

Thus, when angle of incidence is 45°, the reflected ray is normal to the incident ray.

Q. 9. In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is 0.1°. Find the spacing between the two slits.

**Ans.** Angular fringe width 
$$\beta_{\theta} = \frac{\beta}{D} = \frac{\lambda}{d}$$
  
 $\therefore$  Spacing between slits,  $d = \frac{\lambda}{\beta_{\theta}}$ 

Here  $\lambda = (600 \text{ nm} = 600 \times 10^{-9} \text{m}) = 6 \times 10^{-7} \text{m}, \ \beta_{\theta} = 0.1^{\circ} = \frac{0.1 \times \pi}{180}$  radians

$$\therefore \qquad d = \frac{6 \times 10^{-7}}{(0.1\pi/180)} = \frac{6 \times 10^{-7} \times 180}{0.1 \times 3.14} = 3.44 \times 10^{-4} \,\mathrm{m}$$

- Q. 10. A parallel beam of light of 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Calculate the width of the slit. [CBSE (AI) 2013, (F) 2014]
  - Ans. From condition of diffraction,



Provided n = 1,2,3... and n = 0 for central maxima From condition of minima,

 $a \sin \theta = \lambda \ (n=1)$ 

Since the value of  $\lambda$  is very small of the order nm, so

$$a.\theta = \lambda \implies a.\frac{y}{D} = \lambda \qquad \left[ \text{angle} = \frac{\text{arc}}{\text{radius}} \right]$$
  
 $a = \frac{\lambda D}{y} = \frac{500 \times 10^{-9} \times 1}{2.5 \times 10^{-3}} \text{m} = 2 \times 10^{-4} \text{m}$ 



# **Multiple Choice Questions**

Choose and write the correct option(s) in the following questions.

1. Consider a light beam incident from air to a glass slab at Brewster's angle as shown in figure. A polaroid is placed in the path of the emergent ray at point P and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid.

[NCERT Exemplar]

- (*a*) For a particular orientation there shall be darkness as observed through the polaroid.
- (b) The intensity of light as seen through the polaroid shall be independent of the rotation.
- (c) The intensity of light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid.
- (d) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid.
- 2. Two waves having the intensities in the ratio of 9 : 1 produce interference. The ratio of maximum to minimum intensity is
- (a) 10:8(b) 9:1(c) 4:1(d) 2:13. Four independent waves are expressed as<br/>(i)  $y_1 = a_1 \sin \omega t$ (ii)  $y_2 = a_2 \sin 2\omega t$ <br/>(iii)  $y_3 = a_3 \cos \omega t$  and<br/>(iv)  $y_4 = a_4 \sin (\omega t + \pi/3)$ <br/>The interference is possible between
  - (a) (i) and (iii) (b) (i) and (iv) (c) (iii) and (iv)
- 4. Consider sunlight incident on a slit of width 10<sup>4</sup> A. The image seen through the slit shall [NCERT Exemplar]
  - (*a*) be a fine sharp slit white in colour at the center.
  - (b) a bright slit white at the center diffusing to zero intensities at the edges.
  - (c) a bright slit white at the center diffusing to regions of different colours.
  - (*d*) only be a diffused slit white in colour.
- 5. In a Young's double-slit experiment the fringe width is found to be 0.4 mm. If the whole apparatus is dipped in water of refractive index 4/3, without disturbing the arrangement, the new fringe width will be
  - $(a) \quad 0.30 \text{ mm} \qquad (b) \quad 0.40 \text{ mm} \qquad (c) \quad 0.53 \text{ mm} \qquad (d) \quad 0.2 \text{ mm}$

6 In Young's experiment, monochromatic light is used to illuminate the slits *A* and *B*. Interference fringes are observed on a screen placed in front of the slits. Now if a thin glass plate is placed in the path of the beam coming from A, then

- (a) the fringes will disappear
- (b) the fringe width will increase
- (c) the fringe width will decrease
- (d) there will be no change in the fringe width
- 7. In Young's double slit experiment the separation d between the slits is 2 mm, the wavelength  $\lambda$  of the light used is 5896 Å and distance D between the screen and slits is 100 cm. It is found that the angular width of the fringes is 0.20°. To increase the fringe angular width to 0.21° (with same  $\lambda$  and D) the separation between the slits needs to be changed to
  - $(a) 1.8 \text{ mm} \qquad (b) 1.9 \text{ mm} \qquad (c) 2.1 \text{ mm} \qquad (d) 1.7 \text{ mm}$



(d) not possible at all







- 8. In a Young's double slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case [NCERT Exemplar]
  - (*a*) there shall be alternate interference patterns of red and blue.
  - (*b*) there shall be an interference pattern for red distinct from that for blue.
  - (c) there shall be no interference fringes.
  - (d) there shall be an interference pattern for red mixing with one for blue.

9. In a Young's double-slit experiment, the source S and two slits A and B are horizontal, with slit A above slit B. The fringes are observed on a vertical screen K. The optical path length from S to B is increased very slightly (by introducing a transparent material of higher refractive index) and optical path length from S to A is not changed. As a result, the fringe system on K moves

- (a) vertically downwards slightly
- (b) vertically upwards slightly
- (c) horizontally slightly to the left (d) horizontally slightly to the right
- 10. In Young's double-slit experiment, the distance between the slit sources and the screen is 1 m. If the distance between the slits is 2 mm and the wavelength of light used is 600 nm, the fringe width is
  - (a) 3 mm (b) 0.3 mm (*c*) 6 mm (d) 0.6 mm
- 11. Figure shows a standard two slit arrangement with slits  $S_1, S_2, P_1, P_2$  are the two minima points on either side of P. [NCERT Exemplar]



#### At $P_2$ on the screen, there is a hole and behind $P_2$ is a second 2- slit arrangement with slits $S_3$ , $S_4$ and a second screen behind them.

- (a) There would be no interference pattern on the second screen but it would be lighted.
- (*b*) The second screen would be totally dark.
- (c) There would be a single bright point on the second screen.
- (d) There would be a regular two slit pattern on the second screen.
- 12. The Young's double-slit experiment is performed with blue and green lights of wavelengths 4360 Å and 5460 Å respectively. If x is the distance of 4th maxima from the central one, then

(a) 
$$(x)_{blue} = (x)_{green}$$
 (b)  $(x)_{blue} > (x)_{green}$ 

(c) 
$$(x)_{blue} < (x)_{green} = \frac{5460}{4360}$$

- 13. The angular resolution of a 10 cm diameter telescope at a-wavelength 500 nm is of the order of
  - (b) 10<sup>-6</sup> rad (a)  $10^{-4}$  rad
  - $(d) 10^7 \, rad$ (c)  $10^{-3}$  rad
- 14. A telescope has an objective lens of 10 cm diameter and is situated at a distance of 1 km from two objects. The minimum distance between these objects that can be resolved by the telescope, when the mean wavelength of light is 5000 Å is of the order of
  - (a) 5 mm (b) 5 cm
  - (d) 5 m (c) 2.5 m

15. Which of the following phenomenon cannot take place with longitudinal waves (e.g., sound waves)? (b) interference

- (a) reflection
- (c) diffraction (d) polarisation



- 16. Unpolarised light is incident from air on a plane surface of a material of refractive index *n*. At a particular angle of incidence *i*, it is found that the reflected and refracted rays are perpendicular to each other. Which of the following options is correct for this situation?
  - (a) Reflected light is polarised with its electric vector parallel to the plane of incidence
  - (b) Reflected light is polarised with its electric vector perpendicular to the plane of incidence

c) 
$$i = \sin^{-1}\left(\frac{1}{n}\right)$$
 (d)  $i = \tan^{-1}$ 

- 17. An astronomical refracting telescope will have large angular magnification and high angular resolution, when it has an objective lens of
  - (a) small focal length and large diameter
- (b) large focal length and small diameter

 $\left(\frac{1}{n}\right)$ 

- (c) large focal length and large diameter (d) small focal length and small diameter
- 18. A ray of light is incident on the surface of a glass plate at an angle of incidence equal to Brewsters angle  $\phi$ . If *n* represents the refractive index of glass with respect to air, then the angle between the reflected and refracted rays is

(a) 
$$90 + \phi$$
 (b)  $\sin^{-1}(n\cos\phi)$  (c)  $90^{\circ}$  (d)  $90^{\circ} - \sin^{-1}\left(\frac{\sin\phi}{n}\right)$ 

19. Consider the diffraction pattern for a small pinhole. As the size of the hole is increased

[NCERT Exemplar]

- (a) the size decreases(b) the intensity increases(c) the size increases(d) the intensity decreases
- 20. For light diverging from a point source
  - (*a*) the wavefront is spherical.
  - (b) the intensity decreases in proportion to the distance squared.
  - (c) the wavefront is parabolic.
  - (d) the intensity at the wavefront does not depend on the distance.

#### Answers

<b>1.</b> (c)	<b>2.</b> ( <i>c</i> )	<b>3.</b> ( <i>d</i> )	<b>4.</b> ( <i>a</i> )	<b>5.</b> ( <i>a</i> )	<b>6.</b> ( <i>d</i> )
<b>7.</b> ( <i>b</i> )	<b>8.</b> ( <i>c</i> )	<b>9.</b> ( <i>a</i> )	<b>10.</b> ( <i>b</i> )	<b>11.</b> ( <i>d</i> )	<b>12.</b> (c)
<b>13.</b> (b)	<b>14.</b> ( <i>a</i> )	<b>15.</b> ( <i>d</i> )	<b>16.</b> ( <i>a</i> )	<b>17.</b> ( <i>c</i> )	<b>18.</b> ( <i>c</i> )
<b>19.</b> ( <i>a</i> ), ( <i>b</i> )	<b>20.</b> ( <i>a</i> ), ( <i>b</i> )				

# Fill in the Blanks

- 1. A beam of light is incident normally upon a polariser and the intensity of emergent beam is  $I_0$ . The intensity of the emergent beam is found to be unchanged when the polariser is rotated about an axis perpendicular to the pass axis. Incident beam is \_\_\_\_\_\_ in nature.
- 2. The value of Brewster angle depends on the nature of the transparent refracting medium and the \_\_\_\_\_\_ of light used.
- 3. In Young's double slit experiment, the fringe width is given by \_\_\_\_\_
- 4. The phase difference between two waves in \_\_\_\_\_\_ interference is given as an even multiple of  $\pi$ .
- 5. Fringe width is different as separation between two consecutive \_\_\_\_\_ or
- 6. \_\_\_\_\_\_ of light occurs when size of the obstacle of aperture is comparable to the wavelength of light.
- 7. Continuous locus of oscillation with constant phase is called as \_\_\_\_\_\_



[1 mark]

[NCERT Exemplar]

- 8. In interference and \_\_\_\_\_\_, the light energy is redistributed, increases in one region and decreases in other.
- 9. At polarising angle the refracted and reflected rays are \_\_\_\_\_\_ to each other.
- **10.** The tangent of angle of polarization as light ray travels from air to glass is equal to the refractive index. This law is called as \_\_\_\_\_\_.

### Answers

- 1. unpolarised
- **4.** constructive
- 8. diffraction
- 5. maxima, minima

**2.** wavelength

9. perpendicular

#### **3.** $\beta = D\lambda/d$ **6.** Diffraction

**10.** Brewster's law

#### 7. wave-front

[1 mark]

# Very Short Answer Questions

- Q. 1. When monochromatic light travels from one medium to another, its wavelength changes but frequency remains the same. Explain.

