CBSE Class 10 Mathematics Standard

Sample Paper - 10 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. State whether the following rational number will have a terminating decimal expansion or a nonterminating repeating decimal expansion.; $\frac{77}{210}$

OR

Find the simplest form of $\frac{1095}{1168}$.

2. Find the discriminant of equation: $2x^2 - 7x + 6 = 0$
3. Check whether the pair of equations $x + 3y = 6$, $2x - 3y = 12$ is consistent.
4. If a line intersects a circle in two distinct points, what is it called?

5. Find the first four terms of an A.P. whose first term is - 2 and common difference is - 2.

OR

The sum of three numbers in AP is 21 and their product is 231. Find the numbers.

6. For the AP -1.1, - 3.1, -5.1, - 7.1,... write the first term and the common difference.

7. Solve: $(x + 2)(3x - 5) = 0$

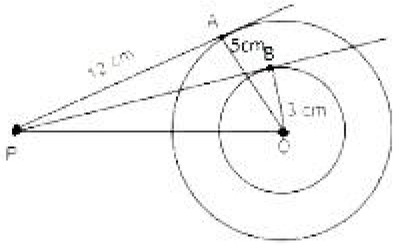
OR

State whether the quadratic equation has two distinct real roots. Justify your answer.

$x^2 - 3x + 4 = 0$

8. What is the angle between a tangent to a circle and the radius through the point of contact? Justify your answer.

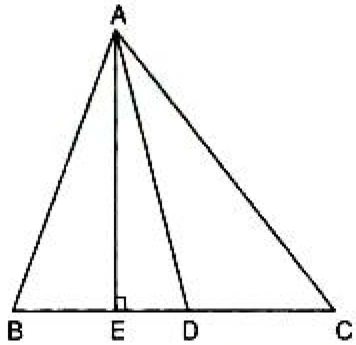
9. Two concentric circles with centre O are of radii 5 cm and 3 cm. From an external point P, two tangents PA and PB are drawn to these circles, respectively. If PA = 12 cm, then find the length of PB



OR

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (in cm) which touches the smaller circle.

10. In Fig. D is the mid-point of side BC and AE ⊥ BC. If BC= a AC= b, AB= c, ED= x, AD = p and AE = h, prove that: $(b^2 - c^2) = 2ax$



11. Which term of the A.P. 21,42,63,84,... is 420?

12. Prove the trigonometric identity:

    (cosecA - sinA) (secA - cosA) (tanA + cotA) = 1

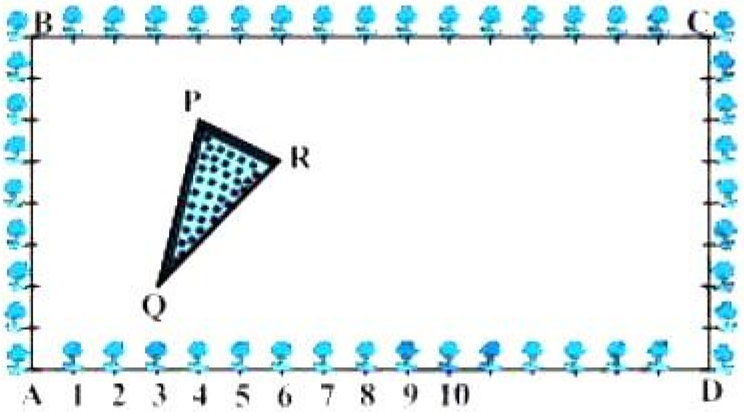
13. Find the value of x, if $\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$.

14. If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced cylinder to that of the original cylinder.

15. Which term of the A.P. 10, 7, 4, ... is -41?

16. A letter of the English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

17. The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar is planted on the boundary of the plot at a distance of 1m from each other. There is a triangular grassy lawn inside the plot as shown in Fig. The students have to sow seeds of flowering plants on the remaining area of the plot.



    i. Considering A as the origin, what are the coordinates of A?
        a. (0, 1)
        b. (1, 0)
        c. (0, 0)
        d. (-1, -1)
    ii. What are the coordinates of P?
        a. (4, 6)
        b. (6, 4)
        c. (4, 5)
        d. (5, 4)
    iii. What are the coordinates of R?
        a. (6, 5)
        b. (5, 6)

c. (6, 0)

d. (7, 4)

iv. What are the coordinates of D?

a. (16, 0)

b. (0, 0)

c. (0, 16)

d. (16, 1)

v. What are the coordinates of P if D is taken as the origin?

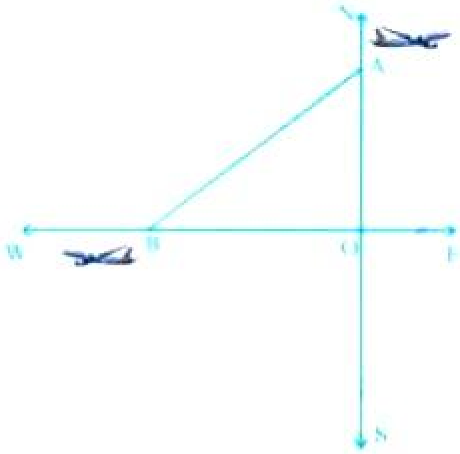
a. (12, 2)

b. (-12, 6)

c. (12, 3)

d. (6, 10)

18. An aeroplane leaves an Airport and flies due north at 300 km/h. At the same time, another aeroplane leaves the same Airport and flies due west at 400 km/h.



