2 Chapter Progressions

The chapter on progressions essentially yields common-sense based questions in examinations.

Questions in the CAT and other aptitude exams mostly appear from either Arithmetic Progressions (more common) or from Geometric Progressions.

The chapter of progressions is a logical and natural extension of the chapter on Number Systems, since there is such a lot of commonality of logic between the problems associated with these two chapters.

ARITHMETIC PROGRESSIONS

Quantities are said to be in arithmetic progression when they increase or decrease by a common difference.

Thus each of the following series forms an arithmetic progression:

3, 7, 11, 15,…

8, 2, –4, –10,…

$a, a + d, a + 2d, a + 3d, \ldots$

The common difference is found by subtracting any term of the series from the next term.

That is, common difference of an $AP = (t_N - t_{N-1})$.

In the first of the above examples the common difference is 4; in the second it is –6; in the third it is d.

If we examine the series $a, a + d, a + 2d, a + 3d, \ldots$ we notice that *in any term the coefficient of d is always less by one than the position of that term in the series*.

Thus the rth term of an arithmetic progression is given by $T_r = a + (r - 1)d$.

If n be the number of terms, and if L denotes the last term or the nth term, we have

$$L = a + (n - 1)d$$

To Find the Sum of the given Number of Terms in an Arithmetic Progression

Let a denote the first term d, the common difference, and n the total number of terms. Also, let L denote the last term, and S the required sum; then

$$S = \frac{n(a+L)}{2} \quad (1)$$

$$L = a + (n - 1)d \quad (2)$$

$$S = \frac{n}{2} \times [2a + (n - 1)d] \quad (3)$$

If any two terms of an arithmetical progression be given, the series can be completely determined; for this data results in two simultaneous equations, the solution of which will give the first term and the common difference.

When three quantities are in arithmetic progression, the middle one is said to be the **arithmetic mean** of the other two.

Thus a is the arithmetic mean between $a - d$ and $a + d$. So, when it is required to arbitrarily consider three numbers in *AP* take $a - d$, a and $a + d$ as the three numbers as this reduces one unknown thereby making the solution easier.

To Find the Arithmetic Mean between any Two given Quantities

Let a and b be two quantities and A be their arithmetic mean. Then since a, A, b, are in *AP*. We must have

$b - A = A - a$

Each being equal to the common difference;

This gives us $A = \frac{(a+b)}{2}$

Between two given quantities it is always possible to insert any number of terms such that the whole series thus formed shall be in *AP*. The terms thus inserted are called the **arithmetic means**.

To Insert a given Number of Arithmetic Means between Two given Quantities

Let a and b be the given quantities and n be the number of means.

Including the extremes, the number of terms will then be $n + 2$ so that we have to find a series of $n + 2$ terms in *AP*, of which a is the first, and b is the last term.

Let d be the common difference;

then b = the $(n + 2)$th term

$= a + (n + 1)d$

Hence, $d = \frac{(b-a)}{(n+1)}$

and the required means are

$$a + \frac{(b-a)}{n-1}, a + \frac{2(b-a)}{n+1}, \cdots a + \frac{n(b-a)}{n+1}$$

Till now we have studied APs in their mathematical context. This was important for you to understand the basic mathematical construct of A.Ps. However, you need to understand that questions on A.P. are seldom solved on a mathematical basis, (Especially under the time pressure that you are likely to face in the CAT and other aptitude exams). In such situations the mathematical processes for solving progressions based questions are likely to fail or at the very least, be very tedious. Hence, understanding the following logical aspects about Arithmetic Progressions is likely to help you solve questions based on APs in the context of an aptitude exam.

Let us look at these issues one by one:

1. Process for finding the nth term of an A.P.

Suppose you have to find the 17th term of the

A.P. 3, 7, 11…….

The conventional mathematical process for this question would involve using the formula.

$T_n = a + (n - 1)\, d$

Thus, for the 17th term we would do

$T_{17} = 3 + (17 - 1) \times 4 = 3 + 16 \times 4 = 67$

Most students would mechanically insert the values for *a*, *n* and *d* and get this answer.

However, if you replace the above process with a thought algorithm, you will get the answer much faster.

The algorithm goes like this:

In order to find the 17th term of the above sequence add the common difference to the first term, sixteen times. (**Note:** Sixteen, since it is one less than 17).

Similarly, in order to find the 37th term of the A.P. 3, 11 …, All you need to do is add the common difference (8 in this case), 36 times.

Thus, the answer is 288 + 3 = 291.

(**Note:** You ultimately end up doing the same thing, but you are at an advantage since the entire solution process is reactionary.)

2. Average of an A.P. and Corresponding terms of the A.P.

Consider the A.P., 2, 6, 10, 14, 18, 22. If you try to find the average of these six numbers you will get: Average = (2 + 6 + 10 + 14 + 18 + 22)/6 = 12

Notice that 12 is also the average of the first and the last terms of the A.P. In fact, it is also the average of 6 and 18 (which correspond to the second and 5th terms of the A.P.). Further, 12 is also the average of the 3rd and 4th terms of the A.P.

(**Note:** In this A.P. of six terms, the average was the same as the average of the 1st and 6th terms. It was also given by the average of the 2nd and the 5th terms, as well as that of the 3rd and 4th terms.)

We can call each of these pairs as "CORRESPONDING TERMS" in an A.P.

What you need to understand is that every A.P. has an average.

And for any A.P., the average of any pair of corresponding terms will also be the average of the A.P.

If you try to notice the sum of the term numbers of the pair of corresponding terms given above:

1st and 6th (so that 1 + 6 = 7)

2nd and 5th(hence, 2 + 5 =7)

3rd and 4th (hence, 3 + 4 = 7)

Note: In each of these cases, the sum of the term numbers for the terms in a corresponding pair is one greater than the number of terms of the A.P.

This rule will hold true for all A.P.s.

For example, if an A.P. has 23 terms then for instance, you can predict that the 7th term will have the 17th term as its corresponding term, or for that matter the 9th term will have the 15th term as its corresponding term. (Since 24 is one more than 23 and 7 + 17 = 9 + 15 = 24.)

3. Process for finding the sum of an A.P.

Once you can find a pair of corresponding terms for any A.P., you can easily find the sum of the A.P. by using the property of averages:

i.e. Sum = Number of terms × Average.

In fact, this is the best process for finding the sum of an A.P. It is much more superior than the process of finding the sum of an A.P. using the expression $\frac{n}{2}(2a+(n-1)d)$.

4. Finding the common difference of an A.P., given 2 terms of an A.P.

Suppose you were given that an A.P. had its 3rd term as 8 and its 8th term as 28. You should visualize this A.P. as – , – , 8 , – , – , –, – , 28.

From the above figure, you can easily visualize that to move from the third term to the eighth term, (8 to 28) you need to add the common difference five times. The net addition being 20, the common difference should be 4.

Illustration: Find the sum of an A.P. of 17 terms, whose 3rd term is 8 and 8th term is 28.

Solution: Since we know the third term and the eighth term, we can find the common difference as 4 by the process illustrated above.

The total = 17 × Average of the A.P.

Our objective now shifts into the finding of the average of the A.P. In order to do so, we need to identify either the 10th term (which will be the corresponding term for the 8th term) or the 15th term (which will be the corresponding term for the 3rd term.)

Again: Since the 8th term is 28 and $d = 4$, the 10th term becomes 28 + 4 + 4 = 36.

Thus, the average of the A.P.

= Average of 8th and 10th terms

= (28 + 36)/2 = 32.

Hence, the required answer is sum of the A.P. = 17 × 32 = 544.

The logic that has applied here is that the difference in the term numbers will give you the number of times the common difference is used to get from one to the other term.

For instance, if you know that the difference between the 7th term and 12th term of an AP is –30, you should realize that 5 times the common difference will be equal to –30. (Since 12 – 7 = 5).

Hence, $d = -6$.

Note: Replace this algorithmic thinking in lieu of the mathematical thinking of:

12^{th} term = $a + 11d$

7^{th} term = $a + 6d$

Hence, difference = –30 = $(a + 11d) - (a + 6d)$

$-30 = 5d$

\ $d = -6$.

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5. Types of APs: Increasing and Decreasing A.P.s.

Depending on whether '*d*' is positive or negative, an A.P. can be increasing or decreasing.

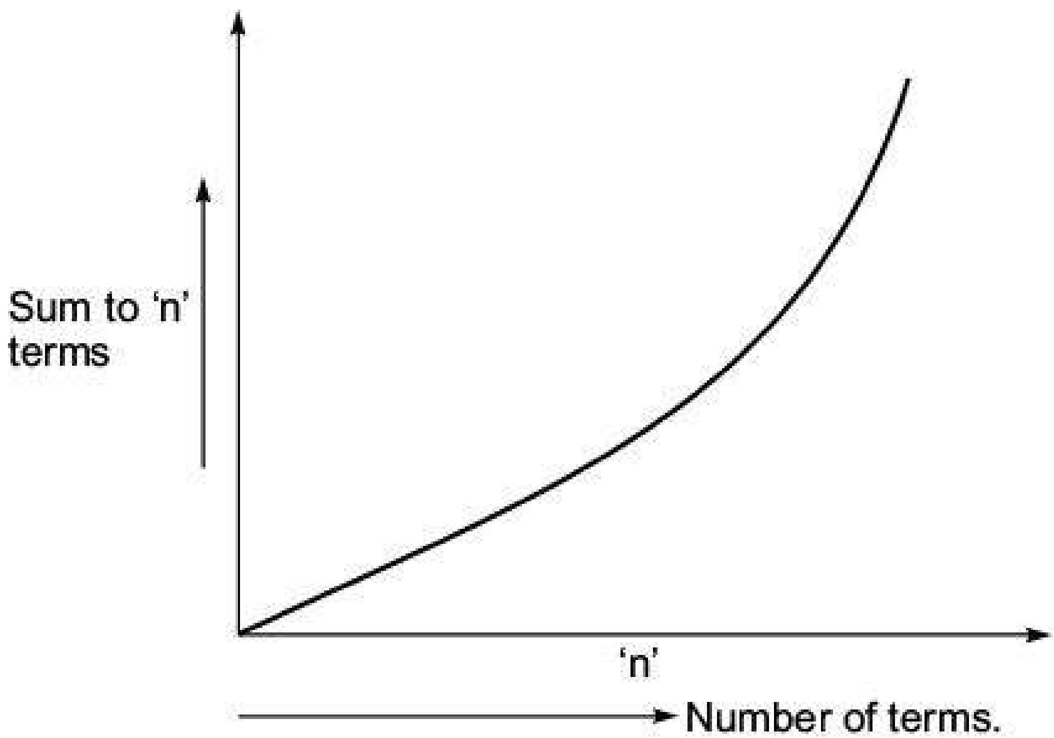
Let us explore these two types of A.P.s further:

(A) Increasing A.P.s:

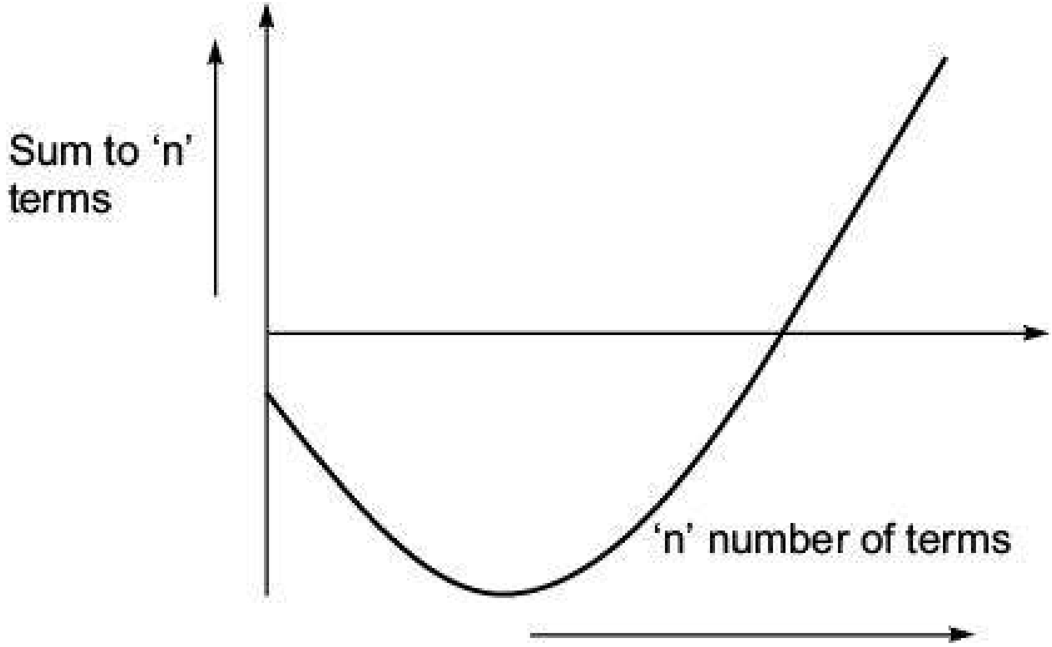
Every term of an increasing AP is greater than the previous term.

Depending on the value of the first term, we can construct two graphs for sum of an increasing A.P.

Case 1: When the first term of the increasing A.P. is positive. In such a case the sum of the A.P. will show a continuously increasing graph which will look like the one shown in the figure below:



Case 2: When the first term of the increasing A.P. is negative. In such a case, the Sum of the A.P. plotted against the number of terms will give the following figure:



The specific case of the sum to n_1 terms being equal to the sum to n_2 terms.

In the series case 2 above, there is a possibility of the sum to '*n*' terms being repeated for 2 values of '*n*'. However, this will not necessarily occur.

This issue will get clear through the following example:

Consider the following series:

Series 1: –12, –8, –4, 0, 4, 8, 12

As is evident the sum to 2 terms and the sum to 5 terms in this case is the same. Similarly, the sum to 3 terms is the same as the sum to 4 terms. This can be written as:

$S_2 = S_5$ and $S_3 = S_4$.

In other words the sum to n_1 terms is the same as the sum to n_2 terms.

Such situations arise for increasing A.P.'s where the first term is negative. But as we have already stated that this does not happen for all such cases.

Consider the following A.P.s.

Series 2: –8, –3, +2, + 7, + 12…

Series 3: –13, –7, –1, + 5, + 11…

Series 4: –12, –6, 0, 6, 12 …

Series 5: –15, –9, –3, + 3, 9, 15 …

Series 6: –20, –12, –4, 4 , 12, …

If you check the series listed above, you will realize that this occurrence happens in the case of Series 1, Series 4, Series 5 and Series 6 while in the case of Series 2 and Series 3 the same value is not repeated for the sum of the Series.

A clear look at the two series will reveal that this phenomenon occurs in series which have what can be called a balance about the number zero.

Another issue to notice is that in Series 4,

$S_2 = S_3$ and $S_1 = S_4$

While in series 5

$S_1 = S_5$ and $S_2 = S_4$.

In the first case (where '0' is part of the series) the sum is equal for two terms such that one of them is odd and the other is even.

In the second case on the other hand (when '0' is not part of the series) the sum is equal for two terms such that both are odd or both are even.

Also notice that the sum of the term numbers which exhibit equal sums is constant for a given A.P.

Consider the following question which appeared in CAT 2004 and is based on this logic:

The sum to 12 terms of an A.P. is equal to the sum to 18 terms. What will be the sum to 30 terms for this series?

Solution: If $S_{12} = S_{18}$, $S_{11} = S_{19\ldots}$ and $S_0 = S_{30}$

But Sum to zero terms for any series will always be 0. Hence $S_{30} = 0$.

Note: The solution to this problem does not take more than 10 seconds if you know this logic

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(B) Decreasing A.P.s.

Similar to the cases of the increasing A.Ps, we can have two cases for decreasing APs –

Case 1– Decreasing A.P. with first term negative.

Case 2– Decreasing A.P. with first term positive.

I leave it to the reader to understand these cases and deduce that whatever was true for increasing A.Ps with first term negative will also be true for decreasing APs with first term positive.

GEOMETRIC PROGRESSION

Quantities are said to be in Geometric Progression when they increase or decrease by a constant factor.

The constant factor is also called the *common ratio* and it is found by dividing any term by the term immediately preceding it.

If we examine the series $a, ar, ar^2, ar^3, ar^4, \ldots$

we notice that in any term the index of r is always less by one than the number of the term in the series.

If n be the number of terms and if l denote the last, or nth term, we have

$l = ar^{n-1}$

When three quantities are in geometrical progression, the middle one is called the geometric mean between the other two. While arbitrarily choosing three numbers in GP, we take a/r, a and a/r. This makes it easier since we come down to two variables for the three terms.

To Find the Geometric Mean between two given Quantities

Let a and b be the two quantities; G the geometric mean. Then since a, G, b are in GP,

$b/G = G/a$

Each being equal to the common ratio

$G^2 = ab$

Hence $G = \sqrt{ab}$

To Insert a given Number of Geometric Means between two given Quantities

Let a and b be the given quantities and n the required number of means to be inserted. In all there will be $n + 2$ terms so that we have to find a series of $n + 2$ terms in GP of which a is the first and b the last.

Let r be the common ratio;

Then b = the $(n + 2)$th term $= ar^{n+1}$;

\ $r^{(n+1)} = \frac{b}{a}$

$$\backslash \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} \qquad (1)$$

Hence the required number of means are $ar, ar^2, \ldots ar^n$, where r has the value found in (1).

To Find the Sum of a Number of Terms in a Geometric Progression

Let a be the first term, r the common ratio, n the number of terms, and S_n be the sum to n terms.

If $r > 1$, then

$$S_n = \frac{a(1-r^n)}{(1-r)} \qquad (1)$$

If $r < 1$, then

$$S_n = \frac{a(1-r^n)}{(1-r)} \qquad (2)$$

Note: It will be convenient to remember both forms given above for S. Number (2) will be used in all cases except when r is positive and greater than **one**.

Sum of an infinite geometric progression when $r < 1$

$$S_\bullet = \frac{a}{(1-r)}$$

Obviously, this formula is used only when the common ratio of the GP is less than one.

