Algebra

Learning Objectives

- Introduction
- Variables
- Equation with One Variable
- Algebra Expressions
- Problems Based on Algebra Equations

Introduction

We live in the world of numbers. We see them everyday, on clocks, in sports, and all over the news. Algebra is all about figuring out the numbers we don't see. In this chapter, we will study about basic algebra and simple problems based on it.

Variables

Algebra is the branch of mathematics that uses letters in place of some unknown numbers we use numerals to represent numbers in arithmetic, in a similar way in Algebra, letters of the alphabet are used to represent numbers and these letters are called variables. Let us look at the puzzle of an unknown number.
?– 7 = 6
Here, we see, 13 - 7 = 6
In algebra, we do not used blank boxes, we use a letter (say a, b, c, x, y, z, etc.). So, we can write it as: x - 7 = 6
Where, x is a variable or unknown number. Now, x - 7 = 6, or, (x - 7) = 6, x - 0 = 7 + 6 or x - 0 = 13. Or, x = 13
Clearly, here, x = 13 stands for the unknown number given in the box above.

We use letter for unknown number because, it is easier to write x than drawing empty boxes and also it is easier to say x than say empty box. If there are several empty boxes, or several unknown, we use different letters for different unknown.

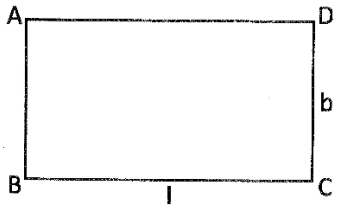
Equation with One Variable

A linear equation in one variable has a single unknown quantity represented by a letter. The process of finding out the variable value that makes the equation true is called 'solving' the equation. Clearly, an equation is a statement that two quantities are equivalent. For example, x + 3 = 5 means that when we added 3 to an unknown value 'x' the answer is equal to 5.

- **Example:**
  Solve for 'x': x - 19 = 34
  Solution: Given equation is: x - 19 = 34
  Now, add 19 to both side of the equation: (x - 19) + 19 = 34 + 19
  Or, x + 0 = 53, or x = 53
  Verification: x - 19 = 34, or 53 -19 = 34.

Equation and Formula

We know that, perimeter of rectangle $=2\times(\text{length}+\text{breadth})=2\times(l+b)$
Then, variable 'l' stands for the length of rectangle and 'b' stands for the breadth of rectangle. Clearly, the letters 'l' and 'b' will represent different numbers for different rectangles.



1. Area of rectangle $=\text{length}\times\text{breadth}=I\times b$

2. The formula, distance = speed × time is written as $D=S\times T$, by making use of the letter 'D' for distance, 'S' for speed and 'V for time.

3. The relation between temperatures in Fahrenheit and Celsius is: $C=\frac{5}{9}(F-32)$

Here, the letter 'C' is used to denote the temperature in degrees in the Celsius scale and 'F' is used to denote the temperature in degrees in the Fahrenheit scale.

4. Formula for simple interest is:

$$\text{Simple Interest} = \frac{\text{Prinidpal}\times\text{Rate of Interest}\times\text{Time}}{100}$$

Or, $1.=\frac{P\times R\times T}{100}$; Here, P, R and T stands for principal, rate of interest and time respectively.

Algebraic Expressions

As we know, an algebraic equation is a complete mathematical sentence that includes a sign of equality (or equal sign).

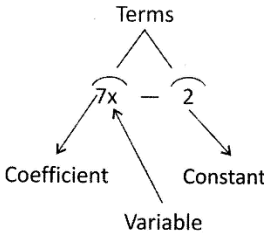
An algebraic expression is not a mathematical sentence. It is a mathematical phrase because it does not have an equal sign.

Algebraic Expression	Algebraic Equation
x + 3 or, 7 + 3	x + 3 = 10 Or, 7 + 3 = 10
3x or 3×6	3x = 18 Or, $3\times6=18$
$2\times(1+7)$ Or, $2(x+7)$	$2\times(1+7)=16$ Or, $2\times(x+7)=16$

An expression is made up of terms, where these terms are separate values in an expression. They can be separated by operations, like plus and minus signs.