   [CBSE Delhi 2011]
- **Ans.** Frequency is the fundamental characteristic of the source emitting waves and does not depend upon the medium. Light reflects and refracts due to the interaction of incident light with the atoms of the medium. These atoms always take up the frequency of the incident light which forces them to vibrate and emit light of same frequency. Hence, frequency remains same.
- Q. 2. Light of wavelength 5000 Å propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected?

[CBSE Delhi 2015]

[CBSE Delhi 2009]

[CBSE Delhi 2009]

Ans.  $5000 \text{ Å} = 5000 \times 10^{-10} = 5 \times 10^{-7} \text{ m}$ Reflected ray: No change in wavelength and frequency. Refracted ray: Frequency remains same, wavelength decreases

Wavelength =  $\lambda' = \frac{\lambda}{n}$ 

- Q. 3. Why are coherent soruces required to create interference of light? [CBSE (F) 2009]
- **Ans.** Coherent sources are required for sustained interference. If sources are incoherent, the intensity at a point will go on changing with time.
- Q. 4. Differentiate between a ray and a wavefront.
- **Ans.** A wavefront is a surface of constant phase. A ray is a perpendicular line drawn at any point on wavefront and represents the direction of propagation of the wave.
- Q. 5. What type of wavefront will emerge from a (*i*) point source and (*ii*) distant light source?
- Ans. (i) Spherical wavefront (ii) Plane wavefront.
- Q. 6. What will be the effect on interference fringes if red light is replaced by blue light?

- **Ans.**  $\beta = \frac{D\lambda}{d}$ , *i.e.*,  $\beta \propto \lambda$ ; the wavelength of blue light is less than that of red light; hence if red light is replaced by blue light, the fringe width decreases, *i.e.*, fringes come closer.
- Q. 7. Unpolarised light of intensity *I* is passed through a polaroid. What is the intensity of the light transmitted by the polaroid? [CBSE (F) 2009]
- **Ans.** Intensity of light transmitted through the polaroid  $=\frac{1}{2}$ .
- Q. 8. If the angle between the pass axes of a polariser and analyser is 45°. Write the ratio of the intensities of original light and the transmitted light after passing through the analyser.

[CBSE Delhi 2009]



<sup>[</sup>CBSE Delhi 2013]

**Ans.** If  $I_0$  is intensity of original light, then intensity of light passing through the polariser  $=\frac{I_0}{2}$ . Intensity of light passing through analyser

$$I = \frac{I_0}{2}\cos^2 45^\circ \implies \frac{I_0}{I} = \frac{2}{\cos^2 45^\circ} = \frac{4}{1}$$

- Q. 9. Which of the following waves can be polarized (i) Heat waves (ii) Sound waves? Give reason to support your answer. [CBSE Delhi 2013]
- **Ans.** Heat waves are transverse or electromagnetic in nature whereas sound wave are not. Polarisation is possible only for transverse waves.
- Q. 10. At what angle of incidence should a light beam strike a glass slab of refractive index  $\sqrt{3}$ , such that the reflected and refracted rays are perpendicular to each other? [CBSE Delhi 2009]
- **Ans.** The reflected and refracted rays are mutually perpendicular at polarising angle; so from Brewster's law

$$i_B = \tan^{-1}(n) = \tan^{-1}(\sqrt{3}) = 60^{\circ}.$$

- Q. 11. How does the fringe width of interference fringes change, when the whole apparatus of Young's experiment is kept in water (refractive index 4/3)? [CBSE Delhi 2011] [HOTS]
- **Ans.** Fringe width,  $\beta = \frac{D\lambda}{d} \Rightarrow \beta \propto \lambda$  for same *D* and *d*. When the whole apparatus is immersed in a transparent liquid of refractive index n = 4/3, the wavelength decreases to  $\lambda' = \frac{\lambda}{n} = \frac{\lambda}{4/3}$ . So, fringe width decreases to  $\frac{3}{4}$  times.
- Q. 12. In what way is the diffraction from each slit related to interference pattern in double slit experiment? [CBSE Bhubaneshwar 2015]
- **Ans.** The intensity of interference fringes in a double slit arrangement is modulated by the diffraction pattern of each slit. Alternatively, in double slit experiment the interference pattern on the screen is actually superposition of single slit diffraction for each slit.
- Q. 13. How does the angular separation between fringes in single-slit diffraction experiment change when the distance of separation between the slit and screen is doubled? [CBSE (AI) 2012]
- **Ans.** Angular separation is  $\theta = \frac{\beta}{D} = \frac{D\lambda/d}{D} = \frac{\lambda}{d}$ Since  $\theta$  is independent of D, angular separation

Since  $\theta$  is independent of *D*, angular separation would remain same.

- Q. 14. In a single-slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band? [CBSE (AI) 2012]
- Ans. In single slit diffraction experiment fringe width is

$$\beta = \frac{2\lambda D}{d}$$

If *d* is doubled, the width of central maxima is halved. Thus size of central maxima is reduced to half. Intensity of diffraction pattern varies with square of slit width. So, when the slit gets double, it makes the intensity four times.

[NCERT Exemplar]

#### Q. 15. What is the shape of the wavefront on earth for sunlight?

- Ans. Spherical with huge radius as compared to the earth's radius so that it is almost a plane.
- Q. 16. Why is the interference pattern not detected, when two coherent sources are far apart? [HOTS]

**Ans.** Fringe width of interference fringes, is given by  $\beta = \frac{D\lambda}{d} \propto \frac{1}{d}$ . If the sources are far apart; *d* is

large; so fringe width ( $\beta$ ) will be so small that the fringes are not resolved and they do not appear separate. That is why the interference pattern is not detected for large separation of coherent sources.

Q. 17. No interference pattern is detected when two coherent sources are infinitely close to each other. Why? [HOTS]



**Ans.** Fringe width of interference fringes is given by  $\beta = \frac{D\lambda}{d} \propto \frac{1}{d}$ . When *d* is infinitely small, fringe width  $\beta$  will be too large. In such a case even a single fringe may occupy the whole field of view. Hence, the interference pattern cannot be detected.

Q. 18. A polaroid (I) is placed in front of a monochromatic source. Another polaroid (II) is placed in front of this polaroid (I) and rotated till no light passes. A third polaroid (III) is now placed in between (I) and (II). In this case, will light emerge from (II)? Explain.

[NCERT Exemplar] [HOTS]

- **Ans.** Only in the special cases when the pass axis of (III) is parallel to (I) or (II) there shall be no light emerging. In all other cases there shall be light emerging because the pass axis of (II) is no longer perpendicular to the pass axis of (III).
- Q. 19. Give reason for the following: The value of the Brewster angle for a transparent medium is different for lights of different colours. [HOTS]
- **Ans.** Brewster's angle,  $i_B = \tan^{-1}(n)$ As refractive index *n* varies as inverse value of wavelength: it

As refractive index n varies as inverse value of wavelength; it is different for lights of different wavelengths (colours), therefore, Brewster's angle is different for lights of different colours.

Q. 20. Two polaroids are placed with their optic axis perpendicular to each other. One of them is rotated through 45°, what is the intensity of light emerging from the second polaroid if  $I_0$  is the intensity of unpolarised light? [CBSE Sample Paper 2017]

**Ans.** 
$$I = \frac{I_0}{2} \cos^2(45)^\circ = \frac{I_0}{4}$$

# Short Answer Questions–I

- Q. 1. When are two objects just resolved? Explain. How can the resolving power of a compound microscope be increased? Use relevant formula to support your answer. [CBSE Delhi 2017]
- Ans. Two objects are said to be just resolved when, in their diffraction patterns, central maxima of one object coincides with the first minima of the diffraction pattern of the second object. Limit of resolution of compound microscope

$$d_{\min} = \frac{1.22\lambda}{2n\sin\beta}$$

Resolving power of a compound microscope is given by the reciprocal of limit of resolution  $(d_{\min})$ .

Therefore, to increase resolving power,  $\lambda$  can be reduced and refractive index of the medium can be increased.

Q. 2. Find the intensity at a point on a screen in Young's double slit experiment where the interfering

waves of equal intensity have a path difference of (i)  $\frac{\lambda}{4}$ , and (ii)  $\frac{\lambda}{3}$ . [CBSE (F) 2017]

**Ans.** 
$$I = I_0 \cos^2 \frac{\phi}{2}$$

(*i*) If path difference =  $\frac{\Lambda}{4}$ 

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[2 marks]

Also, 
$$I = 4I_0 \cos^2 \frac{\Phi}{2} = 4I_0 \cos^2 \frac{\pi}{4} = 2I_0$$

(ii) If 
$$\Delta = \frac{\lambda}{3}$$
  
 $\Rightarrow \quad \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$   
 $\therefore \qquad I = 4I_0 \cos^2 \frac{\phi}{2}$   
 $= 4I_0 \cos^2 \left(\frac{2\pi}{3 \times 2}\right) = I_0$ 

Q. 3. Unpolarised light is passed through a polaroid  $P_1$ . When this polarised beam passes through another polaroid  $P_2$  and if the pass axis of  $P_2$  makes angle  $\theta$  with the pass axis of  $P_1$ , then write the expression for the polarised beam passing through  $P_2$ . Draw a plot showing the variation of intensity when  $\theta$  varies from 0 to  $2\pi$ . [CBSE (AI) 2017]



- Q. 4. A parallel beam of light of wavelength 600 nm is incident normally on a slit of width 'a'. If the distance between the slit and the screen is 0.8 m and the distance of 2nd order maximum from the centre of the screen is 1.5 mm, calculate the width of the slit.
- **Ans.** Given  $\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m} = 6.0 \times 10^{-7} \text{ m}, D = 0.8 \text{ m},$

$$v_2 = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}, n = 2, a = 3$$

Position of  $n^{\text{th}}$  maximum in diffraction of a single slit

$$y_n = \left(n + \frac{1}{2}\right) \frac{\lambda D}{a} \implies a = \left(n + \frac{1}{2}\right) \frac{\lambda D}{y_n}$$

Substituting given values  $a = \left(2 + \frac{1}{2}\right) \frac{6.0 \times 10^{-7} \times 0.8}{1.5 \times 10^{-3}}$ 

$$=\frac{5}{2} \times 4.0 \times 0.8 \times 10^{-4} \text{ m} = 0.8 \times 10^{-3} \text{ m} = 0.8 \text{ mm}$$



Ans.

# Short Answer Questions-II

- Q. 1. Draw the diagrams to show the behaviour of plane wavefronts as they (a) pass through a thin prism, and (b) pass through a thin convex lens and (c) reflect by a concave mirror. [CBSE Bhubaneshwar 2015]
- **Ans.** The behaviour of a thin prism a thin convex lens and a concave mirror are shown in figs. (*a*), (*b*) and (*c*) respectively.



A plane wavefront becomes spherical convergent after reflection Spherical wavefront at radius *R*/2



#### Q. 2. What is the shape of the wavefront in each of the following cases:

#### [CBSE Delhi 2009]

- (a) light diverging from a point source.
- (b) light emerging out of a convex lens when a point source is placed at its focus.
- (c) the portion of a wavefront of light from a distant star intercepted by the earth.
- **Ans.** (*a*) The wavefront will be **spherical** of increasing radius, fig. (*a*).
  - (b) The rays coming out of the convex lens, when point source is at focus, are parallel, so wavefront is **plane**, fig. (b).