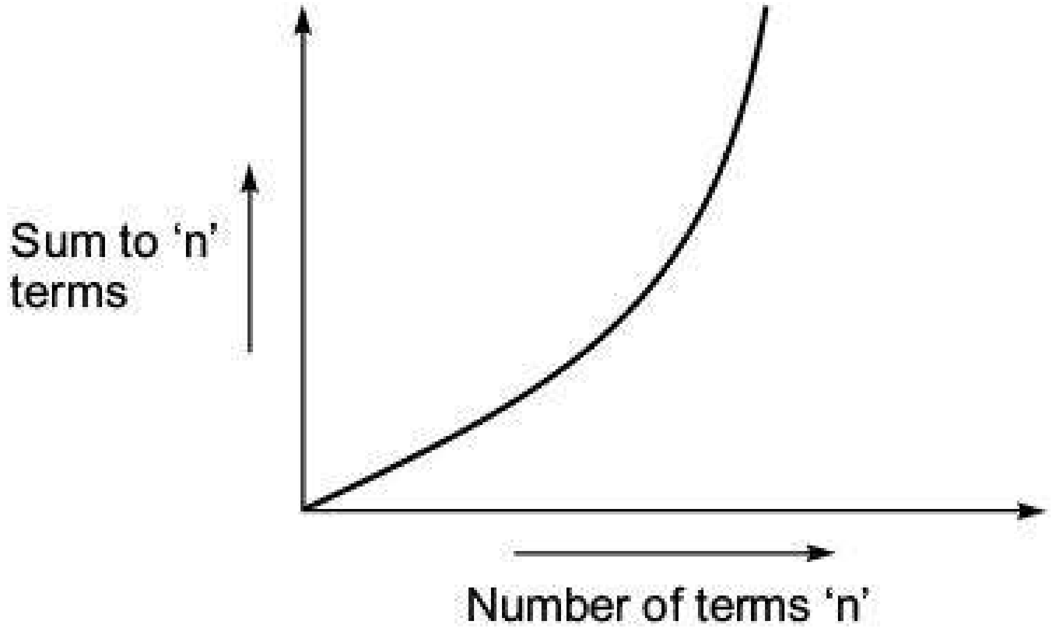
Similar to APs, GPs can also be logically viewed. Based on the value of the common ratio and its first term a G.P. might have one of the following structures:

(1) Increasing GPs type 1:

A G.P. with first term positive and common ratio greater than 1. This is the most common type of G.P.

e.g: 3, 6, 12, 24…(A G.P. with first term 3 and common ratio 2)

The plot of the sum of the series with respect to the number of terms in such a case will appear as follows:



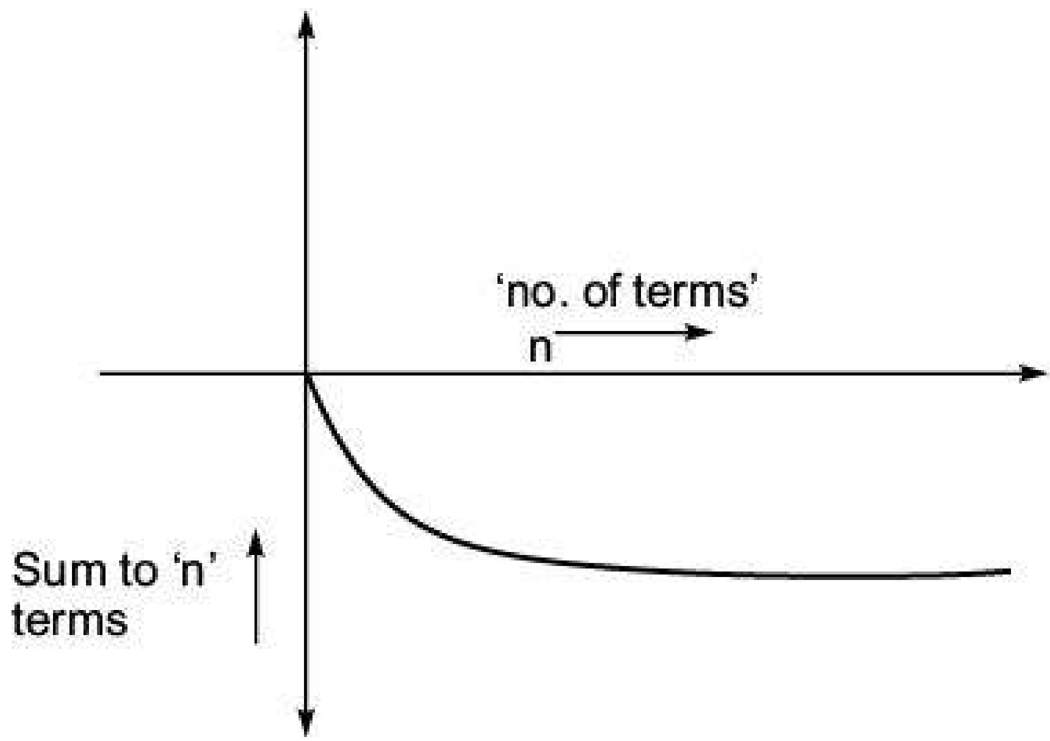
(2) Increasing GPs type 2:

A G.P. with first term negative and common ratio less than 1.

e.g: –8, –4,–2, –1, – ……

As you can see in this GP all terms are greater than their previous terms.

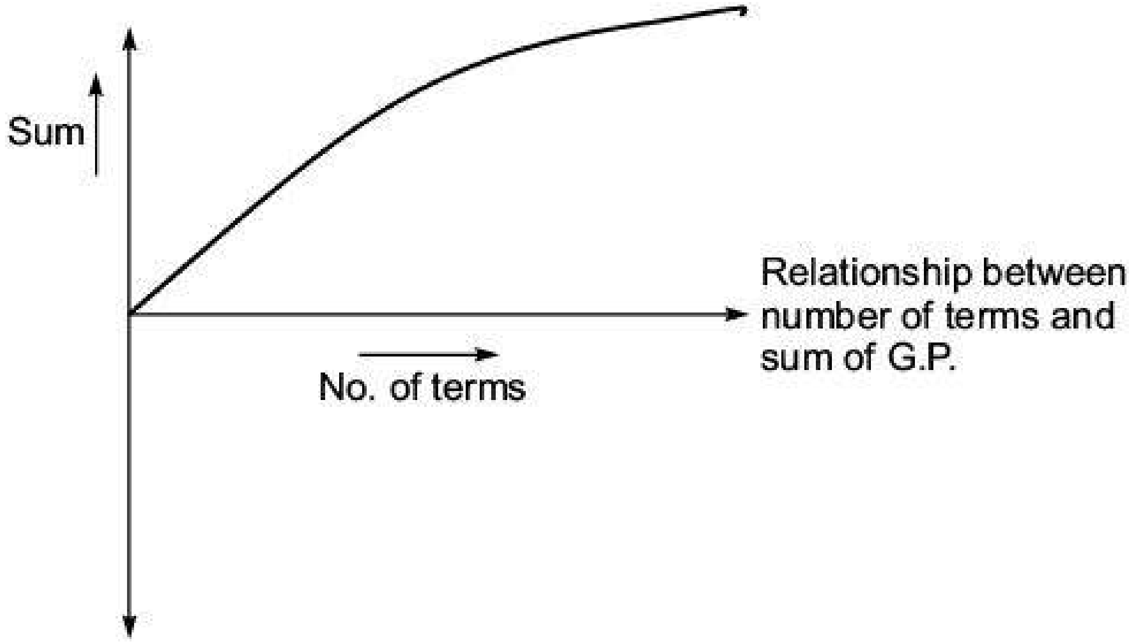
[The following figure will illustrate the relationship between the number of terms and the sum to '*n*' terms in this case]



(3) Decreasing G.Ps type 1:

These GPs have their first term positive and common ratio less than 1.

e.g: 12, 6, 3, 1.5, 0.75 …..

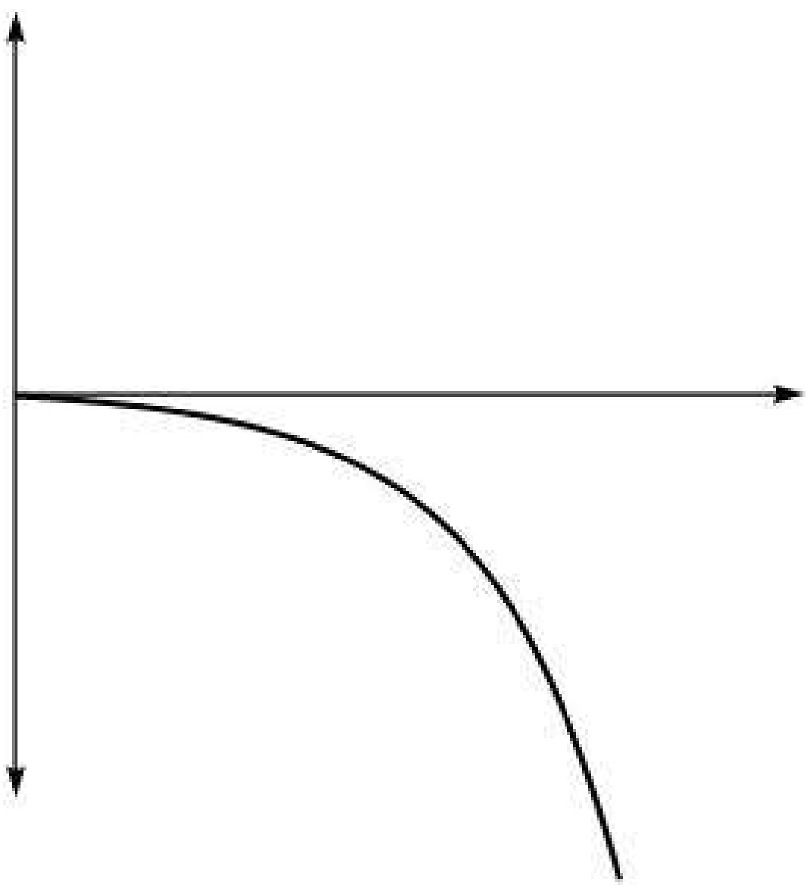


(4) Decreasing GPs type 2:

First term negative and common ratio greater than 1.

e.g: –2, –6, –18 ….

In this case the relationship looks like.



HARMONIC PROGRESSION

Three quantities a, b, c are said to be in Harmonic Progression when $a/c = \frac{(a-b)}{(b-c)}$.

In general, if a, b, c, d are in AP then $1/a$, $1/b$, $1/c$ and $1/d$ are all in HP.

Any number of quantities are said to be in harmonic progression when every three consecutive terms are in harmonic progression.

The reciprocals of quantities in harmonic progression are in arithmetic progression. This can be proved as:

By definition, if *a*, *b*, *c* are in harmonic progression,

$$\frac{a}{c} = \frac{(a-b)}{(b-c)}$$

\ $a(b - c) = c(a - b)$,

dividing every term by *abc*, we get

$$\left[\frac{1}{c} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}\right]$$

which proves the proposition.

There is no general formula for the sum of any number of quantities in harmonic progression. Questions in HP are generally solved by inverting the terms, and making use of the properties of the corresponding AP.

To Find the Harmonic Mean between two given Quantities

Let *a*, *b* be the two quantities, *H* their harmonic mean; then $1/a$, $1/H$ and $1/b$ are in A.P.;

\ $$\frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$$

$$\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$$

i.e. $$H = \frac{2ab}{(a+b)}$$

THEOREMS RELATED WITH PROGRESSIONS

If *A*, *G*, *H* are the arithmetic, geometric, and harmonic means between *a* and *b*, we have

$$A = \left(\frac{a+b}{2}\right) \quad (1)$$

$$G = \sqrt{ab} \quad (2)$$

$$H = \frac{2ab}{(a+b)} \quad (3)$$

Therefore, $A \times H = \frac{(a+b)}{2} \times \frac{2ab}{(a+b)} = ab = G^2$

that is, *G is the geometric mean between A and H.*

From these results we see that

$$A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{(a + b - 2\sqrt{ab})}{2}$$

$$= \left[\frac{(\sqrt{a} - \sqrt{b})}{\sqrt{2}}\right]^2$$

which is positive if a and b are positive. Therefore, the arithmetic mean of any two positive quantities is greater than their geometric mean.

Also from the equation $G^2 = AH$, we see that G is intermediate in value between A and H; and it has been proved that $A > G$, therefore $G > H$ and $A > G > H$.

The arithmetic, geometric, and harmonic means between any two positive quantities are in descending order of magnitude.

As we have already seen in the Back to school section of this block there are some number series which have a continuously decreasing value from one term to the next – and such series have the property that they have what can be defined as the sum of infinite terms. Questions on such series are very common in most aptitude exams. Even though they cannot be strictly said to be under the domain of progressions, we choose to deal with them here.

Consider the following question which appeared in CAT 2003.

Find the infinite sum of the series:

$$1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \ldots\ldots.$$

(a) 27/14 (b) 21/13
(c) 49/27 (d) 256/147

Solution: Such questions have two alternative widely divergent processes to solve them.

The first relies on mathematics using algebraic solving. Unfortunately this process being overly mathematical requires a lot of writing and hence is not advisable to be used in an aptitude exam.

The other process is one where we try to predict the approximate value of the sum by taking into account the first few significant terms. (This approach is possible to use because of the fact that in such series we invariably reach the point where the value of the next term becomes insignificant and does not add substantially to the sum). After adding the significant terms we are in a position to guess the approximate value of the sum of the series.

Let us look at the above question in order to understand the process.

In the given series the values of the terms are:

First term = 1

Second term = 4/7 = 0.57

Third term = 9/63 = 0.14

Fourth term = 16/343 = 0.04

Fifth term = 25/2401 = 0.01

Addition upto the fifth term is approximately 1.76

Options (b) and (d) are smaller than 1.76 in value and hence cannot be correct.

That leaves us with options 1 and 3

Option 1 has a value of 1.92 approximately while option 3 has a value of 1.81 approximately.

At this point you need to make a decision about how much value the remaining terms of the series would add to 1.76 (sum of the first 5 terms)

Looking at the pattern we can predict that the sixth term will be

$36/7^5 = 36/16807 = 0.002$ (approx.)

And the seventh term would be $49/7^6 = 49/117649 = 0.0004$ (approx.).

The eighth term will obviously become much smaller.

It can be clearly visualized that the residual terms in the series are highly insignificant. Based on this judgement you realize that the answer will not reach 1.92 and will be restricted to 1.81. Hence the answer will be option 3.

Try using this process to solve other questions of this nature whenever you come across them. (There are a few such questions inserted in the LOD exercises of this chapter)

Useful Results

1. If the same quantity be added to, or subtracted from, all the terms of an AP, the resulting terms will form an *AP*, but with the same common difference as before.
2. If all the terms of an AP be multiplied or divided by the same quantity, the resulting terms will form an AP, but with a new common difference, which will be the multiplication/division of the old common difference. (as the case may be)
3. If all the terms of a GP be multiplied or divided by the same quantity, the resulting terms will form a GP with the same common ratio as before.
4. If $a, b, c, d, \ldots$ are in GP, they are also in continued proportion, since, by definition,

   $a/b = b/c = c/d = \ldots = 1/r$

   Conversely, a series of quantities in continued proportion may be represented by $x, xr, xr^2, \ldots$
5. If you have to assume 3 terms in *AP*, assume them as

   $a - d, a, a + d$ or as $a, a + d$ and $a + 2d$

   For assuming 4 terms of an AP we use: $a - 3d, a - d, a + d$ and $a + 3d$

   For assuming 5 terms of an *AP*, take them as:

   $a - 2d, a - d, a, a + d, a + 2d$.

   These are the most convenient in terms of problem solving.
6. For assuming three terms of a GP assume them as

   a, ar and ar^2 or as $a/r, a$ and ar
7. To find the sum of the first n natural numbers

   Let the sum be denoted by S; then

   $S = 1 + 2 + 3 + \ldots + n$, is given by

   $$S = \frac{n(n+1)}{2}$$
8. To find the sum of the squares of the first n natural numbers

   Let the sum be denoted by S; then

   $S = 1^2 + 2^2 + 3^2 + \ldots + n^2$

This is given by : $S = \left\{\frac{n(n+1)(2n+1)}{6}\right\}$

9. To find the sum of the cubes of the first n natural numbers.

   Let the sum be denoted by S; then

   $S = 1^3 + 2^3 + 3^3 + \ldots + n^3$

   $S = \left[\frac{n(n+1)}{2}\right]^2$

   Thus, the sum of the cubes of the first n natural numbers is equal to the square of the sum of these numbers.

10. To find the sum of the first n odd natural numbers.

    $S = 1 + 3 + 5 + \ldots + (2n - 1)$ Æ n^2

11. To find the sum of the first n even natural numbers.

    $S = 2 + 4 + 6 + \ldots + 2n$ Æ $n(n + 1)$

    $= n^2 + n$

12. To find the sum of odd numbers £ n where n is a natural number:

    Case A: If n is odd Æ $[(n + 1)/2]^2$

    Case B: If n is even Æ $[n/2]^2$

13. To find the sum of even numbers £ n where n is a natural number:

    Case A: If n is odd Æ $\{(n/2)[(n/2) + 1]\}$

    Case B: If n is even Æ $[(n - 1)/2][(n + 1)/2]$

14. Number of terms in a count:

    - If we are counting in steps of 1 from n_1 to n_2 including both the end points, we get $(n_2 - n_1) + 1$ numbers.
    - If we are counting in steps of 1 from n_1 to n_2 including only one end, we get $(n_2 - n_1)$ numbers.
    - If we are counting in steps of 1 from n_1 to n_2 excluding both ends, we get $(n_2 - n_1) - 1$ numbers.

    Example: Between 16 and 25 both included there are 9 + 1 = 10 numbers.

    Between 100 and 200 both excluded there are 100 – 1 = 99 numbers.

    - If we are counting in steps of 2 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/2] + 1$ numbers.
    - If we are counting in steps of 2 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/2]$ numbers.
    - If we are counting in steps of 2 from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/2] - 1$ numbers.
    - If we are counting in steps of 3 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/3] + 1$ numbers.
    - If we are counting in steps of 3 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/3]$

numbers.

- If we are counting in steps of 3 from n_1 to n_2, excluding both ends, we get $[(n_2 - n_1)/3] - 1$ numbers.

Example: Number of numbers between 100 and 200 divisible by three.

Solution: The first number is 102 and the last number is 198. Hence, answer = (96/3) + 1 = 33 (since both 102 and 198 are included).

Alternately, highest number below 100 that is divisible by 3 is 99, and the lowest number above 200 which is divisible by 3 is 201.

Hence, 201 – 99 = 102 Æ 102/3 = 34 Æ Answer = 34 – 1 = 33 (Since both ends are not included.)

In General

- If we are counting in steps of x from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/x] + 1$ numbers.
- If we are counting in steps of "x" from n_1 to n_2 including only one end, we get $(n_2 - n_1)/x$ numbers.
- If we are counting in steps of "x" from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/x] - 1$ numbers.

For instance, if we have to find how many terms are there in the series 107, 114, 121, 128 ... 254, then we have

(254 – 107)/7 + 1 = 147/7 + 1 = 21 + 1 = 22 terms in the series

Of course, an appropriate adjustment will have to be made when n_2 does not fall into the series. This will be done as follows:

For instance, if we have to find how many terms of the series 107, 114, 121, 128 ... are below 258, then we have by the formula:

(258 – 107)/7 + 1 = 151/7 + 1 = 21.57 + 1. = 22.57. This will be adjusted by taking the lower integral value = 22. Æ The number of terms in the series below 258.