i. Distance travelled by the first aeroplane in 1.5 hours

a. 450 km

b. 300 km

c. 150 km

d. 600 km

ii. Distance travelled by the second aeroplane in 1.5 hours

a. 450 km

b. 300 km

c. 150 km

d. 600 km

iii. Which of the following line segment shows the distance between both the aeroplane?

a. OA
b. AB
c. OB
d. WB

iv. Which aeroplane travelled a long distance and by how many km?
a. Second, 150 km
b. Second, 250 km
c. First, 150 km
d. Fist, 250 km

v. How far apart the two aeroplanes would be after 1.5 hours?
a. 600 km
b. 750 km
c. 300 km
d. 150 km

19. A student noted the number of cars through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below:

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

i. While computing means of the grouped data, we assume that the frequencies are:
a. evenly distributed over all the classes
b. centered at the class marks of the classes
c. centered at the upper limits of the classes
d. centered at the lower limits of the classes

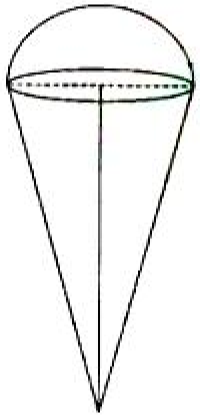
ii. The sum of the lower limits of the median class and modal class is:
   a. 40
   b. 60
   c. 80
   d. 90

iii. Find the mode of the data.
   a. 44.7
   b. 47.7
   c. 54.5
   d. 54.3

iv. Half of (upper-class limit + lower class limit) is:
   a. Class interval
   b. Classmark
   c. Class value
   d. Class size

v. The median of data is:
   a. 44
   b. 43
   c. 41
   d. 42

20. An ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped, conical shape with one end hemispherical, Rectangular shape with one end hemispherical and rectangular brick, etc. One day a child came to his shop and purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm.




By reading the above-given information, find the following:

i. Volume of only hemispherical end of the icecream is:

a. $\frac{1357}{42}\text{cm}^3$

b. $\frac{1375}{42}\text{cm}^3$

c. $\frac{1575}{42}\text{cm}^3$

d. $\frac{1373}{42}\text{cm}^3$

ii. The volume of the ice-cream without hemispherical end is:

a. $\frac{852}{14}\text{cm}^3$

b. $\frac{852}{41}\text{cm}^3$

c. $\frac{825}{41}\text{cm}^3$

d. $\frac{825}{14}\text{cm}^3$

iii. The TSA of the cone is given by:

a. $\pi rl + 2\pi r^2$

b. $\pi rl + 2\pi r$

c. $\pi rl + \pi r^2$

d. $2\pi rl + \pi r^2$

iv. The volume of the whole ice-cream is:

a. $91\frac{2}{3}\text{cm}^3$

b. $91\frac{3}{2}\text{cm}^3$

c. $19\frac{2}{3}\text{cm}^3$

d. $19\frac{3}{2}\text{cm}^3$

v. During the conversion of a solid from one shape to another the volume of the new shape will:

a. increase

b. decrease

c. double

d. remain unaltered

Part-B

21. Express $0.\overline{23}$ as a rational number in simplest form.

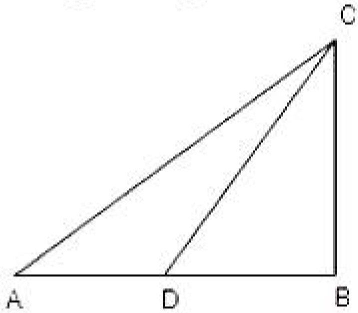
22. Show that A(-3, 2), B(-5, -5), C(2, - 3) and D(4, 4) are the vertices of a rhombus.

OR

A(3, 2) and B(–2, 1) are two vertices of a triangle ABC, whose centroid G has coordinates

$\left(\frac{5}{3}, -\frac{1}{3}\right)$. Find the coordinates of the third vertex C of the triangle.

23. If α and β are zeroes of the polynomial f(x) = x^2- x - k, such that $\alpha - \beta$= 9, find k.
24. In the given figure , if CD = 17 m , BD = 8 m and AD = 4 cm, find the value of AC.

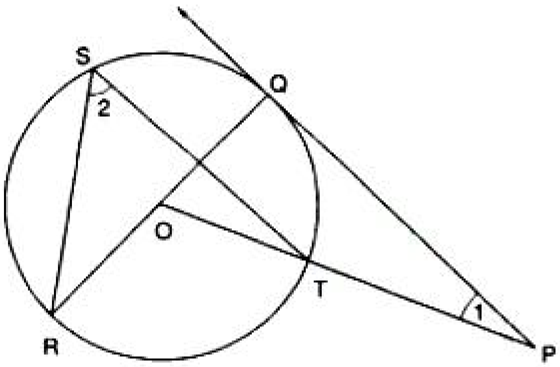


25. If sin(A - B) = $\frac{1}{2}$ and cos(A + B) = $\frac{1}{2}$, 0° < (A + B) < 90° and A > B then find A and B.

OR

Prove that: $\frac{\cot A+\tan B}{\cot B+\tan A} = \cot A \tan B$

26. In figure, PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and QOR is a diameter. If ∠POR = 130° and S is a point on the circle, find ∠1 +∠2.



27. Show that $5 - \sqrt{3}$ is irrational.
28. The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, then find the two numbers.