(c) The wavefront starting from star is spherical. As star is very far from the earth, so the wavefront intercepted by earth is a very small portion of a sphere of large radius; which is **plane** (*i.e.*, wavefront intercepted by earth is plane), fig. (c).

# [3 marks]



- Q. 3. Explain the following, giving reasons:
  - (i) When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency as the incident frequency.
  - (ii) When light travels from a rarer to a denser medium, the speed decreases. Does this decrease in speed imply a reduction in the energy carried by the wave?
  - (iii) In the wave picture of light, intensity of light is determined by the square of the amplitude of the wave. What determines the intensity in the photon picture of light? [CBSE Central 2016]
- (i) Reflection and refraction arise through interaction of incident light with atomic constituents Ans. of matter which vibrate with the same frequency as that of the incident light. Hence frequency remains unchanged.
  - (ii) No; when light travels from a rarer to a denser media, its frequency remains unchanged. According to quantum theory of light, the energy of light photon depends on frequency and not on speed.
  - (iii) For a given frequency, intensity of light in the photon picture is determined by the number of photon incident normally on a crossing an unit area per unit time.
- Q. 4. (a) Write the necessary conditions to obtain sustained interference fringes.
  - Also write the expression for the fringe width.
  - (b) In Young's double slit experiment, plot a graph showing the variation of fringe width versus the distance of the screen from the plane of the slits keeping other parameters same. What information can one obtain from the slope of the curve?
  - (c) What is the effect on the fringe width if the distance between the slits is reduced keeping other parameters same? [CBSE Patna 2015]

#### Ans. (a) Conditions for sustained interference:

- (i) The interfering sources must be coherent *i.e.*, sources must have same frequency and constant initial phase.
- (ii) Interfering waves must have same or nearly same amplitude, so that there may be contrast between maxima and minima.

Fringe width,  $\beta = \frac{D\lambda}{d}$ 

where D = distance between slits and screen.

d = separation between slits.

 $\lambda$  = wavelength of light used.

(b) Information from the slope:

Wavelength,  $\lambda = \text{Slope} \times d = d$ . tan  $\theta$ 

 $\beta = \frac{\lambda D}{d}$ (*c*) **Effect:** From relation,  $\beta \propto \frac{1}{d}$ 

Fringe width,

If distance *d* between the slits is reduced, the size of fringe width will increase.

- Q. 5. For a single slit of width "a", the first minimum of the interference pattern of a monochromatic light of wavelength  $\lambda$  occurs at an angle of  $\frac{\lambda}{a}$ . At the same angle of  $\frac{\lambda}{a}$ , we get a maximum for two narrow slits separated by a distance "a". Explain. [CBSE Delhi 2014]
- **Case I:** The overlapping of the contributions of the wavelets from two halves of a single slit Ans. produces a minimum because corresponding wavelets from two halves have a path difference of  $\lambda/2$ .
  - **Case II:** The overlapping of the wavefronts from the two slits produces first maximum because these wavefronts have the path difference of  $\lambda$ .
- Q. 6. In the experiment on diffraction due to a single slit, show that
  - (i) the intensity of diffraction fringes decreases as the order (n) increases.
  - (ii) angular width of the central maximum is twice that of the first order secondary maximum.

[CBSE (F) 2011]





- Ans. (i) The reason is that the intensity of central maximum is due to constructive interference of wavelets from all parts of slit, the first secondary maximum is due to contribution of wavelets from one third part of slit (wavelets from remaining two parts interfere destructively) the second secondary maximum is due to contribution of wavelets from one fifth part only and so on.
  - (*ii*) For first minima  $a \sin \theta = \lambda$  or  $a\theta = \lambda$   $\tan \theta = \frac{y_1}{D}$  $\Rightarrow \quad \theta = \frac{y_1}{D}$  (for  $\theta$  is small,  $\sin \theta \approx \theta$  and  $\tan \theta \approx \theta$ )

$$\Rightarrow \qquad \theta = \frac{1}{D} \quad \text{(for } \theta \text{ is small, sin } \theta \approx \theta \text{ and } \tan \theta$$
$$\Rightarrow \qquad \frac{ay_1}{D} = \lambda \qquad y_1 = \frac{\lambda D}{a} = y_2$$



Hence the angular width of central maximum =  $2\theta = \frac{2\lambda}{a}$ 

Width of secondary maximum = Separation between *n*th and (n + 1)th minima

For minima  $\theta_n = \frac{n\lambda}{a}$   $\theta_{n+1} = (n+1)\frac{\lambda}{a}$ Angular width of secondary maximum  $= (n+1)\frac{\lambda}{a} - \frac{n\lambda}{a} = \frac{\lambda}{a}$ Hence  $\beta$  = Angular width  $\times D = \frac{\lambda D}{a}$ 

Thus central maximum has twice the angular width of secondary maximum.

- Q. 7. (a) Describe briefly, with the help of suitable diagram, how the transverse nature of light can be demonstrated by the phenomenon of polarisation of light. [CBSE (AI) 2014]
  - (b) When unpolarised light passes from air to a transparent medium, under what condition does the reflected light get polarised? [CBSE Delhi 2011]
- Ans. (a) Light from a source S is allowed to fall normally on the flat surface of a thin plate of a tourmaline crystal, cut parallel to its axis. Only a part of this light is transmitted through A. If now the plate A is rotated, the character of transmitted light remains unchanged. Now another similar plate B is placed at some distance from A such that the axis of B is parallel to that of A. If the light transmitted through A is passed through B, the light is almost completely transmitted through B and no change is observed in the light coming out of B.

If now the crystal A is kept fixed and B is gradually rotated in its own plane, the intensity of light emerging out of B decreases and becomes zero when the axis of B is perpendicular to that of A. If B is further rotated, the intensity begins to increase and becomes maximum when the axes of A and B are again parallel.

Thus, we see that the intensity of light transmitted through B is maximum when axes of A and B are parallel and minimum when they are at right angles.

From this experiment, it is obvious that light waves are transverse and not longitudinal; because, if they were longitudinal, the rotation of crystal *B* would not produce any change in the intensity of light.



- (*b*) The reflected ray is totally plane polarised, when reflected and refracted rays are perpendicular to each other.
- Q. 8. (a) The light from a clear blue portion of the sky shows a rise and fall of intensity when viewed through a polaroid which is rotated. Describe, with the help of a suitable diagram, the basic phenomenon/process which occurs to explain this observation.

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(b) Show how light reflected from a transparent medium gets polarised. Hence deduce Brewster's law.

OR

An unpolarised light is incident on the boundary between two transparent media. State the condition when the reflected wave is totally plane polarised. Find out the expression for the angle of incidence in this case. [CBSE Delhi 2014, 2018, Bhubaneshwar 2015]

Ans. (a) Sun emits unpolarised light, and represented as dots and double arrow. The dots stand for polarisation perpendicular to the plane and double arrow in the polarisation of plane.

> When the unpolarised light strikes on the atmospheric molecules, the electrons in the molecules acquire components of motion in both directions. The charge accelerating parallel to double arrow do not radiate energy



towards the observer, so the component of electric field represented by dots radiate towards the observer.

If the scattered radiations represented by dots is viewed through an artificial polaroid. It shows the variation in its intensity with the rotation of the polaroid.

(*b*) **Condition:** The reflected ray is totally plane polarised, when reflected and refracted rays are perpendicular to each other.

 $\angle BOC = 90^{\circ}$ 

When reflected wave is perpendicular to the refracted wave, the reflected wave is a totally polarised wave. The angle of incidence in this case is called Brewster's angle and is denoted by  $i_B$ .

If r' is angle of reflection and r the angle of refraction, then according to law of reflection  $i_B = r'$ 

and from fig. 
$$r' + 90^{\circ} + r = 180^{\circ}$$

$$\Rightarrow i_B + r = 90^\circ ...(i$$

$$\Rightarrow \qquad r = (90^{\circ} - i_B) \dots (ii$$

From Snell's law, refractive index of second medium relative to first medium (air) say.

$$n = \frac{\sin i_B}{\sin r} = \frac{\sin i_B}{\sin (90^\circ - i_B)} = \frac{\sin i_B}{\cos i_B}$$
$$n = \tan i_B$$

This is known as Brewster's law.

:. Angle of incidence,  $i_B = \tan^{-1}(n)$ .

#### Q. 9. State Brewster's law.

 $\Rightarrow$ 

# The value of Brewster angle for a transparent medium is different for light of different colours.Give reason.[CBSE Delhi 2016]

**Ans. Brewster's Law:** When unpolarised light is incident on the surface separating two media at polarising angle, the reflected light gets completely polarised only when the reflected light and the refracted light are perpendicular to each other.

Now, refractive index of denser (second) medium with respect to rarer (first) medium is given by

$$n = \tan i_B$$
, where  $i_B = \text{polarising angle}$ .

Since refractive index is different for different colour (wavelengths), Brewster's angle is different for different colours.





- Q. 10. Explain why the intensity of light coming out of a polaroid does not change irrespective of the orientation of the pass axis of the polaroid. [CBSE East 2016]
- **Ans.** When unpolarised light passes through a polariser, vibrations perpendicular to the axis of the polaroid are blocked.

Unpolarised light have vibrations in all directions.

Hence, if the polariser is rotated, the unblocked vibrations remain same with reference to the axis of polariser.

Hence for all positions of polaroid, half of the incident light always get transmitted. Hence, the intensity of the light does not change.

- Q. 11. (i) Light passes through two polaroids  $P_1$  and  $P_2$  with axis of  $P_2$  making an angle  $\theta$  with the pass axis of  $P_1$ . For what value of  $\theta$  is the intensity of emergent light zero?
  - (*ii*) A third polaroid is placed between  $P_1$  and  $P_2$  with its pass axis making an angle  $\beta$  with the pass axis of  $P_1$ . Find a value of  $\beta$  for which the intensity of light emerging from  $P_2$  is  $\frac{I_0}{8}$ , where  $I_0$  is the intensity of light on the polaroid  $P_1$ . [CBSE (F) 2011]
- **Ans.** (*i*) At  $\theta = 90^{\circ}$ , the intensity of emergent light is zero.
  - (*ii*) Intensity of light coming out from polariser  $P_1 = \frac{I_0}{2}$

Intensity of light coming out from  $P_3 = \left(\frac{I_0}{2}\right) \cos^2 \beta$ Intensity of light coming out from  $P_2 = \left(\frac{I_0}{2}\right) \cos^2 \beta \cos^2 (90 - \beta)$  $= \frac{I_0}{2} \cdot \cos^2 \beta \cdot \sin^2 \beta = \frac{I_0}{2} \left[\frac{(2\cos\beta \cdot \sin\beta)^2}{(2)^2}\right]$ 

$$I = \frac{I_0}{8} (\sin 2\beta)^2$$

But it is given that intensity transmitted from  $P_2$  is  $I = \frac{I_0}{8}$ 

 $(\sin 2 \beta)^2 = 1$ 

So, 
$$\frac{I_0}{8} = \frac{I_0}{8} (\sin 2\beta)^2$$

or,

$$\sin 2\beta = \sin \frac{\pi}{2} \implies \beta = \frac{\pi}{4}$$

Q. 12. Draw the intensity distribution for (i) the fringes produced in interference, and (ii) the diffraction bands produced due to single slit. Write two points of difference between the phenomena of interference and diffraction. [CBSE (F) 2017]





**Intensity Patterns** 



#### Differences between interference and diffraction

Interference	Diffraction
( <i>a</i> ) It is due to the superposition of two waves coming from two coherent sources.	( <i>a</i> ) It is due to the superposition of secondary wavelets originating from different parts of the same wavefront.
( <i>b</i> ) The width of the interference bands is equal.	(b) The width of the diffraction bands is not the same.
(c) The intensity of all maxima (fringes) is same.	(c) The intensity of central maximum is maximum and goes on decreasing rapidly with increase in order of maxima.

