CHAPTER - 5

"Bending and shear stresses in Beams"

Bending stress in beams :-

Pure bending (B.M. = Constant., A.F. = S.F. = T.M. = 0

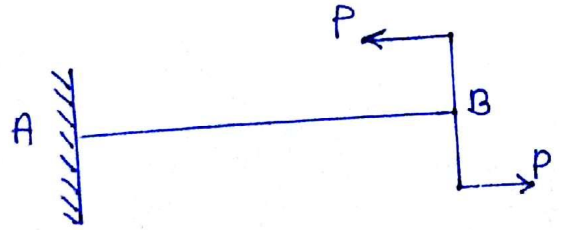
Bending eqⁿs :-

M_R ⎫
M ⎭ P.T.O.

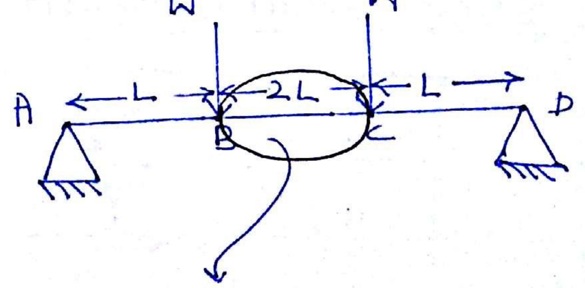
$$\boxed{\frac{M_R}{I_{N.A.}} = \frac{\sigma_b}{Y} = \frac{E}{R}}$$

Pure bending :- A beam or a member is said to be pure bending when it is loaded by two equal and opposite couples in a plane $\perp^r$ to the cross-section of beam (i.e. bending couples) in such a way that the mag. & dirⁿ of bending moment remains Const. through the length of beam.
(i.e. BC = Const., A.F. = S.F. = T.M. = 0)

* Every beam is not under pure bending but a part of beam may be pure bending



Total beam Under pure bending



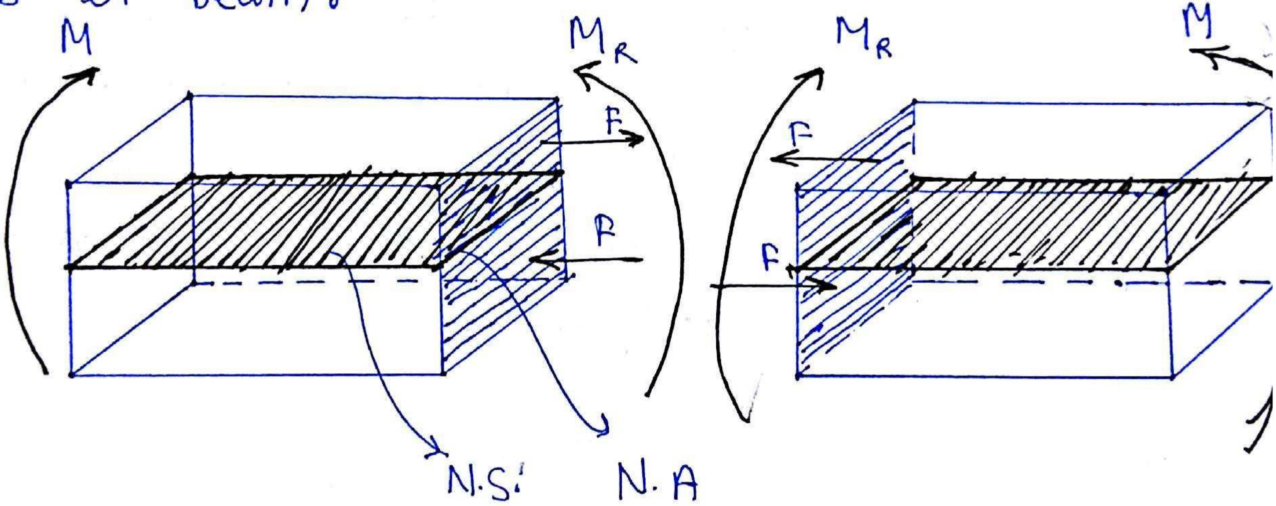
Only BC pure bending.

M_R - moment of resisting or resisting bending couple on X-S/c

M - Bending moment acting on X-S/c of beam

Safe condition for bending is $\boxed{M \leq M_R}$

Resisting bending couple (M_R) is form due to two equal parrell opposite resisting forces develope above ~~the~~ and ~~ab~~ below the neutral axis of beam.



Analysis of Bending Equation:-

$$\boxed{\underbrace{\frac{M_R}{I_{N.A.}}}_{A} = \underbrace{\frac{\sigma_b}{Y}}_{B} = \underbrace{\frac{E}{R}}_{C}} \qquad M \leq M_R$$

Case-1 $\quad A = B$

$$\frac{M_R}{I_{N.A.}} = \frac{\sigma_b}{Y}$$

$$\boxed{\sigma_b = \frac{M_R Y}{I_{N.A}} = \frac{M Y}{I_{N.A.}}} \quad -(1)$$

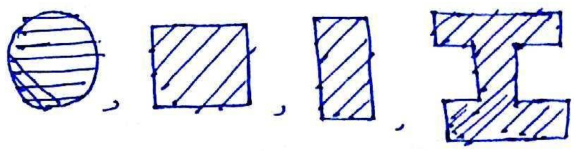
eqn(1) used to determine bending stress (σ_b) developed at a fiber on the x-s/c of a beam when bending moment of that x-s/c is known.

from eqn(1)

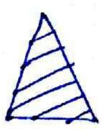
$$\sigma_b \propto Y \quad \left(\because \frac{M}{I_{N.A.}} = \text{constant}\right)$$

$$\boxed{\frac{(\sigma_b)_{Top}}{(\sigma_b)_{BOT}} = \frac{(Y)_{Top}}{(Y)_{BOT.}}} \quad -(2)$$

$$\frac{(\sigma_b)_{Top}}{(\sigma_b)_{BOT.}} = -1$$ for Circular, square, rectange x-s/c



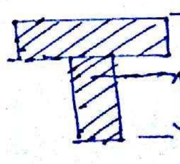
$$\frac{(\sigma_b)_{Top}}{(\sigma_b)_{BOT.}} = -2$$ for, Traingle



$Y_{top} = \frac{2h}{3}$

$Y_{BOT} = \frac{h}{3}$

$$\frac{(\sigma_b)_{Top}}{(\sigma_b)_{BOT}} < -1$$ for, T-section



Y_{top}

$Y_{BOT} = Y_{max.}$

$$\boxed{(\sigma_b)_{max} = \frac{M\,Y_{max}}{I_{N.A.}} \text{ (or) } \frac{M}{Z_{N.A.}}} \quad -③$$

where $Z_{N.A.} = \dfrac{I_{N.A.}}{Y_{max}}$ in $\underline{mm^3}$

$Z_{N.A}$ = Section modulus of X-S/c about its N.A.

