~~Transformation~~ Robotics

Matrix multiplication (Trick)

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_4 & b_4 & c_4 \end{bmatrix} \times \begin{bmatrix} l_1 & l_2 \\ m_1 & m_2 \\ n_1 & n_2 \end{bmatrix} = \begin{bmatrix} a_1 l_1 + b_1 m_1 + c_1 n_1 & a_1 l_2 + b_1 m_2 + c_1 n_2 \\ a_2 l_1 + b_2 m_1 + c_2 n_1 & a_2 l_2 + b_2 m_2 + c_2 n_2 \\ a_3 l_1 + b_3 m_1 + c_3 n_1 & a_3 l_2 + b_3 m_2 + c_3 n_2 \\ a_4 l_1 + b_4 m_1 + c_4 n_1 & a_4 l_2 + b_4 m_2 + c_4 n_2 \end{bmatrix}$$

Step ①:- Second matrix में जितना column होगा First matrix को उतना बार लिखना हैं।

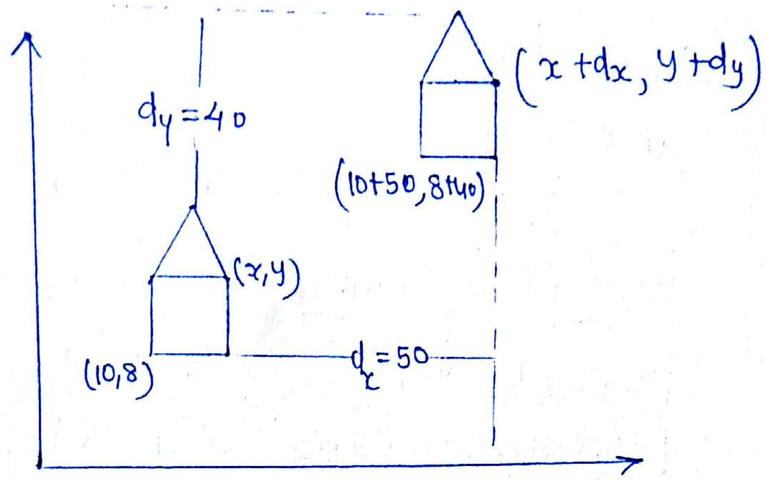
Step ②:- Second matrix का Column में Vertically जो elements मिलेगा उस से Horizontal Multiple करेंगे

Step ③:- Add

Transformation (2D):-

① Translation
② Rotation
③ Scalling
④ shear
⑤ Reflactions.

Translation:-



$$x_{new} = x_{old} + d_x$$

$$y_{new} = y_{old} + d_y$$

In matrix form:-

$$\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix} = \begin{bmatrix} x + d_x \\ y + d_y \end{bmatrix}$$

In homogeneous Cordinate:

$$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

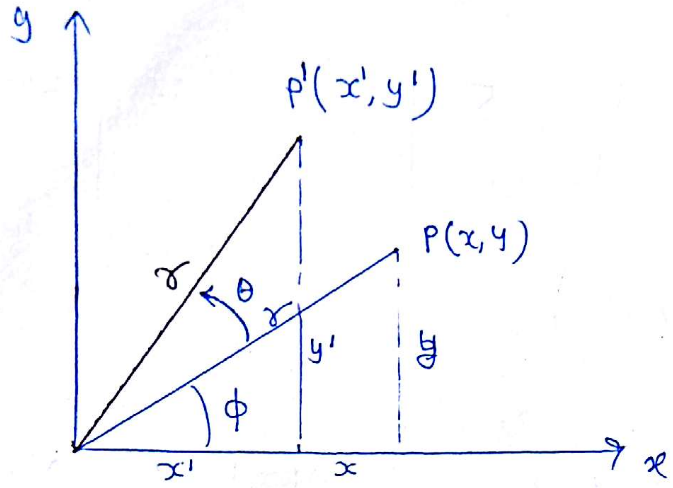
Rotation matrix

Translation Vecter

Scale facter

Prespective Matrix

Rotation :- (About origin)



Rotate P about origion by θ angle.

find x' & y' ?

$$x = r \cos\theta$$

$$y = r \sin\theta$$

$$x' = r\cos(\theta+\phi) = r\cos\theta\cos\phi - r\sin\theta\sin\phi$$

$$y' = r\sin(\theta+\phi) = r\sin\theta\cos\phi + r\cos\theta\sin\phi$$

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

in Matrix form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Therefore transformation matrix of rotation about origin

$$T_r = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

if 'θ' rotates clockwise then Replace $\underline{\theta} \rightarrow \underline{-\theta}$

Gate -2014

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} (0 + -5) \\ (2 + 0) \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}.$$

$$= (-5, 2)$$

Gate 2012

~~Gate - 2016~~

$x' = 5x$

(3, 2)

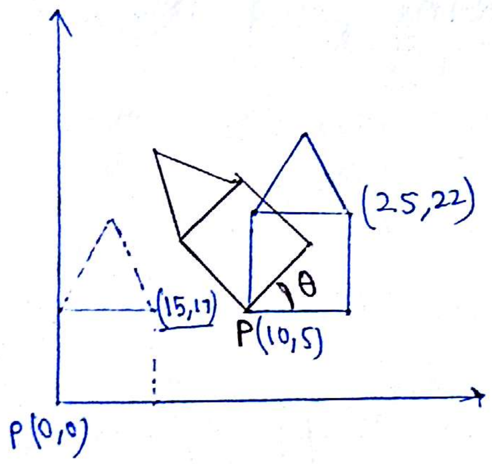
$\cancel{x' = 4 \times 0.8 + 5 \times 0.6} = \cancel{2.4 + 1.2} = 3.6 + 0$

$= 4 \times 0.8 + 2 \times 0.6 \qquad -3 \times 0.6 + 2 \times 0.8$

$3.5 \qquad -1.8 + 1.6 = -0.2+2$

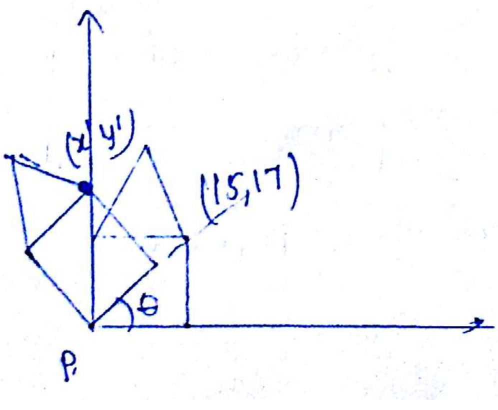
Rotation about any arbitory point 'p'

$$T_r = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



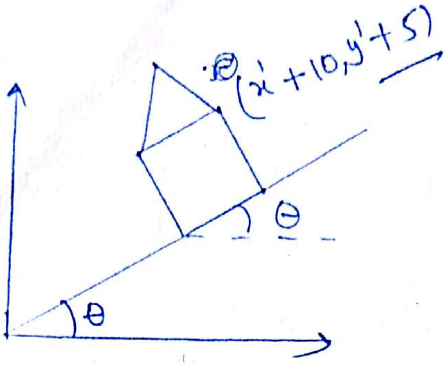
Step ①: Translate point 'p' to origin

Step ②: Rotate 'θ' about origin.