Q. 13. Use Huygen's principle to explain the formation of diffraction pattern due to a single slit illuminated by a monochromatic source of light.

When the width of slit is made double the original width, how this affect the size and intensity<br/>of the central diffraction band?[CBSE Delhi 2012]

**Ans.** According to Huygen's principle, "The net effect at any point due to a number of wavelets is equal to sum total of contribution of all wavelets with proper phase difference. The point *O* is maxima because contribution from each half of the slit  $S_1S_2$  is in phase, *i.e.*, the path difference is zero.

At point P

- (i) If  $S_2P S_1P = n\lambda \Rightarrow$  the point *P* would be minima.
- (*ii*) If  $S_2P S_1P = (2n+1)\frac{\lambda}{2} \implies$  the point would be

maxima but with decreasing intensity.

The width of central maxima =  $\frac{2\lambda D}{c}$ 



When the width of the slit is made double the original width, then the size of central maxima will be reduced to half and intensity will be four times.

- Q. 14. (a) In Young's double slit experiment, two slits are 1 mm apart and the screen is placed 1 m away from the slits. Calculate the fringe width when light of wavelength 500 nm is used.
  - (b) What should be the width of each slit in order to obtain 10 maxima of the double slits pattern within the central maximum of the single slit pattern? [CBSE East 2016]

Ans. (a) Fringe width is given by 
$$\beta = \frac{\lambda D}{d}$$
  
=  $\frac{500 \times 10^{-9} \times 1}{10^{-3}} = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m} = 5 \times 10^{-4} \text{ m}$ 

(b) 
$$\beta_0 = \frac{2\lambda D}{a} = 10\beta$$
  

$$\Rightarrow \qquad a = \frac{2 \times 500 \times 10^{-9} \times 1}{10 \times 5 \times 10^{-4}} = 2 \times 10^{-4} \text{ m}$$

Q. 15. In a double slit experiment, the distance between the slits is 3 mm and the slits are 2 m away from the screen. Two interference patterns can be seen on the screen one due to light with wavelength 480 nm, and the other due to light with wavelength 600 nm. What is the separation on the screen between the fifth order bright fringes of the two interference patterns?

**Ans.** 
$$\beta = \lambda \frac{D}{d}$$

**Case I:** 5<sup>th</sup> bright fringe =  $5\beta_1 = 5\lambda_1 D/d = 5 \times 480 \times 10^{-9} \times 2/3 \times 10^{-3} = 16 \times 10^{-4} \text{ m}$ 



**Case II:** 5<sup>th</sup> bright fringe =  $5\beta_2 = 5\lambda_2 D/d = 5 \times 600 \times 10^{-9} \times 2/3 \times 10^{-3} = 20 \times 10^{-4} \text{ m}$ Distance between two 5th bright fringes =  $(20 - 16) \times 10^{-4} = 4 \times 10^{-4} \text{ m}$ 

- Q. 16. In the diffraction due to a single slit experiment, the aperture of the slit is 3 mm. If monochromatic light of wavelength 620 nm is incident normally on the slit, calculate the separation between the first order minima and the 3<sup>rd</sup> order maxima on one side of the screen. The distance between the slit and the screen is 1.5 m. [CBSE 2019 (55/1/1)]
- Ans. Condition for minima

$$a \sin \theta = n\lambda$$
 ...(i)

and condition for secondary maxima

$$a\sin\theta = \left(n + \frac{1}{2}\right)\lambda$$

The first order minima [n = 1]

$$a \sin \theta = \lambda , \quad \tan \theta = \frac{y_1}{D}$$
  

$$\sin \theta = \frac{\lambda}{a} \implies \theta = \frac{\lambda}{a} , \quad \theta = \frac{y_1}{D} \quad [\because \text{ for } \theta \text{ is small, } \sin \theta \simeq \theta \text{ and } \tan \theta \simeq \theta]$$
  

$$. \quad \frac{y_1}{D} = \frac{\lambda}{a} \implies y_1 = \frac{\lambda D}{a}$$

Also 3rd order maxima

$$a\sin\theta = \left(3 + \frac{1}{2}\right)\lambda \implies a\sin\theta = \frac{7}{2}\lambda$$
  
 $\frac{y_3}{D} = \frac{7}{2}\frac{\lambda}{a} \implies y_3 = \frac{7}{2}\frac{\lambda D}{a}$ 

Distance between first order minima from centre of the central maxima

$$y_1 = \frac{\lambda D}{a}$$

Distance of third order maxima from centre of the central maxima

$$y_3 = \frac{7\lambda D}{2a}$$

Distance between first order minima and third order maxima =  $y_3 - y_1 = \frac{7}{2} \frac{\lambda D}{a} - \frac{\lambda D}{a}$ 

 $\frac{5}{2}$ 

$$= \frac{\lambda D}{a} \left[ \frac{7}{2} - 1 \right] = \frac{\lambda D}{a} \times y_3 - y_1 = \frac{620 \times 10^{-9} \times 1.5}{3 \times 10^{-3}} \times \frac{5}{2}$$
$$= 775 \times 10^{-6} \text{ m} = 7.75 \times 10^{-4} \text{ m}$$

Q. 17. A beam of light consisting of two wavelengths, 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1.4 m away. If the two slits are separated by 0.28 mm, calculate the least distance from the central bright maximum where the bright fringes of the two wavelengths coincide. [CBSE (AI) 2012]

Ans. Given  $\lambda_1 = 800 \text{ nm} = 800 \times 10^{-9} \text{ m}$   $\lambda_2 = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$  D = 1.4 m $d = 0.28 \text{ mm} = 0.28 \times 10^{-3} \text{m}$ 

For least distance of coincidence of fringes, there must be a difference of 1 in order of  $\lambda_1$  and  $\lambda_2$ .

As 
$$\lambda_1 > \lambda_2, n_1 < n_2$$
  
If  $n_1 = n, \quad n_2 = n + 1$   
 $\therefore \quad (y_n)_{\lambda_1} = (y_n + 1)_{\lambda_2} \implies \frac{n D \lambda_1}{d} = \frac{(n+1)D\lambda_2}{d}$ 

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$$\Rightarrow$$

$$n\lambda_1 = (n+1)\lambda_2$$

$$\Rightarrow$$

$$n\lambda_1 = (n+1)\lambda_2$$

$$\Rightarrow$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{600}{800 - 600} = 3$$
$$y_{\min} = \frac{nD\lambda_1}{d} = \frac{3 \times 1.4 \times 800 \times 10^{-9}}{0.28 \times 10^{-3}} = 12000 \times 10^{-6} = 12 \times 10^{-3} \text{ m}$$

- Q. 18. (a) Assume that the light of wavelength 6000 Å is coming from a star. Find the limit of resolution of a telescope whose objective has a diameter of 250 cm.
  - (b) Two slits are made 1 mm apart and the screen is placed 1 m away. What should be the width of each slit to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern? [CBSE Guwahati 2015]
- (a) The limit of resolution of the objective lens in the telescope is Ans.

$$\Delta \theta = \frac{1.22\lambda}{D}$$

Since D = 250 cm = 2.5 m and  $\lambda = 6000 \text{ Å} = 6 \times 10^{-7} \text{ m}$ 

: 
$$\Delta \theta = \frac{1.22 \times 6 \times 10^{-7}}{2.5} = 2.9 \times 10^{-7}$$
 radian

(b) If a is the size of single slit for diffraction pattern then, for first maxima

$$\theta = \frac{\lambda}{a}$$
  $(n = 1)$ 

and angular separation of central maxima in the diffraction pattern

$$\theta' = 2\theta = \frac{2\lambda}{a}$$

The angular size of the fringe in the interference pattern

$$\alpha = \frac{\beta}{D} = \frac{\lambda}{d}$$

If there are 10 maxima within the central maxima of the diffraction pattern, then  $10 \alpha = \theta'$ 

$$10\left(\frac{\lambda}{d}\right) = \frac{2\lambda}{a} \implies a = \frac{d}{5}$$

The distance between two slits is 1 mm.

Size of the single slit 
$$a = \frac{1}{5}$$
mm = **0.2 mm**

- Q. 19. (a) Why are coherent sources necessary to produce a sustained interference pattern?
  - (b) In Young's double slit experiment using monochromatic light of wavelength  $\lambda$ , the intensity of light at a point on the screen where path difference is  $\lambda$ , is K units. Find out

the intensity of light at a point where path difference is  $\frac{\Lambda}{2}$ . [CBSE Delhi 2012]

- (a) This is because coherent sources are needed to ensure that the positions of maxima and Ans. minima do not change with time. If the phase difference between wave, reaching at a point change with time intensity will change and sustained interference will not be obtained.
  - (b) We know

*.*..

$$I = 4 I_0 \cos^2 \frac{\phi}{2}$$

for path difference  $\lambda$ , phase difference  $\phi = 2\pi$ 



Intensity of light = K Hence,  $K = 4I_0 \cos^2 \pi = 4I_0$ For path difference  $\frac{\lambda}{3}$ , Phase difference  $\phi = \frac{2\pi}{3}$ Intensity of light  $I' = 4I_0 \cos^2 \frac{\phi}{2} = 4I_0 \cos^2 \frac{\pi}{3} = I_0$  $\Rightarrow I' = \frac{K}{4}$ 

Q. 20. Distinguish between polarised and unpolarised light. Does the intensity of polarised light emitted by a polaroid depend on its orientation? Explain briefly.

The vibrations in a beam of polarised light make an angle of  $60^{\circ}$  with the axis of the polaroid sheet. What percentage of light is transmitted through the sheet? [CBSE (F) 2016]

**Ans.** A light which has vibrations in all directions in a plane perpendicular to the direction of propagation is said to be unpolarised light. The light from the sun, an incandescent bulb or a candle is unpolarised.

If the electric field vector of a light wave vibrates just in one direction perpendicular to the direction of wave propagation, then it is said to be polarised or linearly polarised light.

Yes, the intensity of polarised light emitted by a polaroid depends on orientation of polaroid. When polarised light is incident on a polaroid, the resultant intensity of transmitted light varies directly as the square of the cosine of the angle between polarisation direction of light and the axis of the polaroid.

$$I \propto \cos^2 \theta$$
 or  $I = I_0 \cos^2 \theta$ 

where  $I_0$  = maximum intensity of transmitted light;

 $\theta$  = angle between vibrations in light and axis of polaroid sheet.

or, 
$$I = I_0 \cos^2 60^\circ = \frac{I_0}{4}$$

Percentage of light transmitted  $= \frac{I}{I_0} \times 100 = \frac{1}{4} \times 100 = 25\%$ 

- Q. 21. Find an expression for intensity of transmitted light when a polaroid sheet is rotated between two crossed polaroids. In which position of the polaroid sheet will the transmitted intensity be maximum? [CBSE Delhi 2015]
- Ans. Let  $P_1$  and  $P_2$  be the crossed polaroids, and no light transmitted through polaroid  $P_2$ .

Let  $I_0$  be the intensity of the polarised light through polaroid  $P_1$ .