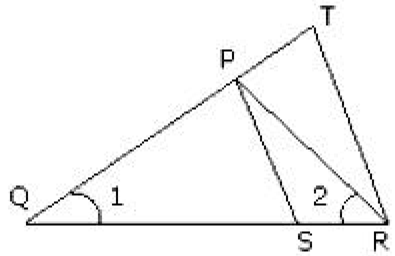
OR

Prove that the equation $x^2(a^2 + b^2)$+ 2x (ac + bd) +$(c^2 + d^2)$= 0 has no real root, if ad $\neq$ bc.

29. Find the zeroes of the polynomial $4x^2 + 5\sqrt{2}x - 3$ by factorisation method and verify the relationship between the zeroes and coefficient of the polynomial.
30. In a $\triangle ABC$, M and N are points on the sides AB and AC respectively such that BM = CN. If $\angle B = \angle C$ then show that $MN \| BC$.

OR

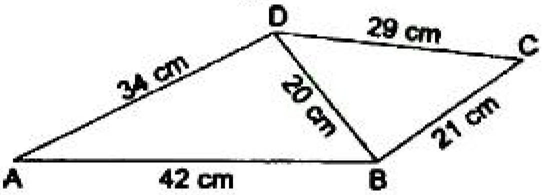
In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and ∠ 1 = ∠ 2, show that △ PQS ≅ △ || QTR



31. A bag contains 31 balls out of which x are red. A ball is drawn at random from the bag. Find the probability that it is
    i. red
    ii. not red.
32. A flagstaff stands on the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flagstaff is 60^{0} and from the same point, the angle of elevation of the top of the tower is 45^{0}. Find the height of the flagstaff.
33. Find the mode of the following distribution:

Class Interval	10 -14	14 - 18	18 - 22	22 - 26	26 - 30	30 - 34	34 - 38	38 - 42
Frequency	8	6	11	20	25	22	10	4

34. Find the area of the quadrilateral ABCD in which AB = 42 cm, BC = 21 cm, CD = 29 cm, DA = 34 cm and diagonal BD = 20 cm.



35. In a ΔABC, ∠C = 3 ∠B = 2 (∠ A + ∠B). Find the three angles.
36. A 1.6 m tall girl stands at a distance of 3.2 m from a lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using
    i. trigonometric ratios
    ii. property of similar triangles.

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Solution

Part-A

1. $\frac{77}{210}=\frac{11}{30}=\frac{11}{2\times3\times5}$ Here, $q=2\times3\times5$
   which is not of the form 2^n5^m.
   So, the rational number $\frac{77}{210}$
   has a non-terminating repeating decimal expansion.

OR

   Prime factors of 1095 and 1168 are
   1095 = 3 × 5 × 73
   $1168=2\times2\times2\times2\times73$
   $\frac{1095}{1168}=\frac{15\times73}{2\times2\times2\times2\times73}$
   $=\frac{15}{16}$

2. Given, $2x^2 - 7x + 6 = 0$
   a = 2, b = -7 and c = 6
   $\therefore$ D = b^2 -4ac
   = $(-7)^2$ - 4(2)(6)
   = 49 - 48
   = 1
3. Given pair of equations,
   x + 3y = 6, 2x - 3y = 12
   Since $\frac{a_1}{a_2}=\frac{1}{2}$ and $\frac{b_1}{b_2}=\frac{3}{-3}$ = -1,
   $\frac{c_1}{c_2}=\frac{-6}{-12}=\frac{1}{2}$
   $\therefore\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$
   $\therefore$ The system is consistent.
4. The line which intersects a circle in two distinct points is called secant.
5. given a_1= -2, common difference d= -2

$a_1 = -2$,

$a_2 = a_1 + d = -2 + (-2) = -4$

$a_3 = a_2 + d = -4 + (-2) = -6$

$a_4 = a_3 + d = =-6 + (-2) = -8$

∴ First four terms are - 2, - 4, - 6, - 8

OR

Let the required numbers be (a-d), a and (a + d).............(1)

Then, according to question, (a - d) + a + (a + d) = 21

⇒ 3a = 21

⇒ a=7.

And, (a - d) × a × (a+d) = 231

⇒a $(a^2 - d^2) = 231$

⇒ $7(49 - d^2) = 231$ [∵ a = 7]

⇒ $7d^2 = 343 - 231 = 112$

⇒ $d^2 = 16$

⇒ d = ±4.

Thus, a = 7 and d = ±4. Now substitute these values of a and d in the above equation (1).

Therefore, the required numbers are(3, 7, 11) or (11, 7, 3).

6. Given progression is: -1.1, - 3.1, -5.1, - 7.1,...

   First term (a) = -1.1

   We know that common difference is the difference between any two consecutive terms of an A.P.

   So, common difference(d) = (-3.1) - (-1.1)

   = -3.1 + 1.1

   = -2

7. We have

   $$(x+2)(3x-5)=0$$

   $$\Rightarrow x+2=0 \; or \; 3x-5=0$$

   ⇒ x = -2 or x = $\frac{5}{3}$

   Hence, the roots of the given equation are -2 and $\frac{5}{3}$

OR

The equation $x^2 - 3x + 4 = 0$ has no real roots.

$\because D = b^2 - 4ac$

$= (-3)^2 - 4(1)(4)$

$= 9 - 16 < 0$

Hence, the roots are imaginary.