Y_{max} = larger of $[Y_{Top}, Y_{BOT.}]$

eqn ③ is used to det. max bending stress $(\sigma_b)_{max}$ on the X-S/c of a beam when B.M. on that X-S/c is know.

From eqn ③, For a given B.M. (M)

$$(\sigma_b)_{max} \propto \frac{1}{Z_{N.A.}}$$

$$Z_{N.A.}\uparrow \Rightarrow (\sigma_b)_{max}\downarrow \Rightarrow \text{Chance of Failure}(\downarrow)$$

Now $\dfrac{\text{eqn ③}}{\text{eqn ①}} \Rightarrow \boxed{\dfrac{(\sigma_b)_{max}}{(\sigma_b)} = \dfrac{Y_{max}}{Y}} \quad -④$

$$\sigma_b = (\sigma_b)_{max}\left[\frac{Y}{Y_{max}}\right]$$

Moment of Resistance :- (M_R)

⇒ Safe Condⁿ for bending w.r.t. strength criterion,

$$\left[(\sigma_b)_{max}\right]_{ind.} \leq \sigma_{per}$$

$$\frac{M}{Z_{N.A.}} \leq \sigma_{per}$$

$$M \leq Z_{N.A.}\,\sigma_{per}$$

$$\boxed{M \leq M_R} \qquad \because \left[Z_{N.A.} \times \sigma_{per} = M_R\right]$$

$$\boxed{M_R = Z_{N.A.}\,\sigma_{per}} \quad \text{--- (5)}$$

eqⁿ (5) is used to compare moment of resistance of given x-s/c of beams.

From eqⁿ (5)

For a given material

$$\boxed{M_R \propto Z_{N.A.}}$$

Higher $Z_{N.A}$ ↑, Higher the M_R ↑ → So load carring capacity increase

* M_R is also know as 'strength of a Beam'

* Higher the M_R, less chance of failure.

Case - II

$B = C$

$$\frac{\sigma_b}{Y} = \frac{E}{R}$$

$$\boxed{\sigma_b = \frac{EY}{R}} \quad - (6)$$

$$\boxed{(\sigma_b)_{max} = \frac{E\,Y_{max}}{R}} \quad - (7)$$

eq^n (6) & (7) are used to det. σ_b & $(\sigma_b)_{max}$ when radius of curvature of. N.S. is know
(Neutral surface).

$$\boxed{(\epsilon_{max})_{long.} = \frac{(\sigma_b)_{max.}}{E} = \frac{Y_{max.}}{R}} \quad - (8)$$

Case III

$A = C$

$$\frac{M_R}{I_{N.A.}} = \frac{E}{R}$$

$$\boxed{R = \frac{E I_{N.A.}}{M_R} = \frac{E I_{N.A.}}{M}}$$

$E I_{N.A.}$ - Flexural rigidity of X-S/c of beam about its neutral axis.

$$EI_{N.A.} = M_R R$$

$$\text{if } R = 1 \Rightarrow \boxed{EI_{N.A.} = M_R}$$

$$\text{if } EI_{N.A.}(\uparrow) \Rightarrow M_R(\uparrow)$$

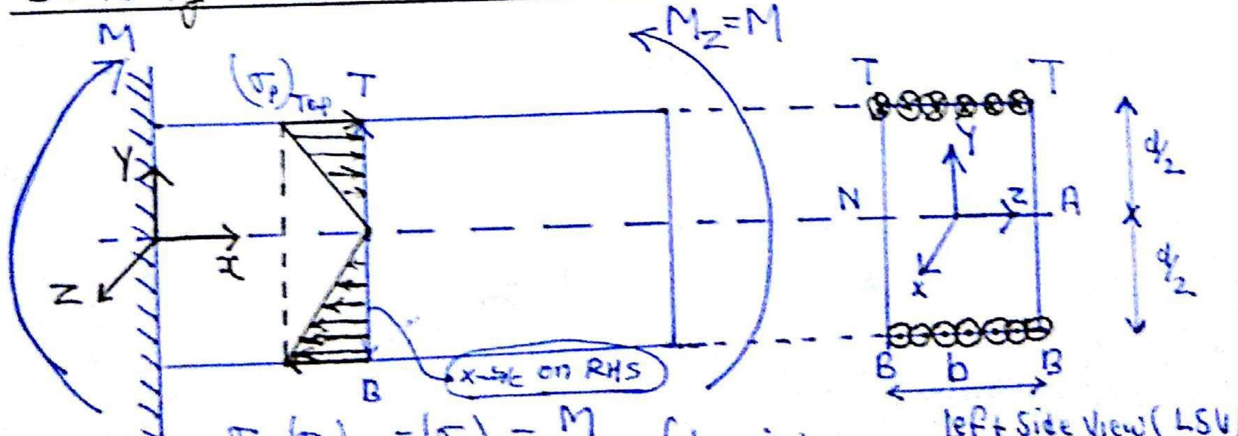
$$\Rightarrow \text{Slope \& def}^n (\downarrow)$$

$$\Rightarrow \text{chances of Failure} (\downarrow)$$

* Flexural Rigidity is moment of resistance offered by a beam for one unit Radius of curvature of neutral surface.

* Section Modulas should be considered in the design of beam based on strength creterion whereas flexural rigidity should be considered in design of beam w.r.t. rigidity creterion

