☆ Signal Flow Graph (SFG): 75

* Purpose:

→ To find the overall TF. of the system.

→ SFG is the graphical representation of the set of linear algebric eqⁿs betⁿ i/p and output.

→ The SFG analysis developed to avoid the mathematical calculation like solving the integro, differential eqⁿs (or) linear algebric eqⁿs.

⇒ The SFG analysis is very easy as compared to solving the mathematical eqⁿs

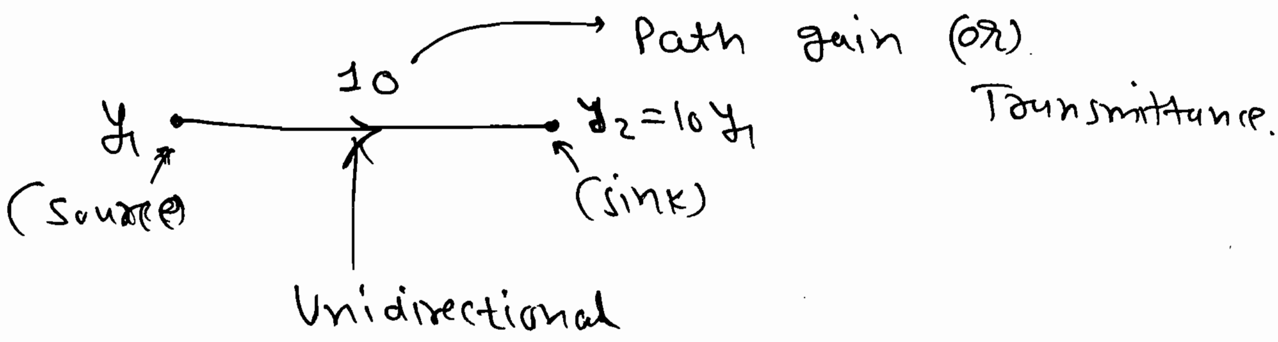
* Construction of SFG to the linear algebric eqⁿs:

① $y_2 = 10 y_1$

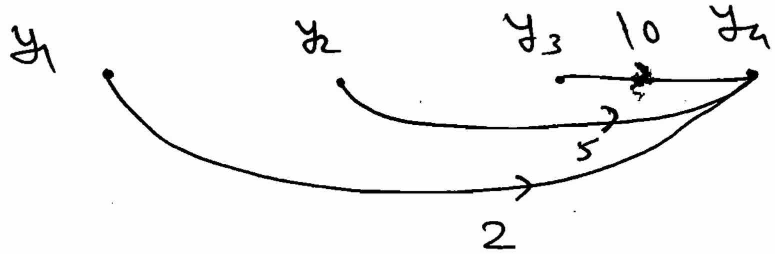
⇒ $y_2 = 10 \cdot y_1$ ← i/p node

o/p node

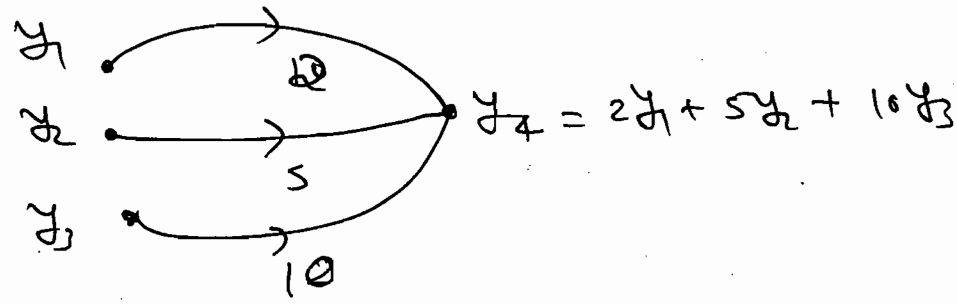
gain

⇒ 

② $y_4 = 2y_1 + 5y_2 + 10y_3$



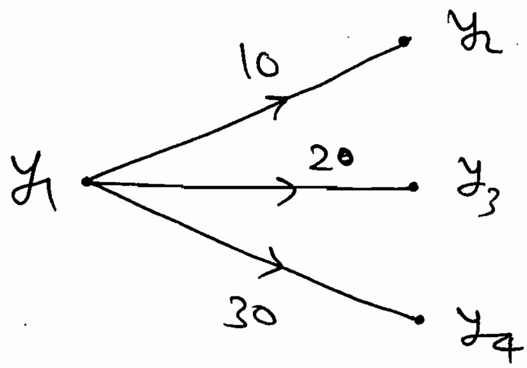
(many to one)



③ $y_2 = 10y_1$.

$y_3 = 20y_1$.

$y_4 = 30y_1$.



(one to many)

* Construct the SFG from the given sets of Linear algebric eqns:

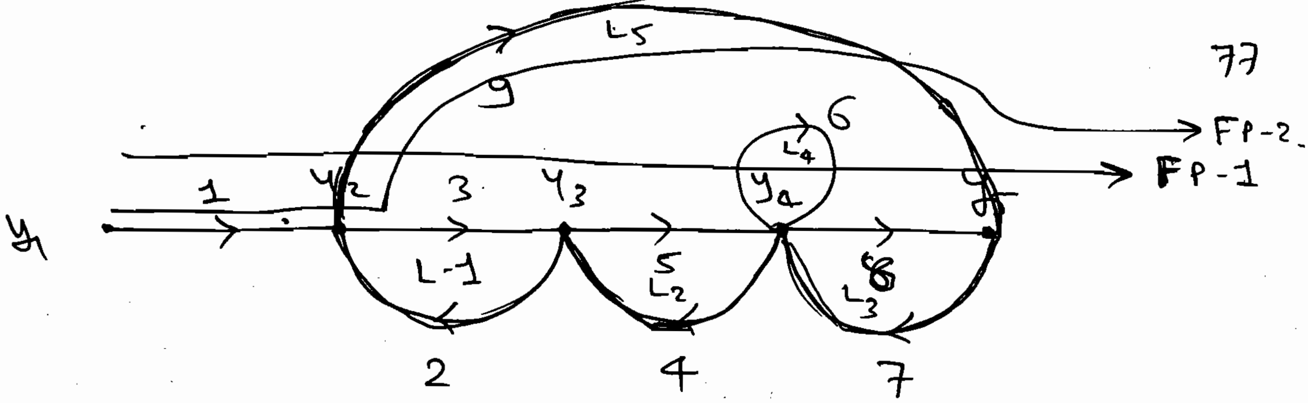
① $y_2 = y_1 + 2y_3$.

$y_3 = 3y_2 + 4y_4$.

$y_4 = 5y_3 + 6y_4 + 7y_5$.

$y_5 = 8y_4 + 9y_2$.

Sol^n:



Q-2 Find the no. of forward paths, no. of individual loops, no. of two non-touching loops to the above signal graph.

Sol^n: Forward path:

$F_1 \rightarrow 1 \cdot 3 \cdot 5 \cdot 8.$

$F_2 \rightarrow 1 \cdot 9.$

→ no. of individual loop:

(node)

$L_1 : 3 \cdot 2 \; (2,3)$ $\quad L_4 : 6 \; (4)$

$L_2 : 5 \cdot 4 \; (3,4)$ $\quad L_5 : 9 \cdot 7 \cdot 4 \cdot 2. \; (2,3,4,5).$

$L_3 : 8 \cdot 7 \; (4,5)$

→ Two non-touching loop. (if common node then touching otherwise non-touching).

⊙ $L_1 \rightarrow$ L_2 ✗, L_3 ✓, L_4 ✓, L_5 ✗

⊙ $L_2 \rightarrow$ L_1 ✗, L_3 ✗, L_4 ✗, L_5 ✗

⊙ $L_3 \rightarrow$ L_1 ✓, L_2 ✗, L_4 ✗, L_5 ✗

⊙ $L_4 \rightarrow$ L_1 ✓, L_2 ✗, L_3 ✗, L_5 ✗

⊙ $L_5 \rightarrow$ L_1 ✗, L_2 ✗, L_3 ✗, L_4 ✗

no·06

So, non-touching loop → 2.

$L_1 L_3$, $L_1 L_4$.

* Loop:

⇒ It is a Path which terminate at the same node where it is started.

* Non-touching Loops:

→ If there is a no common node betⁿ two (OR) more loops then it is said to be the non-touching loop.

* Forward Path:

⇒ It is the Path from input to output.

* Input node:

⇒ A node which has only outgoing branches is called Input node.

* Output node:

⇒ A node which has only incomming branches is called output node.

* Chain (OR) Link node:

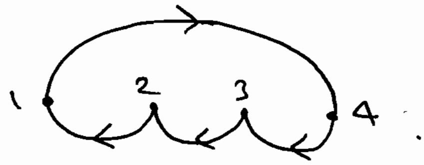
⇒ The node which has both incoming

and outgoing branch.