8. The angle between a tangent to a circle and the radius through the point of contact is 90°. Because radius through the point of contact of the tangent to a circle is perpendicular to the tangent.

9. Two concentric circles with centre O are of radii 5 cm and 3 cm. From an external point P, two tangents PA and PB are drawn to these circles, respectively. If PA = 12 cm, then we have to find the length of PB.

   We know that radius is perpendicular to the tangent at the point of contact, therefore, $AO \perp AP$.

   Now, in right-angled triangle PAO, $\angle PAO = 90^\circ$

   $OP^2 = (PA)^2 + (AO)^2$ [by Pythagoras theorem]

   $\Rightarrow OP = \sqrt{(PA)^2 + (AO)^2}$

   $\Rightarrow OP = \sqrt{169} = 13\ cm$

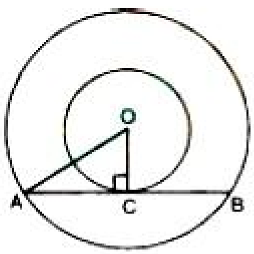
   Similarly, in right-angled triangle PBO, $\angle PBO = 90^\circ$

   $PO^2 = (PB)^2 + (OB)^2$ [by Pythagoras theorem]

   $\Rightarrow PB = \sqrt{(OP)^2 - (OB)^2}$

   $\Rightarrow PB = \sqrt{(13)^2 - (3)^2} = \sqrt{160} = 4\sqrt{10}\ cm$ [$\because$ radius, OB = 3 cm, given]

OR



Now, In $rt. \triangle\ OCA$

$AO^2 = OC^2 + AC^2$

$\Rightarrow AC^2 = 5^2 - 3^2$

$\Rightarrow AC = 4$

We know, $OC \perp AB$

$\therefore AC = BC$

Hence, $AC = 2(4) = 8cm$

10. $b^2 - c^2 = p^2 + ax + \frac{a^2}{4} - \left(p^2 - ax + \frac{a^2}{4}\right) = p^2 - p^2 + ax + ax + \frac{a^2}{4} - \frac{a^2}{4} = 2ax$

11. A.P. = 21, 42, 63, 84........

First term =21 and Common difference(d) = 42 - 21 = 21

Suppose nth term of A.P. is 420

Then, a_n = 420

$\Rightarrow$a + (n - 1)d = 420

$\Rightarrow$21 + (n - 1)$\times$21 = 420

$\Rightarrow$ 21 + 21n - 21 = 420

$\Rightarrow$21n = 420

$\Rightarrow \quad n = \frac{420}{21} = 20$

12. To prove:

(cosecA - sinA) (secA - cosA) (tanA + cotA) = 1

LHS $= (\cos ecA - \sin A)(\sec A - \cos A)(\tan A + \cot A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$$\left[\begin{array}{c}\because \cos ecA = \dfrac{1}{\sin A}, \sec A = \dfrac{1}{\cos A}, \\ \tan A = \dfrac{\sin A}{\cos A}, \cot A = \dfrac{\cos A}{\sin A}\end{array}\right]$$

$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{\sin^2 A+\cos^2 A}{\cos A \sin A}\right)$

$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} \times \frac{1}{\cos A \sin A} \quad \left[\begin{array}{c}\because 1 - \sin^2 A = \cos^2 A, 1 - \cos^2 A = \sin^2 A, \\ \sin^2 A + \cos^2 A = 1\end{array}\right]$

$= \frac{\cos A \times \cos A}{\sin A} \times \frac{\sin A \times \sin A}{\cos A} \times \frac{1}{\cos A \sin A}$

= 1

= RHS

13. Given ,

$\sqrt{3} \tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$

$\Rightarrow \sqrt{3} \tan 2x = \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$\Rightarrow \sqrt{3} \tan 2x = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \sqrt{3}\tan 2x = 1$

$\Rightarrow \tan 2x = \frac{1}{\sqrt{3}}$

$\Rightarrow$ tan 2x = tan30 ($Since, \tan 30^\circ = \sqrt{\frac{1}{3}}$)

$\Rightarrow 2x = 30^\circ$

$\Rightarrow x = 15^\circ$

14. If the radius of original cylinder = r

    Then the radius of the base of a right circular cylinder = $\frac{r}{2}$

    Height of reduced cylinder = Height of original cylinder = h

    Thus,

    $\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{r}{2}\right)^2 h}{\pi r^2 h}$

    $= \frac{1}{4} = 1:4$

15. Given Arithmetic, progression is 10, 7, 4, ...

    Here, $a = 10,\ and\ d = 7 - 10 = -3.$

    Let $a_n = -41$

    $\Rightarrow a + (n-1)d = -41$

    $\Rightarrow 10 + (n-1)(-3) = -41$

    $\Rightarrow 10 - 3n + 3 = -41$

    $\Rightarrow 13 - 3n = -41$

    $\Rightarrow 3n = 41 + 13$

    $\Rightarrow 3n = 54$

    $\Rightarrow n = 18$

    Therefore, -41 is the 18th term of the given A.P.

16. In the English language, there are 26 alphabets.

    Total number of outcomes = 26

    Consonant are 21.