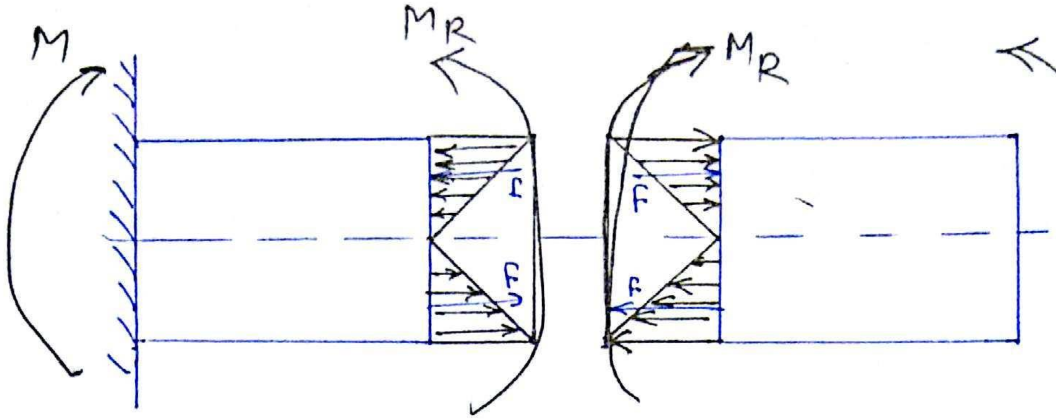
Bending stress Distribution :-



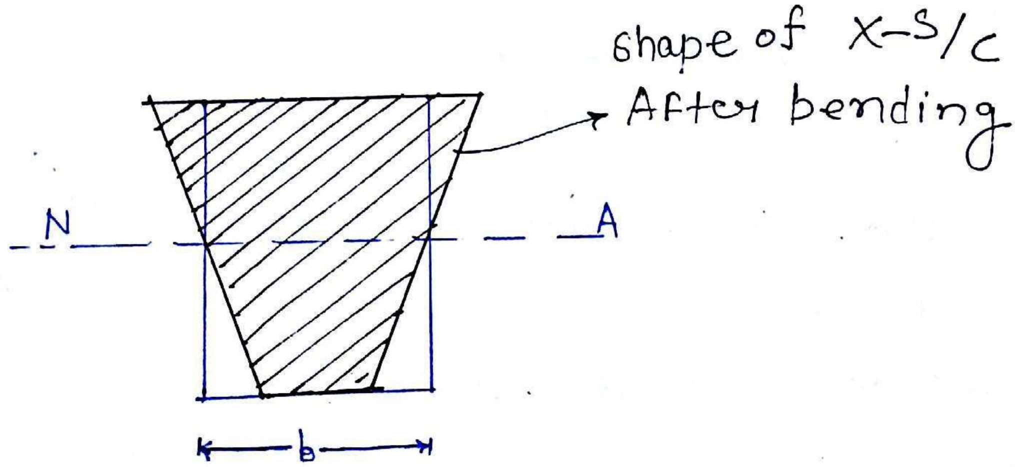
left side view (LSV)
x-s/c in RHS

$$\sigma_x = (\sigma_b)_{max} = (\sigma_b)_{Bot} = \frac{M}{Z_{N.A.}} \text{ (tensile)}$$

$$(\sigma_b)_{max} = (\sigma_b)_{Top} = \frac{M}{Z_{N.A.}} \text{ (comp.)}$$



Imp * shape of X-S/c after pure bending is to trepezoidal (theortically) but lateral strain as assumed to be very small that's why rectangle only.



strain tensor at top fiber

$$\left(\epsilon_{long}\right)_{Top} = \frac{(\sigma_b)_{Top}}{E} = -\epsilon_x$$

$$\left(\epsilon_{lateral}\right)_{Top} = -\mu\left(\epsilon_{long}\right)_{Top}$$

$$\left(\epsilon_{lateral}\right)_{Top} = \mu\,\epsilon_x$$

$$\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0$$

Under sagging bending

Strain tensor $[\epsilon]_{Top} = \begin{bmatrix} -\epsilon_x & 0 & 0 \\ 0 & \mu\epsilon_x & 0 \\ 0 & 0 & \mu\epsilon_x \end{bmatrix}$

$$(\epsilon_{bng})_{top} = \frac{(\sigma_b)_{TOP}}{E} = -\epsilon_x = \frac{-M/Z_{N.A.}}{E} = \frac{-Y_{max}}{R}$$

At bottom

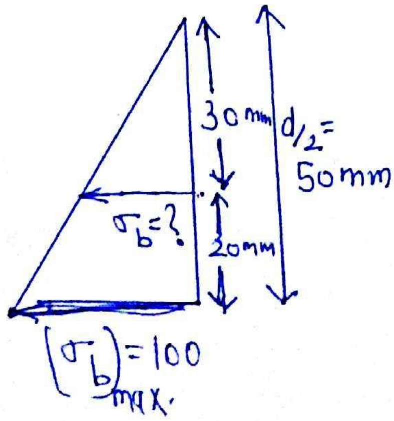
$$[\epsilon]_{BOT} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & -\mu\epsilon_x & 0 \\ 0 & 0 & -\mu\epsilon_x \end{bmatrix}.$$

$$[\sigma_{1D}] = [\sigma_x]_{1\times1}$$

$$= [-(\sigma_b)_{max}]_{1\times1}$$

Quest. $(\sigma_b)_{max} = 100$ MPa, find σ_b at a distance of 20 mm from bottom fiber if $d = 100$ mm

Sol^n



From similar Δ

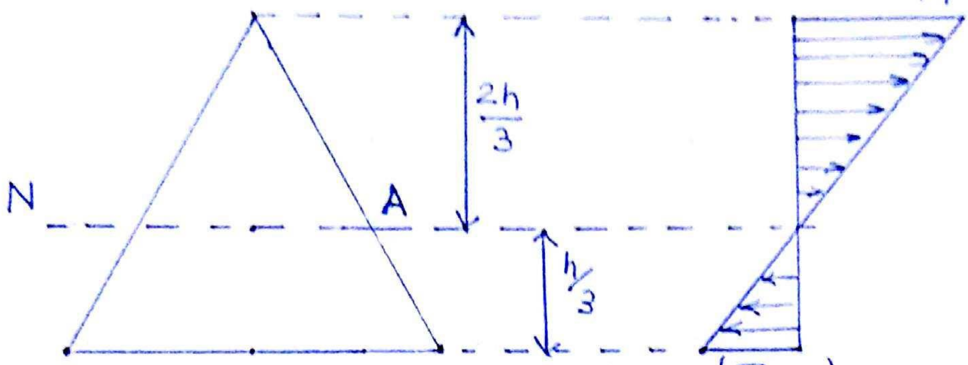
$$\frac{50}{100} = \frac{30}{\sigma_b} \Rightarrow \sigma_b = 60 \text{ MPa}$$