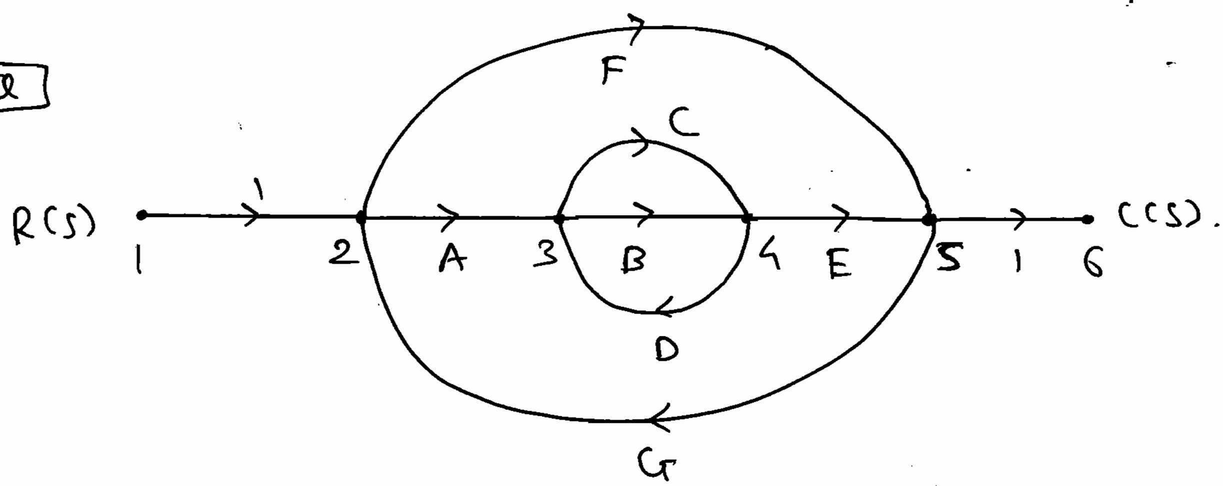
NOTE:

⇒ The condition to select the correct path (or) loop is each node should be touch only once.

⇒ [Whenever many feedback are cascade with only one forward path it forms a loop].

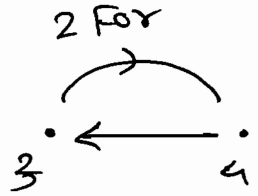


$\boxed{a}$



Sol^n:

D ⇒

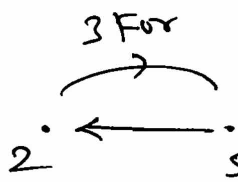


⇒ 2L

$L_1 = BD \rightarrow 3,4$

$L_2 = CD \Rightarrow 3,4.$

G ⇒



⇒ 3L

$L_3 = ABEG \rightarrow 2,3,4,5$

$L_4 = ACEG \rightarrow 2,3,4,5$

$L_5 = FG \rightarrow 2, 5.$

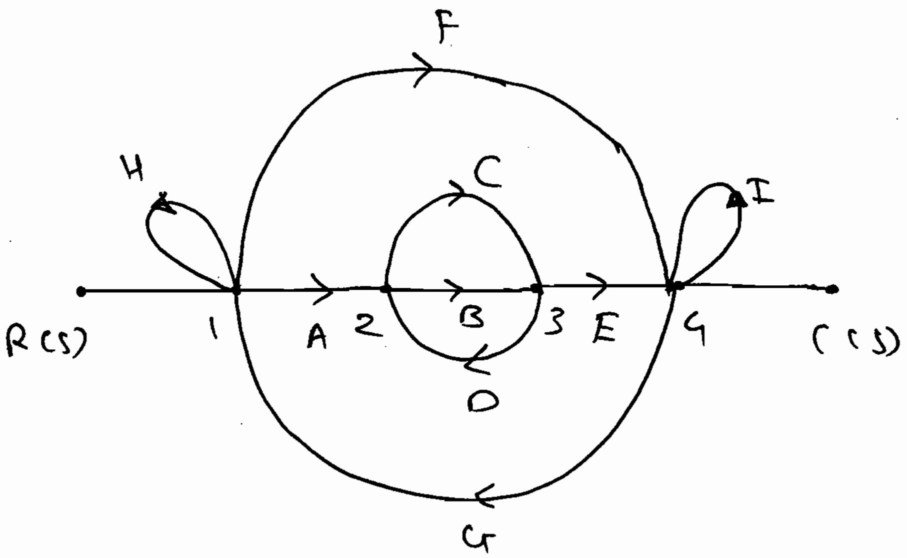
indi → 5L

2 - NTL:

$L_1 L_5$ (3,4,2,5) ✓

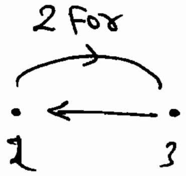
$L_2 L_5$ (3,4,2,5) ✓

$\boxed{Q}$



Soln:

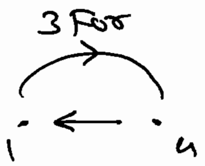
D ⇒



⇒ (2L) → L_1 = BD → 2, 3

→ L_2 = CD → 2, 3.

G ⇒



⇒ (3L) → L_3 = ABEG → 1, 2, 3, 4.

→ L_4 = ACEG → 1, 2, 3, 4.

→ L_5 = FG → 1, 4.

(2L) → L_6: H, 1

→ L_7: I, 4.

2 NTL

~~L_5~~ $L_1 L_5$ = BDFG → 1, 2, 3, 4

$L_1 L_7$ = BDI → 2, 3, 4

$L_2 L_5$ = CDFG → 1, 2, 3, 4

$L_2 L_6$ = CDH → 1, 2, 3.

$L_2 L_7$ = CDI → 2, 3, 4.

$L_1 L_6$ = BDH → 1, 2, 3.

~~$L_3 L_7$ = ABEG I → 1, 2, 3~~

3 NTL

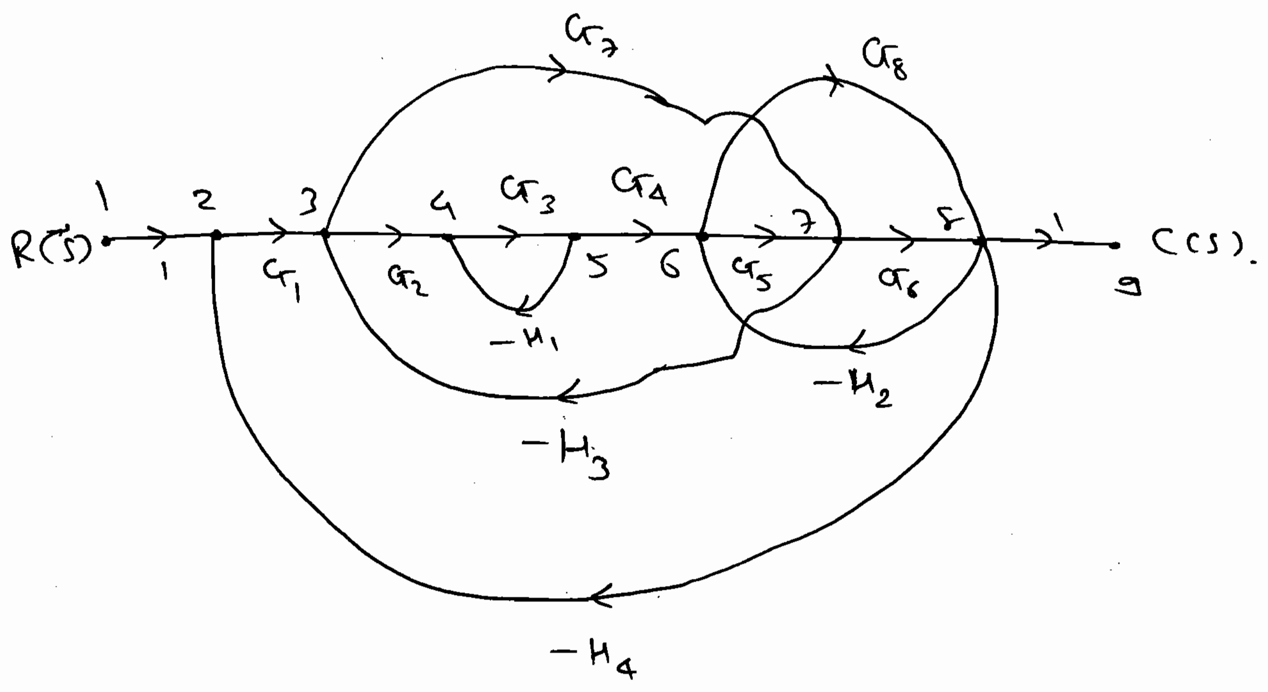
$L_6 L_1 L_7$ = BDIH → 1, 2, 3, 4

$L_6 L_2 L_7$ = CDIH → 1, 2, 3, 4

~~$L_7 L_2 L_6$ = CDHI → 1, 2, 3, 4~~

~~$L_7 L_1 L_6$ = BDHI → 1, 2, 3, 4~~

$\boxed{Q}$



Sol^n:

$H_1 \Rightarrow$ 1 For, $4 \leftarrow 5$ $\Rightarrow$ 1L $\Rightarrow L_1 = -G_3H_1 \rightarrow 4,5.$

$H_2 \Rightarrow$ 2 For, $6 \leftarrow 8$ $\Rightarrow$ 2L $\Rightarrow L_2 = -G_5G_6H_2 \rightarrow 6,7,8.$

$L_3 = -G_8H_2 \rightarrow 6,8.$

$H_3 \Rightarrow$ 2 For, $3 \leftarrow 7$ $\Rightarrow$ 2L $\Rightarrow L_4 = -G_2G_3G_4G_5H_3 \rightarrow 3,4,5,6,7.$

$L_5 = -G_7 \cdot H_3 \rightarrow 3,7.$

$H_4 \Rightarrow$ 3 For, $2 \leftarrow 8$ $\Rightarrow$ 3L $\Rightarrow L_6 = -G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot G_5 \cdot G_6 H_4$

$\rightarrow 2,3,4,5,6,7,8.$

$\rightarrow L_7 = -G_1 \cdot G_7 \cdot G_6 \cdot H_4 \rightarrow 2,3,7,8.$

$\rightarrow L_8 = -G_1 \cdot G_2 G_3 \cdot G_4 \cdot G_8 H_4$

$\rightarrow 2,3,4,5,6,8.$

2NTL ~~3NTL~~

$L_1L_2 \rightarrow 4,5,6,7,8$

$L_1L_3 \rightarrow 4,5,6,7,8$

$L_1L_5 \rightarrow 3,4,5,7$

$L_1L_7 \rightarrow 2,3,4,5,6,7,8.$

$L_3,L_5 \rightarrow 3,6,7,8$

3NTL

$L_1L_3L_5 \rightarrow 3,4,5,6,7,8.$

* Mason's Gain Formula:

Purpose: (i) To find the overall TF of the system.