If another polaroid  $P_3$  is inserted between  $P_1$  and  $P_2$ , and polaroid  $P_3$  is at an angle  $\theta$  with the polaroid  $P_1$ .

Then intensity of light through polaroid  $P_3$  is

$$I_3 = I_0 \cos^2 \theta \qquad \dots (i)$$

If this light  $I_3$  again passes through the polaroid  $P_2$  then

$$I_2 = I_3 \cos^2(90 - \theta)$$
 ...(*ii*)

From equation (1) and (2), we get

$$I_2 = I_0 \cos^2 \theta \cdot \cos^2 (90 - \theta)$$
$$= \frac{I_0}{4} (2 \sin \theta \cos \theta)^2 = \frac{I_0}{4} \sin^2 (2\theta)$$

For maximum value of  $I_2$ ,

si

 $\Rightarrow$ 

$$n \ 2\theta = \pm 1 \qquad \Rightarrow \qquad 2\theta = 90^{\circ}$$
$$\theta = 45^{\circ}$$

It is possible only when polaroid  $P_3$  is placed at angle 45° from each polaroid  $P_1$  (or  $P_2$ ).



- Q. 22. Three identical polaroid sheets  $P_1$ ,  $P_2$  and  $P_3$  are oriented so that the pass axis of  $P_2$  and  $P_3$  are inclined at angles of 60° and 90° respectively with the pass axis of  $P_1$ . A monochromatic source S of unpolarised light of intensity  $I_0$  is kept in front of the polaroid sheet  $P_1$  as shown in the figure. Determine the intensities of light as observed by the observer at O, when polaroid  $P_3$  is rotated with respect to  $P_2$  at angles  $\theta = 30^\circ$  and  $60^\circ$ . [CBSE North 2016]
- Ans. We have, as per Malus's law:  $I = I_0 \cos^2 \theta$   $\therefore$  If the intensity of light, incident on  $P_1$  is  $I_0$ , we have  $I_1$ = Intensity transmitted through  $P_1 = \frac{I_0}{2}$  s  $I_2$ = Intensity transmitted through  $P_2 = \left(\frac{I_0}{2}\right)\cos^2 60^\circ = \frac{I_0}{8}$  $P_1 \qquad P_2 \qquad P_3$

For  $\theta = 30^\circ$ , we have Angle between pass axis of  $P_2$  and  $P_3$ 

$$= (30^{\circ} + 30^{\circ}) = 60^{\circ} \implies I_3 = \frac{I_0}{8} \cos^2 60^{\circ} = \frac{I_0}{32}$$
$$(30^{\circ} - 30^{\circ}) = 0^{\circ} \implies I_3 = \frac{I_0}{8} \cos^2 0^{\circ} = \frac{I_0}{8}$$

or

or

$$\therefore \qquad I_3 \text{ can be either } \frac{I_0}{32} \text{ or } \frac{I_0}{8}.$$

For  $\theta = 60^\circ$ , we have

Angle between pass axis of  $P_2$  and  $P_3$ 

$$= (30^{\circ} + 60^{\circ}) = 90^{\circ} \implies I_3 = \frac{I_0}{8} \cos^2 90^{\circ} = 0$$
  
(30^{\circ} - 60^{\circ}) = -30^{\circ} \implies I\_3 = \frac{I\_0}{8} \cos^2 (-30^{\circ}) = \frac{3I\_0}{32}

 $\therefore$   $I_3$  can be either 0 or  $\frac{\alpha_0}{32}$ .

Q. 23. Two wavelengths of sodium light 590 nm and 596 nm are used, in turn, to study the diffraction taking place at a single slit of aperture 2 × 10<sup>-4</sup> m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in the two cases. [CBSE Delhi 2013]

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Ans. For maxima other than central maxima

$$a.\theta = \left(n + \frac{1}{2}\right)\lambda$$
 and  $\theta = \frac{y}{D}$   
 $\therefore \quad a.\frac{y}{D} = \left(n + \frac{1}{2}\right)\lambda$ 

For light of wavelength  $\lambda_1 = 590$  nm

$$2 \times 10^{-14} \times \frac{y_1}{1.5} = \left(1 + \frac{1}{2}\right) \times 590 \times 10^{-9}$$
$$y_1 = \frac{3}{2} \times \frac{590 \times 10^{-9} \times 1.5}{2 \times 10^{-4}} = 6.64$$

For light of wavelength =596 nm



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$$2 \times 10^{-4} \times \frac{y_2}{1.5} = \left(1 + \frac{1}{2}\right) \times 596 \times 10^{-9}$$
  

$$\Rightarrow \qquad y_2 = \frac{3}{2} \times \frac{596 \times 10^{-9} \times 1.5}{2 \times 10^{-4}} = 6.705 \text{ mm}$$

Separation between two positions of first maxima

 $\Delta y = y_2 - y_1 = 6.705 - 6.64 = 0.065 \text{ mm}$ 

#### Q. 24. (i) State law of Malus.

- (*ii*) Draw a graph showing the variation of intensity (*I*) of polarised light transmitted by an analyser with angle ( $\theta$ ) between polariser and analyser.
- (iii) What is the value of refractive index of a medium of polarising angle 60°?

[CBSE Central 2016]

[CBSE (F) 2011, 2012]

Ans. (*i*) Malus law states that when the pass axis of a polaroid makes an angle  $\theta$  with the plane of polarisation of polarised light of intensity  $I_0$  incident on it, then the intensity of the transmitted emergent light is given by  $I = I_0 \cos^2 \theta$ .



Q. 25. The intensity at the central maxima (O) in a Young's double slit experiment is  $I_0$ . If the distance *OP* equals one-third of the fringe width of the pattern, show that the intensity at



Ans. Fringe width  $(\beta) = \frac{\lambda D}{d}$   $y = \frac{\beta}{3} = \frac{\lambda D}{3d}$ Path diff  $(\Delta P) = \frac{yd}{D} \implies \Delta P = \frac{\lambda D}{3d} \cdot \frac{d}{D} = \frac{\lambda}{3}$   $\Delta \phi = \frac{2\pi}{\lambda} \cdot \Delta P = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{3} = \frac{2\pi}{3}$ Intensity at point  $P = I_0 \cos^2 \Delta \phi$  $= I_0 \left[ \cos \frac{2\pi}{3} \right]^2 = I_0 \left( \frac{1}{2} \right)^2 = \frac{I_0}{4}$ 



Q. 26. Consider a two slit interference arrangements such that the distance of the screen from the slits is half the distance between the slits.



Obtain the value of D in terms of  $\lambda$  such that the first minima on the screen fall at a distance Dfrom the centre O.[CBSE Sample Paper 2017]

- **Ans.**  $T_2P = D + x$ ,  $T_1P = D x$ 
  - $S_1 P = \sqrt{(S_1 T_1)^2 + (P T_1)^2} = \left[D^2 + (D x)^2\right]^{1/2}$   $S_2 P = \left[D^2 + (D + x)^2\right]^{1/2}$ Minima will occur when  $\left[D^2 + (D + x)^2\right]^{1/2} - \left[D^2 + (D - x)^2\right]^{1/2} = \frac{\lambda}{9}$

$$[D^{-1} (D^{-1} x)^{-1}] = [D^{-1} (D^{-1} x)^{-1}]$$
  
If  $x = D, (D^{2} + 4D^{2})^{1/2} - D = \frac{\lambda}{2}$   
$$\Rightarrow \quad D(\sqrt{5} - 1) = \frac{\lambda}{2} \quad \Rightarrow \quad D = \frac{\lambda}{2(\sqrt{5} - 1)}$$

#### Long Answer Questions

Q. 1. Using Huygens' principle, draw a diagram to show propagation of a wavefront originating from a monochromatic point source. Explain briefly.

#### Ans. Propagation of Wavefront from a Point Source:

This principle is useful for determining the position of a given wavefront at any time in the future if we know its present position. The principle may be stated in three parts as follows:

- (*i*) Every point on a given wavefront may be regarded as a source of new disturbance.
- (*ii*) The new disturbances from each point spread out in all directions with the velocity of light and are called the *secondary wavelets*.
- (*iii*) The surface of tangency to the secondary wavelets in forward direction at any instant gives the new position of the wavefront at that time.



Let us illustrate this principle by the following example:

Let *AB* shown in the fig. be the section of a wavefront in a homogeneous isotropic medium at t = 0. We have to find the position of the wavefront at time *t* using Huygens' principle. Let *v* be the velocity of light in the given medium.

- (*a*) Take the number of points 1, 2, 3, ... on the wavefront *AB*. These points are the sources of secondary wavelets.
- (*b*) At time *t* the radius of these secondary wavelets is *vt*. Taking each point as centre, draw circles of radius *vt*.



[5 marks]

(c) Draw a tangent  $A_1B_1$  common to all these circles in the forward direction.

This gives the position of new wavefront at the required time t.

The Huygens' construction gives a backward wavefront also shown by dotted line  $A_2B_2$  which is contrary to observation. The difficulty is removed by assuming that the intensity of the spherical wavelets is not uniform in all directions; but varies continuously from a maximum in the forward direction to a minimum of zero in the backward direction.

The directions which are normal to the wavefront are called rays, *i.e.*, a ray is the direction in which the disturbance is propagated.

#### Q. 2. Define the term wavefront. Using Huygen's wave theory, verify the law of reflection.

#### [CBSE (57/1/1) 2019]

#### Ans. Wavefront: A wavefront is a locus of particles of medium all vibrating in the same phase.

**Law of Reflection:** Let *XY* be a reflecting surface at which a wavefront is being incident obliquely. Let v be the speed of the wavefront and at time t = 0, the wavefront touches the surface *XY* at *A*. After time *t*, the point *B* of wavefront reaches the point *B'* of the surface.

According to Huygen's principle each point of wavefront acts as a source of secondary waves. When the point *A* of wavefront strikes the reflecting surface, then due to presence of reflecting surface, it cannot advance further; but the secondary wavelet originating from point *A* begins to spread in all directions in the first medium with



speed v. As the wavefront AB advances further, its points  $A_1, A_2, A_3$  ... etc. strike the reflecting surface successively and send spherical secondary wavelets in the first medium.

First of all the secondary wavelet starts from point A and traverses distance AA' (= vt) in first medium in time t. In the same time t, the point B of wavefront, after travelling a distance BB', reaches point B' (of the surface), from where the secondary wavelet now starts. Now taking A as centre we draw a spherical arc of radius AA' (= vt) and draw tangent A'B' on this arc from point B'. As the incident wavefront AB advances, the secondary wavelets starting from points between A and B', one after the other and will touch A'B' simultaneously. According to Huygen's principle wavefront A'B' represents the new position of AB, *i.e.*, A'B' is the reflected wavefront corresponding to incident wavefront AB.

Now in right-angled triangles ABB' and AA'B'

$$\angle ABB' = \angle AA'B'$$
 (both are equal to 90°)  
side  $BB' = side AA'$  (both are equal to  $vt$ )

and side AB' is common.

*i.e.*, both triangles are congruent.

 $\therefore \qquad \angle BAB' = \angle AB'A'$ 

*i.e.*, incident wavefront AB and reflected wavefront A'B' make equal angles with the reflecting surface XY. As the rays are always normal to the wavefront, therefore the incident and the reflected rays make equal angles with the normal drawn on the surface XY, *i.e.*,

#### Angle of incidence i = Angle of reflection r

This is the second law of reflection.