    Number of favourable outcomes = 21

    Probability of event happen$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

    The probability of chosen a consonant = $\frac{21}{26}$

17. It can be observed that the coordinates of point P, Q and R are (4, 6), (3, 2), and (6, 5) respectively.

    i. (c) (0, 0)

    ii. (a) (4, 6)

    iii. (a) (6, 5)

iv. (a) (16, 0)

v. (b) (-12, 6)

18. i. (a) 450 km

ii. (d) 600 km

iii. (b) AB

iv. (a) Second, 150 km

v. (b) 750 km

19. i. (b) Centered at the class marks of the classes

ii. (c) 80

iii. (a) 44.7

iv. (b) Classmark

v. (d) 42

20. For cone, Radius of the base (r)

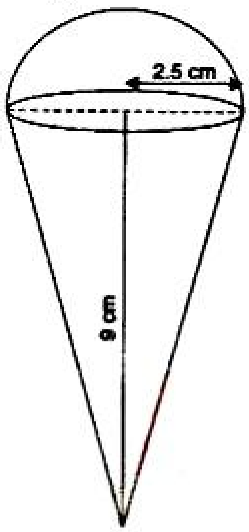
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

i. (a) $\frac{1357}{42}\text{cm}^3$

ii. (d) The volume of the ice-cream without hemispherical end = Volume of the cone $= \frac{825}{14}\text{cm}^3$

iii. (c) The TSA of the cone is given by:

$\pi rl + \pi r^2$

iv. (a) Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere

$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$

$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

v. (d) remain unaltered

Part-B

21. Let $x = 0.\overline{23}$. Then,

x = 0.232323... ...(i)

Therefore, 100x = 23.2323.... ...(ii)

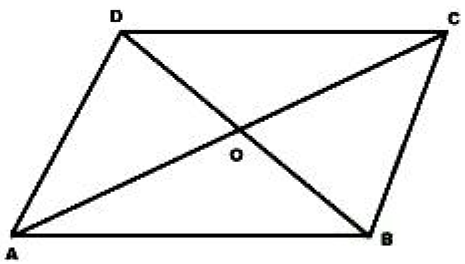
On subtracting (i) from (ii), we get

99x = 23

$\Rightarrow x = \frac{23}{99}$

Hence, $0.\overline{23} = \frac{23}{99}$.

Which is a rational number and in its simplest form.

22. 

We know that all the sides of a rhombus are equal and diagonals are not equal.

i.e. In rhombus ABCD, AB = BC = CD = DA and $AC \neq BD$

Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here A(-3, 2), B(-5, -5), C(2, - 3) and D(4, 4)

AB = $\sqrt{\{-5-(-3)\}^2 + (-5-2)^2} = \sqrt{4+49} = \sqrt{53}$

BC = $\sqrt{\{2-(-5)\}^2 + \{-3-(-5)\}^2} = \sqrt{49+4} = \sqrt{53}$

CD = $\sqrt{(4-2)^2 + \{4-(-3)\}^2} = \sqrt{4+49} = \sqrt{53}$

AD = $\sqrt{\{4-(-3)\}^2 + (4-2)^2} = \sqrt{49+4} = \sqrt{53}$

Thus, AB = CD = BC = AD i.e. all sides are equal.

Now,

AC = $\sqrt{(2+3)^2 + (-3-2)^2}$

$= \sqrt{5^2 + 5^2} == \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$

BD $= \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{9^2 + 9^2} = \sqrt{81 + 81} = 9\sqrt{2}$

Thus, AC $\neq$ BD i.e diagonals are not equal.

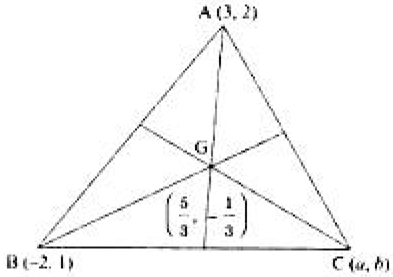
$\therefore$ ABCD is a rhombus.

OR

Let C(a, b) be the third vertex of $\triangle$ABC.

Then, the coordinates of the centroid of $\triangle$ABC are:

$G\left(\frac{3-2+a}{3}, \frac{2+1+b}{3}\right) = \left(\frac{a+1}{3}, \frac{b+3}{3}\right)$ ...(i)



But, the coordinates of the centroid are $G\left(\frac{5}{3}, -\frac{1}{3}\right)$ [Given] ....(ii)

Comparing (i) and (ii), we get

$\frac{a+1}{3} = \frac{5}{3} and \frac{b+3}{3} = \frac{-1}{3}$

$\Rightarrow$ a = 5 - 1 = 4 and b = -1 - 3 = -4

Hence, the coordinates of the third vertex C of $\triangle$ABC are (4, -4)

23. Since α and β are the zeroes of the polynomial, then

$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\Rightarrow \alpha + \beta = -\left(\frac{-1}{1}\right) = 1$.........(i)

Given, $\alpha - \beta = 9$...............(ii)

Solving (i) and (ii), $\alpha = 5, \beta = -4$

$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow \alpha\beta = -k$

$\Rightarrow$ (5)(-4) = -k

$\Rightarrow$ k = 20

So, required value of k is 20

24. Using Pythagoras theorem in $\triangle DBC$,

$CD^2 = BD^2 + BC^2$

$\Rightarrow 17^2 = 8^2 + BC^2$

$\Rightarrow BC^2 = 17^2 - 8^2$

$\Rightarrow BC^2 = 289 - 64$

$\Rightarrow BC^2 = 225$

$\Rightarrow BC = 15$ m.