(ii) To find the ratio of any two nodes.

$$\rightarrow \boxed{\text{overall TF} = \sum_{k=1}^{i} \left(\frac{P_k . \Delta_k}{\Delta} \right).}$$

Where, P_k : K^{th} forward path gain.

$\Delta = 1 - \Sigma$(individual Loop gain)

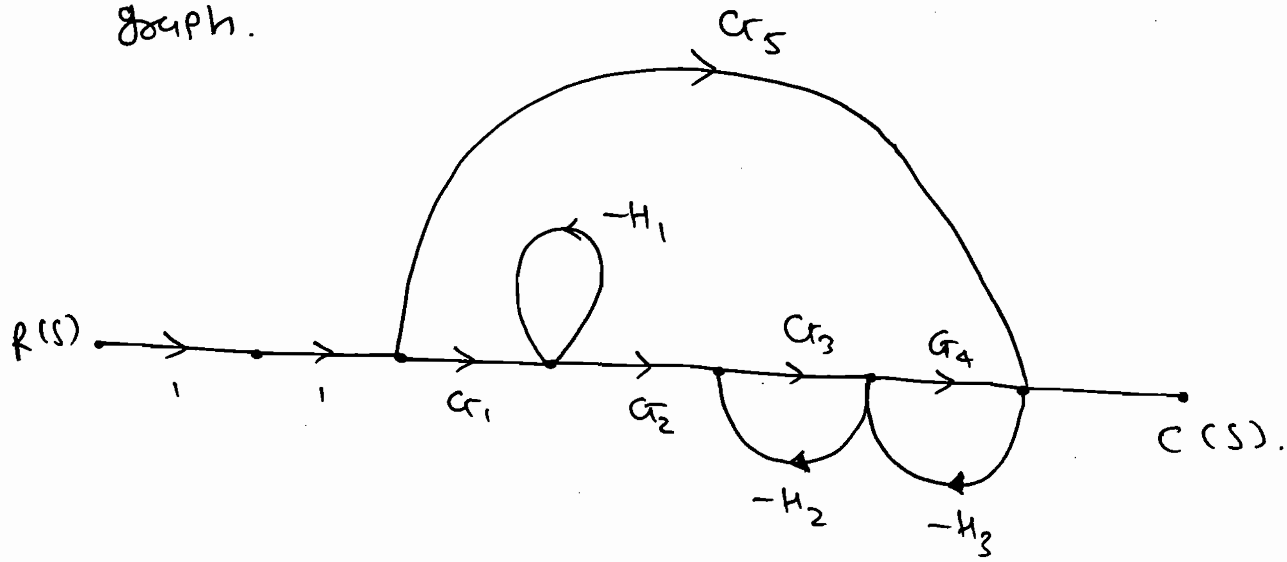
$+ \Sigma$ (Sum of gain product of two non-touching Loop).

$- \Sigma$ (Sum of gain product of three non-touching Loop).

$+ \Sigma$ (sum of gain product of four non-touching Loop). - ....

Δ_k = Δ_k is obtain Δ by removing the Loops touching the k^{th} forward path.

Q Find the TF to the given signal flow graph.



Soln:

F.P.:

$$P_1 = G_1 \cdot G_2 \cdot G_3 \cdot G_4$$

$$P_2 = G_5 .$$

Loops:

$$L_1 = -H_1 .$$

$$L_2 = -G_3 H_2 .$$

$$L_3 = -G_4 H_3 .$$

2 NTL:

$$L_1 L_2 = G_3 H_1 \cdot H_2$$

$$L_1 L_3 = G_4 H_1 \cdot H_3 .$$

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2 + L_1 L_3).$$

$$\therefore \Delta = 1 + H_1 + G_3 H_2 + G_4 H_3 + G_3 H_1 \cdot H_2 + G_4 \cdot H_1 \cdot H_3 .$$

$$\Rightarrow \Delta_1 = 1$$

$$\Delta_2 = 1 - (L_1 + L_2) + (L_1 L_2).$$

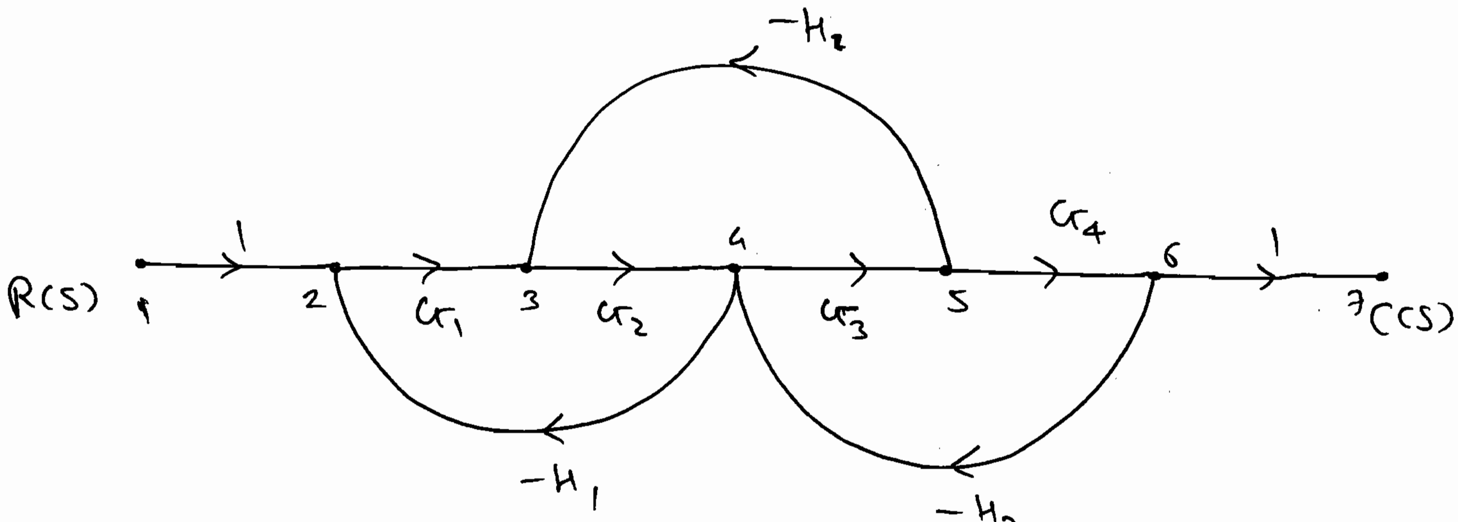
$$\Delta_2 = 1 + H_1 + G_3 H_2 + G_3 H_1 \cdot H_2$$

$$\therefore TF = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_5 (1 + H_1 + G_3 H_2 + G_3 \cdot H_1 \cdot H_2)}{1 + H_1 + G_3 H_2 + G_4 H_3 + G_3 H_1 \cdot H_2 + G_4 \cdot H_1 \cdot H_3} .$$

NOTE:

→ In Δ (or) Δ_k, take the opposite sign for odd no. of non-touching loops and take the same sign for even no. of non-touching loops.

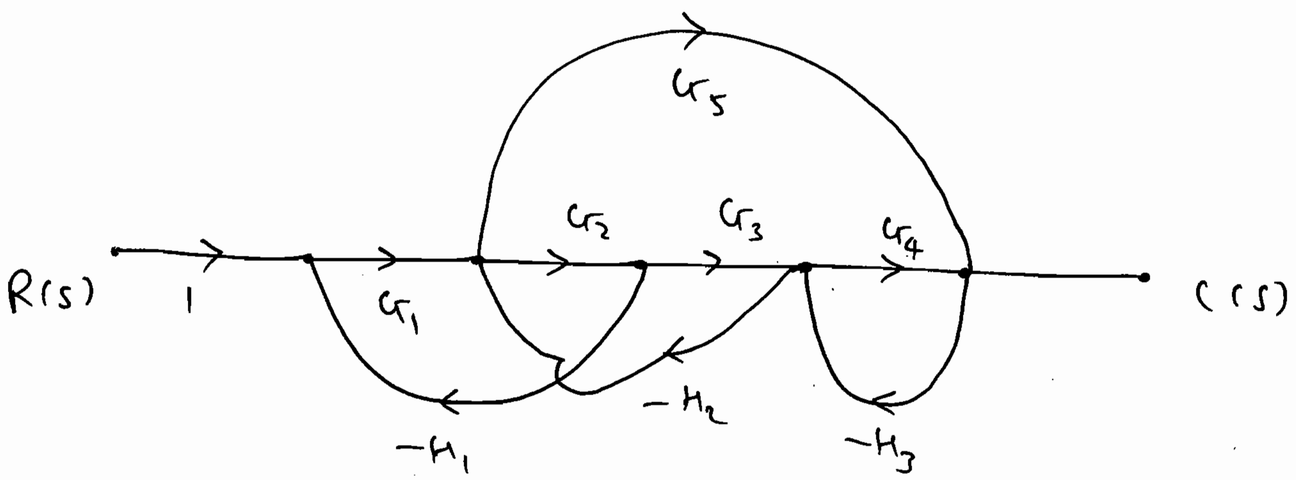
[Q-2] Find the TF.