Since *AB*, *A'B'* and *XY* are all in the plane of paper, therefore the perpendiculars dropped on them will also be in the same plane. Therefore we conclude that *the incident ray, reflected ray and the normal at the point of incidence, all lie in the same plane.* This is the first law of reflection. Thus Huygen's principle explains both the laws of reflection.

Q. 3. (a) How is a wavefront defined? Using Huygen's constructions draw a figure showing the propagation of a plane wave refracting at a plane surface separating two media. Hence verify Snell's law of refraction.

When a light wave travels from rarer to denser medium, the speed decreases. Does itimply reduction its energy? Explain.[CBSE Delhi 2008, 2013, (F) 2011, 2012]



- (b) When monochromatic light travels from a rarer to a denser medium, explain the following, giving reasons:
  - (i) Is the frequency of reflected and refracted light same as the frequency of incident light?
  - (ii) Does the decrease in speed imply a reduction in the energy carried by light wave?

#### [CBSE Delhi 2013]

#### OR

A plane wavefront propagating in a medium of refractive index ' $n_1$ ' is incident on a plane surface making the angle of incidence 'i' as shown in the figure. It enters into a medium of refractive index ' $n_2$ ' ( $n_2 > n_1$ ). Use Huygens' construction of secondary wavelets to trace the propagation of the refracted wavefront. Hence verify Snell's law of refraction. [CBSE (F) 2015]



Huygen's Principle: Refer point 1 of basic concepts.

**Proof of Snell's law of Refraction using Huygen's wave theory:** When a wave starting from one homogeneous medium enters the another homogeneous medium, it is deviated from its path. This phenomenon is called **refraction**. In transversing from first medium to another medium, the frequency of wave remains unchanged but its speed and the wavelength both are changed. Let *XY* be a surface separating the two media '1' and '2'. Let  $v_1$  and  $v_2$  be the speeds of waves in these media.



Suppose a plane wavefront *AB* in first medium is incident obliquely on the boundary surface *XY* and its end *A* touches the surface at *A* at time t = 0 while the other end *B* reaches the surface at point *B'* after time-interval *t*. Clearly  $BB' = v_1 t$ . As the wavefront *AB* advances, it strikes the points between *A* and *B'* of boundary surface. According to Huygen's principle, secondary spherical wavelets originate from these points, which travel with speed  $v_1$  in the first medium and speed  $v_2$  in the second medium.

First of all secondary wavelet starts from A, which traverses a distance  $AA' (= v_2 t)$  in second medium in time t. In the same time-interval t, the point of wavefront traverses a distance  $BB' (= v_1 t)$  in first medium and reaches B', from, where the secondary wavelet now starts. Clearly  $BB' = v_1 t$  and  $AA' = v_2 t$ .

Assuming A as centre, we draw a spherical arc of radius  $AA' (= v_2 t)$  and draw tangent B'A' on this arc from B'. As the incident wavefront AB advances, the secondary wavelets start from points between A and B', one after the other and will touch A'B' simultaneously. According to Huygen's principle A'B' is the new position of wavefront AB in the second medium. Hence A'B' will be the refracted wavefront.

**First law:** As *AB*, *A'B'* and surface *XY* are in the plane of paper, therefore the perpendicular drawn on them will be in the same plane. As the lines drawn normal to wavefront denote the rays, therefore we may say that the incident ray, refracted ray and the normal at the point of incidence all lie in the same plane.

This is the first law of refraction.

**Second law:** Let the incident wavefront AB and refracted wavefront A'B' make angles *i* and *r* respectively with refracting surface *XY*.

In right-angled triangle AB'B,  $\angle ABB' = 90^{\circ}$ 



$$\sin i = \sin \angle BAB' = \frac{BB'}{AB'} = \frac{v_1 t}{AB'} \qquad \dots (i)$$

Similarly in right-angled triangle AA'B',  $\angle AA'B' = 90^{\circ}$ 

$$\therefore \qquad \sin r = \sin \angle AB'A' = \frac{AA'}{AB'} = \frac{v_2 t}{AB'} \qquad \dots (ii)$$

Dividing equation (i) by (ii), we get

*.*..

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \text{constant} \qquad \dots (iii)$$

As the rays are always normal to the wavefront, therefore the incident and refracted rays make angles i and r with the normal drawn on the surface XY *i.e.* i and r are the angle of incidence and angle of refraction respectively. According to equation (*iii*):

The ratio of sine of angle of incidence and the sine of angle of refraction for a given pair of media is a constant and is equal to the ratio of velocities of waves in the two media. This is the second law of refraction, and is called the Snell's law.

(*b*) (*i*) If the radiation of certain frequency interact with the atoms/molecules of the matter, they start to vibrate with the same frequency under forced oscillations.

Thus, the frequency of the scattered light (Under reflection and refraction) equals to the frequency of incident radiation.

(*ii*) No, energy carried by the wave depends on the frequency of the wave, but not on the speed of the wave.

# Q. 4. Use Huygens' principle to show how a plane wavefront propagates from a denser to rarer medium. Hence, verify Snell's law of refraction.

#### [CBSE Allahabad 2015, Sample Paper 2016; 2019 (55/1/1)]

**Ans.** We assume a plane wavefront *AB* propagating in denser medium incident on the interface *PP*' at angle *i* as shown in Fig. Let *t* be the time taken by the wave front to travel a distance *BC*. If  $v_1$  is the speed of the light in medium *I*.

So, 
$$BC = v_1 t$$

In order to find the shape of the refracted wavefront, we draw a sphere of radius  $AE = v_2 t$ , where  $v_2$  is the speed of light in medium II (rarer medium). The tangent plane *CE* represents the refracted wavefront.

In 
$$\triangle ABC$$
,  $\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}$   
and in  $\triangle ACE$ ,  $\sin r = \frac{AE}{AC} = \frac{v_2 t}{AC}$   
 $\therefore \qquad \frac{\sin i}{\sin r} = \frac{BC}{AE} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2}$ 

Let *c* be the speed of light in vacuum

So,  $n_1 = \frac{c}{v_1} \text{ and } n_2 = \frac{c}{v_2}$   $\frac{n_2}{n_1} = \frac{v_1}{v_2}$ From equations (i) and (ii), we have  $\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$   $n_1 \sin i = n_2 \sin r$ 

It is known as Snell's law.



...(i)

...(*ii*)



Q. 5. (a) In Young's double slit experiment, deduce the conditions for (i) constructive, and (ii) destructive interference at a point on the screen. Draw a graph showing variation of the resultant intensity in the interference pattern against position 'X' on the screen.

[CBSE Delhi 2016, (AI) 2012]

(b) Compare and contrast the pattern which is seen with two coherently illuminated narrow slits in Young's experiment with that seen for a coherently illuminated single slit producing diffraction.

#### Ans. (a) Conditions of Constructive and Destructive Interference:

When two waves of same frequency and constant initial phase difference travel in the same direction along a straight line simultaneously, they superpose in such a way that the intensity of the resultant wave is maximum at certain points and minimum at certain other points. The phenomenon of redistribution of intensity due to superposition of two waves of same frequency and constant initial phase difference is called the interference. The waves of same frequency and constant initial phase difference are called **coherent waves**. At points of medium where the waves arrive in the same phase, the resultant intensity is maximum and the interference at these points is said to be **constructive**. On the other hand, at points of medium where the waves arrive in opposite phase, the resultant intensity is minimum and the interference at these points is said to be **destructive**. The positions of maximum intensity are called **maxima** while those of minimum intensity are called **minima**. The interference takes place in sound and light both.

**Mathematical Analysis:** Suppose two coherent waves travel in the same direction along a straight line, the frequency of each wave is  $\frac{\omega}{2\pi}$  and amplitudes of electric field are  $a_1$  and  $a_2$  respectively. If at any time *t*, the electric fields of waves at a point are  $y_1$  and  $y_2$  respectively

and phase difference is  $\phi$ , then equation of waves may be expressed as  $y_1 = a_1 \sin \omega t$  ...(*i*)

$$y_2 = a_2 \sin(\omega t + \phi)$$
 ...(*ii*)

According to Young's principle of superposition, the resultant displacement at that point will be  $y = y_1 + y_2$  ...(*iii*)

Substituting values of  $y_1$  and  $y_2$  from (*i*) and (*ii*) in (*iii*), we get

$$y = a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$

Using trigonometric relation

0		
	$\sin (\omega t + \phi) = \sin \omega t \cos \phi + \cos \omega t \sin \phi ,$	
we get	$y = a_1 \sin \omega t + a_2 (\sin \omega t \cos \phi + \cos \omega t \sin \phi)$	
	$= (a_1 + a_2 \cos \phi) \sin \omega t + (a_2 \sin \phi) \cos \omega t$	( <i>iv</i> )
Let	$a_1 + a_2 \cos \phi = A \cos \theta$	(v)
and	$a_9 \sin \phi = A \sin \theta$	$\dots(vi)$

where *A* and  $\theta$  are new constants.

Then equation (*iv*) gives  $y = A \cos \theta \sin \omega t + A \sin \theta \cos \omega t = A \sin (\omega t + \theta)$  ....(*vii*) This is the equation of the resultant disturbance. Clearly the amplitude of resultant disturbance is *A* and phase difference from first wave is  $\theta$ . The values of *A* and  $\theta$  are determined by (*v*) and (*vi*). Squaring (*v*) and (*vi*) and then adding, we get

$$(a_{1} + a_{2} \cos \phi)^{2} + (a_{2} \sin \phi)^{2} = A^{2} \cos^{2} \theta + A^{2} \sin^{2} \theta$$
  
or  
$$a_{1}^{2} + a_{2}^{2} \cos^{2} \phi + 2a_{1}a_{2} \cos \phi + a_{2}^{2} \sin^{2} \phi = A^{2} (\cos^{2} \theta + \sin^{2} \theta)$$
  
As  $\cos^{2} \theta + \sin^{2} \theta = 1$ , we get  
$$A^{2} = a_{1}^{2} + a_{2}^{2} (\cos^{2} \phi + \sin^{2} \phi) + 2a_{1}a_{2} \cos \phi$$
  
or  
$$A^{2} = a_{1}^{2} + a_{2}^{2} + 2a_{1}a_{2} \cos \phi$$
  
Amplitude,  $A = \sqrt{a_{1}^{2} + a_{2}^{2} + 2a_{1}a_{2} \cos \phi}$ ....(*viii*)

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As the intensity of a wave is proportional to its amplitude in arbitrary units  $I = A^2$  $\therefore$  Intensity of resultant wave

$$=A^{2} = a_{1}^{2} + a_{2}^{2} + 2a_{1}a_{2}\cos\phi \qquad \dots (ix)$$

Clearly the intensity of resultant wave at any point depends on the amplitudes of individual waves and the phase difference between the waves at the point.

**Constructive Interference:** For maximum intensity at any point  $\cos \phi = +1$ 

or phase difference  $\phi = 0, 2\pi, 4\pi, 6\pi$ ...... =  $2n\pi (n = 0, 1, 2, ....)$  ...(x)

The maximum intensity,

1

$$I_{\max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2 \qquad \dots (xi)$$

Path difference  $\Delta = \frac{\lambda}{2\pi} \times \text{Phase difference} = \frac{\lambda}{2\pi} \times 2n\pi = n\lambda$  ...(*xii*)

Clearly the maximum intensity is obtained in the region of superposition at those points where waves meet in the same phase or the phase difference between the waves is even multiple of  $\pi$  or path difference between them is the integral multiple of  $\lambda$  and maximum intensity is  $(a_1 + a_2)^2$  which is greater than the sum intensities of individual waves by an amount  $2a_1a_2$ .