or $$\frac{(\sigma_b)_{max}}{(\sigma_b)} = \frac{(y)_{max}}{y}$$

$$\sigma_b = \frac{100 \times 30}{50} = 60 \text{ MPa}$$

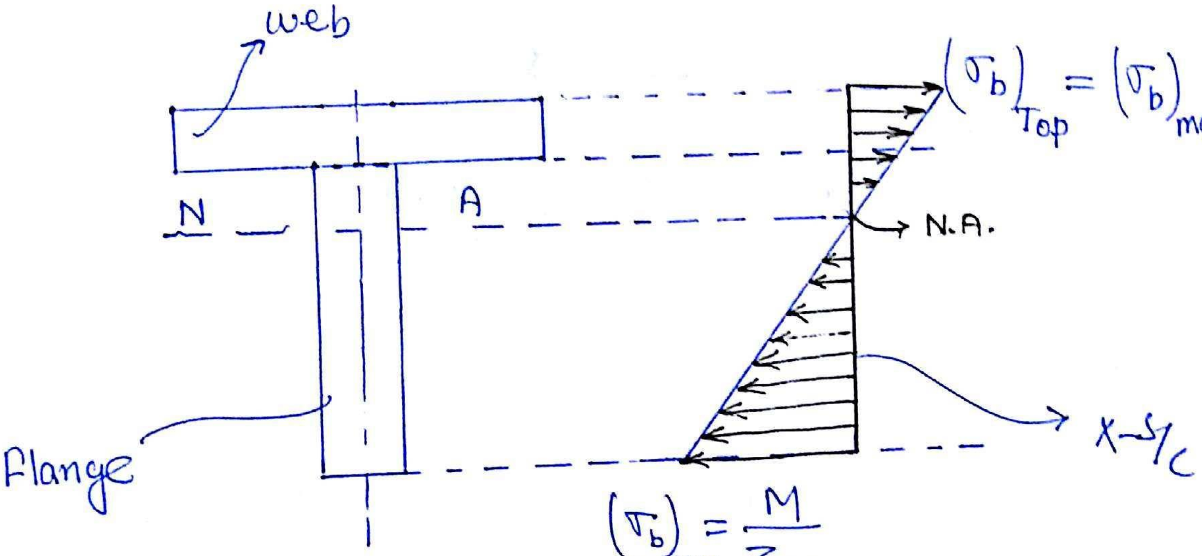
For triangular X-S/c

$$(\sigma_b)_{top} = (\sigma_b)_{max} = \frac{M}{Z_{N.A.}}$$



$$(\sigma_b)_{BOT} = \frac{(\sigma_{max})}{2}$$

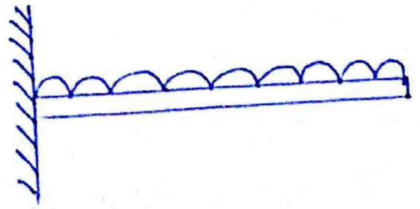
T- Section :-



$$(\sigma_b)_{Top} = (\sigma_b)_{max}\left[\frac{Y_T}{Y_B}\right]$$

$$(\sigma_b)_{max} = \frac{M}{Z_{N.A.}}$$

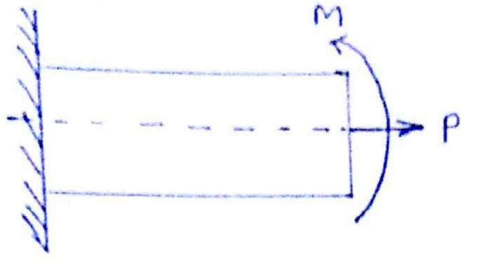
eg.



T section

* Critical point are on bottom fibre at fixed end

eg.



* Critical points are entire bottom surface (M is constant)

$(\sigma_b)_{max} = \frac{M}{Z_{N.A.}}$

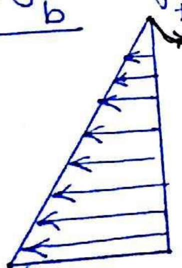
$\sigma_a = P/A$ — only due to Axial load

$(\sigma_b)_{max} = M/Z_{N.A.}$ — only due to bending

when both are acting together! —

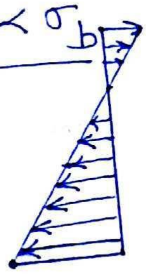
(i) $\sigma_a = \sigma_b$

$\sigma_{top} = 0$ → N.A.



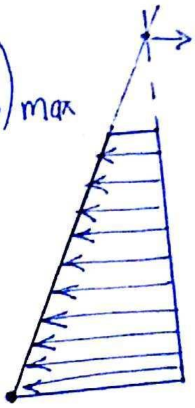
$(\sigma_b)_{BOT} = \sigma_a + (\sigma_b)_{max}$

(ii) $\sigma_a < \sigma_b$



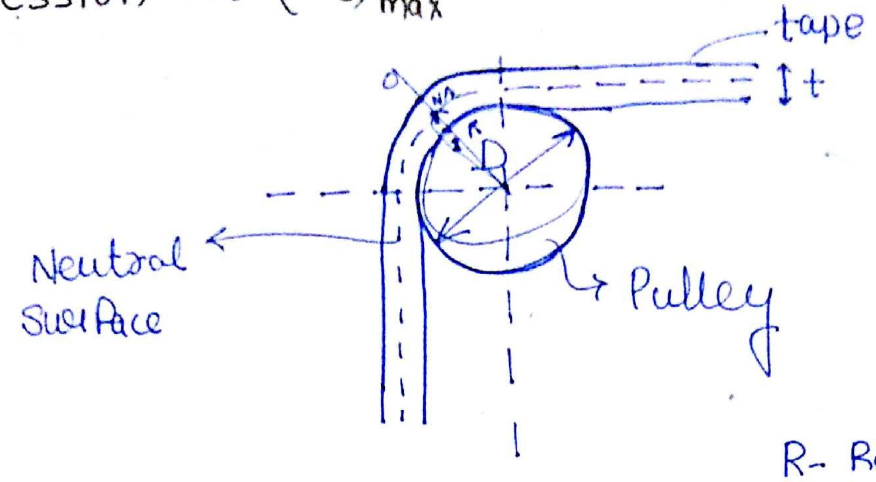
(iii) $\sigma_a > (\sigma_b)_{max}$

→ N.A.

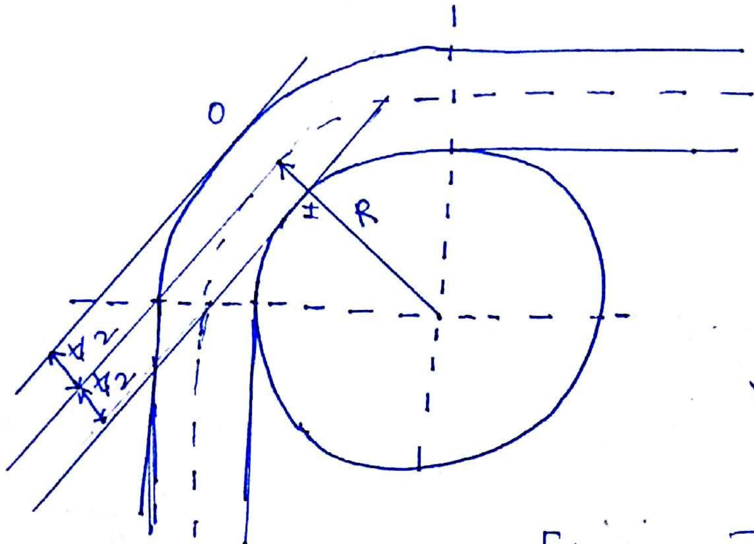


In presence of EAL neutral axis will never coinside with the centroidal axis, Neutral axis may coincides with any one of extreme fiber or any inner fibres lies above of below C.A. or lies outside the plane of X-S/C

Expression for $(\sigma_b)_{max}$ in a flat belt.