Soln:

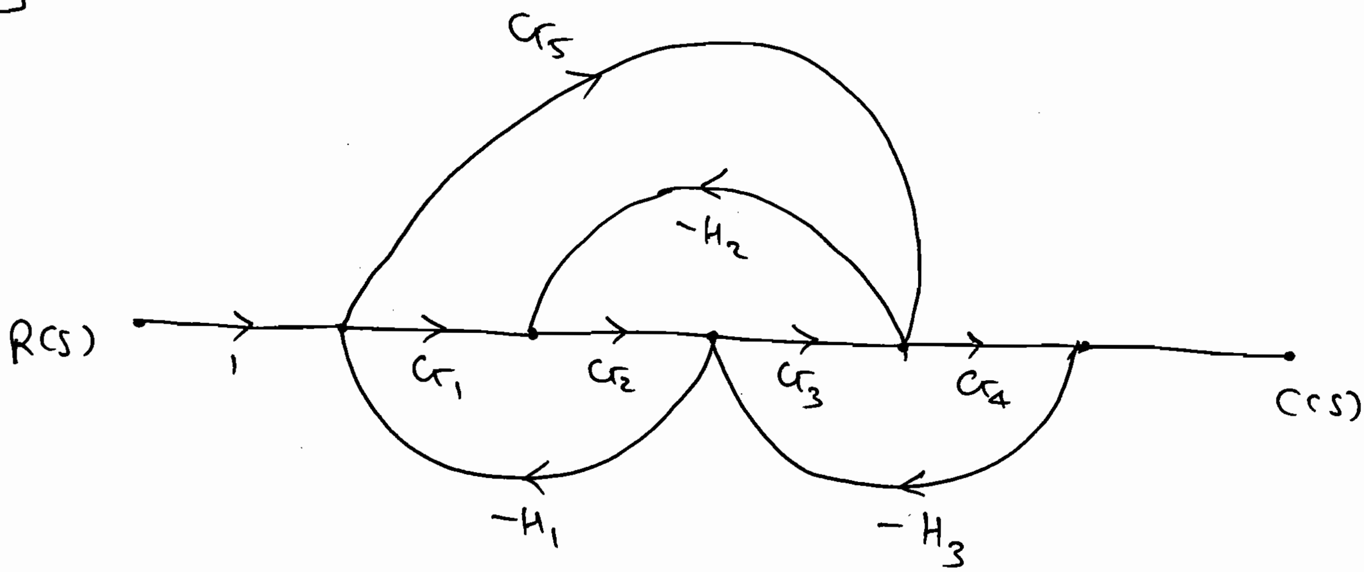
$$TF = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 \,(1).}{1 + G_1 \cdot H_1 + G_3 \cdot H_3 + G_2 \cdot G_3 H_2}$$

[Q]



$$TF = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_1 \cdot G_5}{1 + G_1 \cdot G_2 H_1 + G_2 \cdot G_3 H_2 + G_4 \cdot H_3 \mp G_5 \cdot H_2 \cdot H_3 + G_1 G_2 \cdot G_4 \cdot H_1 \cdot H_3}$$

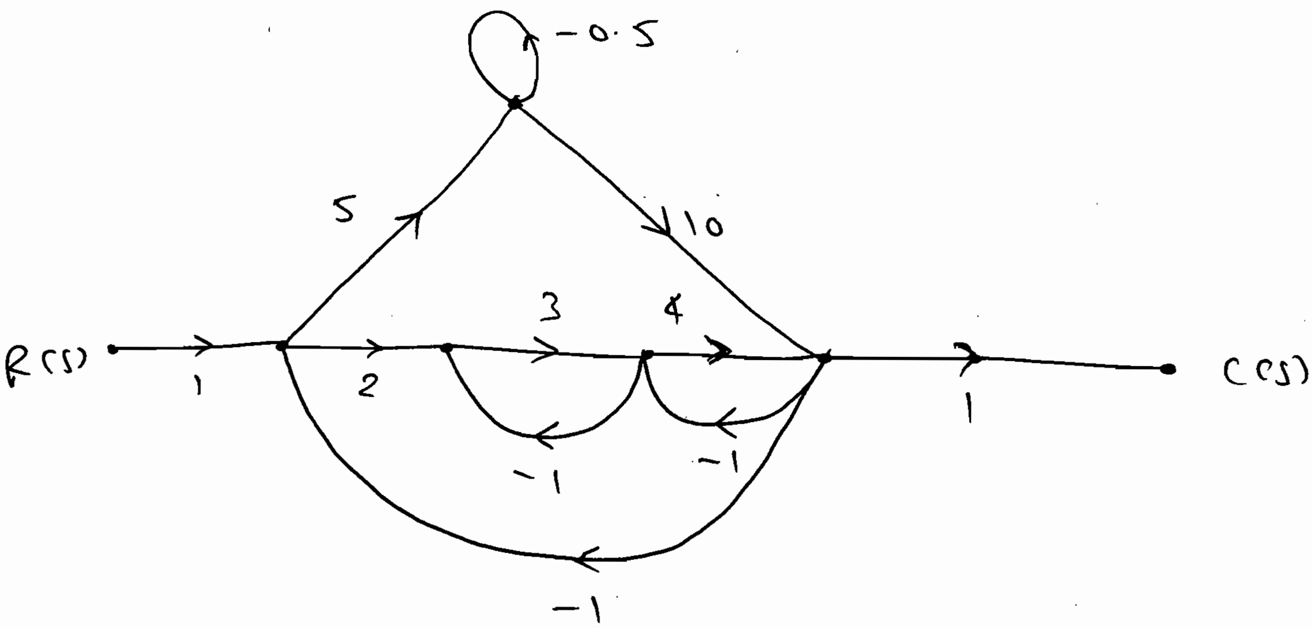
$\boxed{Q}$



Sol^n:

$$\frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_5 G_4}{1 + G_1 G_2 H_1 + G_3 \cdot G_4 \cdot H_3 + G_2 \cdot G_3 H_2 + - G_5 \cdot G_4 \cdot H_3 \cdot H_1 - G_5 \cdot H_2 \cdot G_2 \cdot H_1}$$

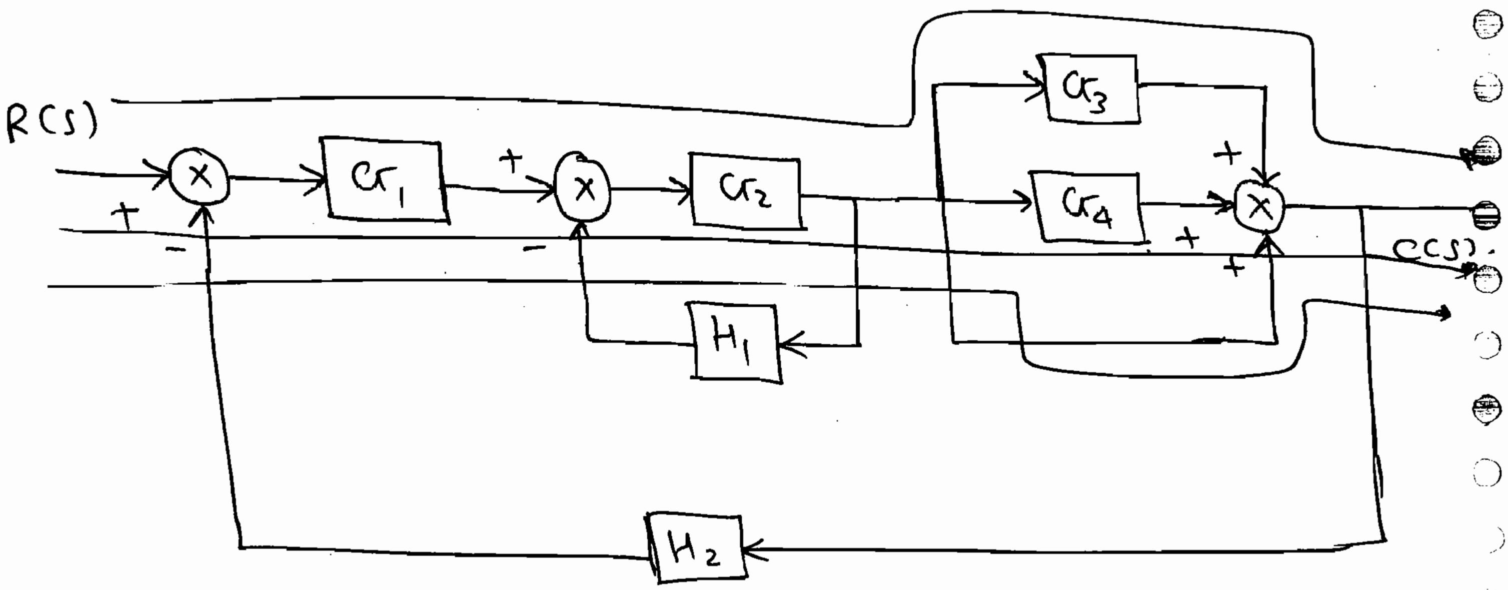
$\boxed{Q}$ Find the TF.