Now, to find AC, apply Pythagoras theorem in $\triangle ABC$

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (AD + DB)^2 + BC^2$

$\Rightarrow AC^2 = (4 + 8)^2 + (15)^2$

$\Rightarrow AC^2 = (12)^2 + (15)^2$

$\Rightarrow AC^2 = 144 + 225$

$\Rightarrow AC^2 = 369$

$\Rightarrow AC = \sqrt{369}$ m

25. sin(A - B) = $\frac{1}{2} \Rightarrow \sin(A - B) = \sin 30^\circ$

$\Rightarrow$ A - B = 30°.........(i)

cos(A + B)= $\frac{1}{2} \Rightarrow \cos(A + B) = \cos 60^\circ$

$\Rightarrow$A + B = 60°.........(ii)

Solving (i) and (ii), we get

$2\text{A} = 90^\circ \Rightarrow \text{A} = 45^\circ$

Putting A = 45° in (i), we get

45° - B= 30° $\Rightarrow$ B = 45° - 30° = 15°

Hence,A = 45°, B = 15°

OR

We have,

L.H.S= $\frac{\cot A + \tan B}{\cot B + \tan A}$

$= \frac{\frac{\cos A}{\sin A}}{\frac{\cos B}{\sin B}} + \frac{\frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A}}$

$= \frac{\frac{(\cos A \cos B + \sin A \sin B}{\sin A \cos B}}{\frac{(\cos A \cos B + \sin A \sin B)}{\sin B \cos A}}$

$= \frac{(\cos A \cos B + \sin A \sin B)}{\sin A \cos B} \times \frac{\sin B \cos A}{(\cos A \cos B + \sin A \sin B)}$

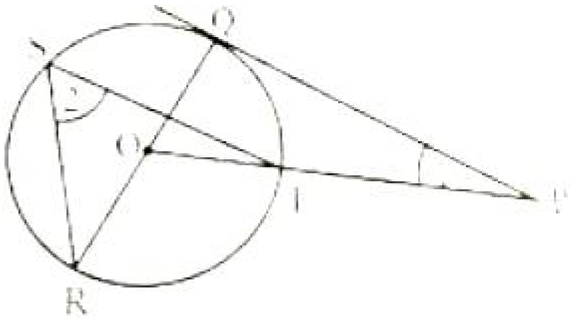
$= \frac{\cos A}{\sin A} \cdot \frac{\sin B}{\cos B}$

$= \cot A \cdot \tan B = R.H.S$

therefore, $\frac{\cot A+\tan B}{\cot B+\tan A}=\cot A\cdot\tan B$

Hence proved.

26. In the given figure



$\angle 2=\frac{1}{2}\angle ROT$ (Angle subtended at the center by the same arc)

$\angle 2=\frac{1}{2}\times 130^{\circ}=65^{\circ}$

$\angle ROT=\angle 1+\angle PQO$

$\angle 1=130^{\circ}-90^{\circ}=40^{\circ}$

$\therefore \quad \angle 1+\angle 2=65^{\circ}+40^{\circ}=105^{\circ}$

27. Let us assume, to the contrary, that $5-\sqrt{3}$ is rational.

That is, we can find coprime numbers a and b (b ≠ 0) such that $5-\sqrt{3}=\frac{a}{b}$

Therefore, $5-\frac{a}{b}=\sqrt{3}$

Rearranging this equation, we get $\sqrt{3}=5-\frac{a}{b}=\frac{5b-a}{b}$

Since a and b are integers, we get $5-\frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational

This contradiction has arisen because of our incorrect assumption that $5-\sqrt{3}$ is rational.

So, we conclude that $5-\sqrt{3}$ is irrational.

28. Let first number be x.

Then, second number = x + 4

According to the question ,

$\frac{1}{x}-\frac{1}{x+4}=\frac{4}{21}$

$\frac{x+4-x}{x(x+4)}=\frac{4}{21}$

$\frac{4}{x^2+4x}=\frac{4}{21}$

$4(x^2+4x)=84$

$\Rightarrow 4x^2+16x-84=0$

$\Rightarrow 4(x^2 + 4x - 21) = 0$

$\Rightarrow (x^2 + 4x - 21) = 0$

$\Rightarrow (x + 7)(x - 3) = 0$

$\Rightarrow x + 7 = 0$ or $x - 3 = 0$

$\Rightarrow x = -7$ or $x = 3$

Therefore, the two numbers are 3 and 7 or -7 and -3

OR

According to question, the given equation is

$x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$.

Let D be the discriminant of this equation.

Therefore, $D = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

$\Rightarrow D = 4[(ac + bd)^2 - (a^2 + b^2)(c^2 + d^2)]$

$\Rightarrow D = 4[a^2c^2 + b^2d^2 + 2ac.bd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$

$\Rightarrow D = 4[2ac.bd - a^2d^2 - b^2c^2] = -4[a^2d^2 + b^2c^2 - 2ad.bc] = -4(ad - bc)^2$

It is given that $ad \neq bc$.

$\Rightarrow ad - bc \neq 0$

$\Rightarrow (ad - bc)^2 > 0$

$\Rightarrow -4(ad - bc)^2 < 0$

$\Rightarrow D < 0$.

Therefore, the given equation has no real root.