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 15 \\ 17 \end{bmatrix}$$

Step ③ Translate back to origional point 'P'



✱ if Rotation Clockwise change $\theta \to -\theta$

$$T_r = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Gate 2016

Q' (3, 2)

$x' = 3(0.8) - 2(-0.6) = 3.6 \to 4.6$

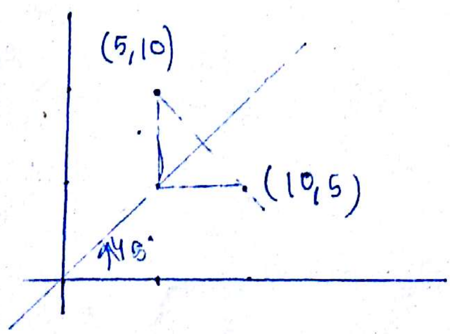
$y' = -3(0.6) + 2(0.8) = -0.2 \to 2.8$

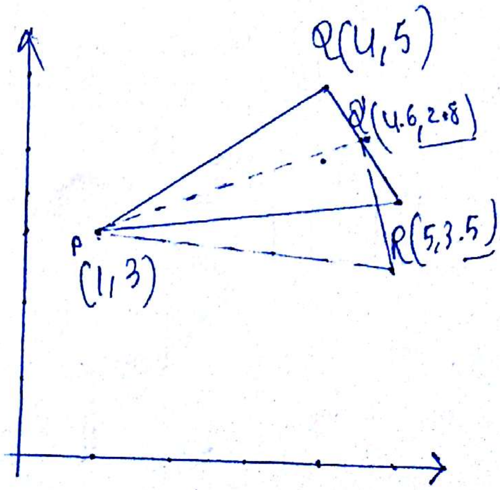
New Coord $x = 3.6 + 1 = 4.6$

$y = -0.2 + 3 = 2.8$ $\quad$ Q(4.6, 2.8)

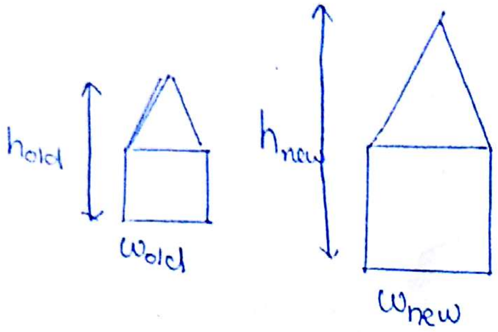
$$\left\{ \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -(-\sin\theta) \\ (-\sin\theta) & \cos\theta \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$$

Gate-2013





Scalling :-



$$S_x = \frac{W_{new}}{W_{old}}$$

$$S_y = \frac{h_{new}}{h_{old}}$$

$$x_{new} = S_x \cdot x_{old}$$

$$x_{new} = S_y \cdot y_{old}.$$

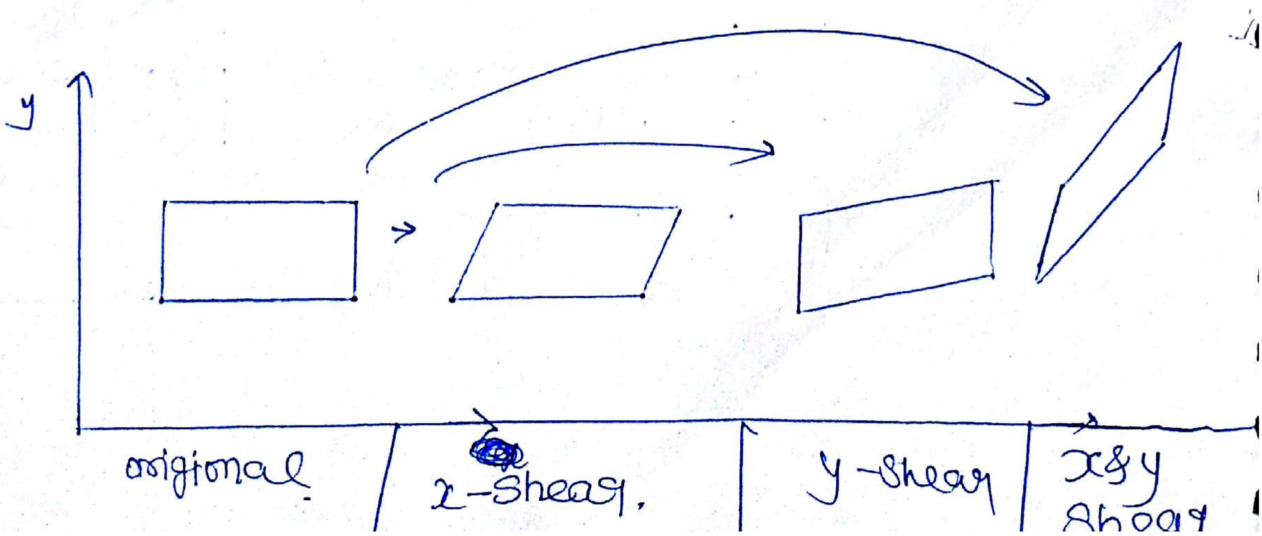
in matrix form :-

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

In homogeneous Coordinate

$$S_{x,y} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Shear



X - shear $T_r = \begin{bmatrix} 1 & \underline{a} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

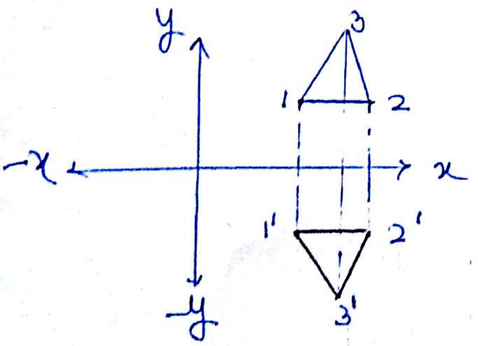
Y - shear $T_r = \begin{bmatrix} 1 & 0 & 0 \\ \underline{b} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

x & y shear $T_r = \begin{bmatrix} 1 & \underline{a} & 0 \\ \underline{b} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now multiple T_r with new Coordinate

Reflection :- Reflection is the mirror image of origional object in other words we can say that it is a rotation operation with 180°.