The student is advised to try and experiment on these principles to get a clear picture.

WORKED-OUT PROBLEMS

Problem 2.1 Two persons—Ramu Dhobi and Kalu Mochi have joined Donkey-work Associates. Ramu Dhobi and Kalu Mochi started with an initial salary of ` 500 and ` 640 respectively with annual increments of ` 25 and ` 20 each respectively. In which year will Ramu Dhobi start earning more salary than Kalu Mochi?

Solution The current difference between the salaries of the two is ` 140. The annual rate of reduction of this dfference is ` 5 per year. At this rate, it will take Ramu Dhobi 28 years to equalise his salary with Kalu Dhobi's salary.

Thus, in the 29th year he will earn more.

This problem should be solved while reading and the thought process should be 140/5 = 28. Hence, answer is 29th year.

Problem 2.2 Find the value of the expression

1 – 6 + 2 – 7 + 3 – 8 +……… to 100 terms

(a) –250 (b) –500

(c) –450 (d) –300

Solution The series (1 – 6 + 2 – 7 + 3 – 8 +……… to 100 terms) can be rewritten as:

fi (1 + 2 + 3 + … to 50 terms) – (6 + 7 + 8 + … to 50 terms)

Both these are *AP*'s with values of *a* and *d* as Æ

$a = 1$, $n = 50$ and $d = 1$ and $a = 6$, $n = 50$ and $d = 1$ respectively.

Using the formula for sum of an *AP* we get:

Æ 25(2 + 49) – 25(12 + 49)

Æ 25(51 – 61) = –250

Alternatively, we can do this faster by considering (1 – 6), (2 – 7), and so on as one unit or one term.

1 – 6 = 2 – 7 = … = –5. Thus the above series is equivalent to a series of fifty –5's added to each other.

So, (1 – 6) + (2 – 7) + (3 – 8) + … 50 terms = –5 × 50 = **–250**

Problem 2.3 Find the sum of all numbers divisible by 6 in between 100 to 400.

Solution Here 1st term = *a* = 102 (which is the 1st term greater than 100 that is divisible by 6.)

The last term less than 400, which is divisible by 6 is 396.

The number of terms in the *AP;* 102, 108, 114…396 is given by [(396 – 102)/6] +1= 50 numbers.

Common difference = $d = 6$

So, $S = 25\ (204 + 294) = 12450$

Problem 2.4 If *x*, *y*, *z* are in *GP*, then $1/(1 + \log_{10}x)$, $1/\ (1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ will be in:

(a) AP (b) GP

(c) HP (d) Cannot be said

Solution Go through the options.

Checking option (a), the three will be in AP if the 2nd expression is the average of the 1st and 3rd expressions. This can be mathematically written as

$2/(1 + \log_{10}y) = [1/(1 + \log_{10}x)] + [1/(1 + \log_{10}z)]$

$$= \frac{[1 + (1 + \log_{10} x) + 1 + (1 + \log_{10} z)]}{[(1 + \log_{10} x)(1 + \log_{10} z)}$$

$$= \frac{[2 + \log_{10} xz]}{(1 + \log_{10} x)\,(1 + \log_{10} z)}$$

Applying our judgement, there seems to be no indication that we are going to get a solution.

Checking option (b)

$[1/(1 + \log_{10}y)]^2 = [1/(1 + \log_{10}x)]\ [1/(1 + \log_{10}z)]$

$= [1/(1 + \log_{10}(x + z) + \log_{10}xz)]$

Again we are trapped and any solution is not in sight.

Checking option (c).

$1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in *HP* then $1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ will be in AP.

So, $\log_{10}x$, $\log_{10}y$ and $\log_{10}z$ will also be in AP.

Hence, $2 \log_{10}y = \log_{10}x + \log_{10}z$

fi$y^2 = xz$ which is given.

So, **(c)** is the correct option.

Alternatively, you could have solved through the following process.

x, *y* and *z* are given as logarithmic functions.

Assume$x = 1$, $y = 10$ and $z = 100$ as *x*, *y*, *z* are in GP

So,$1 + \log_{10}x = 1$, $1 + \log_{10}y = 2$ and $1 + \log_{10}z = 3$

fi Thus we find that since 1, 2 and 3 are in AP, we can assume that

$1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ are in AP

fi Hence, by definition of an HP we have that $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in *HP*.

Hence, option (c) is the required answer.

Author's Note: In my experience I have always found that the toughest equations and factorisations get solved very easily when there are options, by assuming values in place of the variables in the equation. The values of the variables should be taken in such a manner that the basic restrictions put on the variables should be respected. For example, if an expression in three variables *a*, *b* and *c* is given and it is mentioned that $a + b + c = 0$ then the values that you assume for *a*, *b* and *c* should satisfy this restriction. Hence, you should look at values like 1, 2 and –3 or 2, –1, –1 etc.

This process is especially useful in the case where the question as well as the options both contain expressions. Factorisation and advanced techniques of maths are then not required. This process will be very beneficial for students who are weak at Mathematics.

Problem 2.5 Find t_{10} and S_{10} for the following series:

1, 8, 15,…

Solution This is an AP with first term 1 and common difference 7.

$t_{10} = a + (n - 1)\, d = 1 + 9 \times 7 = 64$

$S_{10} = \dfrac{n\,[2a + (n-1)d]}{2}$

$= \dfrac{10\,[21 + (10-1)7]}{2} = 325$

Alternatively, if the number of terms is small, you can count it directly.

Problem 2.6 Find t_{18} and S_{18} for the following series:

2, 8, 32, …

Solution This is a GP with first term 2 and common ratio 4.

$t_{18} = ar^{n-1} = 2.4^{17}$

$S_{18} = \dfrac{a(r^n - 1)}{r-1} = \dfrac{2\,(4^{18} - 1)}{(4-1)}$

Problem 2.7 Is the series 1, 4,… to n terms an *AP*, or a GP, or an HP, or a series which cannot be determined?

Solution *To determine any progression, we should have at least three terms.*

If the series is an *AP* then the next term of this series will be 7

Again, if the next term is 16, then this will be a GP series (1, 4, 16 …)

So, *we cannot determine* the nature of the progression of this series.

Problem 2.8 Find the sum to 200 terms of the series

1 + 4 + 6 + 5 + 11 + 6 +…

(a) 30,200 (b) 29,800

(c) 30,200 (d) None of these

Solution Spot that the above series is a combination of two *AP*s.

The 1st *AP* is (1 + 6 + 11 +…) and the 2nd *AP* is (4 + 5 + 6 +…)

Since the terms of the two series alternate, S = (1 + 6 + 11 +… to 100 terms) + (4 + 5 + 6 + … to 100 terms)

$= \dfrac{100[2 \times 1 + 99 \times 5]}{2} + \dfrac{100[2 \times 4 + 99 \times 1]}{2}$ Æ (Using the formula for the sum of an *AP*)

= 50[497 + 107] = 50[604] = 30200

Alternatively, we can treat every two consecutive terms as one.

So we will have a total of 100 terms of the nature:

(1 + 4) + (6 + 5) + (11 + 6) … Æ 5, 11, 17…

Now, $a = 5$, $d = 6$ and $n = 100$

Hence the sum of the given series is

$S = \frac{100}{2} \times [2 \times 5 + 99 \times 6]$

$= 50[604] = 30200$

Problem 2.9 How many terms of the series –12, –9, –6,… must be taken that the sum may be 54?

Solution Here $S = 54$, $a = –12$, $d = 3$, n is unknown and has to be calculated. To do so we use the formula for the sum of an *AP* and get.

$54 = \frac{[2(-12)+(n-1)3]n}{2}$

or $108 = –24n – 3n + 3n^2$ or $3n^2 – 27n – 108 = 0$

or $n^2 – 9n – 36 = 0$, or $n^2 – 12n + 3n – 36 = 0$

$n(n – 12) + 3(n – 12) = 0$ fi $(n + 3)(n – 12) = 0$

The value of n (the number of terms) cannot be negative. Hence –3 is rejected.

So we have $n = 12$

Alternatively, we can directly add up individual terms and keep adding manually till we get a sum of 54. We will observe that this will occur after adding 12 terms. (In this case, as also in all cases where the number of terms is mentally manageable, mentally adding the terms till we get the required sum will turn out to be much faster than the equation based process.

Problem 2.10 Find the sum of n terms of the series 1.2.4 + 2.3.5 + 3.4.6 + …

(a) $n(n + 1)(n + 2)$

(b) $(n(n + 1)/12)(3n^2 + 19n + 26)$

(c) $((n + 1)(n + 2)(n + 3))/4$

(d) $(n^2 (n + 1)(n + 2)(n + 3))/3$

Solution In order to solve such problems in the examination, the option-based approach is the best. Even if you can find out the required expression mathematically, it is advisable to solve through the options as this will end up saving a lot of time for you. Use the options as follows:

If we put $n = 1$, we should get the sum as 1.2.4 = 8. By substituting $n = 1$ in each of the four options we will get the following values for the sum to 1 term:

Option (a) gives a value of: 6

Option (b) gives a value of: 8

Option (c) gives a value of: 6

Option (d) gives a value of: 8

From this check we can reject the options (a) and (c).

Now put $n = 2$. You can see that up to 2 terms, the expression is 1.2.4 + 2.3.5 = 38

The correct option should also give 38 if we put $n = 2$ in the expression. Since, (a) and (c) have already been rejected, we only need to check for options (b) and (d).

Option (b) gives a value of 38

Option (d) gives a value of 80.

Hence, we can reject option (d) and get (b) as the answer.

Note: **The above process is very effective for solving questions having options. The student should try to keep an eye open for the possibility of solving questions through options. In my opinion, approximately 50–75% of the questions asked in CAT in the QA section can be solved with options (at least partially).**

LEVEL OF DIFFICULTY (I)

1. How many terms are there in the AP 20, 25, 30,… 130.

   (a) 22 (b) 23

   (c) 21 (d) 24

2. Bobby was appointed to Mindworkzz in the pay scale of ` 7000–500–12,500. Find how many years he will take to reach the maximum of the scale.

   (a) 11 years (b) 10 years

   (c) 9 years (d) 8 years

3. Find the 1st term of an AP whose 8th and 12th terms are respectively 39 and 59.

   (a) 5 (b) 6

   (c) 4 (d) 3

4. A number of squares are described whose perimetres are in GP. Then their sides will be in

   (a) AP (b) GP

   (c) HP (d) Nothing can be said

5. There is an *AP* 1, 3, 5…. Which term of this AP is 55?

   (a) 27th (b) 26th

   (c) 25th (d) 28th

6. How many terms are identical in the two *AP*s 1, 3, 5,… up to 120 terms and 3, 6, 9,… up to 80 terms?

   (a) 38 (b) 39

   (c) 40 (d) 41

7. Find the lowest number in an AP such that the sum of all the terms is 105 and greatest term is 6 times the least.

   (a) 5 (b) 10

   (c) 15 (d) (a), (b) & (c)

8. Find the 15th term of the sequence 20, 15, 10, …

   (a) –45 (b) –55

   (c) –50 (d) 0

9. A sum of money kept in a bank amounts to ` 1240 in 4 years and ` 1600 in 10 years at simple Interest. Find the sum.

   (a) ` 800 (b) ` 900

   (c) ` 1150 (d) ` 1000

10. A number 15 is divided into three parts which are in *AP* and the sum of their squares is 83. Find the smallest number.

(a) 5
(b) 3
(c) 6
(d) 8

11. The sum of the first 16 terms of an *AP* whose first term and third term are 5 and 15 respectively is

(a) 600
(b) 765
(c) 640
(d) 680

12. The number of terms of the series 54 + 51 + 48 +… such that the sum is 513 is

(a) 18
(b) 19
(c) Both *a* and *b*
(d) 15

13. The least value of n for which the sum of the series 5 + 8 + 11… n terms is not less than 670 is

(a) 20
(b) 19
(c) 22
(d) 21

14. A man receives ` 60 for the first week and ` 3 more each week than the preceding week. How much does he earn by the 20th week?

(a) ` 1770
(b) ` 1620
(c) ` 1890
(d) ` 1790

15. How many terms are there in the GP 5, 20, 80, 320,… 20480?

(a) 6
(b) 5
(c) 7
(d) 8

16. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and so on. How much will the boy get if he starts working on the 1st of February and finishes on the 20th of February?

(a) 2^{20}
(b) $2^{20} - 1$
(c) $2^{19} - 1$
(d) 2^{19}

17. If the fifth term of a GP is 81 and first term is 16, what will be the 4th term of the GP?

(a) 36
(b) 18
(c) 54
(d) 24

18. The seventh term of a GP is 8 times the fourth term. What will be the first term when its fifth term is 48?

(a) 4
(b) 3
(c) 5
(d) 2

19. The sum of three numbers in a GP is 14 and the sum of their squares is 84. Find the largest number.

(a) 8
(b) 6
(c) 4
(d) 12

20. The first term of an arithmetic progression is unity and the common difference is 4. Which of the

following will be a term of this AP?

(a) 4551 (b) 10091

(c) 7881 (d) 13531

21. How many natural numbers between 300 to 500 are multiples of 7?

(a) 29 (b) 28

(c) 27 (d) 30

22. The sum of the first and the third term of a geometric progression is 20 and the sum of its first three terms is 26. Find the progression.

(a) 2, 6, 18,… (b) 18, 6, 2,…

(c) Both of these (d) None of these

23. If a man saves ` 4 more each year than he did the year before and if he saves ` 20 in the first year, after how many years will his savings be more than ` 1000 altogether?

(a) 19 years (b) 20 years

(c) 21years (d) 18 years

24. A man's salary is ` 800 per month in the first year. He has joined in the scale of 800–40–1600. After how many years will his savings be ` 64,800?

(a) 8 years (b) 7 years

(c) 6 years (d) Cannot be determined

25. The 4th and 10th term of an GP are 1/3 and 243 respectively. Find the 2nd term.

(a) 3 (b) 1

(c) 1/27 (d) 1/9

26. The 7th and 21st terms of an AP are 6 and –22 respectively. Find the 26th term.

(a) –34 (b) –32

(c) –12 (d) –10

27. The sum of 5 numbers in AP is 30 and the sum of their squares is 220. Which of the following is the third term?

(a) 5 (b) 6

(c) 8 (d) 9

28. Find the sum of all numbers in between 10–50 excluding all those numbers which are divisible by 8. (include 10 and 50 for counting.)

(a) 1070 (b) 1220

(c) 1320 (d) 1160

29. The sum of the first four terms of an AP is 28 and sum of the first eight terms of the same AP is 88. Find the sum of the first 16 terms of the AP?

(a) 346 (b) 340

(c) 304

(d) 268

30. Find the general term of the GP with the third term 1 and the seventh term 8.

(a) $(2^{3/4})^{n-3}$

(b) $(2^{3/2})^{n-3}$

(c) $(2^{3/4})^{3-n}$

(d) $(2^{3/4})^{2-n}$

31. Find the number of terms of the series 1/81, –1/27, 1/9,… –729.

(a) 11

(b) 12

(c) 10

(d) 13

32. Four geometric means are inserted between 1/8 and 128. Find the third geometric mean.

(a) 4

(b) 16

(c) 32

(d) 8

33. *A* and *B* are two numbers whose *AM* is 25 and *GM* is 7. Which of the following may be a value of *A*?

(a) 10

(b) 20

(c) 49

(d) 25

34. Two numbers *A* and *B* are such that their *GM* is 20% lower than their *AM*. Find the ratio between the numbers.

(a) 3 : 2

(b) 4 : 1

(c) 2 : 1

(d) 3 : 1

35. A man saves ` 100 in January 2014 and increases his saving by ` 50 every month over the previous month. What is the annual saving for the man in the year 2014?

(a) ` 4200

(b) ` 4500

(c) ` 4000

(d) ` 4100

36. In a nuclear power plant a technician is allowed an interval of maximum 100 minutes. A timer with a bell rings at specific intervals of time such that the minutes when the timer rings are not divisible by 2, 3, 5 and 7. The last alarm rings with a buzzer to give time for decontamination of the technician. How many times will the bell ring within these 100 minutes and what is the value of the last minute when the bell rings for the last time in a 100 minute shift?

(a) 25 times, 89

(b) 21 times, 97

(c) 22 times, 97

(d) 19 times, 97

37. How many zeroes will be there at the end of the expression $(2!)^{2!} + (4!)^{4!} + (8!)^{8!} + (9!)^{9!} + (10!)^{10!} + (11!)^{11!}$?

(a) $(8!)^{8!} + (9!)^{9!} + (10!)^{10!} + (11!)^{11!}$

(b) 10^{101}

(c) $4! + 6! + 8! + 2\,(10!)$

(d) $(0!)^{0!}$

38. The 1^{st}, 8^{th} and 22^{nd} terms of an AP are three consecutive terms of a GP. Find the common ratio of

the GP, given that the sum of the first twenty-two terms of the AP is 385.

(a) Either 1 or1/2 (b) 2

(c) 1 (d) Either 1 or 2

39. The internal angles of a plane polygon are in AP. The smallest angle is 100^o and the common difference is 10^o. Find the number of sides of the polygon.

(a) 8 (b) 9

(c) Either 8 or 9 (d) None of these

40. After striking a floor a rubber ball rebounds $(7/8)^{th}$ of the height from which it has fallen. Find the total distance that it travels before coming to rest, if it is gently dropped from a height of 420 meters.

(a) 2940 (b) 6300

(c) 1080 (d) 3360

41. Each of the series 13 + 15 + 17+…. and 14 + 17 + 20+… is continued to 100 terms. Find how many terms are identical between the two series.

(a) 35 (b) 34

(c) 32 (d) 33

42. Jack and Jill were playing mathematical puzzles with each other. Jill drew a square of sides 8 cm and then kept on drawing squares inside the squares by joining the mid points of the squares. She continued this process indefinitely. Jill asked Jack to determine the sum of the areas of all the squares that she drew. If Jack answered correctly then what would be his answer?

(a) 128 (b) 64

(c) 256 (d) 32

43. How many terms of the series 1 + 3 + 5 + 7 + …… amount to 123454321?

(a) 11101 (b) 11011

(c) 10111 (d) 11111

44. A student takes a test consisting of 100 questions with differential marking is told that each question after the first is worth 4 marks more than the preceding question. If the third question of the test is worth 9 marks. What is the maximum score that the student can obtain by attempting 98 questions?

(a) 19698 (b) 19306

(c) 9900 (d) None of these

45. In an infinite geometric progression, each term is equal to 2 times the sum of the terms that follow. If the first term of the series is 8, find the sum of the series?