29. $4x^2 + 5\sqrt{2}x - 3$

$= 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3$

$= 2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3)$

$= (2\sqrt{2}x - 1)(\sqrt{2}x + 3)$

$\Rightarrow x = \frac{1}{2\sqrt{2}}$ and $x = -\frac{3}{\sqrt{2}}$ are zeroes of the polynomial

If given polynomial is $4x^2 + 5\sqrt{2}x - 3$, then $a = 4$, $b = 5\sqrt{2}$ and $c = -3$

Now, Sum of zeroes $= \frac{1}{2\sqrt{2}} + \frac{-3}{\sqrt{2}} = \frac{1-6}{2\sqrt{2}} = \frac{-5}{2\sqrt{2}}$ .... (i)

Also, $\frac{-b}{a} = \frac{-5\sqrt{2}}{4} = \frac{-5}{2\sqrt{2}}$ .... (ii)

From (i) and (ii)

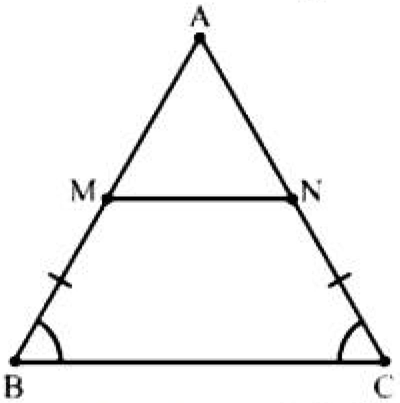
Sum of zeroes = $\frac{-b}{a}$

Product of zeroes = $\frac{1}{2\sqrt{2}} \times \frac{-3}{\sqrt{2}} = \frac{-3}{4}$ .... (iii)

Also, $\frac{c}{a} = \frac{-3}{4}$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

30. 

It is given that in $\triangle ABC$, $\angle B = \angle C$.

Therefore, AB = AC (Sides opposite to equal angle are equal)

Subtracting BM from both sides, we have

AB - BM = AC - BM

$\Rightarrow$ AB - BM = AC - CN ($\because$ BM = CN)

$\Rightarrow$ AM = AN

$\therefore \angle AMN = \angle ANM$ (Angles opposite to equal sides are equal)

Now, in $\triangle ABC$, we have

$\angle A + \angle B + \angle C = 180^0$ ......(i)

(Angle Sum Property of triangle)

Again in $\triangle AMN$,

$\angle A + \angle AMN + \angle ANM = 180^\circ$ .....(ii) (Angle Sum Property of the triangle)

From (i) and (ii), we obtain

$\angle B + \angle C = \angle AMN + \angle ANM$

$\Rightarrow 2\angle B = 2\angle AMN$

$\Longrightarrow \angle B = \angle AMN$

Since, $\angle B$ and $\angle AMN$ are corresponding angles. $\therefore MN \| BC$

OR

Given: , $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$, Proof: As $\angle 1 = \angle 2$,

PQ = PR ....(i) [sides opposite to equal angles are equal]

Also $\frac{QR}{QS} = \frac{QT}{PR}$ (given) ......(ii)

$\Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$ From (i) and (ii)

In $\triangle$PQS and $\triangle$TQR, we have

$\frac{QR}{QS} = \frac{QT}{PQ} = \frac{QS}{QT} \Rightarrow \frac{QR}{QP}$ [From (ii)]

Also $\angle$PQS = $\angle$TQR [Common]

$\therefore \triangle PQS \cong \triangle TQR$ [SAS similarity]

31. Total No of balls = 31

So total no of outcomes = 31

i. **Red ball:**

Let R be the event of getting red ball

No of red balls = x

So no of events favoring R are = x

P(R) = $\frac{x}{31}$

ii. **Not red ball:**

Let N be the event of getting not red ball

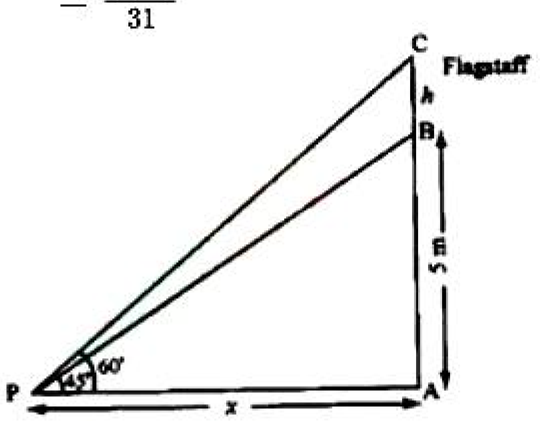
Both events are supplementary.

So P(N) + P(R) = 1

P(N) = 1 - P(R)

= 1 - $\frac{x}{31}$

$= \frac{31-x}{31}$

32. 

Let the height of flagstaff = h m = CB

height of tower = 5 m = AB

Height of top of flagstaff from ground = (h + 5) m = AC

Let distance of point P from tower = x

Using $\triangle PAB, \frac{x}{5} = cot45^0 \rightarrow \frac{x}{5} = 1 \implies x = 5m$

Using $\triangle PAC, \frac{\mathrm{x}}{\mathrm{h+5}} = \cot 60^\circ$

$\Rightarrow \frac{x}{h+5} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{h+5}{\sqrt{3}}$ ....(ii)

From (i) and (ii), we get

$\frac{h+5}{\sqrt{3}} = 5$

$\Rightarrow h + 5 = 5\sqrt{3}$

$\therefore h = 5\sqrt{3} - 5$

= 5(1.73 - 1) = $5 \times .73 = 3.65$ m

$\therefore$ the height of the flagstaff is 3.65 m approx.