$$\frac{C(s)}{R(s)} = \frac{(2\cdot3\cdot4)(1+0.5) + (5\cdot10)(1+3)}{1+3+4+24+50+0.5+(3/2+2) + (50\cdot3) + (0.5\times24)}$$

$$\frac{C(s)}{R(s)} = \frac{236}{248}.$$

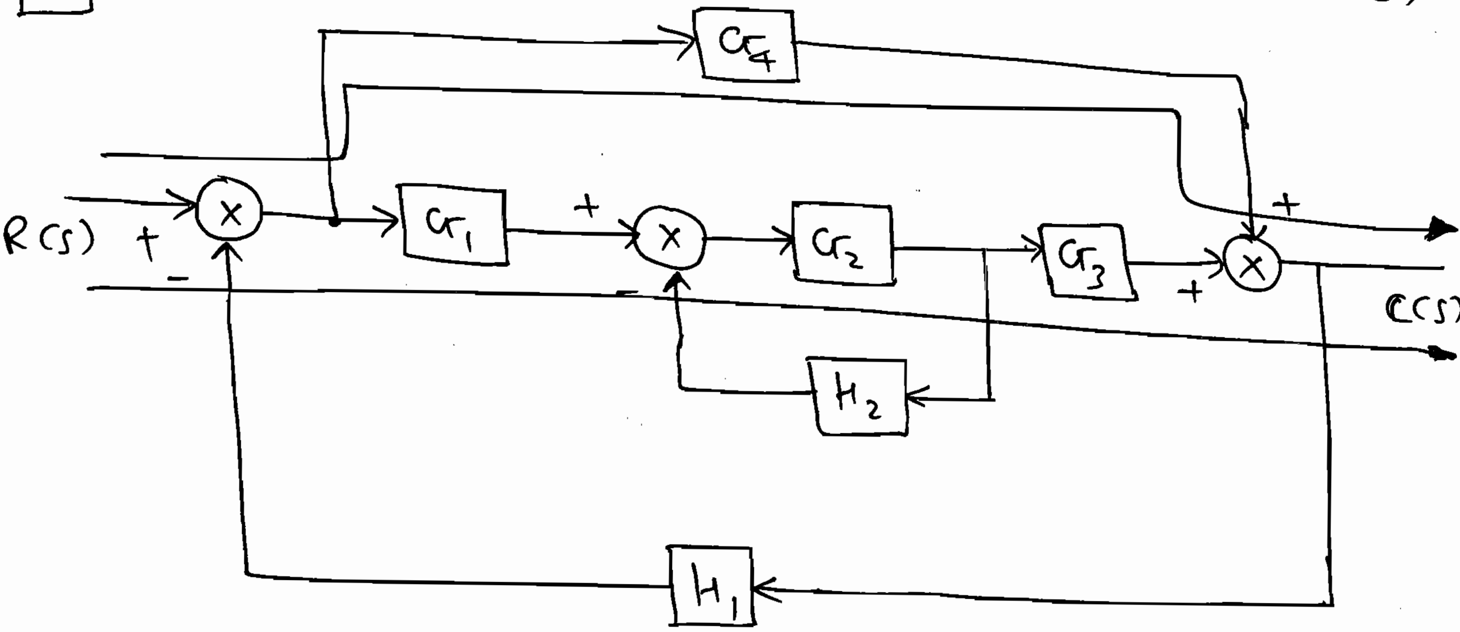
$\boxed{Q}$ Find the TF to the given Block Diagram by using Mason's gain formula.



Sol^n:

$$\frac{C(s)}{R(s)} = \frac{G_1\cdot G_2 G_4 + G_1\cdot G_2\cdot G_3 + G_1\cdot G_2\cdot 1}{1 + G_2H_1 + G_1\cdot G_2\cdot G_3H_2 + G_1\cdot G_2\cdot G_4 H_2 + G_1\cdot G_2\cdot H_2}$$

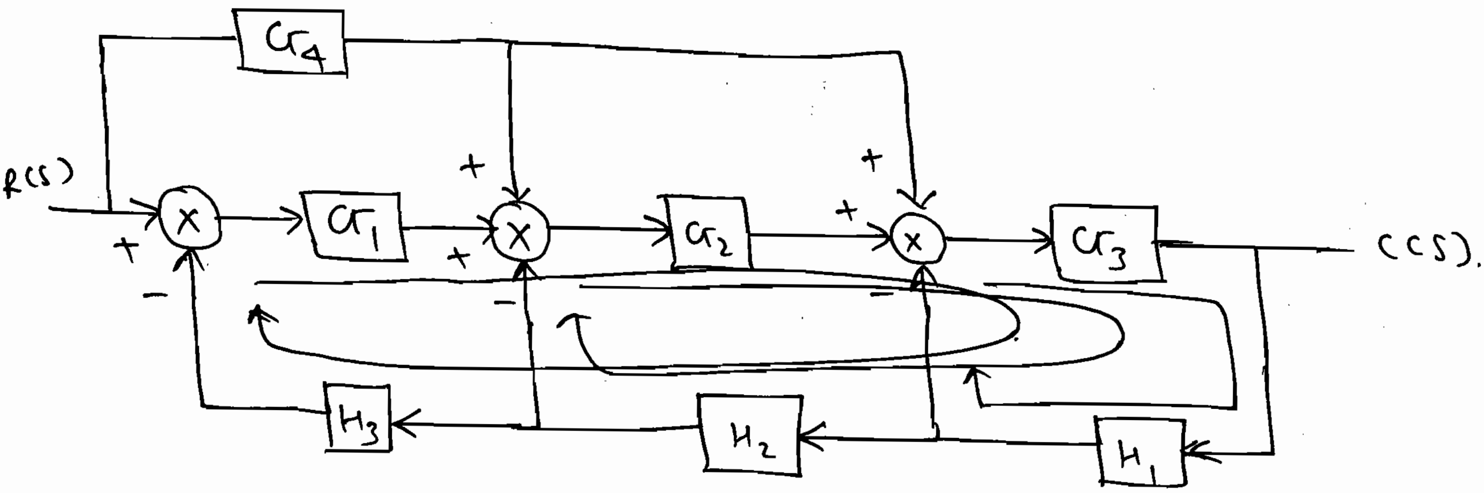
Q



Soln:

$$\frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \cdot G_3 + G_4 (1 + G_2 \cdot H_2)}{1 + G_2 \cdot H_2 + G_1 \cdot G_2 \cdot G_3 H_1 + G_4 H_1 + G_4 \cdot H_1 \cdot G_2 \cdot H_2}$$

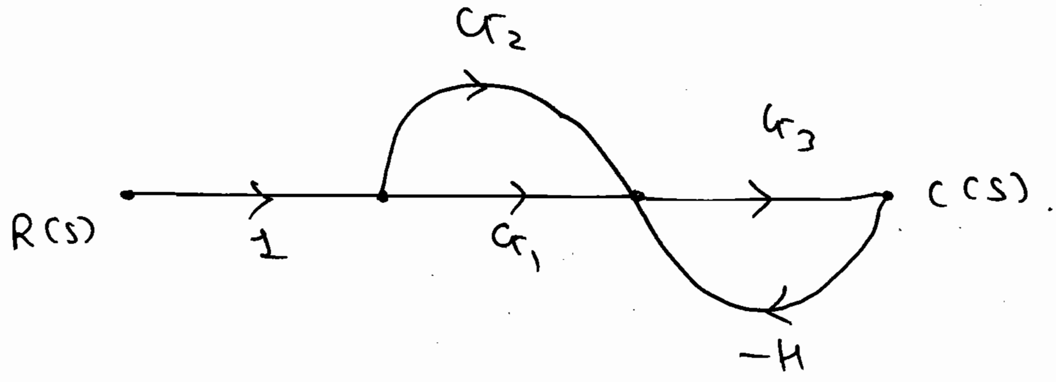
Q



Soln:

$$\frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 \cdot G_3 + G_4 \cdot G_2 \cdot G_3 + G_4 \cdot G_3}{1 + G_3 H_1 + G_2 \cdot G_3 H_1 \cdot H_2 + G_1 \cdot G_2 \cdot G_3 H_1 \cdot H_2 \cdot H_3}$$

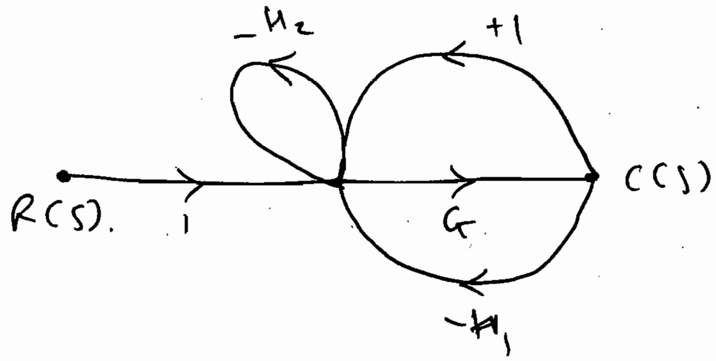
Q



Solⁿ:

$$\frac{C(s)}{R(s)} = \frac{G_1 \cdot G_3 + G_2 \cdot G_3}{1 + G_3 H}.$$

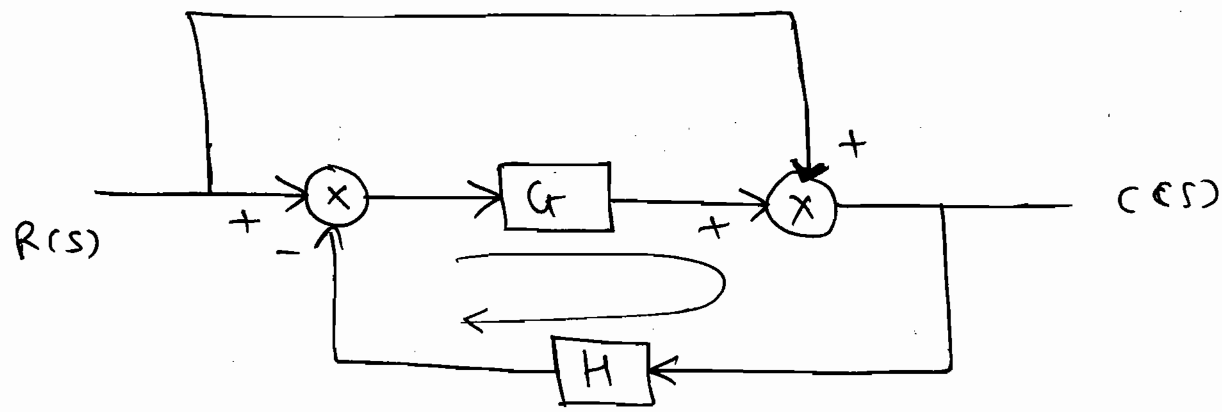
Q



Solⁿ:

$$\frac{C(s)}{R(s)} = \frac{G}{1 + GH_1 + \cancel{H_1} + H_2}$$

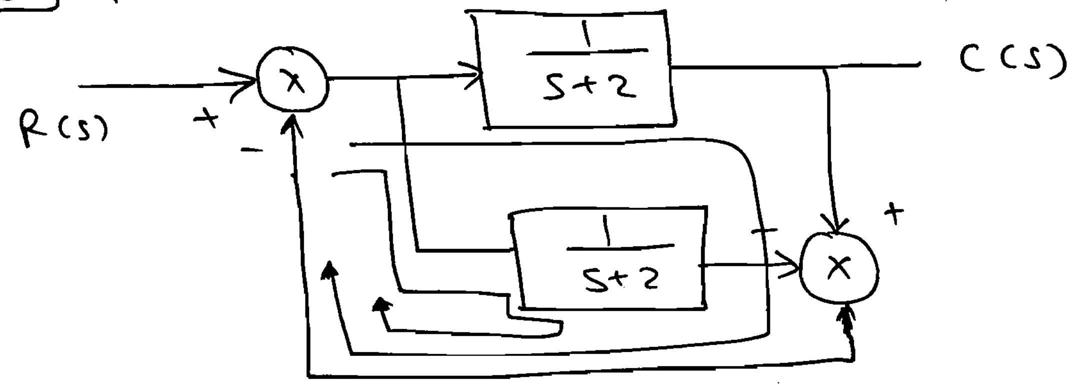
Q *



Solⁿ:

$$\frac{C(s)}{R(s)} = \frac{G + 1}{1 + GH}$$

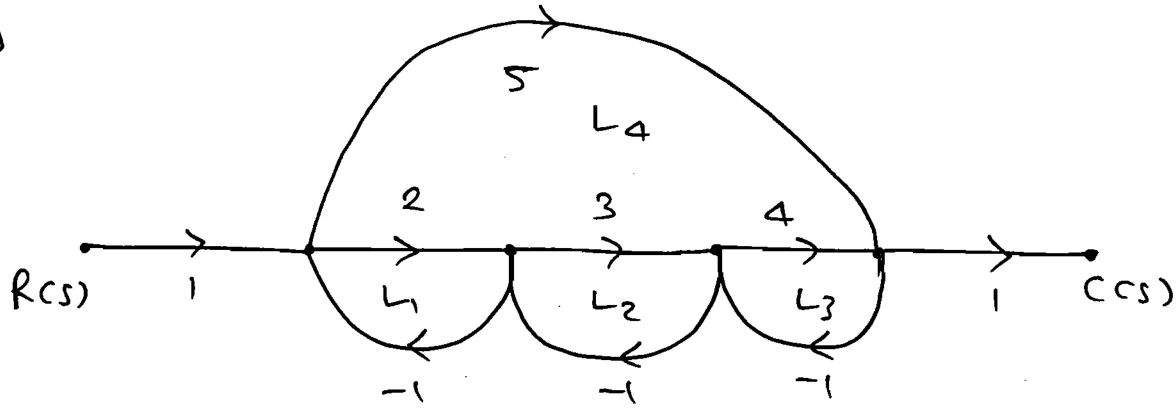
$\boxed{Q}$ *



Sol^n:

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}-\frac{1}{s+2}} = \frac{1}{s+2}.$$

$\boxed{Q}$

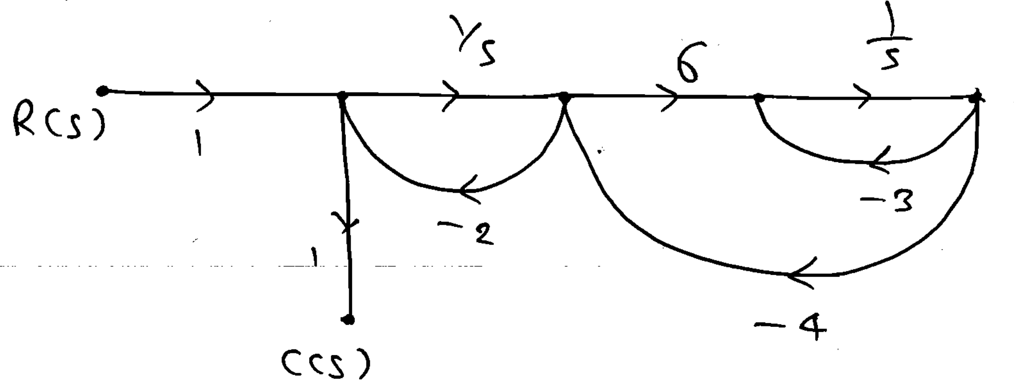


Sol^n:

$$\frac{C(s)}{R(s)} = \frac{(2\cdot3\cdot4) + (5)(1+3)}{1+2+3+4+8+5}$$

$$\frac{C(s)}{R(s)} = \frac{44}{23}.$$

$\boxed{Q}$ ***



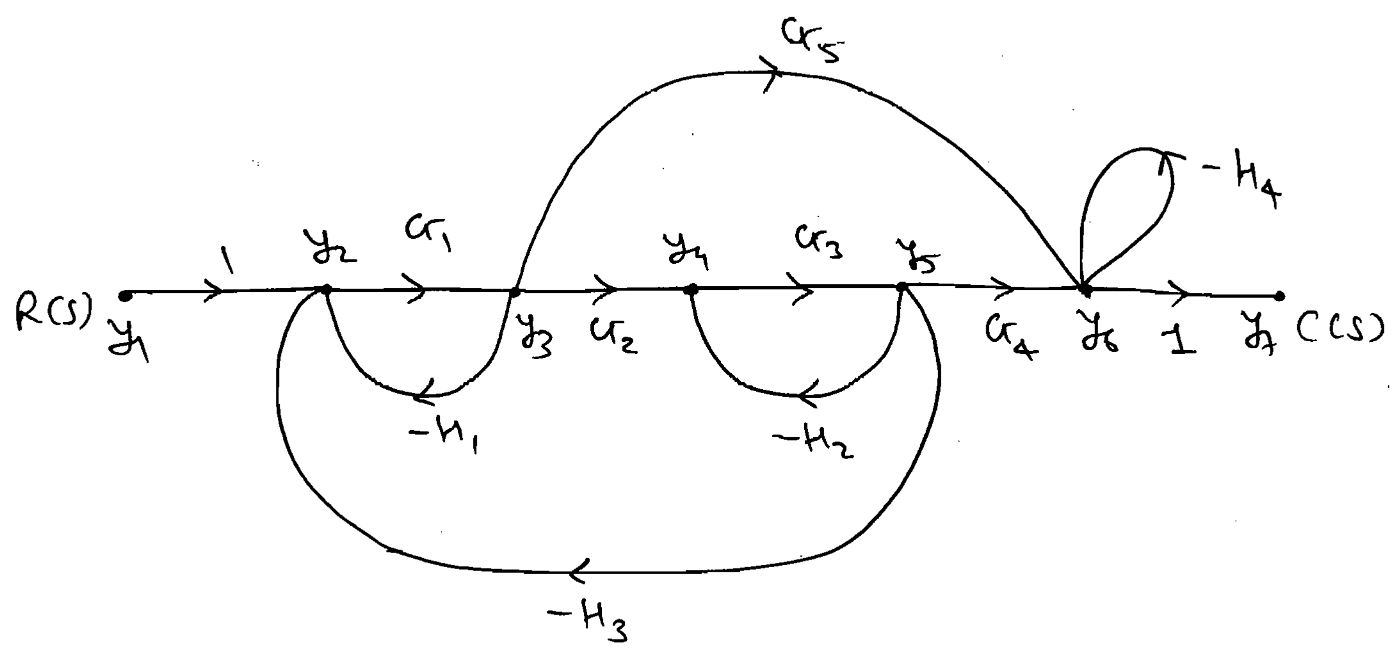
Soln.

$$\frac{C(s)}{R(s)} = \frac{1.1\left(1 + 3/s + \frac{24}{s}\right)}{1 + \frac{2}{s} + 3/s + \frac{24}{s} + 6/s^2}$$

$$\frac{C(s)}{R(s)} = \frac{s(s+27)}{s^2 + 29s + 6}$$

Q Find $\frac{y_6}{y_1}, \frac{y_7}{y_1}, \frac{y_5}{y_1}, \frac{y_2}{y_1}, \frac{y_7}{y_2}, \frac{y_5}{y_3}, \frac{y_5}{y_4}$

and so on ratio of any two nodes.