(A) Reflection about x- axis

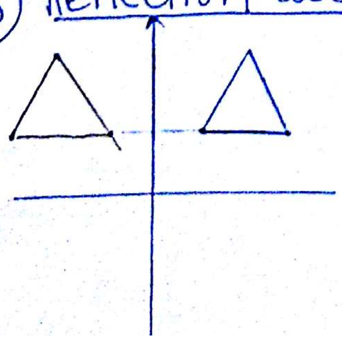


$T_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$1 \times x + 0 \times y = x$

$0 \times x - 1 \times y = -y$

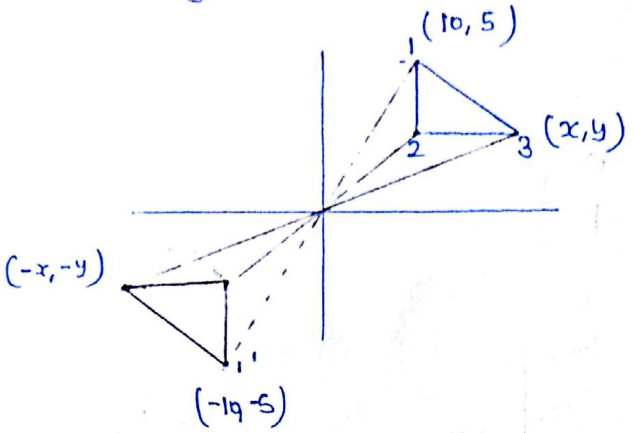
(B) Reflection about Y-axis



$T_x = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

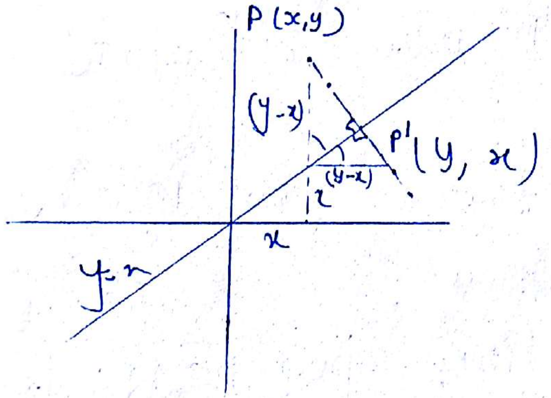
(c) Reflection about origin

* origin works as a point mirror.



$$T_r = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

D) Reflection about $y = x$ line

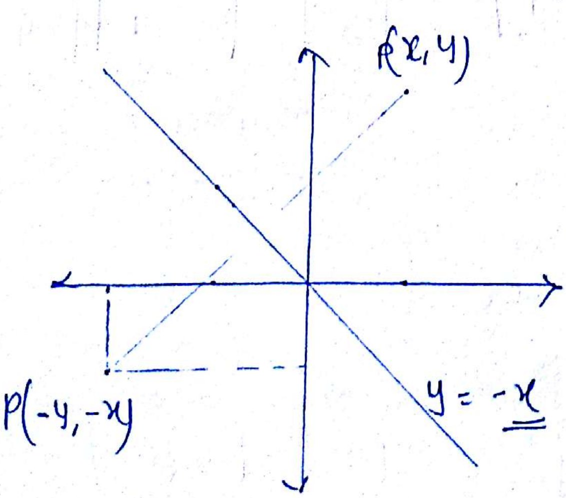


$(x, y) \rightarrow (y, x)$

Mirror Image about $y = \underline{x}$

$$T_{y=x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

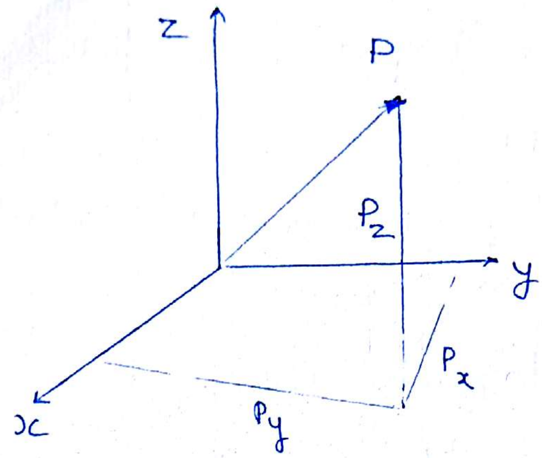
E) Reflection about $y = -x$ line



$$T_{y-x} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

3D Transformation:-

Coordinate Frame:-



Position and orientation of a point P in a Coordinate Frame $\vec{OP}$ is a vector

$$\vec{OP} = P_x \hat{x} + P_y \hat{y} + P_z \hat{z}$$

A Frame space notation is introduce as ^1P to refer to the point P w.r.t. Frame {1} or {xyz} with it's components in the Frame as ^1P_x, ^1P_y, ^1P_z there fore

$$^1P_x = P_x, \quad ^1P_y = P_y, \quad ^1P_z = P_z$$

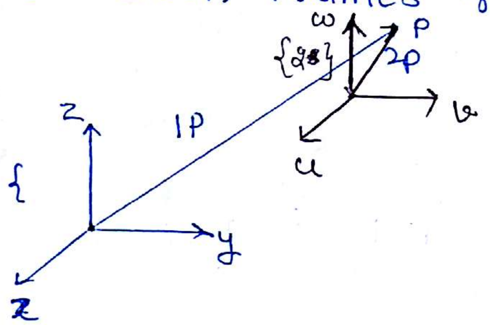
$$^1P = {}^1P_x \hat{i} + {}^1P_y \hat{j} + {}^1P_z \hat{k}$$

In vector matrix notation $^1P = \begin{bmatrix} ^1P_x \\ ^1P_y \\ ^1P_z \end{bmatrix} = \begin{bmatrix} ^1P_x & ^1P_y & ^1P_z \end{bmatrix}^T$

Mapping:- mapping refer to changing the descriptior of a point in space from one frame to another frame.

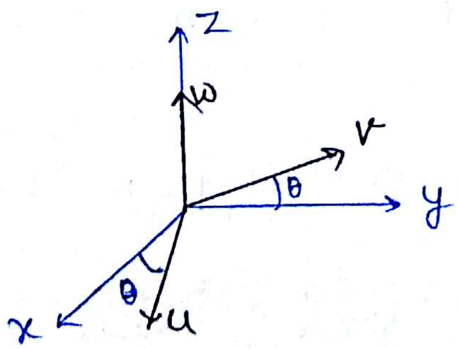
- Mapping changes the description of the point not the point itself.
- The second frame has three posibilities in relation to the first frame.

① Mapping involving translated frames
sencond frame is moved away from the first the axis of both frames remains parallel.