R- Radius of curvature of Neutral Surface.



$$R = \frac{D}{2} + \frac{t}{2}$$

$$Y_{max} = \frac{t}{2}$$

$$\left[(\sigma_b)_{max}\right] = \pm\left[\frac{E\,Y_{max}}{R}\right]$$

$$(\sigma_b)_{max} = \pm\frac{E\,t/2}{\frac{D+t}{2}}$$

$$\boxed{\left[(\sigma_b)_{max}\right]_{O\&I} = \pm\frac{E\,t}{D+t} \simeq \frac{E\,t}{D}}$$

lower E

Small t

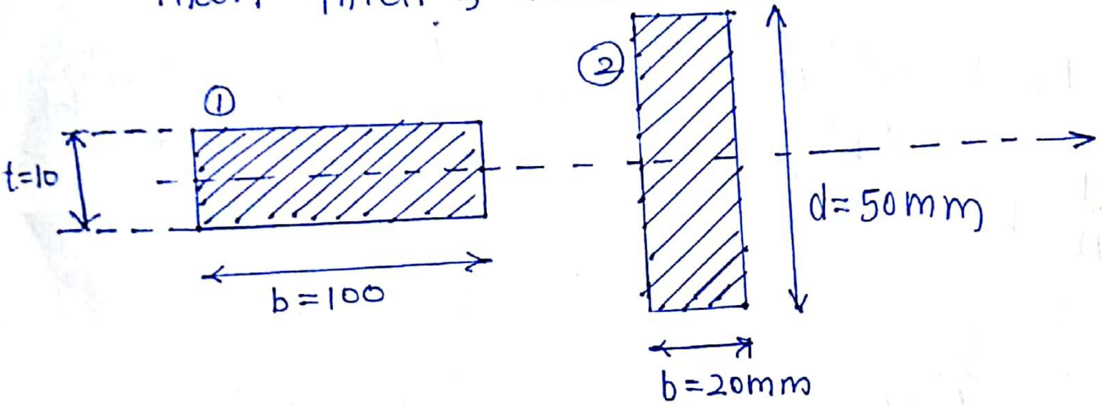
bigger pulley D

Flat belt

$$\boxed{(\sigma_b)_{max} = \frac{Et}{D_{smaller}}} \le (\sigma_{per})_{belt}$$

$$\boxed{D_{smaller} \ge \frac{Et}{\sigma_{per}}}$$

* Thin & wider belts are prefered for power transmission than thick & narrow belt.



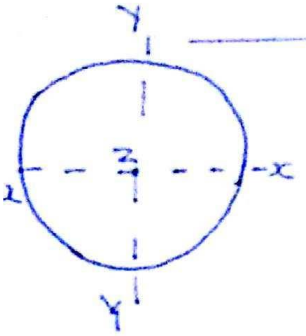
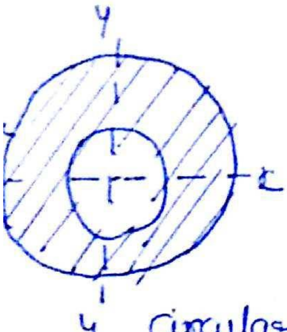
Area & material are same in both.

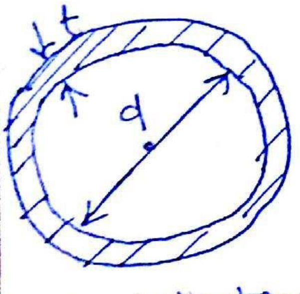
$$\frac{(M_R)_{II}}{(M_R)_I} = \frac{(Z\,\sigma_{per})_{II}}{(Z\,\sigma_{per})_I} = \frac{Z_{II}}{Z_I} = \frac{\frac{1}{6}\times 20\times (50)^2}{\frac{1}{6}\times 100\times (10)^2} = 5$$

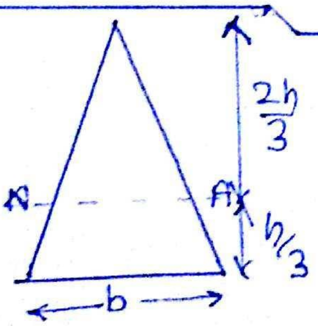
② → best x-s/c for beam because higher moment of resistance (thick & narrow)

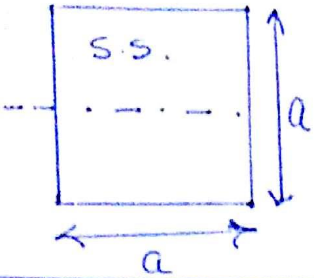
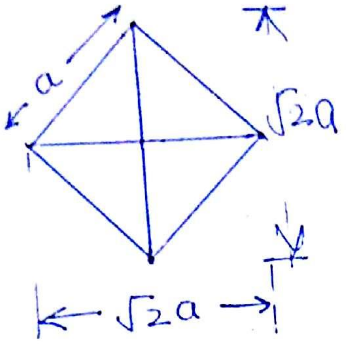
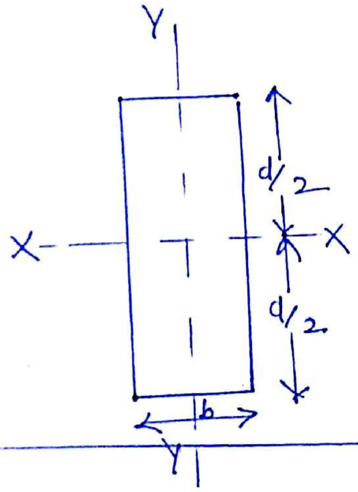
① → best x-s/c for belts because lower M_R (thin & wider)

Geometrical Properties :-

Plane of x-s/c	Area	M.O.I.	$Z_{N.A.} = \frac{I_{N.A.}}{Y_{max}}$	Polar M.O.I. (J)	$Z_p = \frac{J}{R \text{ or } R_o}$
	$\frac{\pi d^2}{4}$	$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$	$\frac{\pi d^3}{32}$	$\frac{\pi d^4}{32}$	$\frac{\pi d^3}{16}$
 Thick circular x-s/c (i.e) $d/t < 20$	$\frac{\pi D^2}{4}(1-k^2)$ $k = \frac{d}{D} < 1$	$I_{xx} = I_{yy} = \frac{\pi D^4}{64}(1-k^4)$	$\frac{\pi D^3}{32}(1-k^4)$	$\frac{\pi D^4}{32}(1-k^4)$	$\frac{\pi D^3}{16}(1-k^4)$