**Destructive Interference:** For minimum intensity at any point  $\cos \phi = -1$ 

In this case the minimum intensity,

$$I_{\min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2 \qquad \dots (xiv)$$

Path difference,  $\Delta = \frac{\lambda}{2\pi} \times$  Phase difference

$$= \frac{\lambda}{2\pi} \times (2n-1)\pi = (2n-1)\frac{\lambda}{2} \qquad \dots (xv)$$

Clearly, the minimum intensity is obtained in the region of superposition at those points where waves meet in opposite phase or the phase difference between the waves is odd

multiple of  $\pi$  or path difference between the waves is odd multiple of  $\frac{\lambda}{2}$  and minimum intensity =  $(a_1 - a_2)^2$  which is less than the sum of intensities of the individual waves by an amount  $2a_1a_2$ .

From equations (*xi*) and (*xiv*) it is clear that the intensity  $2a_1a_2$  is transferred from positions of minima to maxima. This implies that the *interference is based on conservation of energy i.e.*, *there is no wastage of energy*.

**Variation of Intensity** of light with position *x* is shown in fig.



(b) Comparison of two Slit Young's Interference pattern and Single slit diffraction pattern

Both patterns are the result of wave nature of light; both patterns contain maxima and minima. Interference pattern is the result of superposing two coherent wave while the diffraction pattern

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is the superposition of large number of waves originating from each point on a single slit.

**Differences:** (*i*) In Young's two slit experiment; all maxima are of same intensity while in diffraction at a single slit, the intensity of central maximum is maximum and it falls rapidly for first, second order secondary maxima on either side of it.

- (*ii*) In Young's interference the fringes are of equal width while in diffraction at a single slit, the central maximum is twice as wide as other maxima. The intensity falls as we go to successive maxima away from the centre on either side.
- (*iii*) In a single slit diffraction pattern of width *a*, the first minimum occurs at  $\lambda/a$ ; while in

two slit interference pattern of slit separation *a*, we get maximum at the same angle  $\underline{\lambda}$ .

Q. 6. Two harmonic waves of monochromatic light

 $y_1 = a \cos \omega t$  and  $y_2 = a \cos(\omega t + \phi)$ 

are superimposed on each other. Show that maximum intensity in interference pattern is four times the intensity due to each slit. Hence write the conditions for constructive and destructive interference in terms of the phase angle  $\phi$ . [CBSE South 2016]

Ans. The resultant displacement will be given by

$$y = y_1 + y_2$$
  
=  $a \cos \omega t + a \cos(\omega t + \phi)$   
=  $a[\cos \omega t + \cos(\omega t + \phi)]$   
=  $2a \cos(\phi/2) \cos(\omega t + \phi/2)$ 

The amplitude of the resultant displacement is  $2a \cos(\phi/2)$ 

The intensity of light is directly proportional to the square of amplitude of the wave. The resultant intensity will be given by

$$I = 4a^2 \cos^2 \frac{\Phi}{2}$$

:. Intensity =  $4I_0 \cos^2\left(\frac{\Phi}{2}\right)$ , where  $I_0 = a^2$  is the intensity of each harmonic wave

At the maxima,  $\phi = \pm 2n\pi$ 

At the maxima,  $I = 4I_0 = 4 \times \text{intensity}$  due to one slit

$$I = 4I_0 \cos^2\left(\frac{\Phi}{2}\right)$$

For constructive interference, I is maximum.

 $\cos^2\frac{\phi}{2} = 1$ 

It is possible when 
$$\cos^2\left(\frac{\phi}{2}\right) = 1; \frac{\phi}{2} = n\pi; \phi = 2n\pi$$

For destructive interference, I is minimum, *i.e.*, I = 0

It is possible when  $\cos^2\left(\frac{\phi}{2}\right) = 0; \frac{\phi}{2} = \frac{(2n-1)\pi}{2}; \phi = (2n \pm 1)\frac{\pi}{2}$ 

- Q. 7. (a) What are coherent sources of light? State two conditions for two light sources to be coherent.
  - (b) Derive a mathematical expression for the width of interference fringes obtained in Young's double slit experiment with the help of a suitable diagram.

[CBSE Delhi 2011, Panchkula 2015]

(c) If s is the size of the source and b its distance from the plane of the two slits, what should be the criterion for the interference fringe to be seen?

OR



- (a) In Young's double slit experiment, describe briefly how bright and dark fringes are obtained on the screen kept in front of a double slit. Hence obtain the expression for the fringe width.
- (b) The ratio of the intensities at minima to the maxima in the Young's double slit experiment is 9: 25. Find the ratio of the widths of the two slits. [CBSE (AI) 2014]

Ans.

(*a*) Coherent sources are those which have exactly the same frequency and are in this same phase or have a zero or constant difference.

- **Conditions:** (*i*) The sources should be monochromatic and originating from common single source.
  - (*ii*) The amplitudes of the waves should be equal.

Condition for formation of bright and dark fringes.

Suppose a narrow slit *S* is illuminated by monochromatic light of wavelength  $\lambda$ .

The light rays from two coherent sources  $S_1$  and  $S_2$  are reaching a point *P*, have a path difference  $(S_2P - S_1P)$ .



(i) If maxima (bright fringe) occurs at point P, then

$$S_2 P - S_1 P = n\lambda$$
 (*n* = 0, 1, 2, 3 ...)

(*ii*) If minima (dark fringe) occurs at point *P*, then



Light waves starting from *S* and fall on both slits  $S_1$  and  $S_2$ . Then  $S_1$  and  $S_2$  behave like two coherent sources. Spherical waves emanating from  $S_1$  and  $S_2$  superpose on each other, and produces interference pattern on the screen. Consider a point *P* at a distance *x* from O, the centre of screen. The position of maxima (or minima) depends on the path difference.  $(S_2T = S_2P - S_1P)$ .

From right angled  $\Delta S_2 BP$  and  $\Delta S_1 AP$ ,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right] = 2xd$$

 $(S_2P + S_1P)(S_2P - S_1P) = 2xd$ 

 $S_2 P - S_1 P = \frac{2xd}{(S_2 P + S_1 P)}$ 

 $\rightarrow$ 

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In practice, the point P lies very close to O, therefore

$$S_2P + S_1P = 2D$$
  

$$S_2P - S_1P = \frac{2xd}{2D} = \frac{xd}{D}$$
 ... (i)

For constructive interference (Bright fringes)

Path difference,  $\frac{dx}{D} = n\lambda$  where n = 0, 1, 2, 3, ...  $x = \frac{nD\lambda}{d}$ For n = 0,  $x_0 = 0$  for central bright fringe For n = 1,  $x_1 = \frac{D\lambda}{d}$  for 1st bright fringe

For 
$$n = 2$$
,  $x_2 = \frac{2D\lambda}{d}$  for 2nd bright fringe  
For  $n = n$ ,  $x = \frac{nD\lambda}{d}$  nth bright fringe

For n = n,  $x_n = \frac{d}{d}$  nth bright tringe The distance between two consecutive bright fringes is

$$\beta = x_n - x_{n-1} = \frac{nD\lambda}{d} - \frac{(n-1)D\lambda}{d} = \frac{D\lambda}{d}$$

For destructive interference (dark fringes)

Path difference  $\frac{dx}{D} = (2n-1)\frac{\lambda}{2}$   $x = (2n-1)\frac{D\lambda}{2d}$  where n = 1, 2, 3, ...For n = 1,  $x'_1 = \frac{D\lambda}{2d}$  for 1st dark fringe

For n = 2,  $x'_{2} = \frac{3D\lambda}{2d}$  for 2nd dark fringe

For n = n,  $x'_n = (2n-1)\frac{D\lambda}{2d}$  for *n*th dark fringe. The distance between two consecutive dark fringe is

$$B' = (2n-1)\frac{D\lambda}{2d} - \{2(n-1)-1\}\frac{D\lambda}{2d} = \frac{D\lambda}{d}$$

The distance between two consecutive bright or dark fringes is called fringe width (w).

 $\therefore \qquad \text{Fringe width} = \frac{D\lambda}{d}$ 

The expression for fringe width is free from n. Hence the width of all fringes of red light are broader than the fringes of blue light.

(b) Intensity of light (using classical theory) is given as

$$I \propto (\text{Width of the slit})$$

$$\propto (\text{Amplitude})^2$$

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{25}{9} \implies \frac{a_1 + a_2}{a_1 - a_2} = \frac{5}{3} \Rightarrow \frac{a_1}{a_2} = \frac{4}{12}$$

Intensity ratio

$$\frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2} \qquad \qquad \Rightarrow \qquad \frac{I_1}{I_2} = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$$

(c) The condition for the interference fringes to be seen is

$$\frac{s}{b} < \frac{\lambda}{d}$$

where s is the size of the source and b is the distance of this source from plane of the slit.



Q. 8. What is interference of light? Write two essential conditions for sustained interference pattern to be produced on the screen.

Draw a graph showing the variation of intensity versus the position on the screen in Young's experiment when (a) both the slits are opened and (b) one of the slits is closed.

What is the effect on the interference pattern in Young's double slit experiment when:

(*i*) screen is moved closer to the plane of slits?

(ii) separation between two slits is increased?

Explain your answer in each case.

**Ans. Interference of light:** When two waves of same frequency and constant initial phase difference travel in the same direction along a straight line simultaneously, they superpose in such a way that the intensity of the resultant wave is maximum at certain points and minimum at certain other points. This phenomenon of redistribution of energy due to superposition of two waves of same frequency and constant initial phase difference is called interference.

**Conditions for Sustained Interference of Light Waves** 

To obtain sustained (well-defined and observable) interference pattern, the intensity must be maximum and zero at points corresponding to constructive and destructive interference. For the purpose following conditions must be fulfilled:

- (i) The two interfering sources must be coherent and of same frequency, i.e., the sources should emit light of the same wavelength or frequency and their initial phase should remain constant. If this condition is not satisfied the phase difference between the interfering waves will vary continuously. As a result the resultant intensity at any point will vary with time being alternately maximum and minimum, just like the phenomenon of beats in sound.
- *(ii) The interfering waves must have equal amplitudes.* Otherwise the minimum intensity will not be zero and there will be general illumination.



The variation of intensity *I* versus the position *x* on the screen in Young's experiment.

Fringe width,  $\beta = \frac{D\lambda}{d}$ 

- (*i*)  $\beta \propto D$ , therefore with the decrease of separation between the plane of slits and screen, the fringe width decreases.
- (*ii*) On increasing the separation between two slits (*d*), the fringe separation decreases as  $\beta$  is inversely proportional to  $d(i.e.,\beta \propto \frac{1}{d})$ .
- Q. 9. What is diffraction of light? Draw a graph showing the variation of intensity with angle in a<br/>single slit diffraction experiment. Write one feature which distinguishes the observed pattern<br/>from the double slit interference pattern.[CBSE (F) 2013]

How would the diffraction pattern of a single slit be affected when:

- (i) the width of the slit is decreased?
- (ii) the monochromatic source of light is replaced by a source of white light?



Ans. Diffraction of Light: When light is incident on a narrow opening or an obstacle in its path, it is bent at the sharp edges of the obstacle or opening. This phenomenon is called diffraction of light. For graph refer point 5 of basic concepts.