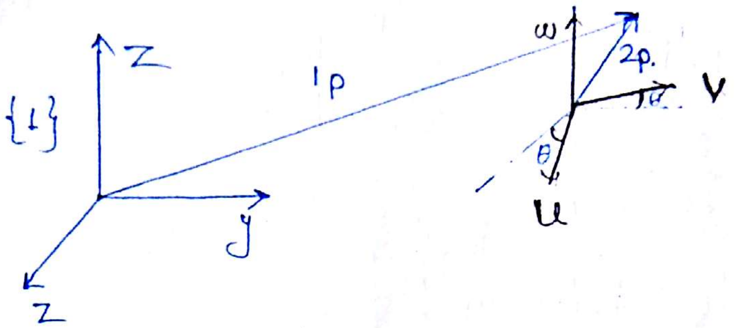


x-axis ∥ u-axis
y axis ∥ v-axis
z axis ∥ w-axis

② Mapping involving rotated frame
Second frame is rotated wrt the first the origin of both the frame is same.
In robotics this is reffered as changing the orientation



③ Second frame is rotated wrt the first and moved away from it i.e., the second frame is translated and it's orientation is also changed.



Homogeneous transformation matrix:- (HTM)

$$HTM = \left[\begin{array}{c|c} \begin{matrix}\text{Rotation} \\ \text{Matrix} \\ (3\times3)\end{matrix} & \begin{matrix}\text{translation} \\ \text{Vector} \\ (3\times1)\end{matrix} \\ \hline \begin{matrix}\text{Perspective} \\ \text{transformation} \\ (1\times3)\end{matrix} & \begin{matrix}\text{scale factor } (\sigma) \\ (1\times1)\end{matrix} \end{array}\right]_{4\times4}$$

This is generalised homogeneous transformation matrix has above four sub-matrices

~~Pres Percept~~

→ Perspective transformation matrix is used in vision System and is set to zero vector wherever No perspective view are Involve.

→ The scale factor (σ) has non zero positive values and is called Global scalling parameter $\sigma > 1$ is use full for reducing

$\sigma > 1$ is useful for reducing

$0 < \sigma < 1$ is useful for enlarging

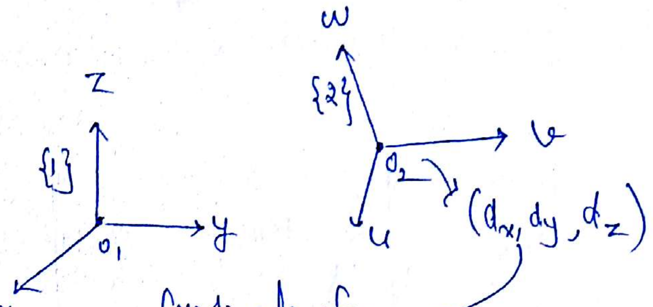
In robotics $\sigma = 1$ is always.

$$HTM = {}^{1}T_{2} = \begin{bmatrix} {}^{1}R_{2} & {}^{1}O_{2} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

$${}^{1}R_{2} = \begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{x}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \hat{z}\cdot\hat{w} \end{bmatrix}$$

$${}^{1}D_{2} = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

$${}^{1}T_{2} = \begin{bmatrix} {}^{1}R_{2} & {}^{1}D_{2} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$



Coordinate of O_2 wrt O_1

if ${}^{2}p$ is known

$$\boxed{{}^{1}p = {}^{1}T_{2} \cdot {}^{2}p}$$

if ${}^{1}p$ is known $\boxed{{}^{2}p = {}^{2}T_{1} \cdot {}^{1}p}$

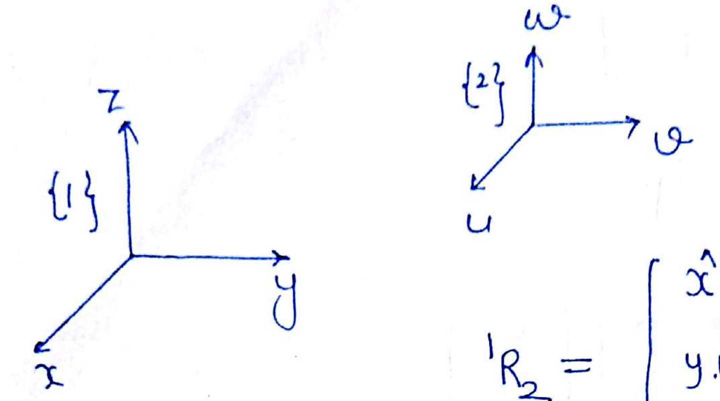
$${}^{2}T_{1} = \left[{}^{1}T_{2}\right]^{-1}$$

if vector is involved:-

$${}^{1}T_{2} \Rightarrow HTM$$

[New vector] = [HTM] [old vector]

Only translation



$$^1R_2 = \begin{bmatrix} \hat{x}.\hat{u} & \hat{x}.\hat{v} & x.\hat{w} \\ y.\hat{u} & y.\hat{v} & y.\hat{w} \\ z.\hat{u} & z.\hat{v} & z.\hat{w} \end{bmatrix}$$

$$^1R_2 = \begin{bmatrix} \cos 0 & \cos 90^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos 0 & \cos 90 \\ \cos 90^\circ & \cos 90^\circ & \cos 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$HTM = {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

eg (Pure translation) Consider a frame {2} which is obtain from frame {1} by translating it two units along y and 1 unit along z find HTM. Also find ^1p if ^2p is $= [0\ 2\ 3]^T$

Sol^n

$$HTM = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1T_2$$

$$^1T_2 = \begin{bmatrix}1&0&0&0\\0&1&0&2\\0&0&1&1\\0&0&0&1\end{bmatrix}$$

Now $^1p = {}^1T_2 \cdot {}^2p$ $\qquad$ $^2p = [0\ 2\ 3]^T$

$$^1p = \begin{bmatrix}1&0&0&0\\0&1&0&2\\0&0&1&1\\0&0&0&1\end{bmatrix}\begin{bmatrix}0\\2\\3\\1\end{bmatrix}$$

$\rightarrow$ multiflication के चक्कर मे आयेग

$$^1p = \begin{bmatrix}0\\4\\4\\1\end{bmatrix}$$

बेकार है

$$^1p = [0\ \ 4\ \ 4]^T$$

जुगाड़ से :-

$$^2p = [0\ \ 2\ \ 3]^T$$

$$\text{translation} = 0\ \ 2\ \ 1$$

$$^1p = [0\ \ 4\ \ 4]^T$$ Ans

If Vector involve:-

Q48 For the vector $\vec{V} = 25\hat{i} + 10\hat{j} + 20\hat{k}$ perform a translation by a distance 8 in the x dir^n 5 in y dir^n & 0 in z dirn. Find HTM & New Vector.

$$HTM = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[\text{New Vector}] = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 25 \\ 16 \\ 20 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 33 \\ 15 \\ 20 \\ 1 \end{bmatrix} \text{---} \underline{X}$$

$$V_{New} = 33\hat{i} + 15\hat{j} + 20\hat{k}$$

ट्रिक से (for only translation)