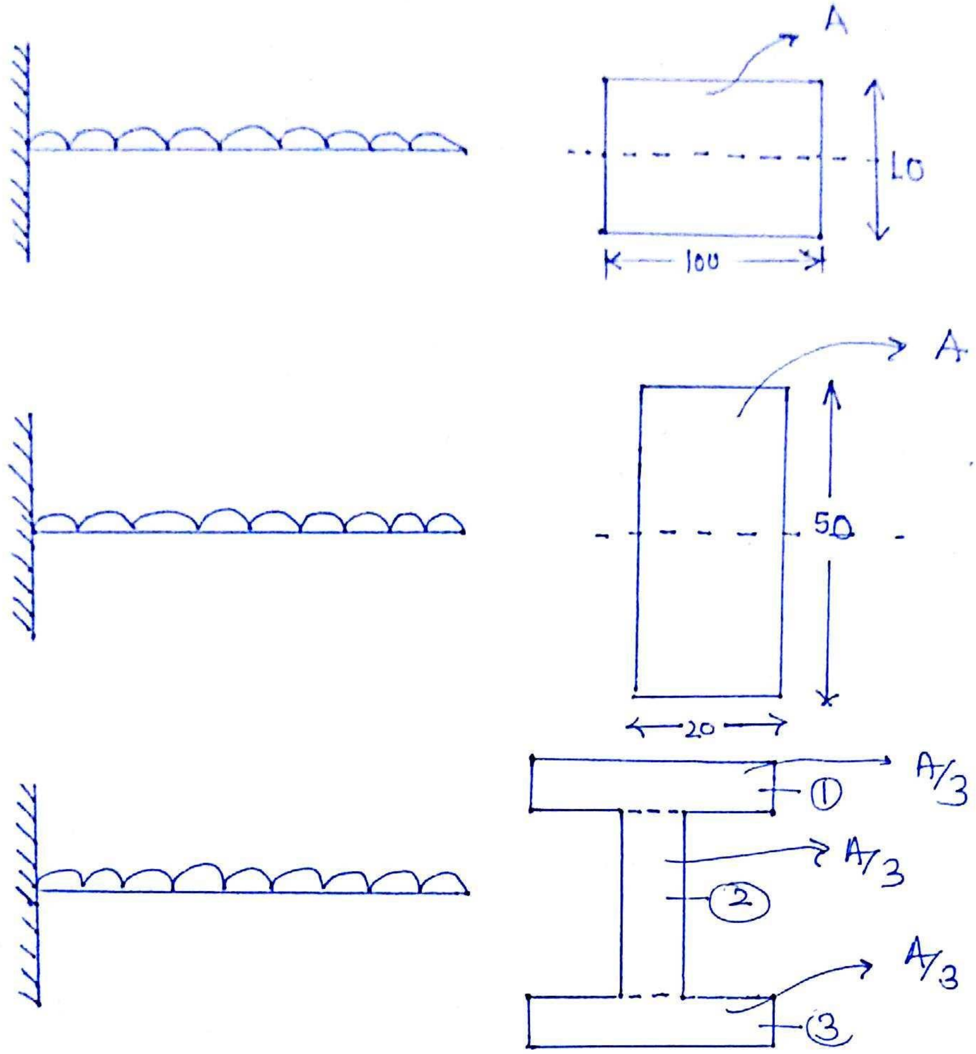
	Area	MOI	$Z_{N.A.}$	J (Polar MOI)	$Z_p = \frac{J}{R}$
 Thin circular x-s/c	$\pi d t$ $d/t > 20$	$\left(\frac{\pi d^3}{8}\right) t$	$\frac{\pi d^2}{4} t$ $\because Y_{max} = \frac{d}{2} + t \simeq \frac{d}{2}$	$\frac{\pi d^3}{4} t$	$\frac{\pi d^2}{2} t$ $\because R_o = \frac{d}{2} + t \simeq \frac{d}{2}$

X-S/c	Area	MOI ($I_{N.A.}$)	$Z_{N.A.} = \frac{I_{N.A.}}{Y_{max}}$
	$\frac{bh}{2}$	$I_{N.A} = \frac{bh^3}{32}$ $I_{base} = \frac{bh^3}{12}$	$Y_{max} = 2h/3$ $Z_{N.A} = \frac{bh^2}{24}$

	a^2	$I_{xx} = I_{yy} = \frac{a^4}{12}$ $I_{N.A} = \frac{a^4}{4}$	$Z_{N.A.} = \frac{a^3}{6}$
	a^2	$I_{xx} = I_{yy} = \frac{a^4}{12}$	$Z_{N.A.} = \frac{a^3}{6\sqrt{2}}$
	bd	$I_{xx} = bd^3/12$ $I_{yy} = db^3/12$	$Z_{x-x} = Z_{N.A} = \frac{bd^2}{6}$ $Z_{yy} = \frac{db^2}{6}$

* A Square cross section with side are Vertical & Horizontal (SS) is 41.4 % stronger the a Square X-S/c with diagonal are vertical & H.Z (S.D) under bending. w.r.t. strength criterion.

because $(Z)_{S.S} = \sqrt{2}\,(Z)_{S.D}$.



$$A_I = A_{II} = A_{III} = 1000 \text{ mm}^2$$

$$(M_{max})_I = (M_{max})_{II} = (M_{max})_{III} = \frac{wL^2}{2}$$

$$Z_{III} > Z_{II} > Z_I$$

$$\left[(\sigma_b)_{max}\right]_{III} < \left[(\sigma_b)_{max}\right]_{II} < \left[(\sigma_b)_{max}\right]_I$$

$\because$ for a given B.M.