(a) 12 (b) 32/3

(c) 34/3 (d) Data inadequate

46. What is the maximum sum of the terms in the arithmetic progression 25, 24 ½ , 24, ………..?

(a) 637 ½ (b) 625

(c) 662 ½ (d) 650

47. An equilateral triangle is drawn by joining the midpoints of the sides of another equilateral triangle. A third equilateral triangle is drawn inside the second one joining the midpoints of the sides of the second equilateral triangle, and the process continues infinitely. Find the sum of the perimeters of all the equilateral triangles, if the side of the largest equilateral triangle is 24 units.

(a) 288 units (b) 72 units

(c) 36 units (d) 144 units

48. The sum of the first two terms of an infinite geometric series is 18. Also, each term of the series is seven times the sum of all the terms that follow. Find the first term and the common ratio of the series respectively.

(a) 16, 1/8 (b) 15, 1/5

(c) 12, 1/2 (d) 8, 1/16

49. Find the 33rd term of the sequence: 3, 8, 9, 13, 15, 18, 21, 23…

(a) 93 (b) 99

(c) 105 (d) 83

50. For the above question, find the sum of the series till 33 terms.

(a) 728 (b) 860

(c) 1595 (d) 1583

LEVEL OF DIFFICULTY (II)

1. If a times the ath term of an AP is equal to b times the bth term, find the $(a + b)$th term.
   (a) 0 (b) $a^2 – b^2$
   (c) $a – b$ (d) 1
2. A number 20 is divided into four parts that are in AP such that the product of the first and fourth is to the product of the second and third is 2 : 3. Find the largest part.
   (a) 12 (b) 4
   (c) 8 (d) 9
3. Find the value of the expression: 1 – 4 + 5 – 8 … to 50 terms.
   (a) –150 (b) –75
   (c) –50 (d) 75
4. If a clock strikes once at one o'clock, twice at two o'clock and twelve times at 12 o'clock and again once at one o'clock and so on, how many times will the bell be struck in the course of 2 days?
   (a) 156 (b) 312
   (c) 78 (d) 288
5. What will be the maximum sum of 44, 42, 40, … ?
   (a) 502 (b) 504
   (c) 506 (d) 500
6. Find the sum of the integers between 1 and 200 that are multiples of 7.
   (a) 2742 (b) 2842
   (c) 2646 (d) 2546
7. If the mth term of an AP is $1/n$ and nth term is $1/m$, then find the sum to mn terms.
   (a) $(mn – 1)/4$ (b) $(mn + 1)/4$
   (c) $(mn + 1)/2$ (d) $(mn – 1)/2$
8. Find the sum of all odd numbers lying between 100 and 200.
   (a) 7500 (b) 2450
   (c) 2550 (d) 2650
9. Find the sum of all integers of 3 digits that are divisible by 7.
   (a) 69,336 (b) 71,336
   (c) 70,336 (d) 72,336
10. The first and the last terms of an AP are 107 and 253. If there are five terms in this sequence, find the sum of sequence.
    (a) 1080 (b) 720

(c) 900 (d) 620

11. Find the value of 1 – 2 – 3 + 2 – 3 – 4 +… + upto 100 terms.

(a) –694 (b) –626

(c) –624 (d) –549

12. What will be the sum to n terms of the series 8 + 88 + 888 +…?

(a) $\frac{8(10^n - 9n)}{81}$ (b) $\frac{8(10^{n+1} - 10 - 9n)}{81}$

(c) $8(10^{n-1} - 10)$ (d) $8(10^{n+1} - 10)$

13. If a, b, c are in GP, then log a, log b, log c are in

(a) AP (b) GP

(c) HP (d) None of these

14. After striking the floor, a rubber ball rebounds to 4/5th of the height from which it has fallen. Find the total distance that it travels before coming to rest if it has been gently dropped from a height of 120 metres.

(a) 540 metres (b) 960 metres

(c) 1080 metres (d) 1020 metres

15. If x be the first term, y be the nth term and p be the product of n terms of a GP, then the value of p^2 will be

(a) $(xy)^{n-1}$ (b) $(xy)^n$

(c) $(xy)^{1-n}$ (d) $(xy)^{n/2}$

16. The sum of an infinite GP whose common ratio is numerically less than 1 is 32 and the sum of the first two terms is 24. What will be the third term?

(a) 2 (b) 16

(c) 8 (d) 4

17. What will be the value of $x^{1/2}.x^{1/4}.x^{1/8}$… to infinity.

(a) x^2 (b) x

(c) $x^{3/2}$ (d) x^3

18. Find the sum to n terms of the series

1.2.3 + 2.3.4 + 3.4.5 +…

(a) $(n + 1)(n + 2)(n + 3)/3$

(b) $n(n + 1)(2n + 2)(n + 2)/4$

(c) $n(n + 1)(n + 2)$

(d) $n(n + 1)(n + 2)(n + 3)/4$

19. Determine the first term of the geometric progression, the sum of whose first term and third term is 40 and the sum of the second term and fourth term is 80.

(a) 12 (b) 16

(c) 8 (d) 4

20. Find the second term of an AP if the sum of its first five even terms is equal to 15 and the sum of the first three terms is equal to –3.

(a) –3 (b) –2

(c) –1 (d) 0

21. The sum of the second and the fifth term of an AP is 8 and that of the third and the seventh term is 14. Find the eleventh term.

(a) 19 (b) 17

(c) 15 (d) 16

22. How many terms of an AP must be taken for their sum to be equal to 120 if its third term is 9 and the difference between the seventh and the second term is 20?

(a) 6 (b) 9

(c) 7 (d) 8

23. Four numbers are inserted between the numbers 4 and 39 such that an AP results. Find the biggest of these four numbers.

(a) 31.5 (b) 31

(c) 32 (d) 30

24. Find the sum of all three-digit natural numbers, which on being divided by 5, leave a remainder equal to 4.

(a) 57,270 (b) 96,780

(c) 49,680 (d) 99,270

25. The sum of the first three terms of the arithmetic progression is 30 and the sum of the squares of the first term and the second term of the same progression is 116. Find the seventh term of the progression if its fifth term is known to be exactly divisible by 14.

(a) 36 (b) 40

(c) 43 (d) 22

26. *A* and *B* set out to meet each other from two places 165 km apart. *A* travels 15 km the first day, 14 km the second day, 13 km the third day and so on. *B* travels 10 km the first day, 12 km the second day, 14 km the third day and so on. After how many days will they meet?

(a) 8 days (b) 5 days

(c) 6 days (d) 7 days

27. If a man saves ` 1000 each year and invests at the end of the year at 5% compound interest, how much will the amount be at the end of 15 years?

(a) ` 21,478 (b) ` 21,578

(c) ` 22,578

(d) ` 22,478

28. If sum to n terms of a series is given by (n + 8), then its second term will be given by

(a) 10

(b) 9

(c) 8

(d) 1

29. If A is the sum of the n terms of the series 1 + 1/4 + 1/16 + … and B is the sum of 2n terms of the series 1 + 1/2 + 1/4 + …, then find the value of A/B.

(a) 1/3

(b) 1/2

(c) 2/3

(d) 3/4

30. A man receives a pension starting with ` 100 for the first year. Each year he receives 90% of what he received the previous year. Find the maximum total amount he can receive even if he lives forever.

(a) ` 1100

(b) ` 1000

(c) ` 1200

(d) ` 900

31. The sum of the series represented as:

1/1 × 5 + 1/5 × 9 + 1/9 × 13 ….. + 1/221 × 225 is

(a) 28/221

(b) 56/221

(c) 56/225

(d) None of these

32. The sum of the series

1/($\sqrt{2}$ + $\sqrt{1}$) + 1/($\sqrt{2}$ + $\sqrt{3}$) +……………+ 1/($\sqrt{120}$ + $\sqrt{121}$) is:

(a) 10

(b) 11

(c) 12

(d) None of these

33. Find the infinite sum of the series 1/1 + 1/3 + 1/6 + 1/10 + 1/15 ….

(a) 2

(b) 2.25

(c) 3

(d) 4

34. The sum of the series 5 × 8 + 8 × 11 + 11 × 14 upto n terms will be:

(a) $(n + 1)[3(n + 1)^2 + 6(n + 1) + 1] – 10$

(b) $(n + 1)[3(n + 1)^2 + 6(n + 1) + 1] + 10$

(c) $(n + 1)[3(n + 1) + 6(n + 1)^2 + 1] – 10$

(d) $(n + 1)[3(n + 1) + 6(n + 1)^2 + 1] + 10$

35. The sum of the series: ½ + 1/6 + 1/12 + 1/20 + …1/156 + 1/182 is:

(a) 12/13

(b) 13/14

(c) 14/13

(d) None of these

36. For the above question 35, what is the sum of the series if taken to infinite terms:

(a) 1.1

(b) 1

(c) 14/13 (d) None of these

Directions for Questions 37 to 39: Answer these questions based on the following information.

There are 250 integers a_1, a_2.......a_{250}, not all of them necessarily different. Let the greatest integer of these 250 integers be referred to as Max, and the smallest integer be referred to as Min. The integers a_1 through a_{124} form sequence *A*, and the rest form sequence *B*. Each member of *A* is less than or equal to each member of *B*.

37. All values in *A* are changed in sign, while those in *B* remain unchanged. Which of the following statements is true?

(a) Every member of *A* is greater than or equal to every member of *B*.

(b) Max is in *A*.

(c) If all numbers originally in *A* and *B* had the same sign, then after the change of sign, the largest number of *A* and *B* is in *A*.

(d) None of these

38. Elements of *A* are in ascending order, and those of *B* are in descending order. a_{124} and a_{125} are interchanged. Then which of the following statements is true?

(a) A continues to be in ascending order.

(b) B continues to be in descending order

(c) A continues to be in ascending order and B in descending order

(d) None of the above

39. Every element of *A* is made greater than or equal to every element of *B* by adding to each element of *A* an integer *x*. Then, *x* cannot be less than:

(a) 2^{10}

(b) the smallest value of *B*

(c) the largest value of *B*

(d) (Max-Min)

40. Rohit drew a rectangular grid of 529 cells, arranged in 23 Rows and 23 columns, and filled each cell with a number. The numbers with which he filled each cell were such that the numbers of each row taken from left to right formed an arithmetic series and the numbers of each column taken from top to bottom also formed an arithmetic series. The seventh and the seventeenth numbers of the fifth row were 47 and 63 respectively, while the seventh and the seventeenth numbers of the fifteenth row were 53 and 77 respectively. What is the sum of all the numbers in the grid?

(a) 32798 (b) 65596

(c) 52900 (d) None of these

41. How many three digit numbers have the property that their digits taken from left to right form an Arithmetic or a Geometric Progression?

(a) 15 (b) 36

(c) 20 (d) 42

Directions for Questions 42 and 43: These questions are based on the following data.

At Burger King—a famous fast food centre on Main Street in Pune, burgers are made only on an automatic burger making machine. The machine continuously makes different sorts of burgers by adding different sorts of fillings on a common bread. The machine makes the burgers at the rate of 1 burger per half a minute. The various fillings are added to the burgers in the following manner. The 1st, 5th, 9th, ... burgers are filled with a chicken patty; the 2nd, 9th, 16th, ... burgers with vegetable patty; the 1st, 5th, 9th, ... burgers with mushroom patty; and the rest with plain cheese and tomato fillings.

The machine makes exactly 660 burgers per day.

42. How many burgers per day are made with cheese and tomato as fillings?

(a) 424 (b) 236

(c) 237 (d) None of these

43. How many burgers are made with all three fillings Chicken, vegetable and mushroom?

(a) 23 (b) 24

(c) 25 (d) 26

44. An arithmetic progression P consists of *n* terms. From the progression three different progressions P_1 P_2 and P_3 are created such that P_1 is obtained by the 1st, 4th ,7th...... terms of *P*, P_2 has the 2nd, 5th, 8th,......terms of *P* and P_3 has the 3rd, 6th, 9th, ...terms of P. It is found that of P_1, P_2 and P_3 two progressions have the property that their average is itself a term of the original Progression *P*. Which of the following can be a possible value of *n*?

(a) 20 (b) 26

(c) 36 (d) Both (a)and (b)

45. For the above question, if the Common Difference between the terms of P_1 is 6, what is the common difference of *P*?

(a) 2 (b) 3

(c) 6 (d) Cannot be determined

LEVEL OF DIFFICULTY (III)

1. If in any decreasing arithmetic progression, sum of all its terms, except for the first term, is equal to –36, the sum of all its terms, except for the last term, is zero, and the difference of the tenth and the sixth term is equal to – 16, then what will be first term of this series?

   (a) 16 (b) 20

   (b) –16 (d) –20

2. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99, and except for the sixth term, 89. Find the third term of the progression if the sum of the first and the fifth term is equal to 10.

   (a) 15 (b) 5

   (c) 8 (d) 10

3. Product of the fourth term and the fifth term of an arithmetic progression is 456. Division of the ninth term by the fourth term of the progression gives quotient as 11 and the remainder as 10. Find the first term of the progression.

   (a) – 52 (b) – 42

   (c) – 56 (d) – 66

4. A number of saplings are lying at a place by the side of a straight road. These are to be planted in a straight line at a distance interval of 10 meters between two consecutive saplings. Mithilesh, the country's greatest forester, can carry only one sapling at a time and has to move back to the original point to get the next sapling. In this manner he covers a total distance of 1.32 kms. How many saplings does he plant in the process if he ends at the starting point?

   (a) 15 (b) 14

   (c) 13 (d) 12

5. A geometric progression consists of 500 terms. Sum of the terms occupying the odd places is P_1 and the sum of the terms occupying the even places is P_2. Find the common ratio.

   (a) P_2/P_1 (b) P_1/P_2

   (c) $P_2 + P_1/P_1$ (d) $P_2 + P_1/P_2$

6. The sum of the first ten terms of the geometric progression is S_1 and the sum of the next ten terms (11th through 20th) is S_2. Find the common ratio.

   (a) $(S_1/S_2)^{1/10}$ (b) $-(S_1/S_2)^{1/10}$

   (c) $\pm\sqrt[10]{S_2/S_1}$ (d) $(S_1/S_2)^{1/5}$

7. The first and the third terms of an arithmetic progression are equal, respectively, to the first and the third term of a geometric progression, and the second term of the arithmetic progression exceeds the second term of the geometric progression by 0.25. Calculate the sum of the first five terms of the arithmetic progression if its first term is equal to 2.

(a) 2.25 or 25

(b) 2.5 or 27.5

(c) 1.5

(d) 3.25

8. If (2 + 4 + 6 +… 50 terms)/(1 + 3 + 5 +… *n* terms) = 51/2, then find the value of *n* .

(a) 12

(b) 13

(c) 9

(d) 10

9. (666…. *n* digits)^2 + (888… *n* digits) is equal to

(a) $(10^n - 1) \times \frac{4}{9}$

(b) $(10^{2n} - 1) \times \frac{4}{9}$

(c) $\frac{4(10^n - 10^{n-1} - 1)}{9}$

(d) $\frac{4(10^n + 1)}{9}$

10. The interior angles of a polygon are in *AP*. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

(a) 7

(b) 8

(c) 9

(d) 10

11. Find the sum to *n* terms of the series 11 + 103 + 1005 + …

(a) $\frac{10(10^n - 1)}{9} + 1$

(b) $\frac{10(10^n - 1)}{9} + n$

(c) $\frac{10(10^n - 1)}{9} + n^2$

(d) $\frac{10(10^n + 1)}{11} + n^2$

12. The sum of the first term and the fifth term of an *AP* is 26 and the product of the second term by the fourth term is 160. Find the sum of the first seven terms of this *AP*.

(a) 110

(b) 114

(c) 112

(d) 116

13. The sum of the third and the ninth term of an *AP* is 10. Find a possible sum of the first 11 terms of this *AP*.

(a) 55

(b) 44

(c) 66

(d) 48

14. The sum of the squares of the fifth and the eleventh term of an *AP* is 3 and the product of the second and the fourteenth term is equal it *P*. Find the product of the first and the fifteenth term of the *AP*.

(a) (58*P* – 39)/45

(b) (98*P* + 39)/72

(c) (116*P* – 39)/90

(d) (98*P* + 39)/90

15. If the ratio of harmonic mean of two numbers to their geometric mean is 12 : 13, find the ratio of the numbers.

(a) 4/9 or 9/4

(b) 2/3 or 3/2

(c) 2/5 or 5/2 (d) 3/4 or 4/5

16. Find the sum of the series $1.2 + 2.2^2 + 3.2^3 + \ldots + 100.\ 2^{100}$.

(a) $100.2^{101} + 2$ (b) $99.2^{100} + 2$

(c) $99.2^{101} + 2$ (d) None of these

17. The sequence $[x_n]$ is a *GP* with $x_2/x_4 = 1/4$ and $x_1 + x_4 = 108$. What will be the value of x_3?

(a) 42 (b) 48

(c) 44 (d) 56

18. If x, y, z are in GP and a^x, b^y and c^z are equal, then a, b, c are in

(a) *AP* (b) *GP*

(c) *HP* (d) None of these

19. Find the sum of all possible whole number divisors of 720.

(a) 2012 (b) 2624

(c) 2210 (d) 2418

20. Sum to n terms of the series $\log m + \log m^2/n + \log m^3/n^2 + \log m^4/n^3 \ldots$ is

(a) $\log \left(\frac{m^{n+1}}{n^{n-1}}\right)^{\frac{n}{2}}$ (b) $\log \left(\frac{n^{n-1}}{m^{n+1}}\right)^{\frac{n}{2}}$

(c) $\log \left(\frac{m^{n}}{n^{n}}\right)^{\frac{n}{2}}$ (d) $\log \left(\frac{m^{1-n}}{n^{1-m}}\right)^{\frac{n}{2}}$

21. The sum of first 20 and first 50 terms of an *AP* is 20 and 2550. Find the eleventh term of a *GP* whose first term is the same as the *AP* and the common ratio of the *GP* is equal to the common difference of the *AP*.

(a) 560 (b) 512

(c) 1024 (d) 3072

22. If three positive real numbers x, y, z are in *AP* such that $xyz = 4$, then what will be the minimum value of y?

(a) $2^{1/3}$ (b) $2^{2/3}$

(c) $2^{1/4}$ (d) $2^{3/4}$

23. If a_n be the nth term of an *AP* and if $a_7 = 15$, then the value of the common difference that would make $a_2a_7a_{12}$ greatest is

(a) 3 (b) 3/2

(c) 7 (d) 0

24. If $a_1, a_2, a_3 \ldots a_n$ are in *AP*, where $a_i > 0$, then what will be the value of the expression

$1/(\sqrt{a_1} + \sqrt{a_2}) + 1/(\sqrt{a_2} + \sqrt{a_3}) + 1/(\sqrt{a_3} + \sqrt{a_4}) + \ldots$ to n terms?