Soln: (i) $\frac{y_6}{y_1}$

$$\rightarrow \frac{y_6}{y_1} = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_1 \cdot G_5 (1 + G_3 H_2)}{\left(\begin{aligned} &1 + G_1 \cdot H_1 + G_3 \cdot H_2 + H_4 + G_1 \cdot G_2 \cdot G_3 \cdot H_3 \\ &+ G_1 \cdot H_1 \cdot G_3 \cdot H_2 + G_1 \cdot H_1 H_4 + G_3 \cdot H_2 \cdot H_4 \\ &+ G_1 \cdot H_1 \cdot G_3 \cdot H_2 \cdot H_4 \end{aligned}\right)}$$

Δ

(ii) $\frac{y_7}{y_1}$

$\rightarrow$ $\cancel{y_6 =}$ $y_7 = 1 \cdot y_6.$

$$\therefore \frac{y_7}{y_1} = \frac{y_6}{y_1}.$$

$$\therefore \frac{y_7}{y_1} = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_1 \cdot G_5 (1 + G_3 H_2)}{\Delta}$$

(iii) $\frac{y_5}{y_1}$

$$\rightarrow \frac{y_5}{y_1} = \frac{G_1 \cdot G_2 \cdot G_3 \, (1 + H_4)}{\Delta}$$

(iv) $\frac{y_2}{y_1}$

$$\rightarrow \frac{y_2}{y_1} = \frac{1 \, (1 + G_3 H_2 + H_4 + G_3 H_2 \cdot H_4)}{\Delta}$$

(v) $\frac{y_7}{y_2}$

$\rightarrow$ <u>NOTE</u>: $\rightarrow$ The Mason's gain formula gives the ratio w.r.t. input only. It can not gives the ~~nodes~~ ratio directly w.r.t. middle nodes.

$$\rightarrow \frac{y_7}{y_2} = \frac{y_7 / y_1}{y_2 / y_1}$$

$$\rightarrow \frac{y_7}{y_2} = \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 + G_1 \cdot G_5 (1 + G_3 H_4)}{(1 + G_3 \cdot H_2 + H_4 + G_3 \cdot H_2 \cdot H_4)}$$

$$\rightarrow \frac{y_5}{y_3} = \frac{y_5 | y_1}{y_3 | y_1}$$

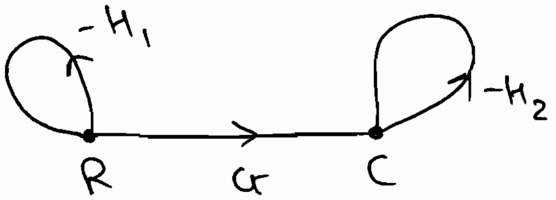
$$\frac{y_5}{y_3} = \frac{G_1 . G_2 . G_3 (1 + H_4)}{G_1 (1 + H_4 + G_3 H_2 + G_3 H_2 H_4)}.$$

$$\rightarrow \frac{y_5}{y_4} = \frac{y_5 | y_1}{y_4 | y_1}$$

$$\frac{y_5}{y_4} = \frac{G_1 . G_2 . G_3 (1 + H_4)}{G_1 . G_2 (1 + H_4)}$$

$$* \rightarrow \frac{y_5}{y_4} = G_3 .$$

$\boxed{Q}$ * * *

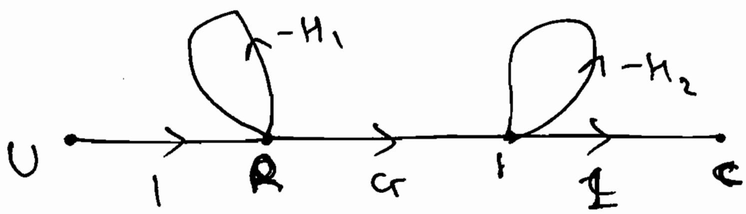


H.B

Soln:

NOTE: In the above signal flow graph R is not input node. In this case we require to assume a dummy input node with path gain of 1 as shown in fig.

$\Rightarrow$



M-I :

$$\frac{C}{R} = \frac{C|U}{R|U} = \frac{G}{1+H_2}.$$

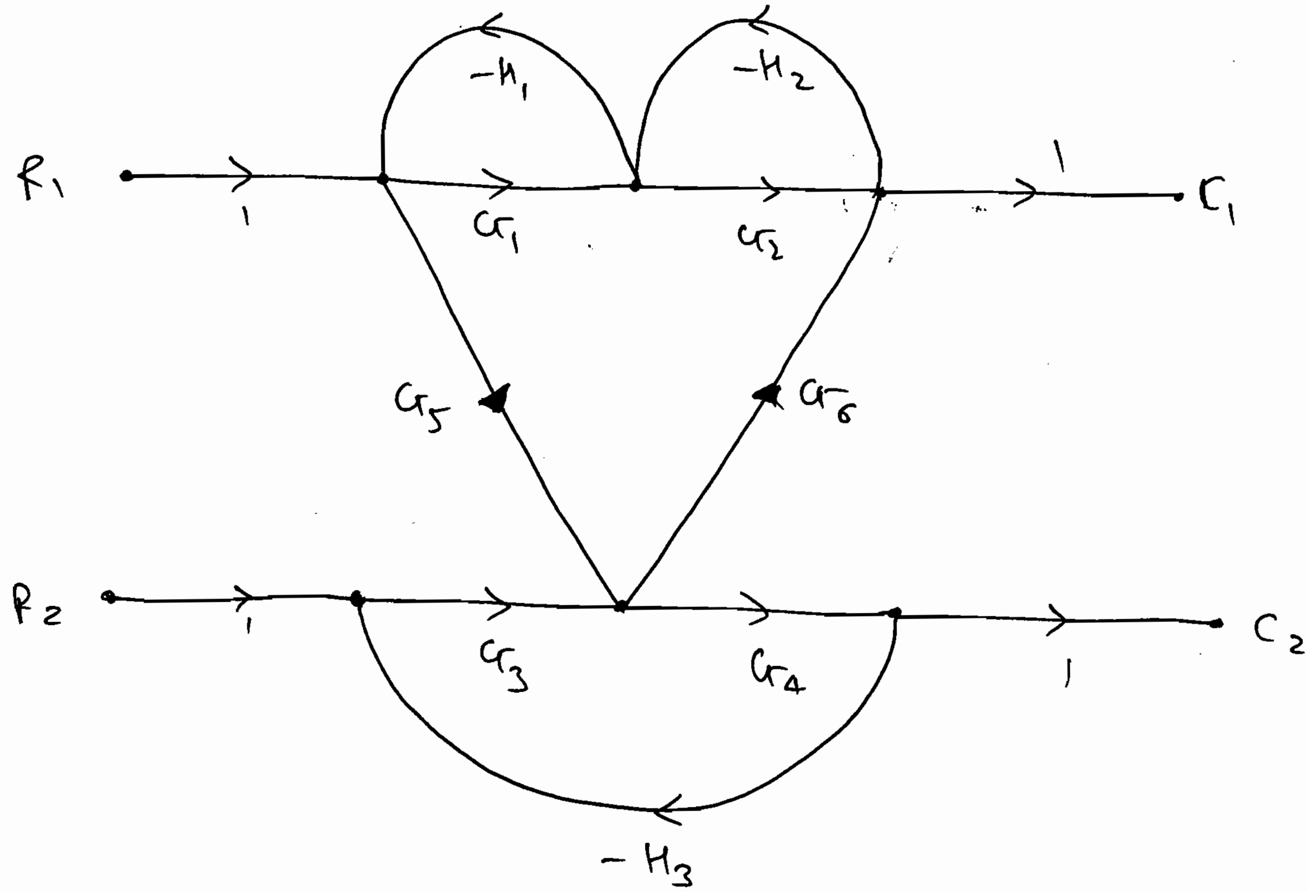
M-II: o/p node eqn.

$$C = RG - CH_2.$$

$$\therefore C(1+H_2) = RG$$

$$\therefore \frac{C}{R} = \frac{G}{1+H_2}.$$

Q Find $C_1|R_1$, $C_1|R_2$, $C_2|R_1$, $C_2|R_2$ to the given multi-input multi o/p system.



Soln:

$$\frac{C_1}{R_1} = \frac{G_1 \cdot G_2(1 + G_3G_4 \cdot H_3) + G_5 \cdot G_6}{1 + G_1 \cdot H_1 + G_2 \cdot H_2 + G_3G_4 \cdot H_3 - G_5G_6 \cdot H_1 \cdot H_2 + G_1 \cdot H_1 \cdot G_3 \cdot G_4 \cdot H_3 + G_2H_2 \cdot G_3 \cdot G_4 \cdot H_3}$$

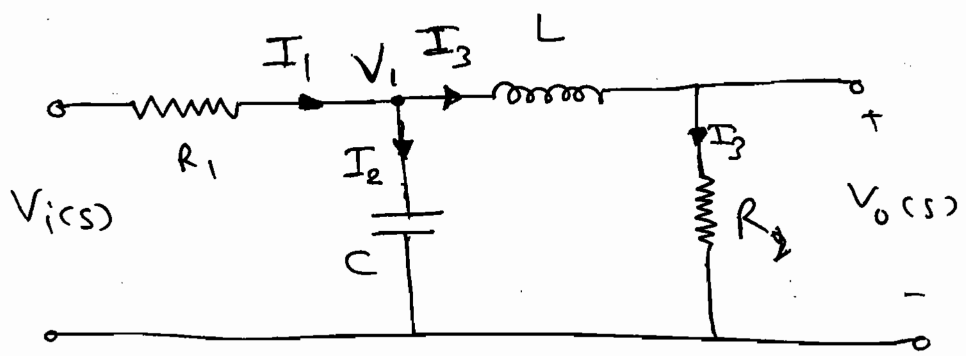
$\rightarrow \dfrac{C_1}{R_2} = \dfrac{G_3 \cdot G_6 \, (1 + G_1 \cdot H_1)}{\Delta}$.