$$(\sigma_b)_{max} \propto \frac{1}{Z}$$

$$(\sigma_{per})_I = (\sigma_{per})_{II} = (\sigma_{per})_{III}$$

$$(M_R)_{III} > (M_R)_{II} > (M_R)_I$$

$\because$ for given material

$M_R \propto Z_{N.A}$

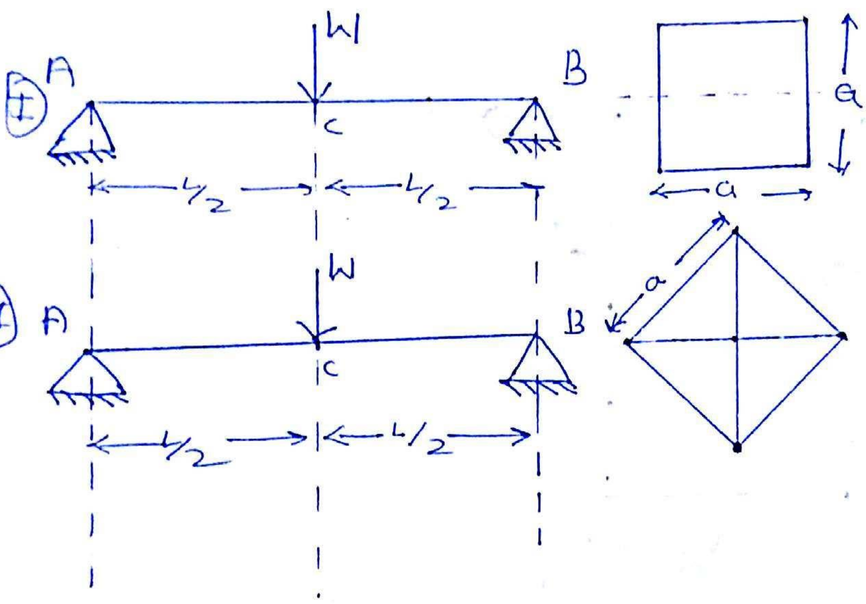
* Best x-s/c is a x-s/c which having it's most weight part away from the N.A. because for a given material $M_R \propto Z_{N.A.}$

⇒ 'I' section is best as compare to others

Quest. For two SSB as shown in fig determine the following (i) Ratio of max. bending stress

(ii) Ratio of max. deflection

(iii) Ratio of moment of resistance (M_R)



(a) $$\frac{[(\sigma_b)_{max}]_I}{[(\sigma_b)_{max}]_{II}} = \frac{Z_{II}}{Z_I} = \frac{Z_{S.D.}}{Z_{S.S}} = \frac{1}{\sqrt{2}}$$ $\left[\because (M_{max})_1 = (M_{max})_2 = \frac{WL}{4}\right.$

$\sigma_b \propto \frac{1}{Z}$ for a BM

(b) $$\frac{(Y_{max})_I}{(Y_{max})_{II}} = \frac{\left(\frac{WL^3}{48EI}\right)_I}{\left(\frac{WL^3}{48EI}\right)_{II}} = \frac{I_{II}}{I_I} = \frac{I_{S.D.}}{I_{SS.}} = 1$$

$\left[I_{S.D.} = I_{S.S} = \frac{a^4}{12}\right]$

(c) $$\frac{(M_R)_I}{(M_R)_{II}} = \frac{(Z.\sigma_{per})_I}{(Z.\sigma_{per})_{II}} = \frac{Z_1}{Z_2} = \frac{Z_{S.S}}{Z_{S.D.}} = \sqrt{2}$$

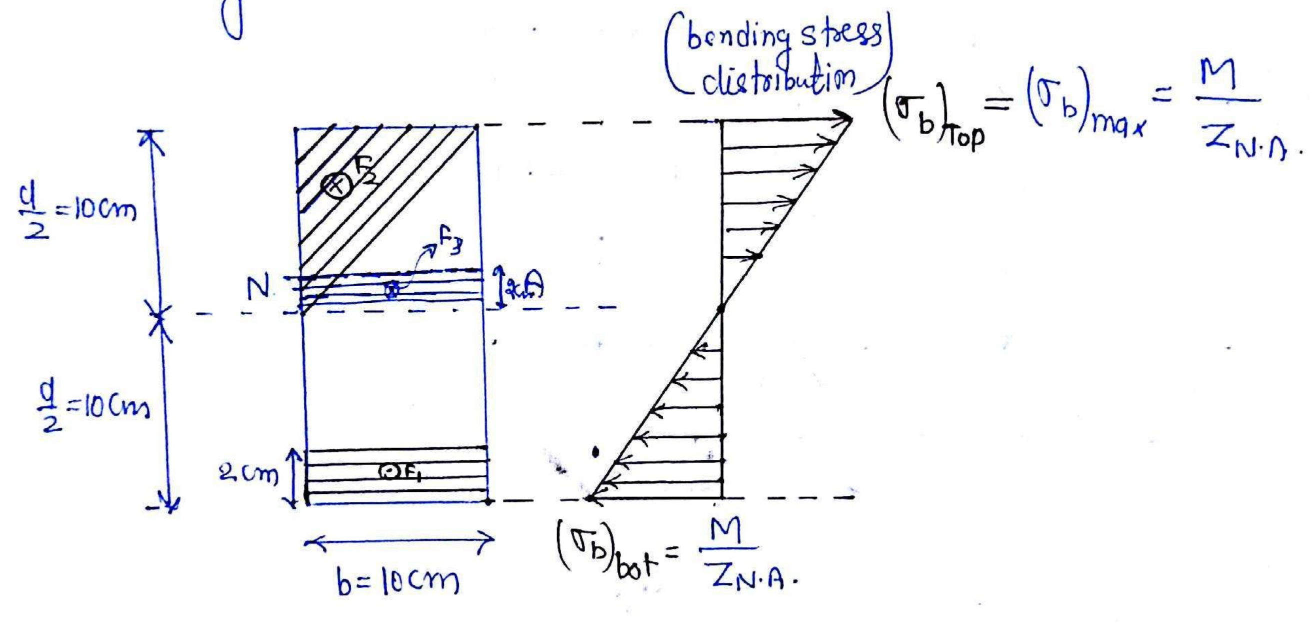
Question X-S/C of a beam under sagging bending as shown in fig determine

ⓐ max σ_b developed on X-S/C if Bending moment on X-S/C is 20 KN m

ⓑ bending stress develope at a fiber located at a distance of 2 cm from the bottom fiber

ⓒ Resisting tensile force develope on the rectangular hatched area as shown in fig

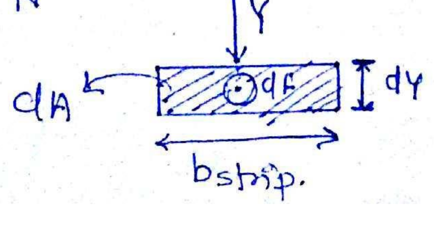
(d) Resisting comp. force developed on the triangular hatched area as shown



(a) $(\sigma_b)_{max} = \frac{M}{Z_{N.A.}} = \frac{20 \times 10^3 \times 10^3}{\frac{1}{6} \times 100 \times (200)^2} = 30 \, MPa$

(b) $\sigma_b = (\sigma_b)_{max} \left[\frac{Y}{Y_{max}} \right] = 30 \left[\frac{8}{10} \right] = 24 \, MPa$