(a) $(1 - n)/(\sqrt{a_1} + \sqrt{a_n})$

(b) $(n - 1)/(\sqrt{a_1} + \sqrt{a_n})$

(c) $(n - 1)/(\sqrt{a_1} - \sqrt{a_n})$

(d) $(1 - n)/(\sqrt{a_1} + \sqrt{a_n})$

25. If the first two terms of a *HP* are 2/5 and 12/13 respectively, which of the following terms is the largest term?

(a) 4th term (b) 5th term

(c) 6th term (d) 7th term

26. One side of a staircase is to be closed in by rectangular planks from the floor to each step. The width of each plank is 9 inches and their height are successively 6 inches, 12 inches, 18 inches and so on. There are 24 planks required in total. Find the area in square feet.

(a) 112.5 (b) 107

(c) 118.5 (d) 105

27. The middle points of the sides of a triangle are joined forming a second triangle. Again a third triangle is formed by joining the middle points of this second triangle and this process is repeated infinitely. If the perimetre and area of the outer triangle are *P* and *A* respectively, what will be the sum of perimetres of triangles thus formed?

(a) $2P$ (b) P^2

(c) $3P$ (d) $p^2/2$

28. In Problem 27, find the sum of areas of all the triangles.

(a) $\frac{4}{5}$A (b) $\frac{4}{3}$A

(c) $\frac{3}{4}$A (d) $\frac{5}{4}$A

29. A square has a side of 40 cm. Another square is formed by joining the mid-points of the sides of the given square and this process is repeated infinitely. Find the perimetre of all the squares thus formed.

(a) $160(1 + \sqrt{2})$ (b) $160(2 + \sqrt{2})$

(c) $160(2 - \sqrt{2})$ (d) $160(1 - \sqrt{2})$

30. In problem 29, find the area of all the squares thus formed.

(a) 1600 (b) 2400

(c) 2800 (d) 3200

31. The sum of the first *n* terms of the arithmetic progression is equal to half the sum of the next *n* terms of the same progression. Find the ratio of the sum of the first 3*n* terms of the progression to

the sum of its first n terms.

(a) 5 (b) 6

(c) 7 (d) 8

32. In a certain colony of cancerous cells, each cell breaks into two new cells every hour. If there is a single productive cell at the start and this process continues for 9 hours, how many cells will the colony have at the end of 9 hours? It is known that the life of an individual cell is 20 hours.

(a) $2^9 - 1$ (b) 2^{10}

(c) 2^9 (d) $2^{10} - 1$

33. Find the sum of all three-digit whole numbers less than 500 that leave a remainder of 2 when they are divided by 3.

(a) 49637 (b) 39767

(c) 49634 (d) 39770

34. If a be the arithmetic mean and b, c be the two geometric means between any two positive numbers, then $(b^3 + c^3) /abc$ equals

(a) $(ab)^{1/2}/c$ (b) 1

(c) a^2c/b (d) None of these

35. If p, q, r are three consecutive distinct natural numbers then the expression $(q + r - p)(p + r - q)(p + q - r)$ is

(a) Positive (b) Negative

(c) Non-positive (d) Non-negative

ANSWER KEY

Level of Difficulty (I)

1. (b)	2. (a)	3. (c)	4. (b)
5. (d)	6. (c)	7. (d)	8. (c)
9. (d)	10. (b)	11. (d)	12. (c)
13. (a)	14. (a)	15. (c)	16. (b)
17. (c)	18. (b)	19. (a)	20. (c)
21. (a)	22. (c)	23. (a)	24. (d)
25. (c)	26. (b)	27. (b)	28. (a)
29. (c)	30. (a)	31. (a)	32. (d)
33. (c)	34. (b)	35. (b)	36. (c)
37. (d)	38. (b)	39. (c)	40. (b)
41. (d)	42. (a)	43. (d)	44. (d)
45. (a)	46. (a)	47. (d)	48. (a)
49. (b)	50. (c)		

Level of Difficulty (II)

1. (a)	2. (c)	3. (b)	4. (b)
5. (c)	6. (b)	7. (c)	8. (a)
9. (c)	10. (c)	11. (b)	12. (b)
13. (a)	14. (c)	15. (b)	16. (d)
17. (b)	18. (d)	19. (c)	20. (c)
21. (a)	22. (d)	23. (c)	24. (d)
25. (b)	26. (c)	27. (b)	28. (d)
29. (c)	30. (b)	31. (c)	32. (a)
33. (a)	34. (a)	35. (b)	36. (b)
37. (d)	38. (a)	39. (d)	40. (a)
41. (d)	42. (a)	43. (b)	44. (d)
45. (a)			

Level of Difficulty (III)

1. (a)	2. (b)	3. (d)	4. (d)
5. (a)	6. (c)	7. (b)	8. (d)
9. (b)	10. (c)	11. (c)	12. (c)
13. (a)	14. (d)	15. (d)	16. (d)
17. (b)	18. (b)	19. (d)	20. (a)
21. (d)	22. (b)	23. (d)	24. (b)
25. (d)	26. (a)	27. (a)	28. (b)
29. (b)	30. (d)	31. (b)	32. (d)
33. (b)	34. (d)	35. (d)	

Hints

Level of Difficulty (II)

2. $a + (a + d) + (a + 2d) + (a + 3d) = 20$
   and $a(a + 3d) = (a + d)(a + 2d)$
4. Calculate the sum of an AP with first term 1, common difference 1 and last term 12. Multiply this sum by 4 for 2 days.
5. The maximum sum will occur when the last term is either 2 or 0.
6. Visualise the AP as 7, 14…196.
9. The AP is 105, 112 …994.
10. The common difference is $\frac{146}{5} = 29.2$.
11. See the terms of the series in 33 blocks of 3 each. This will give the AP – 4, –5, –6…–33. Further, the hundredth term will be 34.
12. Solve through options.
14. The first drop is 120 metres. After this the ball will rise by 96 metres and fall by 96 meters. This

process will continue in the form of an infinite GP with common ratio 0.8 and first term 96.

The required answer will be got by

120 + 96 * 1.25 * 2

15. Take any GP and solve by using values.
18. Solve by using values to check options.
22. The difference between the seventh and third term is given by

$(a + 6d) – (a + d)$

23. $\frac{(39 - 4)}{5} = 7.$
27. The required answer will be by adding 20 terms of the GP starting with the first term as 1000 and the common ratio as 1.05.
30. Visualise it as an infinite GP with common ratio 0.9.

Level of Difficulty (III)

1. Difference between the tenth and the sixth term = –16

or $(a + 9d) – (a + 5d) = –16$

Æ $d = –4$

2. Sum of the first term and the fifth term = 10

or $a + a + 4d = 10$

or $a + 2d = 5$ (1)

and, the sum of all terms of the AP except for the 1st term = 99

or $9a + 45d = 99$

$a + 5d = 11$ (2)

Solve (1) and (2) to get the answer.

3. The second statement gives the equation as $a + 8d = 2(a + 3d) + 6$

or $a – 2d = 6$

Now, use the options to find the value of d, and put these values to check the equation obtained from the first statement.

i.e. $(a + 2d)(a + 5d) = 406$

4. To plant the 1st sapling, Mithilesh will cover 20 m; to plant the 2nd sapling he will cover 40 m and so on. But for the last sapling, he will cover only the distance from the starting point to the place where the sapling has to be planted.
5. Assume a series having a few number of terms e.g.

1, 2, 4, 8, 16, 32…

Now sum of all the terms at the even places = 42 (P_2)

and sum of all the terms at the odd places = 21 (P_1)

common ratio of this series = $\frac{42}{21} = 2 = P_2/P_1$.

6. Use the same process as illustrated above.

7. Check the options by putting $n = 1, 2, \ldots$ and then equate it with the original equation given in question.

9. For 1 term, the value should be:

$6^2 + 8 = 44$

Only option (b) gives 44 for $n = 1$

10. Sum of the AP for n sides = Sum of interior angles of a polygon of n sides.

$$\frac{n}{2} \times (2a + (n - 1)d) = (2n - 4) \times 90$$

where $a = 120°$ and $d = 5°$.

11. Solve using options to check for the correct answer.

12. $a + (a + 4d) = 26$ and $(a + d)(a + 3d) = 160$

Alternatively, you can try to look at the factors of 160 and create an AP such that it meets the criteria.

Thus, 160 can be written as

2×80

4×40

8×20

10×16

and so on.

If we consider 8×20, then one possibility is that $d = 6$ and the first and fifth terms are 2 and 26. But $2 + 26$ π 28. Hence, this cannot be the correct factors.

Try 10×16. This will give you,

$a = 7$, $d = 3$ and 5th term = 19

and $7 + 19 = 26$ satisfies the condition

13. A possible AP satisfying this condition is

0, 1, 2, 3, 4, 5, 6, 7, 8, …

14. Assume the fifth term as $(a + 4d)$, the eleventh term as $(a + 10d)$, the second term as $(a + d)$, the fourteenth term as $(a + 13d)$, the first term as (a) and the fifteenth term as $(a + 14d)$.

Then, First term × Fifteenth term

$= a \times (a + 14d) = a^2 + 14ad$

Also $(a + d)(a + 13d) = P$

and $(a + 4d)^2 + (a + 10d)^2 = 3$

16. The solution (from the options) has got something to do with either 2^{100} or 2^{101} for 100 terms. Hence, for 3 terms recreate the options and crosscheck with the actual sum.

For 3 terms: Sum = 2 + 8 + 24 = 34.

(a) $100 \times 2^{101} + 2$ for 100 terms becomes $3 \times 2^4 + 2$ for 3 terms.

$= 48 + 2 = 50$ π 34. Hence is not correct.

(b) $99 \times 2^{100} + 2$ for 100 terms becomes $2 \times 2^3 + 2$ for 3 terms.

But this does not give 34. Hence is not correct.

(c) $99 \times 2^{101} + 2$ Æ $2 \times 2^4 + 2 = 34$

(d) $100 \times 2^{100} + 2$ Æ $3 \times 2^3 + 2 \pi 34$.

Hence, option (c) is correct.

17. $r = 2$ and $a + ar^3 = 108$.
20. Solve by checking options and using principles of logarithms.
21. The sum of the first 20 terms will be

    $a + (a + d) + (a + 2d) \ldots + (a + 19d)$

    i.e.$20a + 190d = 400$

    Similarly, use the sum to fifty terms.
23. For a product $a \times b \times c$ to be maximum, given $a + b + c =$ constant, the condition is $a = b = c$.
27. The length of sides of successive triangles form a GP with common ratio 1/3.
28. The area of successive triangles form an infinite GP with a common ratio 1/4.
29. Common ration = $1/\sqrt{2}$.
31. Ration of sum of the first $2n$ terms to the first n terms is equal to 3.

    Thus,

    $$\frac{2n\left(\dfrac{a_1 + a_{2n}}{2}\right)}{n\left(\dfrac{a_1 + a_n}{2}\right)} = 3$$

    Solve to get, $2a = (n + 1)d$.

    Put values for a and n to get a value for d and check for the conditions given in the question.
33. Visualize the AP and solve using standard formulae.
34. Take any values for the numbers.

    Say, the two positive numbers are 1 and 27.

    Then, $a = 14$, $b = 3$ and $c = 9$.

Solutions and Shortcuts

Level of Difficulty (I)

1. In order to count the number of terms in the AP, use the short cut:

   [(last term – first term)/ common difference] + 1. In this case it would become:

   [(130 – 20)/5] +1 = 23. Option (b) is correct.
2. 7000 – 500 – 12500 means that the starting scale is 7000 and there is an increment of 500 every year. Since, the total increment required to reach the top of his scale is 5500, the number of years required would be 5500/500 = 11. Option (a) is correct.
3. Since the 8th and the 12th terms of the AP are given as 39 and 59 respectively, the difference between the two terms would equal 4 times the common difference. Thus we get $4d = 59 - 39 = 20$. This gives us $d = 5$. Also, the 8th term in the AP is represented by $a + 7d$, we get:

   $a + 7d = 39$ Æ $a + 7 \times 5 = 39$ Æ $a = 4$. Option (c) is correct.
4. If we take the sum of the sides we get the perimeters of the squares. Thus, if the side of the

respective squares are a_1, a_2, a_3, a_4… their perimeters would be $4a_1$, $4a_2$, $4a_3$, $4a_4$. Since the perimeters are in GP, the sides would also be in GP.

5. The number of terms in a series are found by:

$$\frac{\text{Difference between first and last terms}}{\text{Common Difference}} + 1$$

6. The first common term is 3, the next will be 9 (Notice that the second common term is exactly 6 away from the first common term. 6 is also the LCM of 2 and 3 which are the respective common differences of the two series.)

   Thus, the common terms will be given by the A.P 3, 9, 15 ….., last term. To find the answer you need to find the last term that will be common to the two series.

   The first series is 3, 5, 7 … 239

   While the second series is 3, 6, 9 ….. 240.

   Hence, the last common term is 237.

   Thus our answer becomes $\frac{237-3}{6} + 1 = 40$

7. Trying Option (a),

   We get least term 5 and largest term 30 (since the largest term is 6 times the least term).

   The average of the A.P becomes (5 + 30)/2 = 17.5

   Thus, 17.5 × n = 105 gives us:

   to get a total of 105 we need n = 6 i.e. 6 terms in this A.P. That means the A.P. should look like:

   5, _ , _ , _, _, 30.

   It can be easily seen that the common difference should be 5. The A.P, 5, 10, 15, 20, 25, 30 fits the situation.

   The same process used for option (b) gives us the A.P. 10, 35, 60. (10 + 35 + 60 = 105) and in the third option 15, 90 (15 + 90 = 105).

   Hence, all the three options are correct.

8. The first term is 20 and the common difference is –5, thus the 15th term is:

   20 + 14 × (–5) = –50. Option (c) is correct.

9. The difference between the amounts at the end of 4 years and 10 years will be the simple interest on the initial capital for 6 years.

   Hence, 360/6 = 60 =(simple interest.)

   Also, the Simple Interest for 4 years when added to the sum gives 1240 as the amount.

   Hence, the original sum must be 1000.

10. The three parts are 3, 5 and 7 since $3^2 + 5^2 + 7^2 = 83$. Since, we want the smallest number, the answer would be 3. Option (b) is correct.

11. $a = 5$, $a + 2d = 15$ means $d = 5$. The 16th term would be $a + 15d = 5 + 75 = 80$. The sum of the series would be given by:

    [16/2] × [5 + 80] = 16 × 42.5 = 680. Option (d) is correct.

12. Use trial and error by using various values from the options.

If you find the sum of the series till 18 terms the value is 513. So also for 19 terms the value of the sum would be 513. Option (c) is correct.

13. Solve this question through trial and error by using values of n from the options:

    For 19 terms, the series would be 5 + 8 + 11 + …. + 59 which would give us a sum for the series as 19 × 32 = 608. The next term (20th term of the series) would be 62. Thus, 608 + 62 = 670 would be the sum to 20 terms. It can thus be concluded that for 20 terms the value of the sum of the series is not less than 670. Option (a) is correct.

14. His total earnings would be 60 + 63 + 66 + … + 117 = ` 1770. Option (a) is correct.

15. The series would be 5, 20, 80, 320, 1280, 5120, 20480. Thus, there are a total of 7 terms in the series. Option (c) is correct.

16. Sum of a G.P. with first term 1 and common ratio 2 and no. of terms 20.

    $\frac{1\times(2^{20}-1)}{(2-1)} = 2^{20} - 1$

17. $16r^4 = 81$ Æ $r^4 = 81/16$ Æ $r = 3/2$. Thus, 4th term = $ar^3 = 16 \times (3/2)^3 = 54$. Option (c) is correct.

18. In the case of a G.P. the 7th term is derived by multiplying the fourth term thrice by the common ratio. (**Note:** this is very similar to what we had seen in the case of an A.P.)

    Since, the seventh term is derived by multiplying the fourth term by 8, the relationship.

    $r^3 = 8$ must be true.

    Hence, $r = 2$

    If the fifth term is 48, the series in reverse from the fifth to the first term will look like:

    48, 24, 12, 6, 3. Hence, option (b) is correct.

19. Visualising the squares below 84, we can see that the only way to get the sum of 3 squares as 84 is: $2^2 + 4^2 + 8^2 = 4 + 16 + 64$. The largest number is 8. Option (a) is correct.

20. The series would be given by: 1, 5, 9… which essentially means that all the numbers in the series are of the form $4n + 1$. Only the value in option (c) is a $4n + 1$ number and is hence the correct answer.

21. The series will be 301, 308, …….. 497

    Hence, Answer = $\frac{196}{7} + 1 = 29$

22. The answer to this question can be seen from the options. Both 2, 6, 18 and 18, 6, 2 satisfy the required conditions- viz: GP with sum of first and third terms as 20. Thus, option (c) is correct.

23. We need the sum of the series 20 + 24 + 28 to cross 1000. Trying out the options, we can see that in 20 years the sum of his savings would be: 20 + 24 + 28 + … + 96. The sum of this series would be 20 × 58 =1160. If we remove the 20th year we will get the series for savings for 19 years. The series would be 20 + 24 + 28 + …. 92. Sum of the series would be 1160 – 96 = 1064. If we remove the 19th year's savings the savings would be 1064 – 92 which would go below 1000. Thus, after 19 years his savings would cross 1000. Option (a) is correct.

24. The answer to this question cannot be determined because the question is talking about income and asking about savings. You cannot solve this unless you know the value of the expenditure the man incurs over the years. Thus, "Cannot be Determined" is the correct answer.

25. Similar to what we saw in question 18,

4th term × r^6 = 10th term.

The 4th term here is $3^{-1}$and the tenth term is 3^5.

Hence $3^{-1} \times r^6 = 3^5$

Gives us: $r = 3$.

Hence, the second term will be given by (fourth term/r^2)

$^1/_3 \times {}^1/_3 \times {}^1/_3 = {}^1/_{27}$

[**Note:** To go forward in a G.P. you multiply by the common ratio, to go backward in a G.P. you divide by the common ratio.]

26. $a + 6d = 6$ and $a + 20d = -22$. Solving we get $14d = -28$ Æ $d = -2$. 26th term = 21st term + $5d$ = –22 + 5 × (–2) = –32. Option (b) is correct.

27. Since the sum of 5 numbers in AP is 30, their average would be 6. The average of 5 terms in AP is also equal to the value of the 3rd term (logic of the middle term of an AP). Hence, the third term's value would be 6. Option (b) is correct.