There are three parts of most expressions which are as follow:

Terms



Variable: It is the unknown number x in the given expression above.

Coefficient: It is the number that is multiplied by the variable x. In the above expression, x is multiplied by 7, so 7 is the coefficient of x.

Constant: It is a fixed number that does not change. In the above expression, 2 is a constant.

- An expression like 'x + y' indicates the sum of two numbers represented by x and y. It means any number x plus any number y. For example. If x = 3 and y = 7 then x + y = 3 + 7 = 10. Clearly, x represents 3 and y represents 7.
- The expression x - y indicates that the number represented by y is subtracted from the number represented by x, For example, if x = 12 and y = 4, then x - y = 12 - 4 = 8.
  Where, x represents the number 12 and y represents the number 4.

- The expression xy or $x \times y$ indicates that the number represented by x is multiplied by the number represented by y.
  For example, if x = 7 and y = 4, then $x \times y = xy = 7 \times 4 = 28$.
  Where, x represents 7 and y represents 4.
- The expression $x \div y$, or, $\frac{x}{y}$ indicates that the number represented by x is divided by the number represented by y.
  For example, if x = 27 and y = 3, then, $x \div y = \frac{x}{y} = \frac{27}{3} = 9$
  Where, x represents 27 and y represents 3.
- An expression like x^2 and x^3 means $(x) \times (x)$ and $(x) \times (x) \times (x)$ respectively.
  For example, if x = 3 then $x^2 = (x) \times (x) = 3 \times 3 = 9$ and
  $x^3 = (x) \times (x) \times (x) = 3 \times 3 \times 3 = 27$.

- **Example:**

1. Square of 'x' plus four times 'y' is:

(a) $x^2 + 4y$ (b) $(x + 4y)^2$

(c) $x^2 + 4y^2$ (d) $x + 4y^2$

(e) None of these

Answer: (a)

Explanation: Here, square of $'x' = x^2$ and four times $y = 4y$

So, (square of 'x') plus (four times 'Y') $= x^2 + 4y$

2. 5x - 2y means:

(a) Five times 'Y' minus two times 'x'.

(b) Five times and two times of the difference of 'x' and 'y'.

(c) Five times 'x' minus two times 'y'.

(d) Two times 'y' minus five times 'x'.

(e) None of these

Answer: (c)

Explanation: $5x - 2y = 5 \times x - 2 \times y =$ Five times 'x' - Two times 'y'.

So, 5x - 2y means five times Y minus two times 'y'.

Evaluating Expression

We can find the value of a given expression by substituting the value of the variable in the expression. Look at the following examples.

- **Example:**

1. If a = 3, b = 4 and c = 7, then find the value of $\frac{\mathbf{c - 2a}}{\mathbf{2b}}$

Solution: $\frac{c - 2a}{2b} = \frac{7 - 2 \times 3}{2 \times 4} = \frac{7 - 6}{8} = \frac{1}{8}$.

2. If x = 7 and y = 4 then find the value of expression: $5xy^2 + xy - 3$.

Solution: $5xy^2 + xy - 3 = (5 \times x \times y \times y) + (x \times y - 3)$

$= (5 \times 7 \times 4 \times 4) + (7 \times 4 = -3) = 560 + 28 - 3 = 588 - 3 = 585$.

3. Sum of two odd numbers is 40. If one of them is 17, then find the other odd number.

(a) 19 (b) 17

(c) 23 (d) 27

(e) None of these

Answer: (c)

Explanation: Let be the other odd number is x.

Given that odd number = 17

Since, the sum of both odd numbers is 40.
Hence, x + 17 = 40, or (x + 17) - 17 = 40 - 17, or, x + 0 = 23, or, x = 23
So, odd numbers are 23 and 17.