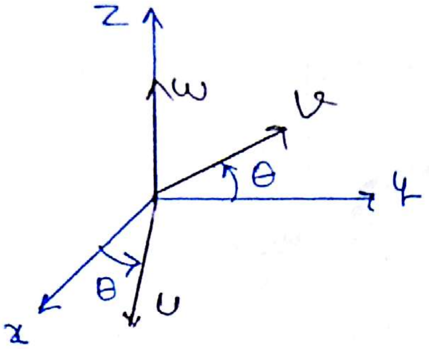
$$V_{old} = 25\hat{i} + 10\hat{j} + 20\hat{k}$$
$$\quad\; 8 \quad\; 5 \quad\; 0$$

$$\Downarrow$$

$$V_{new} = 33\hat{i} + 15\hat{j} + 20\hat{k}$$

Rotation

Principle Axis Rotation :-



about z

$$ {}^1R_2 = R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

जिसके w.r.t घुमाया उसको ⊥ & rows, Colum 0, 0

$$ {}^1T_2 = \left[\begin{array}{c|c} \underset{R}{\overset{\parallel}{{}^1R_2}} & \begin{matrix}0\\0\\0\end{matrix} \\ \hline \begin{matrix}0 & 0 & 0\end{matrix} & 1 \end{array}\right] $$

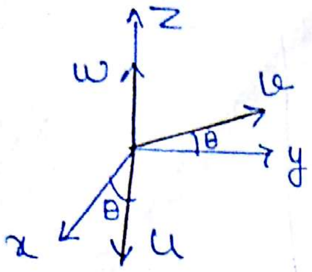
$$ \boxed{{}^1R_2 = R_z(\theta) \text{ or } R_{z,\theta} \text{ or } R(z,\theta)} $$

rotation about x $\quad R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$

about y $\quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

बीच मे घुसने से sin का sign change होगा

How ?



$$R_z(\theta) = {}^1R_2 = \begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{x}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \hat{z}\cdot\hat{w} \end{bmatrix}$$

$$R_z(\theta) = {}^1R_2 = \begin{bmatrix} \cos\theta & \cos(90+\theta) & \cos 90^\circ \\ \cos(90-\theta) & \cos\theta & \cos 90^\circ \\ \cos 90 & \cos 90 & \cos 0 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ques [Pure rotation] Frame {2} obtain from the frame 1 by rotating it about it z- axis by an angle 30° find HTM.

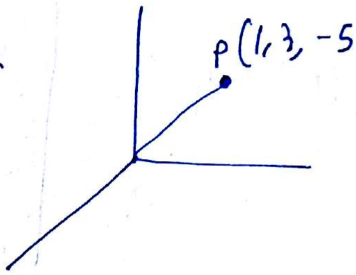
$$HTM = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 0 & 0 \\ \sin 30^\circ & \cos 30^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= {}^1T_2$$

Gate-2016 about Z axis

$$HTM = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 2 \\ \sin\theta & \cos\theta & 0 & 3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~New position~~ = translation



translation

$$New\ P = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -9 \\ 1 \end{bmatrix}$$

$= \cancel{3\ 6}$ $\boxed{Or}$

$$\begin{array}{rrr} 1 & 3 & -5 \\ +\ 2 & 3 & -4 \\ \hline 3 & 6 & -9 \end{array}$$

Now Rotation

$$New\ P = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ -9 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \\ -9 \end{bmatrix}$$

Ans

(ESE only)

Translation and Rotation Combined
(only for frames)

Ques Frame{2} is rotated wrt to frame{1} about the x axis by an angle of 60°. the position of origin of frame{2} as seen from frame{1} is ${}^1D_2 = [7\ 5\ 7]^T$ obtain the transformation matrix ${}^1T_2 = ?$
Also find the description of point 1 in frame{1} if ${}^2p = [2\ 4\ 6]^T$

$$
{}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & \cos 60 & -\sin 60 & 5 \\ 0 & \sin 60 & \cos 60 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0.5 & -0.866 & 5 \\ 0 & 0.866 & 0.5 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1p = {}^1T_2 \cdot {}^2p = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0.5 & -0.866 & 5 \\ 0 & 0.866 & 0.5 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 1.804 \\ 13.464 \\ 1 \end{bmatrix}
$$

$$
{}^1p = [9\ \ 1.80\ \ 13.467]^T
$$

Note: Fundametal rotation matrixes $\{R_x(\theta) / R_y(\theta) / R_z(\theta)\}$ can be multiplied together to represent a sequence of finite rotation.

For eg:- The overall rotation matrix representing a rotation of angle θ_1 about x axis follow by a rotation of angle θ_2 about y axis can be obtain by multiplying

$$\boxed{{}^1R_2 = R_y(\theta) \cdot R_x(\theta)}$$

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बाद वाला पहले}

& then put in HTM

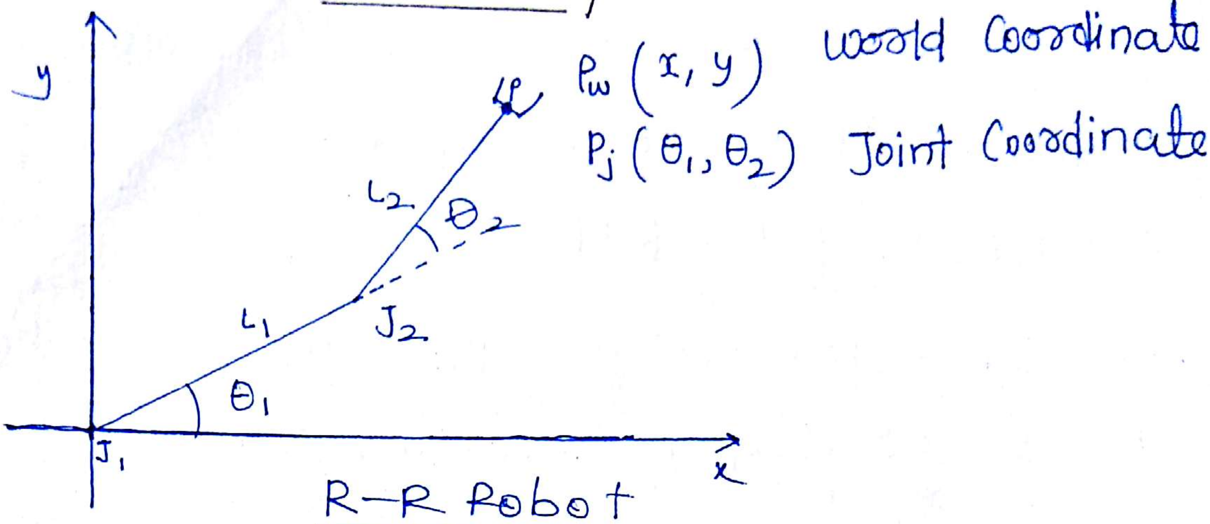
$${}^1R_2 = \begin{bmatrix} C_2 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_2 & 0 & C_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_1 & -S_1 \\ 0 & S_1 & C_1 \end{bmatrix}$$

$${}^1R_2 = \begin{bmatrix} C_2 & S_1 S_2 & C_1 S_2 \\ 0 & C_1 & -S_1 \\ -S_2 & S_1 C_2 & C_1 C_2 \end{bmatrix}$$

$$\left\{ \begin{aligned} S_i &= S\theta_i = \sin\theta_i \\ C_i &= C\theta_i = \cos\theta_i \end{aligned} \right.$$