28. The answer will be given by:

[10 + 11 + 12 + …….. + 50] – [16 + 24 + … + 48]

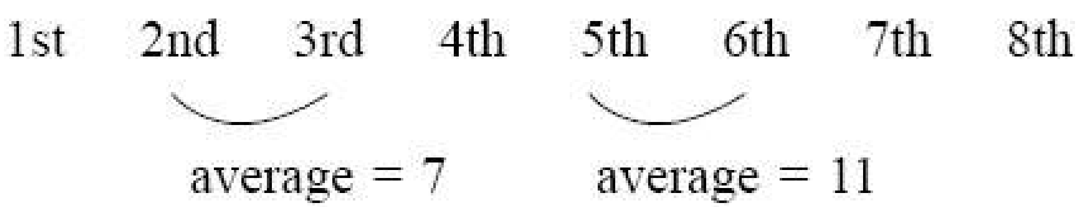
= 41 × 30 – 32 × 5

= 1230 – 160 = 1070.

29. Think like this:

The average of the first 4 terms is 7, while the average of the first 8 terms must be 11.

Now visualize this :



Hence, $d = 4/2 = 2$ [Note: understand this as a property of an A.P.]

Hence, the average of the 6th and 7th terms = 15 and the average of the 8th and 9th term = 19

But this (19) also represents the average of the 16 term A.P.

Hence, required answer = 16 × 19 = 304.

30. Go through the options. The correct option should give value as 1, when $n = 3$ and as 8 when $n = 8$.

Only option (a) satisfies both conditions.

31. The series is: 1/81, –1/27, 1/9, –1/3, 1, –3, 9, –27, 81, –243, 729. There are 11 terms in the series. Option (a) is correct.

32. $1/8 \times r^5 = 128$ Æ $r^5 = 128 \times 8 = 1024$ Æ $r = 4$. Thus, the series would be 1/8, 1/2, 2, 8, 32, 128. The third geometric mean would be 8. Option (d) is correct.

33. AM = 25 means that their sum is 50 and GM = 7 means their product is 49. The numbers can only be 49 and 1. Option (c) is correct.

34. Trial and error gives us that for option (b):

With the ratio 4:1, the numbers can be taken as $4x$ and $1x$. Their AM would be $2.5x$ and their GM would be $2x$. The GM can be seen to be 20% lower than the AM. Option (b) is thus the correct answer.

35. The total savings would be given by the sum of the series:

100 + 150 + 200 + 650 = 12 × 375 = ` 4500. Option (b) is correct.

36. In order to find how many times the alarm rings we need to find the number of numbers below 100 which are not divisible by 2,3, 5 or 7. This can be found by:

100 – (numbers divisible by 2) – (numbers divisible by 3 but not by 2) – (numbers divisible by 5 but not by 2 or 3) – (numbers divisible by 7 but not by 2 or 3 or 5).

Numbers divisible by 2 up to 100 would be represented by the series 2, 4, 6, 8, 10..100 Æ A total of 50 numbers.

Numbers divisible by 3 but not by 2 up to 100 would be represented by the series 3, 9, 15, 21… 99 Æ A total of 17 numbers. Note use short cut for finding the number of number in this series :

[(last term – first term)/ common difference] + 1 = [(99 – 3)/6] + 1 = 16 + 1 = 17.

Numbers divisible by 5 but not by 2 or 3: Numbers divisible by 5 but not by 2 up to 100 would be represented by the series 5, 15, 25, 35…95 Æ A total of 10 numbers. But from these numbers, the numbers 15, 45 and 75 are also divisible by 3. Thus, we are left with 10 – 3 = 7 new numbers which are divisible by 5 but not by 2 and 3.

Numbers divisible by 7, but not by 2, 3 or 5:

Numbers divisible by 7 but not by 2 upto 100 would be represented by the series 7, 21, 35, 49, 63, 77, 91 Æ A total of 7 numbers. But from these numbers we should not count 21, 35 and 63 as they are divisible by either 3 or 5. Thus a total of 7 – 3 = 4 numbers are divisible by 7 but not by 2, 3 or 5.

37. For looking at the zeroes in the expression we should be able to see that the number of zeroes in the third term onwards is going to be very high. Thus, the number of zeroes in the expression would be given by the number of zeroes in:

$4 + 24^{24}$. 24^{24} has a unit digit 6. Hence, the number of zeroes in the expression would be 1. Option (d) is correct.

38. Since the sum of 22 terms of the AP is 385, the average of the numbers in the AP would be 385/22 = 17.5. This means that the sum of the first and last terms of the AP would be 2 × 17.5 = 35. Trial and error gives us the terms of the required GP as 7, 14, 28. Thus, the common ratio of the GP can be 2.

39. The sum of the interior angles of a polygon are multiples of 180 and are given by $(n – 1) × 180$ where n is the number of sides of the polygon. Thus, the sum of interior angles of a polygon would be a member of the series: 180, 360, 540, 720, 900, 1080, 1260

The sum of the series with first term 100 and common difference 10 would keep increasing when we take more and more terms of the series. In order to see the number of sides of the polygon, we should get a situation where the sum of the series represented by 100 + 110 + 120… should become a multiple of 180. The number of sides in the polygon would then be the number of terms in the series 100, 110, 120 at that point.

If we explore the sums of the series represented by

100 + 110 + 120…

We realize that the sum of the series becomes a multiple of 180 for 8 terms as well as for 9 terms.

It can be seen in: 100 + 110 + 120 + 130 + 140 + 150 + 160 + 170 = 1080

Or 100 + 110 + 120 + 130 + 140 + 150 + 160 + 170 + 180 = 1260.

40. The sum of the total distance it travels would be given by the infinite sum of the series:

    420 × 8/1 + 367.5 × 8/1 = 3360 + 2940 = 6300. Option (b) is correct.

41. The two series till their hundredth terms are 13, 15, 17….211 and 14, 17, 20…311. The common terms of the series would be given by the series 17, 23, 29….209. The number of terms in this series of common terms would be 192/6 + 1 = 33. Option (d) is correct.

42. The area of the first square would be 64 sq cm. The second square would give 32, the third one 16 and so on. The infinite sum of the geometric progression 64 + 32 + 16 + 8… = 128. Option (a) is correct.

43. It can be seen that for the series the average of two terms is 2, for 3 terms the average is 3 and so on. Thus, the sum to 2 terms is 2^2, for 3 terms it is 3^2 and so on. For 11111 terms it would be 11111^2 = 123454321. Option (d) is correct.

44. The maximum score would be the sum of the series 9 + 13 + …. + 389 + 393 + 397 = 98 × 406/2 = 19894. Option (d) is correct.

45. The series would be 8, 8/3, 8/9 and so on. The sum of the infinite series would be 8/(1 – 1/3) = 8 × 3/2 = 12. Option (a) is correct.

46. The maximum sum would occur when we take the sum of all the positive terms of the series.

    The series 25, 24.5, 24, 23.5, 23, …….. 1, 0.5, 0 has 51 terms. The sum of the series would be given by:

    n × average = 51 × 12.5 = 637.5

    Option (a) is correct.

47. The side of the first equilateral triangle being 24 units, the first perimeter is 72 units. The second perimeter would be half of that and so on.

    72, 36, 18 …

    The infinite sum of this series = 72(1 – 1/2) = 144. Option (d) is correct.

48. Solve using options. Option (a) fits the situation as 16 + 2 + 2/8 + 2/64 meets the conditions of the question. Option (a) is correct.

49. The 33rd term of the sequence would be the 17th term of the sequence 3, 9, 15, 21 ….

    The 17th term of the sequence would be 3 + 6 × 16 = 99.

50. The sum to 33 terms of the sequence would be:

    The sum to 17 terms of the sequence 3, 9, 15, 21, …99 + The sum to 16 terms of the sequence 8, 13, 18, 83.

    The required sum would be 17 × 51 + 16 × 45.5 = 867 + 728 = 1595.

Level of Difficulty (II)

1. Identify an A.P. which satisfies the given condition.

   Suppose we are talking about the second and third terms of the A.P.

   Then an A.P. with second term 3 and third term 2 satisfies the condition.

a times the *a*th term = *b* times the *b*th term.

In this case the value of $a = 2$ and $b = 3$.

Hence, for the $(a + b)^{th}$ term, we have to find the fifth term.

It is clear that the fifth term of this A.P. must be zero.

Check the other three options to see whether any option gives 0 when $a = 2$ and $b = 3$.

Since none of the options b, c or d gives zero for this particular value, option (a) is correct.

2. Since the four parts of the number are in *AP* and their sum is 20, the average of the four parts must be 5. Looking at the options for the largest part, only the value of 8 fits in, as it leads us to think of the *AP* 2, 4, 6, 8. In this case, the ratio of the product of the first and fourth (2 × 8) to the product of the first and second (4 × 6) are equal. The ratio becomes 2:3.

3. View: 1 – 4 + 5 – 8 + 9 – 12 . . . . . 50 terms as

   (1 – 4) + (5 – 8) + (9 – 12) …… 25 terms.

   Hence, –3 + –3 + –3 . . . . 25 terms

   = 25 × – 3 = –75.

4. In the course of 2 days the clock would strike 1 four times, 2 four times, 3 four times and so on. Thus, the total number of times the clock would strike would be:

   4 + 8 + 12 + ..48 = 26 × 12 = 312. Option (b) is correct.

5. Since this is a decreasing A.P. with first term positive, the maximum sum will occur upto the point where the progression remains non – negative.

   44, 42, 40 ….. 0

   Hence, 23 terms × 22 = 506.

6. The sum of the required series of integers would be given by 7 + 14 + 21 + ….196 = 28 × 101.5 = 2842. Option (b) is correct.

7. A little number juggling would give you 2nd term is 1/3 and 3rd term is ½ is a possible situation that satisfies the condition.

   The A.P. will become:

   1/6, 1/3, 1/2, 2/3, 5/6, 1

   or in decimal terms, 0.166, 0.333, 0.5, 0.666, 0.833, 1

   Sum to 6 terms = 3.5

   Check the option with $m = 2$ and $n = 3$. Only option (c) gives 3.5. Hence, must be the answer.

8. 101 + 103 + 105 + … 199 = 150 × 50 = 7500

9. The required sum would be given by the sum of the series 105, 112, 119, …. 994. The number of terms in this series = (994 – 105)/7 + 1 = 127 + 1 = 128. The sum of the series = 128 × (average of 105 and 994) = 70336. Option (c) is correct.

10. 5 × average of 107 and 253 = 5 × 180 = 900. Option (c) is correct.

11. The first 100 terms of this series can be viewed as:

    (1 – 2 –3) + (2 – 3 –4) + ... + (33 – 34 – 35) + 34

    The first 33 terms of the above series (indicated inside the brackets) will give an A.P.: –4, –5, –6 .... –36

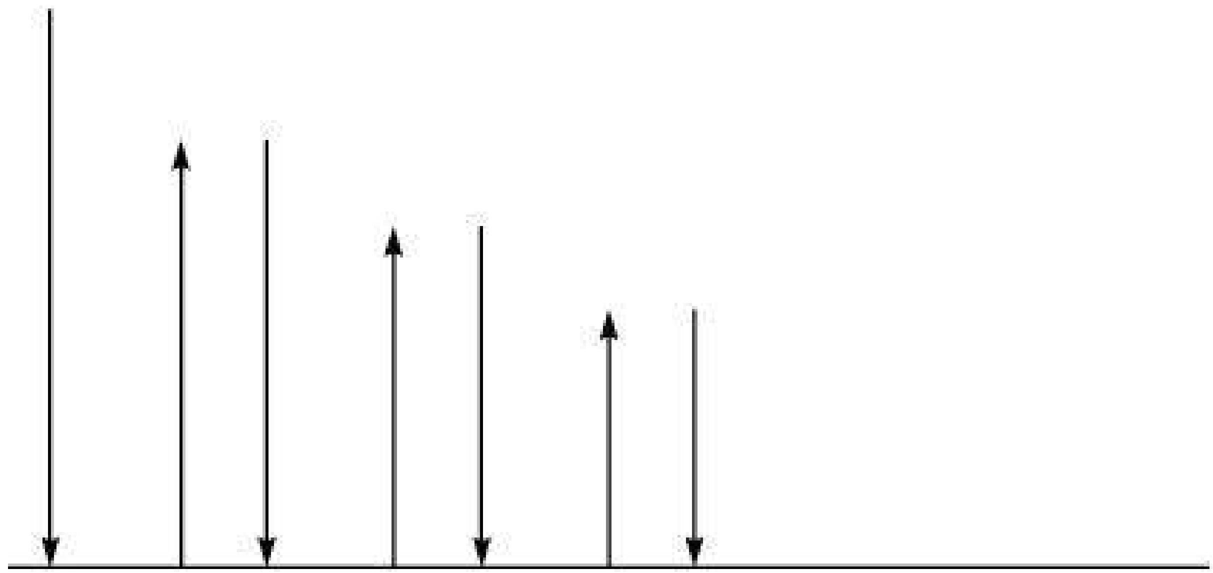
    Sum of this A.P. = 33 × –20 = –660

Answer = –660 + 34 = –626

12. Solve this one through trial and error. For $n = 2$ terms the sum upto 2 terms is equal to 96. Putting $n = 2$, in the options it can be seen that for option (b) the sum to two terms would be given by $8 \times (1000 - 10 - 18)/81 = 8 \times 972/81 = 8 \times 12 = 96$.

13. If we take the values of a, b, and c as 10,100 and 1000 respectively, we get log a, log b and log c as 1, 2 and 3 respectively. This clearly shows that the values of log a, log b and log c are in AP.

14. The path of the rubber ball is:



In the figure above, every bounce is 4/5th of the previous drop.

In the above movement, there are two infinite G.Ps (The GP representing the falling distances and the GP representing the Rising distances.)

The required answer: (Using $a/(1–r)$ formula)

$$\frac{120}{1/5} + \frac{96}{1/5} = 1080$$

15. Solve this for a sample GP. Let us say we take the GP as 2, 6, 18, 54. x, the first term is 2, let $n = 3$ then the 3rd term $y = 18$ and the product of 3 terms $p = 2 \times 6 \times 18 = 216 = 6^3$. The value of $p^2 = 216 \times 216 = 6^6$.

Putting these values in the options we have:

Option (a) gives us $(xy)^{n-1} = 36^2$ which is not equal to the value of p^2 we have from the options

Option (b) gives us $(xy)^n = 36^3 = 6^6$ which is equal to the value of p^2 we have from the options.

It can be experimentally verified that the other options yield values of p^2 which are different from 6^6 and hence we can conclude that option (b) is correct.

16. Trying to plug in values we can see that the infinite sum of the GP 16, 8, 4, 2… is 32 and hence the third term is 4.

17. The expression can be written as $x^{(1/2 + 1/4 + 1/8 + 1/16\ldots..)} = x^{\text{INFINTE SUM OF THE GP}} = x^1$. Option (b) is correct.

18. For $n = 1$, the sum should be 6. Option (b), (c) and (d) all give 6 as the answer.

For $n = 2$, the sum should be 30.

Only option d gives this value. Hence must be the answer.

19. From the facts given in the question it is self evident that the common ratio of the GP must be 2 (as the sum of the 2nd and 4th term is twice the sum of the first and third term). After realizing this, the question is about trying to match the correct sums by taking values from the options.

The GP formed from option (c) with a common ratio of 2 is: 8,16,32,64 and this GP satisfies the conditions of the problem- sum of 1st and 3rd term is 40 and sum of 2nd and 4th term is 80.

20. Since the sum of the first five even terms is 15, we have that the 2nd, 4th, 6th, 8th and 10th term of the AP should add up to 15. We also need to understand that these 5 terms of the AP would also be in an AP by themselves and hence, the value of the 6th term (being the middle term of the AP) would be the average of 15 over 5 terms. Thus, the value of the 6th term is 3. Also, since the sum of the first three terms of the AP is –3, we get that the 2nd term would have a value of –1. Thus, the AP can be visualized as:

_, -1, _,_,_,3,….

Thus, it is obvious that the AP would be –2, –1, 0, 1, 2, 3. The second term is –1. Thus, option (c) is correct.

21. Second term = $a + d$, Fifth term = $a + 4d$; third term = $a + 2d$, seventh term = $a + 6d$.

Thus, $2a + 5d = 8$ and $2a + 8d = 14$ Æ $d = 2$ and $a = –1$.

The eleventh term = $a + 10d = –1 + 20 = 19$. Thus, option (a) is correct.

22. If the difference between the seventh and the second term is 20, it means that the common difference is equal to 4. Since, the third term is 9, the *AP* would be 1, 5, 9, 13, 17, 21, 25, 29 and the sum to 8 terms for this *AP* would be 120. Thus, option (d) is correct.

23. $5d = 35$ Æ $d = 7$. Thus, the numbers are 4, 11, 18, 25, 32, 39. The largest number is 32. Option (c) is correct.

24. Find sum of the series:

104, 109, 114 999

Average × n = 551.5 × 180 = 99270

25. Since the sum of the first three terms of the *AP* is 30, the average of the *AP* till 3 terms would be 30/3 = 10. The value of the second term would be equal to this average and hence the second term is 10. Using the information about the sum of squares of the first and second terms being 116, we have that the first term must be 4. Thus, the AP has a first term of 4 and a common difference of 6. The seventh term would be 40. Thus, option (b) is correct.

26. The combined travel would be 25 on the first day, 26 on the second day, 27 on the third day, 28 on the fourth day, 29 on the fifth day and 30 on the sixth day. They meet after 6 days. Option (c) is correct.

27. This is a calculation intensive problem and you are not supposed to know how to do the calculations in this question mentally. The problem has been put here to test your concepts about whether you recognize how this is a question of GPs. If you feel like, you can use a calculator/ computer spreadsheet to get the answer to this question.

The logic of the question would hinge on the fact that the value of the investment of the fifteenth year would be 1000. At the end of the 15th year, the investment of the 14th year would be equal to 1000 × 1.05, the 13th year's investment would amount to 1000×1.05^2 and so on till the first year's investment which would amount to 1000×1.05^{14} after 15 years.

Thus, you need to calculate the sum of the GP:

1000, 1000 × 1.05, 1000 × 1.05^2, 1000 × 1.05^3 for 15 terms.

28. Since, sum to n terms is given by (n + 8),

    Sum to 1 terms = 9

    Sum to 2 terms = 10

    Thus, the 2nd term must be 1.

29. Solve this question by looking at hypothetical values for n and $2n$ terms. Suppose, we take the sum to 1 (n = 1) term of the first series and the sum to 2 terms ($2n$ = 2) of the second series we would get A/B as 1/1.5 = 2/3.

    For n = 2 and $2n$ = 4 we get A = 1.25 and B = 1.875 and A/B = 1.25/1.875 = 2/3.

    Thus, we can conclude that the required ratio is always constant at 2/3 and hence the correct option is (c).

30. We need to find the infinite sum of the GP: 100, 90, 81…(first term = 100 and common ratio= 0.9)

    We get: Infinite sum of the series as 1000. Thus, option (b) is correct.