$\rightarrow \dfrac{C_2}{R_1} = \dfrac{G_5 \cdot G_4 \, (1 + G_2 \cdot H_2)}{\Delta}$,

$\rightarrow \dfrac{C_2}{R_2} = \dfrac{G_3 \cdot G_4 \, (1 + G_1 H_1 + G_2 H_2)}{\Delta}$.

* Construction of SFG to Electrical N/w:

Q Draw the SFG.



Sol^n:

→ Select the branch current and node voltages.

→ Apply Laplace transform to the N/w variable and elements.

→ Write the eq^ns for unknown currents and unknown voltages.

$\rightarrow \quad I_1(s) = \dfrac{V_i(s) - V_1(s)}{R_1}$. —— ①

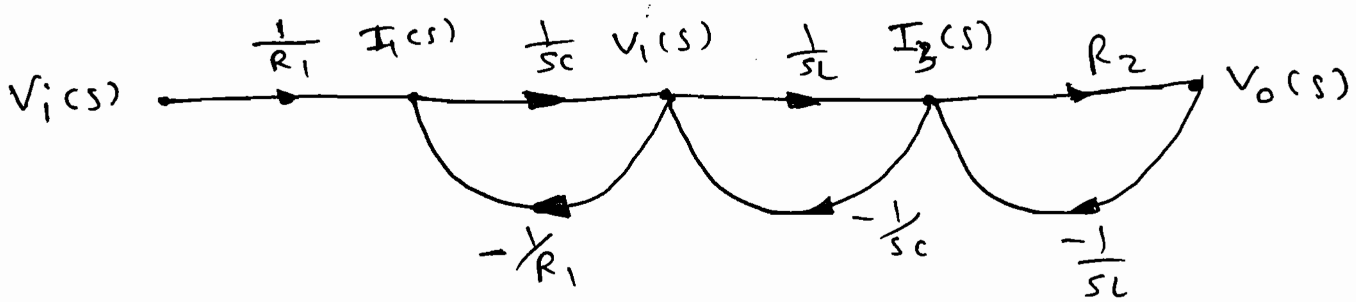
$$\therefore \quad V_1(s) = I_2(s) \cdot \frac{1}{sC}.$$

But $\quad I_2(s) = I_1(s) - I_3(s).$

$$\rightarrow \therefore \quad V_1(s) = \frac{1}{sC}\left(I_1(s) - I_3(s)\right) \quad \text{--- (2)}$$

$$\rightarrow \quad I_3(s) = \frac{V_1(s) - V_0(s)}{sL}. \quad \text{--- (3)}$$

$$\rightarrow \quad V_0(s) = R_2 \cdot I_3(s). \quad \text{--- (4)}$$



$$\Rightarrow \quad \boxed{TF_{E-N/w} = TF_{B-D} \text{ (or) } TF_{SFG}}$$

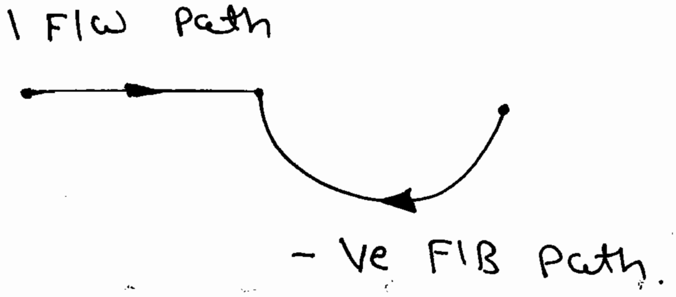
* <u>Procedure to draw SFG directly.</u>

→ The nodes in a SFG are nothing but the variable along the series path. (branch).

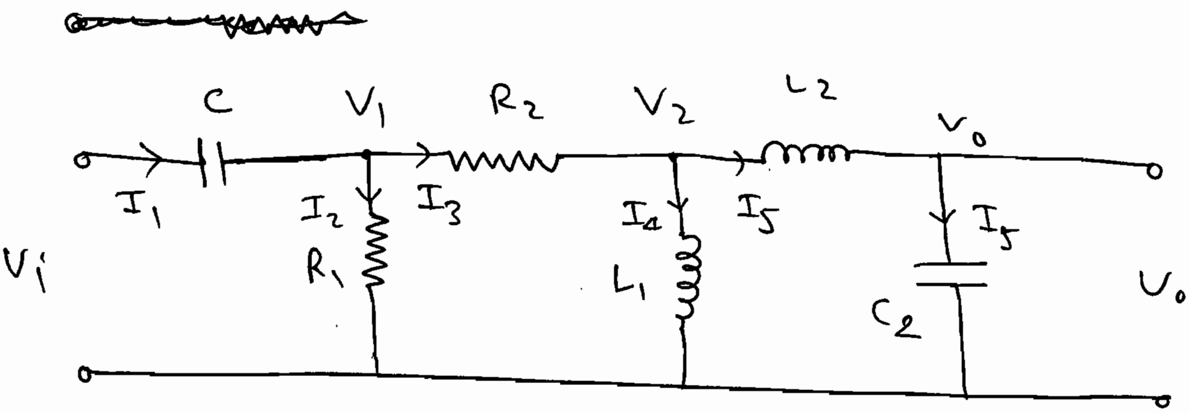
→ Each elements in electrical N/w gives the 1 forward path and 1 -ve feedback path. except the last element. The last element is giving the only forward path.

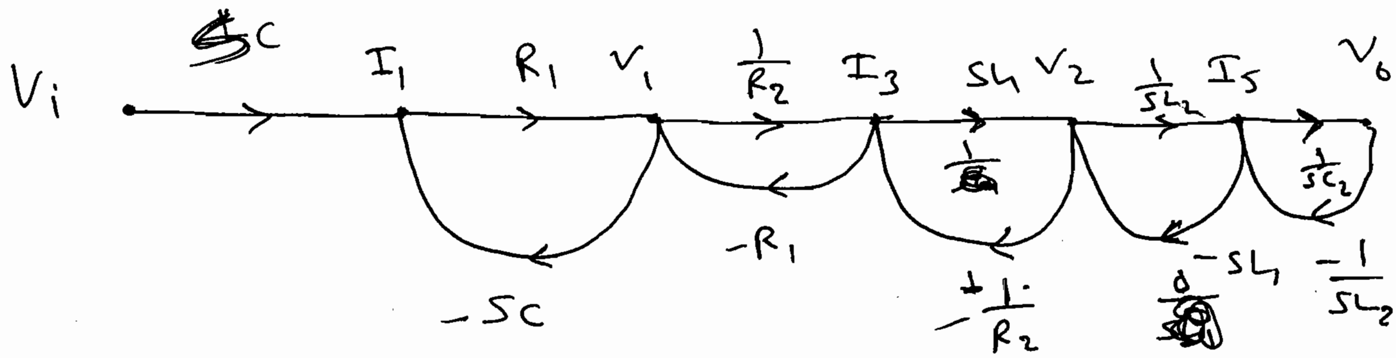
→ Take inverse of the impedence to the series branch elements as a path gain and take the same impedence for shunt branch elements as a path gain.

⇒

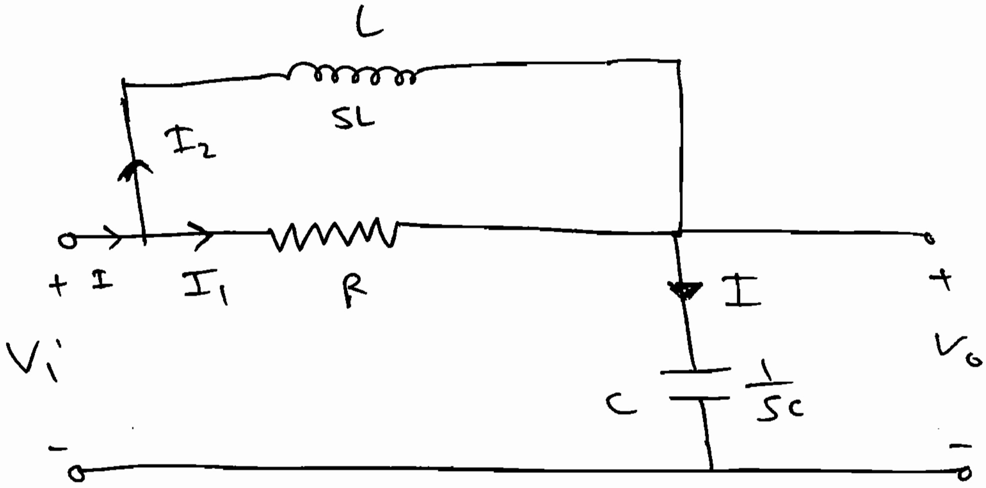


$\boxed{a}$





Q ***



Solⁿ:

$$V_o = \frac{1}{SC} \cdot I.$$

$$I = I_1 + I_2$$

$$I = \frac{V_i - V_o}{R} + \frac{V_i - V_o}{SL}.$$