Commonly Asked Question

1. Which one of the following is the equation for the statement given "Half of a number (x) subtracted from 20 is 15"

(a) $15-\frac{x}{2}=20$ (b) $\frac{x}{2}-15=20$

(c) $20-\frac{x}{2}=15$ (d) $\frac{x}{2}-20=15$

(e) None of these

Answer: (c)

Explanation: Given number = x Half of the number $x=\frac{1}{2}x$

Half of the number $\times$ subtracted from $20=20-\frac{1}{2}x$

So, half of a number (x) subtracted from 20 is 15 means $20-\frac{1}{2}x=15$ or, $20-\frac{x}{2}=15$

2. if a = 4, b = 5 and c = 8 then the value of $\frac{a^2-b}{2c}$ is:

(a) $\frac{16}{21}$ (b) $\frac{1}{8}$

(c) $\frac{11}{16}$ (d) $\frac{16}{11}$

(e) None of these

Answer: (c)

Explanation: $\frac{a^2-b}{2c}=\frac{a\times a-b}{2\times c}=\frac{4\times 4-5}{2\times 8}=\frac{16-5}{16}=\frac{11}{16}$

3. Find the value of x in the given equation.

$\frac{2x-5}{3}=4$

(a) $\frac{17}{2}$ (b) $\frac{15}{2}$

(c) $\frac{17}{3}$ (d) $\frac{15}{3}$

(e) None of these

Answer: (a)

Explanation: $\frac{2x-5}{3}=4$, *or*, $\frac{2x-5}{\not 3}\times \not 3=4\times 3$

Or, 2x - 5 = 12, or, (2x - 5) + 5 = 12 + 5

Or, 2x = 17, or, $\frac{2x}{2}=\frac{17}{2}$, or, $x=\frac{17}{2}$

4. If four times of a number is 56, then the number is:

(a) 7 (b) 21

(c) $\frac{7}{2}$ (d) 14

(e) None of these

Answer: (d)

Explanation: Let be the number is x.

Now, four times of the number = 4x

According to question, $4x = 56$, or, $\frac{4x}{4} = \frac{56}{4}$, *or*, $x = 14$

5. Find the value of m in the given equation.

3(2m - 1) + 4 = 16

(a) $\frac{5}{4}$ (b) $\frac{5}{2}$

(c) 5 (d) $\frac{1}{5}$

(*e*) None of these

Answer: (b)

Explanation: 3(2m - 1) + 4 = 16, or, 3(2m -1) + 4 - 4 = 16 - 4

Or, 3(2m - 1) = 12, or, $\frac{3(2m-1)}{3} = \frac{12}{3}$

Or, 2m - 1 = 4, or, 2m - 1 + 1 = 4 + 1, or 2m = 5

Or, $\frac{2m}{2} = \frac{5}{2}$, *or* $m = \frac{5}{2}$

6. If x = 2 and y = 5, then find the value of $x^3 + y^2$.

(a) 31 (b) 27

(c) 32 (d) 33

(*e*) None of these

Answer: (d)

Explanation: Given that x = 2 and y = 5.

Now, $x^3 + y^2 = (x)\times(x)\times(x) + (y)\times(y) = (2\times2\times2) + (5\times5) = 8 + 25 = 33$.

7. Simplify: $\frac{15a^2b}{5a}$

(a) 3a (b) 3b

(c) 3ab (d) $3a^2b$

(*e*) None of these

Answer: (c)

Explanation: $\frac{15a^2b}{5a} = \frac{15\times a^2\times b}{5\times a} = \frac{3\times \cancel{5}\times \cancel{a}\times a\times b}{\cancel{5}\times \cancel{a}} = \frac{3\times a\times b}{1} = 3ab$

8. Coefficient of x in $3x^2 + 4x + 1$ is:

(a) 3 (b) 4

(c) 1 (d) 8

(*e*) None of these

Answer: (b)

Explanation: Given equation is $3x^2 + 4x + 1$

Coefficient of x^2 is 3 and Coefficient of x is 4, Constant is 1.