31. Questions such as these have to be solved on the basis of a reading of the pattern of the question. The sum upto the first term is: 1/5. Upto the second term it is 2/9 and upto the third term it is 3/13. It can be easily seen that for the first term, second term and third term the numerators are 1, 2 and 3 respectively. Also, for 1/5 – the 5 is the second value in the denominator of 1/1 × 5 (the first term); for 2/9 also the same pattern is followed—as 9 comes out of the denominator of the second term of series and for 3/13 the 13 comes out of the denominator of the third term of the series and so on. The given series has 56 terms and hence the correct answer would be 56/225.

32. Solve this on the same pattern as Question 31 and you can easily see that for the first term sum of the series is $\sqrt{2}-1$, for 2 terms we have the sum as $\sqrt{3}-1$ and so on. For the given series of 120 terms the sum would be $\sqrt{121}-1$ = 10.

    Option (a) is correct.

33. If you look for a few more terms in the series, the series is:

    1, 1/3, 1/6, 1/10, 1/15, 1/21, 1/28, 1/36, 1/45, 1/55, 1/66, 1/78, 1/91, 1/105, 1/120, 1/136, 1/153 and so on. If you estimate the values of the individual terms it can be seen that the sum would tend to 2 and would not be good enough to reach even 2.25.

    Thus, option (a) is correct.

34. Solve this using trial and error. For 1 term the sum should be 40 and we get 40 only from the first option when we put n = 1. Thus, option (a) is correct.

35. For this question too you would need to read the pattern of the values being followed. The given sum has 13 terms.

    It can be seen that the sum to 1 term = ½

    Sum to 2 terms = 2/3

    Sum to 3 terms = ¾

    Hence, the sum to 13 terms would be 13/14.

36. The sum to infinite terms would tend to 1 because we would get (infinity)/(infinity + 1).

37. All members of A are smaller than all members of B. In order to visualize the effect of the change

in sign in A, assume that A is {1, 2, 3…124} and B is {126, 127…250}. It can be seen that for this assumption of values neither options (a), (b) or (c) is correct.

38. If elements of A are in ascending order a_{124} would be the largest value in A. Also a_{125} would be the largest value in B. On interchanging a_{124} and a_{125}, A continues to be in ascending order, but B would lose its descending order arrangement since a_{124} would be the least value in B. Hence, option *a* is correct.

39. Since the minimum is in A and the maximum is in B, the value of x cannot be *less than Max-Min.*

40. It is evident that the whole question is built around Arithmetic progressions. The 5th row has an average of 55, while the 15th row has an average of 65. Since even column wise each column is arranged in an *AP* we can conclude the following:

    1st row – average 51 – total = 23 × 51

    2nd row – average 52 – total 23 × 52…..

    23rd row – average 73 – total 23 × 73

    The overall total can be got by using averages as:

    23 × 23 × 62 = 32798

41. The numbers forming an *AP* would be:

    123, 135, 147, 159, 210, 234, 246, 258, 321, 345, 357, 369, 432, 420, 456, 468, 543, 531, 567, 579, 654, 642, 630, 678, 765, 753, 741, 789, 876, 864, 852, 840, 987, 975, 963 and 951. A total of 36 numbers.

    If we count the GPs we get:

    124, 139, 248, 421, 931, 842—a total of 6 numbers.

    Hence, we have a total of 42 3 digit numbers where the digits are either APs or GPs.

    Thus, option (d) is correct.

42. Total burgers made = 660

    Burgers with chicken and mushroom patty = 165 (Number of terms in the series 1, 5, 9…657)

    Burgers with vegetable patty = 95 (Number of terms in the series 2, 9, 16, …660)

    Burgers with chicken, mushroom and vegetable patty = 24 (Number of terms in the series 9, 37, 65….653)

    Required answer = 660 – 165 – 95 + 24 = 424

43. From the above question, we have 24 such burgers.

44. The key to this question is what you understand from the statement- 'for two progressions out of P_1, P_2 and P_3 the average is itself a term of the original progression P.' For option (a) which tells us that the Progression P has 20 terms, we can see that P_1 would have 7 terms, P_2 would have 7 terms and P_3 would have 6 terms. Since, both P_1 and P_2 have an odd number of terms we can see that for P_1 and P_2 their 4th terms (being the middle terms for an *AP* with 7 terms) would be equal to their average. Since, all the terms of P_1, P_2 and P_3 have been taken out of the original *AP P*, we can see that for P_1 and P_2 their average itself would be a term of the original progression P. This would not occur for P_3 as P_3 has an even number of terms. Thus, 20 is a correct value for n.

    Similarly, if we go for n = 26 from the second option we get:

P_1, P_2 and P_3 would have 9, 9 and 8 terms respectively and the same condition would be met here too.

For n = 36 from the third option, the three progressions would have 12 terms each and none of them would have an odd number of terms.

Thus, option (d) is correct as both options (a) and (b) satisfy the conditions given in the problem.

45. Since, P_1 is formed out of every third term of P, the common difference of P_1 would be three times the common difference of P. Thus, the common difference of P would be 2.

HOW TO THINK IN PROBLEMS ON BLOCK I

1. A number x is such that it can be expressed as $a + b + c = x$ where a, b, and c are factors of x. How many numbers below 200 have this property?

(a) 31 (b) 32

(c) 33 (d) 5

Solution: In order to think about this question you need to think about whether the number can be divided by the initial numbers below the square root like 2, 3, 4, 5… and so on.

Let us say, if we think of a number that is not divisible by 2, in such a case if we take the number to be divisible by 3, 5 and 7, then the largest factors of x that we will get would be $\frac{x}{3}, \frac{x}{5}$ and $\frac{x}{7}$.

Even in this situation, the percentage value of these factors as a percentage of x would only be: for $\frac{x}{3}$ = 33.33%, $\frac{x}{5}$ = 20% and $\frac{x}{7}$ = 14.28%

Hence, if we try to think of a situation where $a = \frac{x}{3}$, $b = \frac{x}{5}$, and $c = \frac{x}{7}$ the value of $a + b + c$ would give us only (33.33% + 20% + 14.28%) = 67.61% of x, which is not equal to 100%

Since the problem states that $a + b + c = x$, the value of $a + b + c$ should have added up to 100% of x.

The only situation, for this to occur would be if

$a = \frac{x}{2}$ = 50% of x $b = \frac{x}{3}$ = 33.33% of x and

$c = \frac{x}{6}$ = 16.66% of x.

This means that 2, 3 and 6 should divide x necessarily. In other words x should be a multiple of 6.The multiples of 6 below 200 are 6, 12, 18 … 198, a total of 33 numbers.

Hence the correct answer is c.

2. Find the sum of all 3 digit numbers that leave a remainder of 3 when divided by 7.

(a) 70821 (b) 60821

(c) 50521 (d) 80821

Solution: In order to solve this question you need to visualise the series of numbers, which satisfy this condition of the remainder 3 when divided by 7.

The first number in 3 digits for this condition is 101 and the next will be 108, followed by 115, 122 …

This series would have its highest value in 3 digits as 997.

The number of terms in this series would be 129 (using the logic that for any AP, the number of terms is given by $\frac{D}{d} + 1$)

Also the average value of this series is the average of the first and the last term i.e., the average of 101 and 997. Hence the required sum = 549 × 129 = 70821.

Hence the correct Option is (a).

3. How many times would the digit 6 be used in numbering a book of 639 pages?

(a) 100 (b) 124

(c) 150 (d) 164

Solution: In order to solve this question you should count the digit 6 appearing in units digit, separately from the instances of the digit 6 appearing in the tens place and appearing in the hundreds place.

When you want to find out the number of times 6 appears in the unit digit, you will have to make a series as follows: 6, 16, 26, 36 … 636.

It should be evident to you that the above series has 64 terms because it starts from 06, 16, 26 … and continues till 636. The digit 6 will appear once in the unit digit for each of these 64 numbers.

Next you need to look at how many times the digit 6 appears in the ten's place.

In order to do this we will need to look at instances when 6 appears in the tens place. These will be in 6 different ranges 60s, 160s, 260s, 360s, 460s, 560s and in each of these ranges there are 10 numbers each with exactly one instance of the digit 6 in the tens place, a total of 60 times.

Lastly, we need to look at the number of instances where 6 appears in the hundreds place. For this we need to form the series 600, 601, 602, 603 … 639. This series will have 40 numbers each with exactly one instance of the digit 6 appearing in the hundreds place.

Therefore the required answer would be 64 + 60 + 40 = 164.

Hence, Option (d) is correct.

4. A number written in base 3 is 100100100100100100. What will be the value of this number in base 27?

(a) 999999 (b) 900000

(c) 989999 (d) 888888

The number can be visualised as:

1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
3^{17}	3^{16}	3^{15}	3^{14}	3^{13}	3^{12}	3^{11}	3^{10}	3^9	3^8	3^7	3^6	3^5	3^4	3^3	3^2	3^1	3^0

Now in the base 27, we can visualise this number as:

—	—	—	—	—	—
27^5	27^4	27^3	27^2	27^1	27^0

When you want to write this number in Base 27 the unit digit of the number will have to account for the value of 3^2 in the above number. Since the unit digit of the number in Base 27 will correspond to 27^0 we would have to use 27^0, 9 times to make a value 3^2. The number would now become:

—	—	—	—	—	9

27^5	27^4	27^3	27^2	27^1	27^0

Similarly, the number of times that we will have to use 27^1 in order to make the value of 3^5 (or 243) will be $\frac{243}{27} = 9$. Hence the second last digit of the number will also be 9.

Similarly, to make a value of 3^8 using 27^2 the number of times we will have to use 27^2 will be given by [illegible] = 9. The number would now look as:

—	—	—	9	9	9
27^5	27^4	27^3	27^2	27^1	27^0

It can be further predicted that each of subsequent 1's in the original number will equal 9 in the number, which is being written in Base 27. Since the original number in Base 3 has 18 digits, the left most '1' in that number will be covered by a number corresponding to 27^5 in the new number (i.e. 3^{15}). Hence the required number will be a 6-digit number 999999. Hence, Option (a) is correct.

5. Let $x = 1640$, $y = 1728$ and $z = 448$. How many natural numbers are there that divide at least one amongst x, y, z.

(a) 47 (b) 48

(c) 49 (d) 50

Solution: 1640 can be prime factorised as $2^3 \times 5^1 \times 41^1$. This number would have a total of 16 factors.

Similarly, 1728 can be prime factorised as $2^6 \times 3^3$. Hence, it would have 28 factors. While $448 = 2^6 \times 7^1$ would have 14 factors. Thus there are a total of (16 + 28 + 14) = 58 factors amongst x, y and z.

However, some of these factors must be common between x, y, z. Hence, in order to find the number of natural numbers that would divide at least one amongst x, y, z, we will need to account for double and triple counted numbers amongst these 58 numbers (by reducing the count by 2 for each triple counted number and by reducing the count by 1 for each double counted number).

The number of cases of triple counting would be for all the common factors of (x, y, z). This number can be estimated by finding the HCF of x, y and z and counting the number of factors of the HCF. The HCF of 1640, 1728 and 448 is 8 and hence the factors of 8, i.e., 1, 2, 4 and 8 itself must have got counted in each of the 3 counts done above. Thus each of these 4 numbers should get subtracted twice to remove the triple count. This leaves us with $58 - 4 \times 2 = 50$ numbers.

We still need to eliminate numbers that have been counted twice, i.e., numbers, which belong to the factors of any two of these numbers (while counting this we need to ensure that we do not count the triple counted numbers again). This can be visualised in the following way:

Number of common factors that are common to only 1640 and 1728 and not to 448 (x and y but not z):

$1640 = 2^3 \times 5^1 \times 41^1$

$1728 = 2^6 \times 3^3$

It can be seen from these 2 standard forms of the numbers that the highest common factors of these 2 numbers is 8. Hence there is no new number to be subtracted for double counting in this case.

The case of 1640 and 448 is similar because $1640 = 2^3 \times 5^1 \times 41^1$ while $448 = 2^6 \times 7^1$ and HCF = 2^3 and hence they will not give any more numbers as common factors apart from 1, 2, 4 and 8.

Thus there is no need of adjustment for the pair 1640 and 448.

Finally when we look for 1728 and 448, we realise that the HCF = 2^6 = 64 and hence the common factors between 448 and 1728 are 1, 2, 4, 8, 16, 32, 64. But we are looking for factors which are common for 1728 and 448 but not common to 1640 to estimate the double counting error for this case.

Hence we can eliminate the number 1, 2, 4, 8 from this list and conclude that there are only 3 numbers 16, 32 and 64 that divide both 1728 and 448 but do not divide 1640.

If we subtract these numbers once each, from the 50 numbers we will end up with 50 – 3 × 1 = 47.

The complete answer can be visualised as 16 + 28 + 14 – 4 × 2 – 3 × 1 = 47.

Hence, Option (a) is correct.

6. How many times will the digit 6 be used when we write all the six digit numbers?

(a) 5,50,000 (b) 5,00,000

(c) 4,50,000 (d) 4,00,000

Solution: When we write all 6-digit numbers we will have to write all the numbers from 100000 to 999999, a total of 9 lac numbers in 6 digits without omitting a single number. There will be a complete symmetry and balance in the use of all the digits. However the digit 0 is not going to be used in the leftmost place.

Using this logic we can visualise that when we write 9 lakh, 6-digit numbers, the units place, tens place, hundreds place, thousands place, ten thousands place and lakh place – Each of these places will be written 9 lakh times. Thinking about the units place, we can think as follows: In writing the units place 9 lakh times (once for every number) we will be using the digit 0, 1, 2, 3 … 9 an equal number of times. Hence any particular digit like 6 would get used in the units digit a total number of 90000 times (9 lakh/10). The same logic will continue for tens, hundreds, thousands and ten thousands, i.e., the digit 6 will be used (9 lakh/10) = 90000 times in each of these places. (Note here that we are dividing by 10 because we have to equally distribute 9 lakh digits amongst the ten digits 0, 1, 2, 3,4, 5, 6, 7, 8, 9.)

Finally for the lakh place since "0" is not be used in the lakh place as it is the leftmost digit of the number, the number of times the digit 6 will be used would be 9 lakh/9 = 1 lakh. Hence the next time you are solving the problem of this type, you should solve directly using $\frac{9\text{ lakh}}{9} + \frac{9\text{ lakh}}{10} \times 5$ = 5,50,000.

Hence, Option (a) is the correct answer.

BLOCK REVIEW TESTS

Review Test 1

1. Lata has the same number of sisters as she has brothers, but her brother Shyam has twice as many sisters as he has brothers. How many children are there in the family?

   (a) 7 (b) 6

   (c) 5 (d) 3

2. How many times does the digit 6 appear when you count from 11 to 100?

   (a) 9 (b) 10

   (c) 19 (d) 20

3. If m < n, then

   (a) $m .m < n . n$

   (b) $m.m > n . n$

   (c) $m . n. n < n. m . m$

   (d) $m . m . m < n. n . n$

4. A square is drawn by joining the midpoints of the side of a given square. A third square is drawn in side the second square in the same way and this process is continued indefinitely. If a side of the first square is 8 cm, the sum of the areas of all the squares (in sq. cm) is

   (a) 128 (b) 120

   (c) 96 (d) None of these

5. Find the least number which when divided by 6, 15, 17 leaves a remainder 1, but when divided by 7 leaves no remainder.

   (a) 211 (b) 511

   (c) 1022 (d) 86

6. The number of positive integers not greater than 100, which are not divisible by 2, 3 or 5 is

   (a) 26 (b) 18

   (c) 31 (d) None of these

7. The smallest number which when divided by 4, 6 or 7 leaves a remainder of 2, is

   (a) 44 (b) 62

   (c) 80 (d) 86

8. An intelligence agency decides on a code of 2 digits selected from 0, 1, 2,….9. But the slip on which the code is hand-written allows confusion between top and bottom, because these are indistinguishable. Thus, for example, the code 91 could be confused with 16. How many codes are there such that there is no possibility of any confusion?

   (a) 25 (b) 75

(c) 80 (d) None of these

9. Suppose one wishes to find distinct positive integers x, y such that $(x + y)/xy$ is also a positive integer. Identify the correct alternative.

   (a) This is never possible

   (b) This is possible and the pair (x, y) satisfying the stated condition is unique.

   (c) This is possible and there exist more than one but a finite number of ways of choosing the pair(x, y).

   (d) This is possible and the pair (x, y) can be chosen in infinite ways.

10. A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, little finger 5, then reversed direction, calling the ring finger 6, middle finger 7, index finger 8, thumb 9, then back to the index finger for 10, middle finger for 11, and so on. She counted up to 1994. She ended on her.

    (a) thumb (b) index finger

    (c) middle finger (d) ring finger

11. 139 persons have signed up for an elimination tournament. All players are to be paired up for the first round, but because 139 is an odd number one player gets a bye, which promotes him to the second round, without actually playing in the first round. The pairing continues on the next round, with a bye to any player left over. If the schedule is planned so that a minimum number of matches is required to determine the champion, the number of matches which must be played is

    (a) 136 (b) 137

    (c) 138 (d) 139

12. The product of all integers from 1 to 100 will have the following numbers of zeros at the end.

    (a) 20 (b) 24

    (c) 19 (d) 22

13. There are ten 50 paise coins placed on a table. Six of these show tails four show heads. A coin chosen at random and flipped over (<u>not</u> tossed). This operation is performed seven times. One of the coins is then covered. Of the remaining nine coins five show tails and four show heads. The covered coin shows

    (a) a head (b) a tail

    (c) more likely a head (d) more likely a tail

14. A five digit number is formed using digits 1, 3, 5, 7 and 9 without repeating any one of them. What is the sum of all such possible numbers?

    (a) 6666600 (b) 6666660

    (c) 6666666 (d) None

15. From each of two given numbers, half the smaller number is subtracted. Of the resulting numbers the larger one is three times as large as the smaller. What is the ratio of the two numbers?

    (a) 2:1 (b) 3:1

(c) 3:2 (d) None

16. If the harmonic mean between two positive numbers is to the inverse of their geometric mean as 12:13; then the numbers could be in the ratio

(a) 12:13 (b) 1/12:1/13

(c) 4:9 (d) 2:3

17. Fourth term of an arithmetic progression is 8. What is sum of the first 7 terms of the arithmetic progression?

(a) 7 (b) 64

(c) 56 (d) Cannot be determined

18. It takes the pendulum of a clock 7 seconds to strike 4 o'clock. How much time will it take to strike 11 o'clock?

(a) 18 seconds (b) 20 seconds

(c) 19.25 seconds (d) 23.33 seconds

19. Along a road lie an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is

(a) 35 (b) 15

(c) 29 (d) 31

20. What is the smallest number which when increased by 5 is completely divisible by 8, 11 and 24?

(a) 264 (b) 259

(c) 269 (d) None of these

21. Which is the least number that must be subtracted from 1856, so that the remainder when divided by 7, 12 and 16 will leave the same remainder 4.