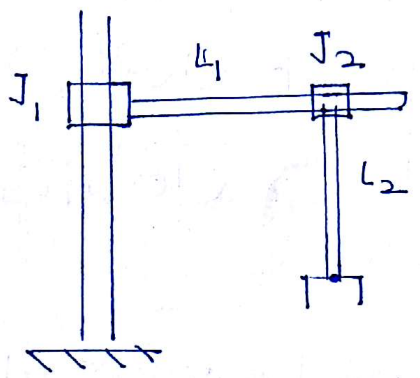
instead of $\cos\theta$, $\sin\theta$ we use c_1, c_2, S_1, S_2 in Robotics

Direct and inverse kinematics



R–R Robot

The kinematic of the R-R Robot are more difficult to analyse P–P Robot



P-P Robot

- We will make frequent use of this Configration and extension of it.
- Now we will analys in 2D cases only considering the R-R manupulator in 2-D.
- The position of the end of the arm may be represented in a no. of ways

→ One way is to ~~an~~ utilise the two joint angle θ_1 & θ_2 this is known as ~~joint space~~ JOINT SPACE representation.

→ Another way to define the arm position in WORLD SPACE. This involve the use of a Cartesian Coordinate system i.e. external to the robot. The origin of cartesian axis system is often located in robot base $P_w(x, y)$

→ This concept of a point defination in world space can be extended to 3-D $P_w(x, y, z)$

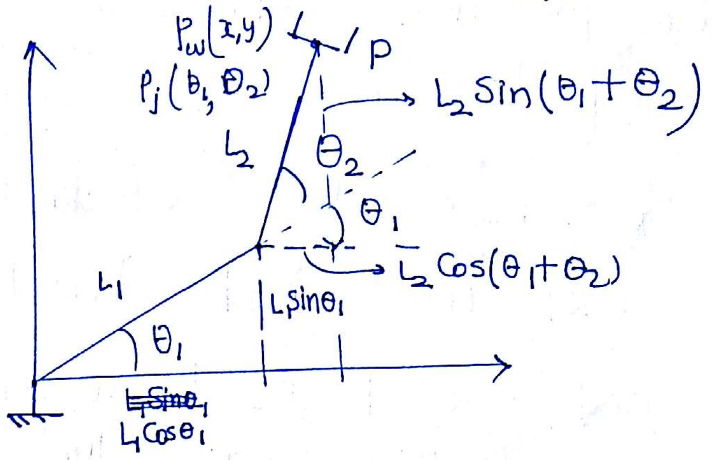
World space is useful when the robot must communicate with other machines because other machines may not have a detail understanding of robot kinematics.

→ In order to use both representation we must be able to transfor from one to other.

→ Going from joint space to world space is called the forward transformation or direct kinematics. (i.e. θ_1, θ_2 given $x = ?, y = ?$) $(\theta_1, \theta_2) \longrightarrow (x, y)$

→ Going from world space to joint space is called the reverse transformation or inverse kinematics. (i.e. x, y Given $\theta_1, \theta_2 = ?$) $(x, y) \longrightarrow (\theta_1, \theta_2)$

Direct kinematics of 2 DoF are:-



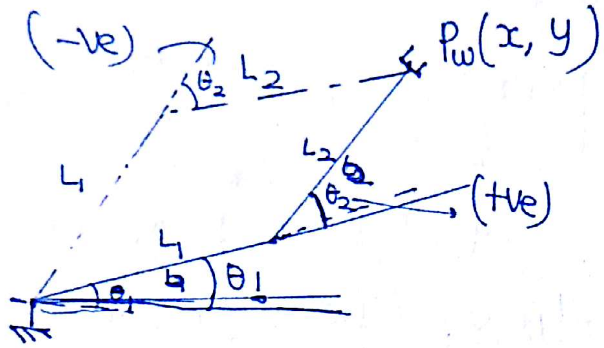
$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

Inverse kinematic of 2 DoF are:-

→ Drive the joint angles given the end of Arm position in world space

→ The typical situation is where the robots Controller must compute the joint angles require to move it's end of arm. to a point in space. define by the point coordinates.



2 ways to place robot at $P_w(x,y)$

→ For the two link manipulator we have develope there are two possible configration for reaching the point (x, y)

→ This is so because the relation b/w the joint angle and the end effector coordinate involve sin and cos term. hence we can get two solution when we solve the two equation as given before.

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

→ Some straighty must be develope to select the appropiate configration.

→ Eg:- in the PUMA robot control languge VAL there is a set of commands called ABOVE and BELOW the determine wheather the elbow is to make an angle θ_2 i.e. $\boxed{\theta_2 \lessgtr 0}$ Greater / less than

let θ_2 is +ive

$$x_p = L_1 C_1 + L_2 C_1 C_2 - L_2 S_1 S_2$$
$$y = L_1 S_1 + L_2 S_1 C_2 + L_2 C_1 S_2$$

$$x^2 + y^2 = (L_1 C_1 + L_2 C_1 C_2 - L_2 S_1 S_2)^2 + (L_1 S_1 + L_2 S_1 C_2 + L_2 C_1 S_2)^2$$

$$x^2+y^2 = (L_1 C_1)^2 + (L_2 C_1 C_2 - L_2 S_1 S_2)^2 + 2(L_1 C_1)(L_2 C_1 C_2 - L_2 S_1 S_2)$$
$$+ (L_1 S_1)^2 + (L_2 S_1 C_2 + L_2 C_1 S_2)^2 + 2(L_1 S_1)(L_2 S_1 C_2 + L_2 C_1 S_2)$$

$$x^2+y^2 = L_1^2 C_1^2 + L_2^2 C_1^2 C_2^2 + L_2^2 S_1^2 S_2^2 + 2L_1 L_2 C_1^2 C_2 - 2L_1 C_1 S_1 S_2$$
$$+ L_1^2 S_1^2 + L_2^2 S_1^2 C_2^2 + L_2^2 C_1^2 S_2^2 + 2L_2 S_1 C_2 + L_2 C_1 S_2$$
$$+ 2L_1 S_1 L_2 S_1 C_2 + 2L_1 S_1 L_2 C_1 S_2$$

$$x^2 = L_1^2 C_1^2 + L_2^2 C_1^2 C_2^2 + L_2^2 S_1^2 S_2^2 + 2L_1 L_2 C_1^2 C_2 - 2L_2^2 \cancel{C_1} C_2 S_1 S_2 - \cancel{2L_1} L_2 S_1 S_2$$
$$y^2 = L_1^2 S_1^2 + L_2^2 S_1^2 C_2^2 + L_2^2 C_1^2 S_2^2 + 2L_1 L_2 S_1^2 C_2 + 2L_1^2 \cancel{C_1} C_2 S_1 S_2 + 2L_1 L_2 C_1 S_1 S_2$$