For graph refer point 5 of basic concepts.

In an interference pattern all the maxima have the same intensity while in diffraction pattern the maxima are of different intensities. For example in Young's double slit experiment all maxima are of the same intensity and in diffraction at a single slit, the central maximum have the maximum intensity and it falls rapidly for first, second orders secondary maxima on either side of it.

(*i*) When the width of the slit is decreased: From the relation sin  $\theta = \frac{\lambda}{a}$ , we find that if the width of the slit (*a*) is decreased, then for a given wavelength, sin  $\theta$  is large and hence  $\theta$  is

large. Hence diffraction maxima and minima are quite distant on either side of  $\theta$ .

- (*ii*) With monochromatic light, the diffraction pattern consists of alternate bright and dark bands. If white light is used central maximum is white and on either side, the diffraction bands are coloured.
- Q. 10. Describe diffraction of light due to a single slit. Explain formation of a pattern of fringes obtained on the screen and plot showing variation of intensity with angle θ in single slit diffraction.
   [CBSE Delhi 2010, (F) 2013, (AI) 2014]
- **Ans.** Diffraction of light at a single slit: When monochromatic light is made incident on a single slit, we get diffraction pattern on a screen placed behind the slit. The diffraction pattern contains bright and dark bands, the intensity of central band is maximum and goes on decreasing on both sides.

**Explanation:** Let *AB* be a slit of width '*a*' and a parallel beam of monochromatic light is incident on it. According to Fresnel the diffraction pattern is the result of superposition of a large number of waves, starting from different points

of illuminated slit.

Let  $\theta$  be the angle of diffraction for waves reaching at point *P* of screen and *AN* the perpendicular dropped from *A* on wave diffracted from *B*.

The path difference between rays diffracted at points *A* and *B*,

$$\Delta = BP - AP = B.$$

In  $\triangle ANB$ ,  $\angle ANB = 90^{\circ}$  and  $\angle BAN = \theta$ 

$$\therefore \qquad \sin \theta = \frac{BN}{AB} \text{ or } BN = AB \sin \theta$$

As 
$$AB$$
 = width of slit =  $a$ 

 $\therefore \text{ Path difference,} \quad \Delta = a \sin \theta \qquad \dots (i)$ 

To find the effect of all coherent waves at *P*, we have to sum up their contribution, each with a different phase. This was done by Fresnel by rigorous calculations, but the main features may be explained by simple arguments given below:

At the central point *C* of the screen, the angle  $\theta$  is zero. Hence the waves starting from all points of slit arrive in the same phase. This gives maximum intensity at the central point *C*.

**Minima:** Now we divide the slit into two equal halves *AO* and *OB*, each of width  $\frac{a}{2}$ . Now for every point,  $M_1$  in *AO*, there is a corresponding point  $M_2$  in *OB*, such that  $M_1 M_2 = \frac{a}{2}$ ; then path difference between waves arriving at *P* and starting from  $M_1$  and  $M_2$  will be  $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$ . (*ii*) This





means that the contributions from the two halves of slit *AO* and *OB* are opposite in phase and so cancel each other. Thus equation (*ii*) gives the angle of diffraction at which intensity falls to zero. Similarly it may be shown that the intensity is zero for sin  $\theta = \frac{n\lambda}{a}$ , with *n* as integer. Thus the general condition of **minima** is

$$a\sin\theta = n\lambda$$
 ...(iii)

Secondary Maxima: Let us now consider angle  $\theta$  such that

$$\sin \theta = \theta = \frac{3\lambda}{2a}$$

which is midway between two dark bands given by



Let us now divide the slit into three parts. If we take the first two parts of slit, the path difference between rays diffracted from the extreme ends of the first two parts

$$\frac{2}{3}a\sin\theta = \frac{2}{3}a \times \frac{3\lambda}{2a} = \lambda$$

Then the first two parts will have a path difference of  $\frac{\lambda}{2}$  and cancel the effect of each other. The remaining third part will contribute to the intensity at a point between two minima. Clearly there will be a maxima between first two minima, but this maxima will be of much weaker intensity than central maximum. This is called *first secondary maxima*. In a similar manner we can show that there are secondary maxima between any two consecutive minima; and the intensity of maxima will go on decreasing with increase of order of maxima. In general the position of *n*th maxima will be given by

$$a\sin\theta = \left(n + \frac{1}{2}\right)\lambda,$$
  $[n = 1, 2, 3, 4, ...]$  ...(*iv*)

The intensity of secondary maxima decreases with increase of order n because with increasing n, the contribution of slit decreases.

For n = 2, it is one-fifth, for n = 3, it is one-seventh and so on.

- Q. 11. (a) What is linearly polarized light? Describe briefly using a diagram how sunlight is polarised.
  - (b) Unpolarised light is incident on a polaroid. How would the intensity of transmitted light change when the polaroid is rotated? [CBSE (AI) 2013]
  - Ans. (a) Molecules in air behave like a dipole radiator. When the sunlight falls on a molecule, dipole molecule does not scatter energy along the dipole axis, however the electric field vector of light wave vibrates just in one direction perpendicular to the direction of the propagation. The light wave having direction of electric field vector in a plane is said to be linearly polarised.



In figure, a dipole molecule is lying along *x*-axis. Molecules behave like dipole radiators and scatter no energy along the dipole axis.



The unpolarised light travelling along x-axis strikes on the dipole molecule get scattered along y and z directions. Light traversing along y and z directions is plane polarised light.

(*b*) When unpolarised light is incident on a polaroid, the transmitted light has electric vibrations in the plane consisting of polaroid axis and direction of wave propagation as shown in Fig.



If polaroid is rotated the plane of polarisation will change, however the intensity of transmitted light remain unchanged.

- Q. 12. (i) Distinguish between unploarised light and linearly polarised light. How does one get linearly polarised light with the help of a polaroid?
  - (*ii*) A narrow beam of unpolarised light of intensity  $I_0$  is incident on a polaroid  $P_1$ . The light transmitted by it is then incident on a second polaroid  $P_2$  with its pass axis making angle of 60° relative to the pass axis of  $P_1$ . Find the intensity of the light transmitted by  $P_2$ . [CBSE Delhi 2017]
- Ans. (i) Unpolarised Light: The light having vibrations of electric field vector in all possible directions perpendicular to the direction of wave propagation is called the ordinary (or unpolarised) light.
   Plane (or Linearly) Polarised Light: The light having vibrations of electric field vector in only one direction perpendicular to the direction of propagation of light is called plane (or linearly) polarised light.

When unpolarised light wave is incident on a polaroid, then the electric vectors along the direction of its aligned molecules get absorbed; the electric vector oscillating along a direction perpendicular to the aligned molecules, pass through. This light is called linearly polarised light.

(ii) According to Malus' Law:

$$I = I_0 \cos^2 \theta$$

∴

 $I' = \frac{I_0}{2} \cos^2 \theta$ , where  $I_0$  is the intensity of unpolarised light.

Given,  $\theta = 60^{\circ}$ 

$$I' = \frac{I_0}{2} \cos^2 60^\circ = \frac{I_0}{2} \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$$



# **Self-Assessment Test**

#### Time allowed: 1 hour

- 1. Choose and write the correct option in the following questions.
  - (*i*) The ratio of resolving powers of an optical microscope for two wavelengths  $\lambda_1 = 4000$  Å and  $\lambda_2 = 6000$  Å is
    - (a) 9:4
      (b) 3:2
      (c) 16:81
      (d) 8:27
  - (*ii*) Two polaroids  $P_1$  and  $P_2$  are placed with their axis perpendicular to each other. Unpolarised light  $I_0$  is incident on  $P_1$ . A third polaroid  $P_3$  is kept in between  $P_1$  and  $P_2$  such that its axis makes an angle 45° with that of  $P_1$ . The intensity of transmitted light through  $P_2$  is

(a) 
$$\frac{I_0}{4}$$
 (b)  $\frac{I_0}{8}$   
(c)  $\frac{I_0}{16}$  (d)  $\frac{I_0}{2}$ 

- (*iii*) A linear aperture whose width is 0.02 cm is placed immediately in front of a lens of focal length 60 cm. The aperture is illuminated normally by a parallel beam of wavelength  $5 \times 10^{-5}$  cm. The distance of the first dark band of the differaction pattern from the centre of the screen is
  - (a) 0.10 cm (b) 0.25 cm (c) 0.20 cm (d) 0.15 cm

#### 2. Fill in the blanks.

- (*i*) A point source produces spherical wavefronts, a line source produces cylindrical wavefronts and a parallel beam of light have \_\_\_\_\_\_ wavefronts.
- (*ii*) The minimum distance between the objects which can just be seen as separated by the optical instrument is known as the \_\_\_\_\_\_ of the instrument.
- **3.** Define a wavefront.
- State the reason, why two independent sources of light cannot be considered as coherent sources.
- 5. How does the fringe width, in Young's double-slit experiment, change when the distance of separation between the slits and screen is doubled?
- 6. Find the intensity at a point on a screen in Young's double slit experiment where the interfering

waves of equal intensity have a path difference of (i)  $\frac{\lambda}{4}$ , and (ii)  $\frac{\lambda}{3}$ .

- 7. Define the resolving power of a microscope. How is this affected when
  - (i) the wavelength of illuminating radiations is decreased, and
  - (*ii*) the diameter of the objective lens is decreased?

Justify your answer.

8. A parallel beam of light of 600 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1.2 m away. It is observed that the first minimum is at a distance of 3 mm from the centre of the screen. Calculate the width of the slit.
2



 $(2 \times 1 = 2)$ 

1

2

2

Max. marks: 30

 $(3 \times 1 = 3)$ 

**9.** The figure shows a modified Young's double slit experimental set-up. Here  $SS_2 - SS_1 = \lambda/4$ .



- (a) Write the condition for constructive interference.
- (b) Obtain an expression for the fringe width.
- **10.** (*a*) If one of two identical slits producing interference in Young's experiment is covered with glass, so that the light intensity passing through it is reduced to 50%, find the ratio of the maximum and minimum intensity of the fringe in the interference pattern.
  - (b) What kind of fringes do you expect to observe if white light is used instead of monochromatic light?
- **11.** Answer the following:
  - (*a*) In what way is diffraction from each slit related to the interference pattern in a double slit experiment?
  - (*b*) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain, why.
  - (c) How does the resolving power of a microscope depend on (i) the wavelength of the light used and (ii) the medium used between the object and the objective lens?
- **12.** (*a*) Using the phenomenon of polarisation, show how transverse nature of light can be demonstrated.
  - (b) Two polaroids  $P_1$  and  $P_2$  are placed with their pass axes perpendicular to each other. Unpolarised light of intensity  $I_0$  is incident on  $P_1$ . A third polaroid  $P_3$  is kept in between  $P_1$  and  $P_2$  such that its pass axis makes an angle of 30° with that of  $P_1$ . Determine the intensity of light transmitted through  $P_1$ ,  $P_2$  and  $P_3$ .
- 13. Describe diffraction of light due to a single slit. Explain formation of a pattern of fringes obtained on the screen and plot showing variation of intensity with angle  $\theta$  in single slit diffraction. 5

# Answers

- **1.** (i) (b) (ii) (b) (iii) (d)
- **2.** (*i*) plane (*ii*) limit of resolution

**8.** 
$$2.4 \times 10^{-4}$$
m **12.**  $\frac{I_0}{2}, \frac{3I_0}{32}, \frac{3I_0}{8}$ 