(a) 137 (b) 1361

(c) 140 (d) 172

22. Two positive integers differ by 4 and sum of their reciprocals is 10/21. Then one of the numbers is

(a) 3 (b) 1

(c) 5 (d) 21

23. $5^6 - 1$ is divisible by

(a) 13 (b) 31

(c) 5 (d) None of these

24. For the product $n(n + 1)(2n + 1)$. n Œ N, which one of the following is necessarily false?

(a) It is always even

(b) Divisible by 3

(c) Always divisible by the sum of the square of first *n* natural numbers

(d) Never divisible by 237

25. The remainder obtained when a prime number greater than 6 is divided by 6 is

(a) 1 or 3 (b) 1 or 5

(c) 3 or 5 (d) 4 or 5

Review Test 2

Directions for Questions 1 to 4: Four sisters Suvarna, Tara, Uma and Vibha playing a game such that the loser doubles the money of each of the other player. They played four games and each sister lost one game in alphabetical order. At the end of fourth game each sister had ` 32.

1. Who started with the lowest amount?

   (a) Suvarna (b) Tara

   (c) Uma (d) Vibha

2. Who started with the highest amount?

   (a) Suvarna (b) Tara

   (c) Uma (d) Vibha

3. What was the amount with Uma at the end of the second round?

   (a) 36 (b) 72

   (c) 16 (d) None of these

4. How many rupees did Suvarna start with?

   (a) 60 (b) 34

   (c) 66 (d) 28

5. If n is an integer, how many values of n will give an integral value of $(16n^2+7n+6)/n$?

   (a) 2 (b) 3

   (c) 4 (d) None of these

6. A student instead of finding the value of $7/8^{th}$ of a number found the value of $7/18^{th}$ of the number. If his answer differed from the actual one by 770. Find the numbers.

   (a) 1584 (b) 2520

   (c) 1728 (d) 1656

7. P and Q are two integers such that $PQ = 8^2$. Which of the following cannot be the value of $P+Q$?

   (a) 20 (b) 65

   (c) 16 (d) 35

8. If m and n are integers divisible by 5, which of the following is not necessarily true?

   (a) $m - n$ is divisible by 5

   (b) $m^2 - n^2$ is divisible by 25

   (c) $m + n$ is divisible by 10

   (d) None of the above

9. Which of the following is true?

   (a) $7^{3^2} = (7^3)^2$ (b) $7^{3^2} > (7^3)^2$

   (c) $7^{3^2} < (7^3)^2$ (d) None of these

10. P, Q and R are three consecutive odd numbers in ascending order. If the value of three times P is three less than two times R. find the value of R.

(a) 5 (b) 7

(c) 9 (d) 11

11. ABC is a three-digit number in which $A > 0$. The value of ABC is equal to the sum of the factorials of its three digits. What is the value of B?

(a) 9 (b) 7

(c) 4 (d) 2

12. A, B and C are defined as follows:

$A = (2.000004) \prod [(2.000004)^2 + (4.000008)]$

$B = (3.000003) \prod [(3.000003)^2 + (9.000009)]$

$C = (4.000002) \prod [(4.000002)^2 + (8.000004)]$

Which of the following is true about the value of the above three expressions?

(a) All of them lie between 0.18 and 0.20

(b) A is twice C

(c) C is the smallest

(d) B is the smallest

13. Let $x < 0.50$, $0 < y < 1$, $z > 1$. Given a set of numbers, the middle number, when they arranged in ascending order is called the median. So the median of the numbers x, y and z would be

(a) less than one (b) between 0 and 1

(c) greater than one (d) cannot say

14. let a, b, c d, and e be integers such that $a = 6b = 12c$, and $2b = 9d = 12e$. Then which of the following pairs contains a number that is not an integer?.

(a) $\left(\frac{a}{27}, \frac{b}{e}\right)$ (b) $\left(\frac{a}{36}, \frac{c}{e}\right)$

(c) $\left(\frac{a}{3}, \frac{bd}{9}\right)$ (d) $\left(\frac{a}{7}, \frac{c}{d}\right)$

15. If a, $a + 2$, and $a + 4$ are prime numbers, then the number of possible solutions for a is:

(a) One (b) Two

(c) Three (d) More than three

16. Let x and y be positive integers such that x is prime and y is composite. Then,

(a) $y - x$ cannot be an even integer

(b) xy cannot be an even integer

(c) $(x + y)$ cannot be an even integer

(d) None of the above statement is true

17. Let n (>1) be a composite natural number such that the square root of n is not an integer. Consider the following statements:

A: n has a factor which is greater than 1 and less than square root n

B: n has a factor which is greater than square root n but less than n

Then

(a) Both A and B are false

(b) Both A and B are true

(c) A is false but B is true

(d) A is true and B is false

18. What is the remainder when 4^{96} is divided by 6?

(a) 0 (b) 2

(c) 3 (d) 4

19. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?

(a) 646 (b) 676

(c) 683 (d) 797

20. The infinite sum $1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}$ + – – – – –

(a) $\frac{27}{14}$ (b) $\frac{29}{13}$

(c) $\frac{49}{27}$ (d) $\frac{256}{147}$

21. If the product of n positive real numbers is unity, then their sum is necessarily:

(a) a multiple of n (b) equal to $n+\frac{1}{n}$

(c) never less than n (d) None of these

22. How many three digit positive integer, with digits x, y and z in the hundred's, ten's and unit's place respectively, exist such that $x < y$, $z < y$ and x π 0?

(a) 245 (b) 285

(c) 240 (d) 320

23. How many even integers n, where $100 \leq n \leq 200$, are divisible neither by seven nor by nine?

(a) 40 (b) 37

(c) 39 (d) 38

24. A positive whole number M less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the

three cases the leading digit in 1. Then M equals:

(a) 31 (b) 63

(c) 75 (d) 91

25. In a certain examinations paper, there are n questions. For $j = 1, 2 \ldots.n$. there are 2^{n-1} students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is:

(a) 12 (b) 11

(c) 10 (d) 9

Review Test 3

1. In 1936, I was as old as the number formed by the last two digits of my year of birth. Find the date of birth of my father who is 25 years older to me.

   (a) 1868
   (b) 1893
   (c) 1902
   (d) 1900
   (e) Can’t be determined

2. Find the total number of integral solutions of the equation $(407)x – (ddd)y = 2589$, where ‘*ddd*’ is a three-digit number.

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) Can’t be determined

3. Find the digit at the ten’s place of the number $N = 7^{281} \times 3^{264}$.

   (a) 0
   (b) 1
   (c) 6
   (d) 5
   (e) None of these

4. Raju went to a shop to buy a certain number of pens and pencils. Raju calculated the amount payable to the shopkeeper and offered that amount to him. Raju was surprised when the shopkeeper returned him ` 24 as balance. When he came back home, he realized that the shopkeeper had actually transposed the number of pens with the number of pencils. Which of the following is certainly an invalid statement?

   (a) The number of pencils that Raju wanted to buy was 8 more than the number of pens.
   (b) The number of pens that Raju wanted to buy was 6 less than the number of pencils.
   (c) A pen cost ` 4 more than a pencil.
   (d) None of the above.

5. HCF of 384 and a^5b^2 is 16ab. What is the correct relation between *a* and *b*?

   (a) $a = 2b$
   (b) $a + b = 3$
   (c) $a – b = 3$
   (d) $a + b = 5$

6. In ancient India, 0 to 25 years of age was called Brahmawastha and 25 to 50 was called Grahastha. I am in Grahastha and my younger brother is also in Grahastha such as the difference in our ages is 6 years and both of our ages are prime numbers. Also twice my brother’s age is 31 more than my age. Find the sum of our ages.

   (a) 80
   (b) 68
   (c) 70
   (d) 71

7. Volume of a cube with integral sides is the same as the area of a square with integral sides. Which of these can be the volume of the cube formed by using the square and its replicas as the 6 faces?

   (a) 19683
   (b) 512

(c) 256 (d) Both (a) and (b)

8. Let A be a two-digit number and B be another two-digit number formed by reversing the digits of A. If A + B + (Product of digits of the number A) = 145, then what is the sum of the digits of A?

(a) 9 (b) 10

(c) 11 (d) 12

9. When a two-digit number N is divided by the sum of its digits, the result is Q. Find the minimum possible value of Q.

(a) 10 (b) 2

(c) 5.5 (d) 1.9

10. A one-digit number, which is the ten's digit of a two digit number X, is subtracted from X to give Y which is the quotient of the division of 999 by the cube of a number. Find the sum of the digits of X.

(a) 5 (b) 7

(c) 6 (d) 8

11. After Yuvraj hit 6 sixes in an over, Geoffery Boycott commented that Yuvraj just made 210 runs in the over. Harsha Bhogle was shocked and he asked Geoffery which base system was he using? What must have been Geoffery's answer?

(a) 9 (b) 2

(c) 5 (d) 4

12. Find the ten's digit of the number 7^{2010}.

(a) 0 (b) 1

(c) 2 (d) 4

13. Find the HCF of 481 and the number 'aaa' where 'a' is a number between 1 and 9 (both included).

(a) 73 (b) 1

(c) 27 (d) 37

14. The number of positive integer valued pairs (x, y), satisfying $4x - 17y = 1$ and $x < 1000$ is:

(a) 59 (b) 57

(c) 55 (d) 58

15. Let a, b, c be distinct digits. Consider a two digit number 'ab' and a three digit number 'ccb' both defined under the usual decimal number system. If $(ab)^2 = ccb$ and $ccb > 300$ then the value of b is:

(a) 1 (b) 0

(c) 5 (d) 6

16. The remainder 7^{84} is divided by 342 is:

(a) 0 (b) 1

(c) 49 (d) 341

17. Let x, y and z be distinct integers, x and y are odd and positive, and z is even and positive. Which one of the following statements can't be true?

(a) $(x - z)^2y$ is even (b) $(x - z)y^2$ is odd

(c) $(x - y)y$ is odd (d) $(x - y)^2z$ is even

18. A boy starts adding consecutive natural numbers starting with 1. After some time he reaches a total of 1000 when he realizes that he has made the mistake of double counting 1 number. Find the number double counted.

(a) 44 (b) 45

(c) 10 (d) 12

19. In a number system the product of 44 and 11 is 1034. The number 3111 of this system, when converted to decimal number system, becomes:

(a) 406 (b) 1086

(c) 213 (d) 691

20. Ashish is given ` 158 in one rupee denominations. He has been asked to allocate them into a number of bags such that any amount required between Re 1 and ` 158 can be given by handing out a certain number of bags without opening them. What is the minimum number of bags required?

(a) 11 (b) 12

(c) 13 (d) None of these

Review Test 4

1. Find the number of 6-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 once such that the 6-digit number is divisible by its unit digit.

   (a) 648 (b) 528

   (c) 728 (d) 128

2. Which is the highest 3-digit number that divides the number 11111….1(27 times) perfectly without leaving any remainder?

   (a) 111 (b) 333

   (c) 666 (d) 999

3. W1, W2, … , W7 are 7 positive integral values such that by attaching the coefficients of +1, 0 and -1 to each value available and adding the resultant values, any number from 1 to 1093 (both included) could be formed. If W1, W2, … , W7 are in ascending order, then what is the value of W3?

   (a) 10 (b) 9

   (c) 0 (d) 1

4. What is the unit digit of the number $63^{25} + 25^{63}$?

   (a) 3 (b) 5

   (c) 8 (d) 2

5. Find the remainder when $(2222^{5555} + 5555^{2222})$ is divided by 7.

   (a) 1 (b) 0

   (c) 2 (d) 5

6. What is the number of nines used in numbering a 453 page book?

   (a) 86 (b) 87

   (c) 84 (d) 85

7. How many four digit numbers are divisible by 5 but not by 25?

   (a) 2000 (b) 8000

   (c) 1440 (d) 9999

8. The sum of two integers is 10 and the sum of their reciprocals is 5/12. What is the value of larger of these integers?

   (a) 7 (b) 5

   (c) 6 (d) 4

9. Saurabh was born in 1989. His elder brother Siddhartha was also born in the 1980's such that the last two digits of his year of birth form a prime number P. Find the remainder when $(P^2 + 11)$ is divided by 5.

   (a) 0 (b) 1

(c) 2
(d) 3

10. The HCF of x and y is H. Find the HCF of $(x - y)$ and $(x^3 + y^3)/(x^2 - xy + y^2)$.

(a) H – 1
(b) H^2
(c) H
(d) H + 1

11. 4 bells toll together at 9:00 A.M. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?

(a) 3
(b) 5
(c) 6
(d) 9

12. What power of 210 will exactly divide 142!

(a) 22
(b) 11
(c) 34
(d) 33

13. Find the total numbers between 122 and 442 that are divisible by 3 but not by 9.

(a) 70
(b) 71
(c) 72
(d) 73

14. If 146! is divisible by 6^n, then find the maximum value of n.

(a) 74
(b) 75
(c) 76
(d) 70

15. If we add the square of the digit in the tens place of a positive two-digit number to the product of the digits of that number, we shall get 52, and if we add the square of the digit in the units place to the same product of the digits, we shall get 117. Find the two-digit number.

(a) 18
(b) 39
(c) 49
(d) 28

16. Find the smallest natural number n such that $(n + 1)n[(n - 1)!]$ is divisible by 990.

(a) 2
(b) 4
(c) 10
(d) 11

17. If x, y and z are odd, even and odd respectively, then $(x^2 - yz^2 + y^3)$ and $(x^2 + y^2 + z^2)$ are respectively:

(a) Odd & Odd
(b) Even & Odd
(c) Odd & Even
(d) Odd & Odd

18. A two digit number N has its digits reversed to form another two digit number M. What could be the unit digit of M if product of M and N is 574?

(a) 1
(b) 3
(c) 6
(d) 9

19. For what relation between b and c is the number abcacb divisible by 7, if $b > c$?

(a) $b + c = 7$
(b) $b = c + 7$

(c) $2bc = 7$

(d) $c = 7b$

20. What is the remainder when a^6 is divided by $(a + 1)$?

(a) $a + 1$

(b) a

(c) 0

(d) 1

Review Test 5

1. What is the last digit of 62^43^54^65^76^87?

(a) 2 (b) 4

(c) 6 (d) 8

2. $N = 99^3 - 36^3 - 63^3$, how many factors does N have?

(a) 51 (b) 96

(c) 128 (d) 192

3. Find the highest power of 2 in 1! + 2! + 3! + 4!..............600!

(a) 1 (b) 494

(c) 3. 0 (d) 256

4. 100! is divisible by 160^n...what is the max. integral value of *n*?

(a) 19 (b) 24

(c) 26 (d) 28

5. What is the sum of the digits of the decimal form of the product $2^{999} * 5^{1001}$?

(a) 2 (b) 4

(c) 5 (d) 7

6. What is the remainder when 1*1 + 11*11 + 111*111 + 1111*1111 +......+ (2001 times 1)*(2001 times 1) is divided by 100 ?

(a) 99 (b) 22

(c) 01 (d) 21

7. What is the remainder when 789456123 is divided by 999?

(a) 123 (b) 369

(c) 963 (d) 189

8. What is the total number of the factors of 16!

(a) 2016 (b) 1024

(c) 3780 (d) 5376

9. Find the sum of the first 125 terms of the sequence 1, 2, 1, 3, 2, 1, 4, 3, 2, 1, 5, 4, 3, 2...

(a) 616 (b) 460

(c) 750 (d) 720

10. Umesh purchased a Tata Nano recently, but the faulty car odometer of Tata Nano proceeds from digit 4 to digit 6, always skipping the digit 5, regardless of position. If the odometer now reads 003008 (starting with 000000), how many km has Nano actually travelled?

(a) 2100 (b) 1999

(c) 2194 (d) 2195

11. What is the number of consecutive zeroes in the end of 1000!?

(a) 248
(b) 249
(c) 250
(d) 251

12. Mr. Ramlal lived his entire life during the 1800s. In the last year of his life, Ramlal stated: Once I was x years old in the year x^2. He was born in the year

(a) 1822
(b) 1851
(c) 1853
(d) 1806

13. Find the unit's digit of LCM of $13^{501} - 1$ and $13^{501} + 1$

(a) 2
(b) 4
(c) 5
(d) 8

14. If you were to add all odd numbers between 1 and 2007 (both inclusive), the result would be

(a) A perfect square
(b) Divisible by 2008
(c) Multiple of 251
(d) All of the above

15. Find the remainder when $971(30^{99} + 61^{100}) * (1148)^{56}$ is divided by 31

(a) 25
(b) 0
(c) 11
(d) 21

16. What is the remainder when 2^{100} is divided by 101?

(a) 1
(b) 100
(c) 0
(d) 99

17. Find the last two digits of 2^{134}

(a) 04
(b) 84
(c) 24
(d) 64

18. Find the remainder when $(10^3 + 9^3)^{1000}$ by 12^3

(a) 01
(b) 11
(c) 1001
(d) 1727

19. The number of factors of the number 3000 are:

(a) 16
(b) 32
(c) 24
(d) 28

20. If N! has 73 zeroes at the end then find the value of N?

(a) 295
(b) 300
(c) 290
(d) Not possible

ANSWER KEY

Review Test 1

1. (a)	2. (c)	3. (d)	4. (a)
5. (b)	6. (a)	7. (d)	8. (c)
9. (a)	10. (b)	11. (c)	12. (b)
13. (a)	14. (a)	15. (a)	16. (c)
17. (c)	18. (d)	19. (d)	20. (b)
21. (d)	22. (a)	23. (b)	24. (d)
25. (b)			

Review Test 2

1. (d)	2. (a)	3. (b)	4. (c)
5. (c)	6. (a)	7. (d)	8. (c)
9. (b)	10. (c)	11. (c)	12. (d)
13. (b)	14. (d)	15. (a)	16. (d)
17. (b)	18. (d)	19. (b)	20. (c)
21. (c)	22. (c)	23. (c)	24. (d)
25. (a)			

Review Test 3

1. (b)	2. (a)	3. (c)	4. (d)
5. (d)	6. (a)	7. (d)	8. (c)
9. (d)	10. (a)	11. (d)	12. (d)
13. (d)	14. (a)	15. (a)	16. (b)
17. (a)	18. (c)	19. (a)	20. (d)

Review Test 4

1. (a)	2. (d)	3. (b)	4. (c)
5. (b)	6. (d)	7. (c)	8. (c)
9. (a)	10. (c)	11. (b)	12. (a)
13. (b)	14. (d)	15. (c)	16. (c)
17. (c)	18. (a)	19. (b)	20. (d)

Review Test 5

1. (a)	2. (b)	3. (c)	4. (a)
5. (d)	6. (c)	7. (b)	8. (d)
9. (c)	10. (c)	11. (b)	12. (d)
13. (b)	14. (d)	15. (b)	16. (a)
17. (b)	18. (a)	19. (b)	20. (d)