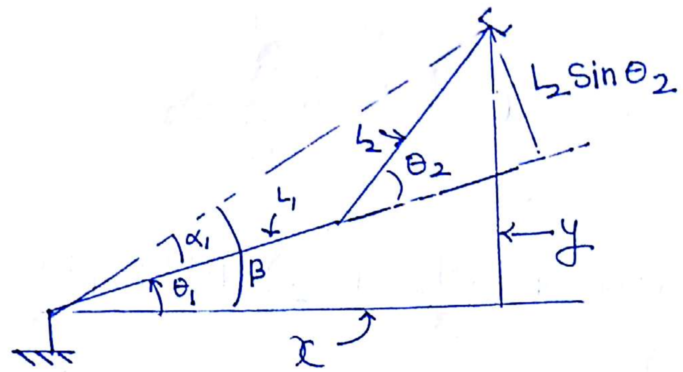
$$x^2+y^2 = L_1^2 + L_2^2 C_2^2 + L_2 S_2^2 + 2L_1 L_2 C_2$$

$$x^2+y^2 = L_1^2 + L_2^2 + 2L_1L_2C_2$$

$$\Downarrow$$

$$\boxed{\cos\theta_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}}$$

θ_2 is known



$$\tan\beta = \frac{y}{x} \qquad \tan\alpha = \frac{L_2\sin\theta_2}{L_1+L_2\cos\theta_2}$$

$$\boxed{\theta_1 = \beta - \alpha}$$

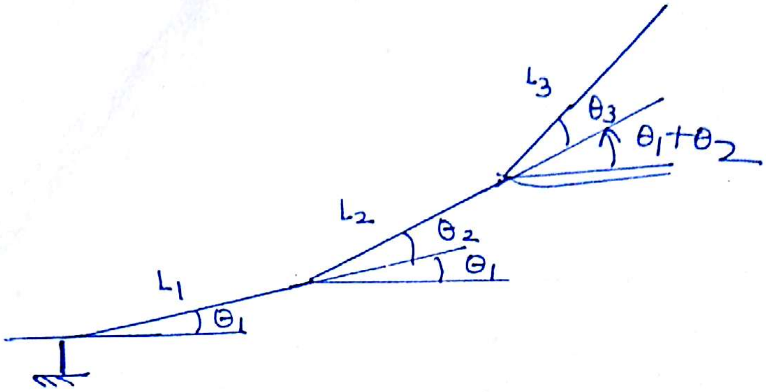
$$\tan\theta_1 = \frac{\tan\beta - \tan\alpha}{1+\tan\beta\tan\alpha}$$

$$\tan\theta_1 = \frac{\frac{y}{x} - \frac{L_2S_2}{L_1+L_2C_2}}{1+\frac{y}{x}\,\frac{L_2S_2}{L_1+L_2C_2}}$$

☆

$$\boxed{\tan\theta_1 = \frac{y(L_1+L_2\cos\theta_2)-xL_2S_2}{x(L_1+L_2\cos\theta_2)+yL_2S_2}}$$

Direct & Indirect kinematic of 3 DoF Arm



$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) + L_3 \cos(\theta_1+\theta_2+\theta_3)$$

$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2) + L_3 \sin(\theta_1+\theta_2+\theta_3)$$

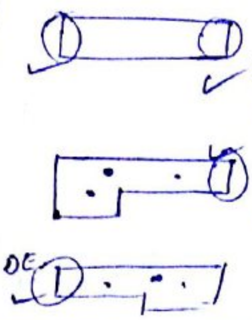
Imp.

4- Axis Scara Robotics

DH Notation (Denavit - Hartenberg Notation) 1 link का 4 parameters होगा

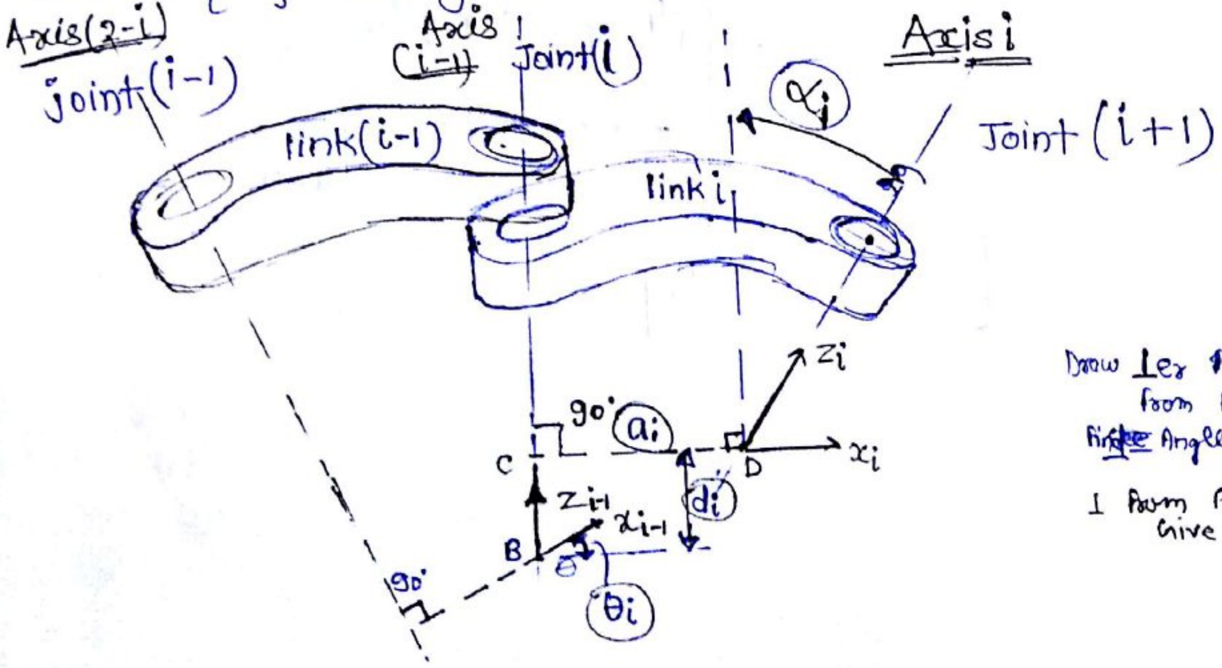
→ The defination of a manipulator with four joint link parameters for each link and a systematic procedure for assigning right handed orthonormal Cordinate frames (x, y, z direction) one to each link in an open kinematic chain is known as DH notation.