33. The modal class is 26 - 30, as it has the maximum frequency.

$\therefore x_k = 26, h = 4, f_k = 25, f_{k-1} = 20, f_{k+1} = 22$

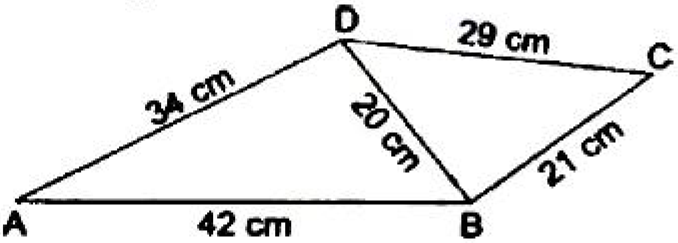
Mode, $M_0 = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$

$= 26 + \left(\frac{(25-20)}{(2\times25-20-22)} \times 4\right)$

$= 26 + \frac{5}{2}$

$= 26 + 2.5 = 28.5$

34. Area of quad. ABCD = Area of $\triangle$ABD + Area of $\triangle$DBC



For area of $\triangle$ABD

Let a = 42cm, b = 34 cm, and c = 20 cm

s = $\frac{a+b+c}{2} = \frac{(42+34+20)}{2}$ cm = 48

Then, (s - a) = 6, (s - b) = 14 and (s - c) = 28

Area of $\triangle$ABD = $\sqrt{s \times (s-a)(s-b)(s-c)}$

= $\sqrt{48 \times 6 \times 14 \times 28}$ cm^2

= 336 cm^2

For area of $\triangle$DBC

a = 29 cm, b = 21 cm, c = 20 cm

s = $\frac{a+b+c}{2} = \frac{(29+21+20)}{2}$ cm = 35 cm

Area of $\triangle$DBC = $\sqrt{s \times (s-a)(s-b)(s-c)}$ sq. units

= $\sqrt{35 \times 6 \times 14 \times 15}$ cm^2

= 210 cm^2

Area of quad. ABCD = Area of $\triangle$ABC + Area of $\triangle$DBC

= (336 + 210) cm^2 = 546 cm^2

35. $\angle C = 3\angle B = 2(\angle A + \angle B)$

Taking $3\angle B = 2(\angle A + \angle B)$

$\Rightarrow \angle B = 2\angle A$

$\Rightarrow 2\angle A - \angle B = 0$ .... (1)

We know that the sum of the measures of all angles of a triangle is 180°.

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle B + 3\angle B = 180°$

$\Rightarrow \angle A + 4\angle B = 180°$ .... (2)

Multiplying equation (1) by 4, we obtain:

$8\angle A - 4\angle B = 0$ ...... (3)

Adding equations (2) and (3), we get

$9\angle A = 180°$

$\Rightarrow \angle A = 20°$

From eq. (2), we get,

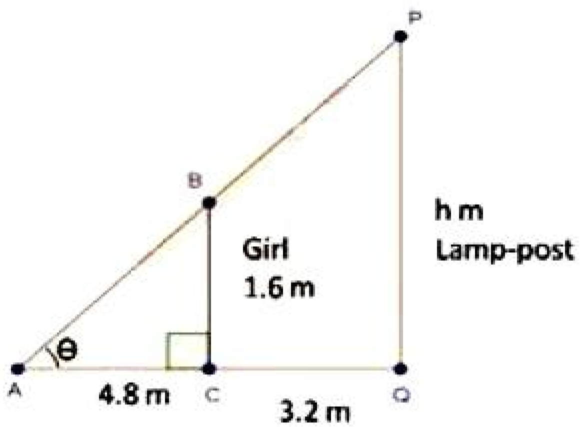
$20° + 4\angle B = 180°$

$\Rightarrow \angle B = 40°$

And $\angle C = 3 \times 40° = 120°$

Hence the measures of $\angle A$, $\angle B$ and $\angle C$ are 20°, 40° and 120° respectively.

36.



Suppose BC be the height of girl of length 1.6 m

Let PQ be the height of lamp - post of length = h m

Suppose AC be the length of shadow = 4.8 m

Distance between girl and lamp - post = 3.2 m

(i) Using trigonometric ratios

In right ΔACB

$\tan\theta = \frac{BC}{AC}$

$\Rightarrow \tan\theta = \frac{1.6}{4.8} = \frac{1}{3}$ ...(1)

In right ΔPQA

$\tan\theta = \frac{PQ}{AQ}$

$\Rightarrow \tan\theta = \frac{h}{8}$ ...(2)

Compare equation (1) and equation (2)

$\frac{h}{8} = \frac{1}{3}$

$\Rightarrow h = \frac{8}{3}m$ = 2.67 meter

(ii) Using similar triangles

In ΔACB and ΔAQP

$\angle A = \angle A$

and, $\angle ACB$ and $\angle AQP$ [each 90^{0}]

Then, $\Delta ACB - \Delta AQP$ [by AA similarity]

$\therefore \frac{AC}{AQ} = \frac{BC}{PQ}$ [c.p.c.t]

$\Rightarrow \frac{4.8}{8} = \frac{1.6}{h}$

$\Rightarrow h = \frac{1.6\times 8}{4.8} = \frac{8}{3}m$ = 2.67 M

Hence, length of the pole is 2.67 m(nearly)