→ A Frame {i} is rigidly attached to distal end of link i and it moves with link i. An n-DOF manipulator will have (n+1) Frames with the Frame{0} or base frame acting as the reference inertial Frame and Frame {n} being the tool Frame.



prismatic both are distal end

CG से दूर वाला distal end



Draw ⊥er from Axis(i-1)(C) from Axis(i) D

Find Angle α_i

⊥ from Axis(2-i) Give axis (x_{i-1})

(a) link length a_i :- Distance measured along x_i axis from the point of intersection of x_i axis with (z_{i-1}) axis (point C) to the origion of frame i, i.e. distance CD

(b) Link twist (α_i): Angle between Z_{i-1} and Z_i axises measured about x_i axis in the right hand sence

(c) Joint Distance (d_i):- Distance measured along Z_{i-1} axis from the origion of frame {i-1} (point B) to the intersection of x_i axis with the Z_{i-1} axis point C ~~at~~ i.e. distance BC.

(d) Joint Angle (θ_i) :- Angle between x_{i-1} and x_i axis measured about Z_{i-1} axis in the right hand sense.

Scara Robot kinematics :-

- scara has four(4) DOF.

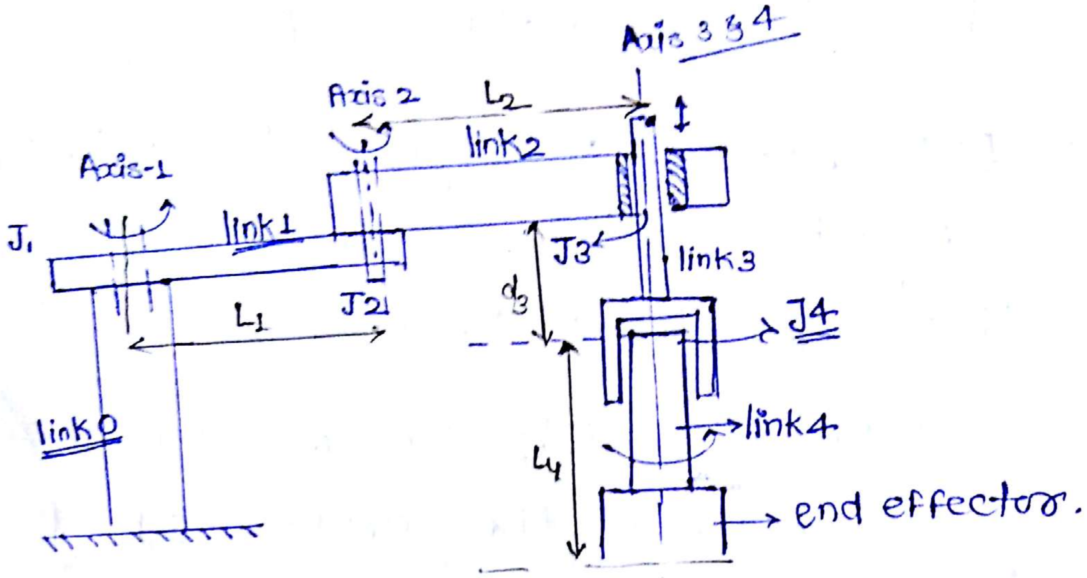


Fig. 4 - DOF scara robot

line diagram :-

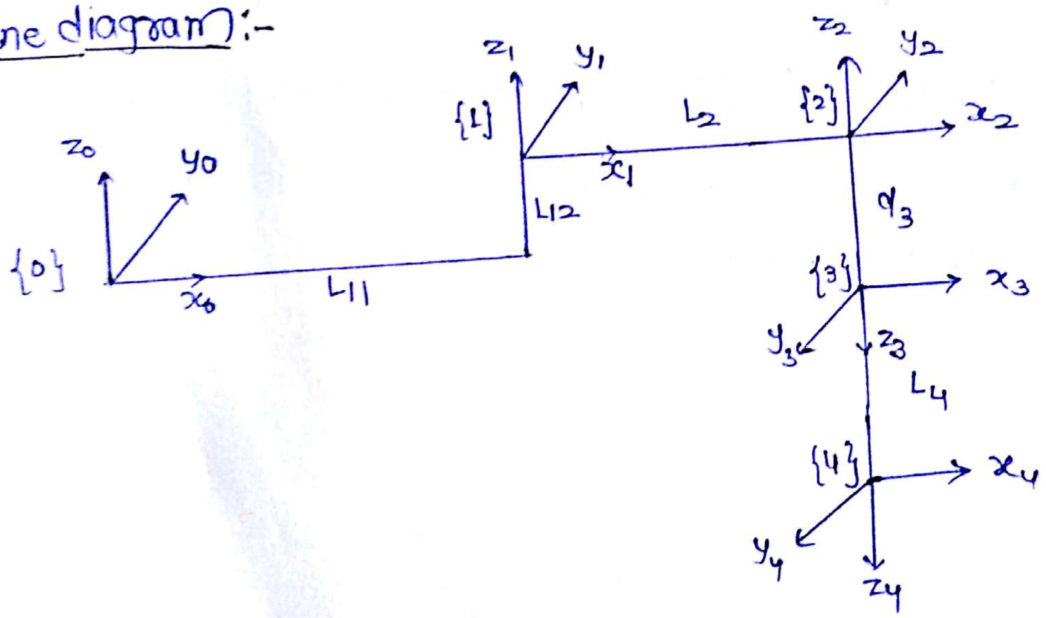


Fig. 4-Axis Scara robot

DH parameter

Parameter (Home position से क्या change आएगा)

link i	a_i	α_i	d_i	θ_i	q_i	$\cos\theta_i$	$\sin\theta_i$	$\cos\alpha_i$	$\sin\alpha_i$
1	L_{11}	0	L_{12}	θ_1	θ_1	C_1	S_1	1	0
2	L_2	0	0	θ_2	θ_2	C_2	S_2	1	0
3	0	180	d_3	0	d_3	1	0	-1	0
4	0	0	L_4	θ_4	θ_4	C_4	S_4	1	0

Now make HTM (Homogeneous transpose matrix)

$$^0T_1(\theta_1) = \begin{bmatrix} C_1 & -S_1 & 0 & L_{11}C_1 \\ S_1 & C_1 & 0 & L_{11}S_1 \\ 0 & 0 & 1 & L_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2(\theta_2) = \begin{bmatrix} C_2 & -S_2 & 0 & L_2C_2 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^2T_3(d_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\theta_3 = 180$ होने से unit matrix change होगा $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$$^3T_4(\theta_4) = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^{0}T_4 = {}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_3 \times {}^{3}T_4 $$

$$ {}^{0}T_1 = \left[\begin{array}{ccc|c} +C_{124} & +S_{124} & 0 & L_2 C_{12} + L_{11} C_1 \\ +S_{124} & -C_{124} & 0 & L_2 S_{12} + L_{11} S_1 \\ 0 & 0 & -1 & L_{12} + d_3 - L_4 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

$$ \left\{ \begin{array}{l} S_{124} = \sin(\theta_1 + \theta_2 - \theta_4) \\ C_{124} = \cos(\theta_1 + \theta_2 - \theta_4) \end{array} \right\} $$