(c) F_1 = IRTF developed on rect. hatched are (Internal Resisting tensile force)



$$dF = \sigma_b (dA)$$

$$\int dF = \int \frac{MY}{I_{N.A.}} \left[b_{strip} \, dY \right]$$

$$dF = \int_{Y_1}^{Y_2} \left(\frac{MY}{I_{N.A.}}\right) (b_{strip}) (dY)$$

$$\boxed{F = \frac{M}{I_{N.A.}} \int_{Y_1}^{Y_2} Y (b_{strip}) \, dY}$$

$$F_1 = \left[\frac{20 \times 10^6}{\frac{100 \times (200)^3}{12}} \right] \int_{80}^{100} (100) \, Y \, dY$$

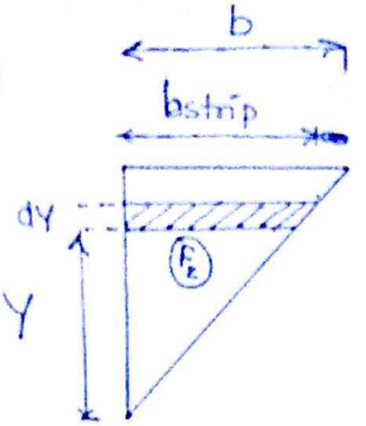
$$F_1 = \frac{24 \times 10^7}{8 \times 10^8} \times 100 \left[\frac{Y^2}{2} \right]_8^{10}$$

$$F_1 = 54 \text{ kN}$$

Second method (Valid when width is constant on given hatched area)

$$F_1 = \sigma_{avg}(A) = \left(\frac{24+30}{2}\right)(100 \times 20) = 54 \text{ kN}$$

(d)



$$\frac{b}{(d/2)} = \frac{(b)_{strip}}{Y}$$

$$(b_{strip}) = \frac{2bY}{d} = \frac{2 \times 100 \times Y}{200}$$

$$b_{strip} = Y$$

$$F_2 = \frac{M}{I_{N.A.}} \int_{Y_1}^{Y_2} (Y)(Y)\, dY$$

$$F_2 = \frac{20 \times 10^6}{\frac{100 \times (200)^3}{12}} \left[\frac{Y^3}{3} \right]_0^{100} \Rightarrow$$

$$\Rightarrow \quad F_2 = 100\,KN$$

Second method not Valid

$$F_3 = \left(\frac{0+6}{2}\right)(100 \times 20) = 6\,KN$$

Area are same but

$\rightarrow F_1 = 9F_3$ beacause area of F_1 is far away from N.A. then F_3 area.

Ques. 6 A rectangular beam is to be cut from a circular log of wood of diameter D. The sides of the strongest rectangular section will be in the ratio.

workbook

Sol^n

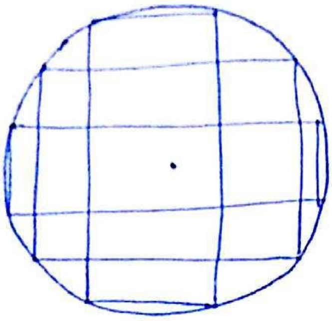
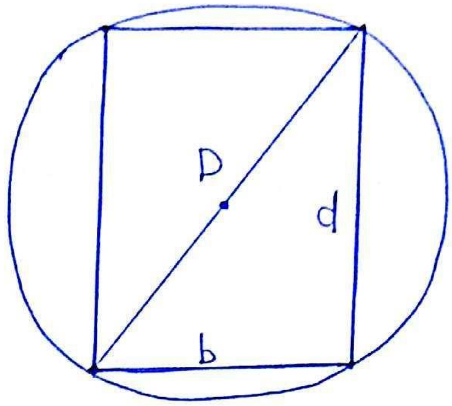


Fig. X-S/c of circular log of wood of dia (D)

Consider a rectangular X-S/c



strongest rect. X-S/c

$\because \rightarrow Z$ should be maximum

$$Z_{N.A.} = \frac{bd^2}{6} \quad -(I)$$

$$b^2 + d^2 = D^2 \Rightarrow d^2 = D^2 - b^2 \quad -(II)$$

$$Z_{N.A.} = \frac{b}{6}[D^2 - b^2] = \frac{bD^2}{6} - \frac{b^3}{6} \quad -(III)$$

if $Z_{N.A.}$ should be max. $\frac{d(Z_{N.A.})_{max}}{db} = 0$

$$\frac{d}{db}\left[\frac{bD^2}{6} - \frac{b^3}{6}\right] = 0 \Rightarrow \frac{D^2}{6} - \frac{3b^2}{6} = 0$$

$$\boxed{\frac{b}{D} = \frac{1}{\sqrt{3}}} \quad -(A)$$

From eq^n (II) $d^2 = D^2 - D^2/3$

$$\boxed{\frac{d}{D} = \sqrt{\frac{2}{3}}} \quad -(B)$$

$$\frac{(A)}{(B)} \Rightarrow \boxed{\frac{b}{d} = \frac{1}{\sqrt{2}} \Rightarrow d = \sqrt{2}\,b}$$

Beams of Uniform section! -

→ A beam is said to be a beam of uniform strength when bending stess develope at every X-S/c of the beam remains same.

eg. - Prismatic beam under pure bending

→ In presence TSL beam of non uniform strength because bending stess varying from X-S/c to X-S/c.

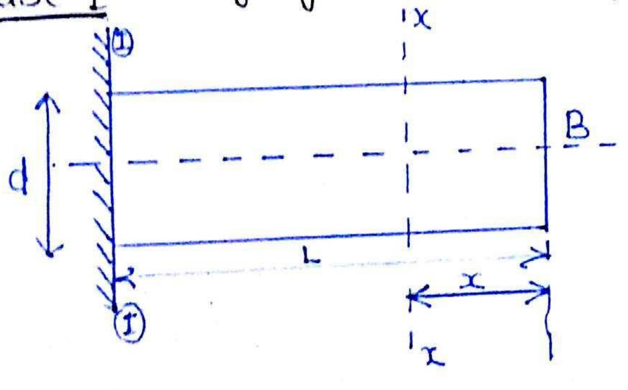
→ To obtain a beam of uniform strength in prensece of TSL non prismatic beam should be use.

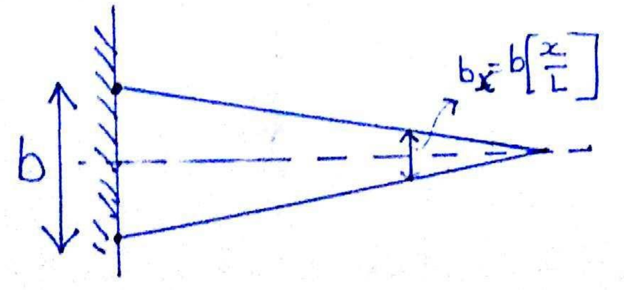
→ In presence of TSL rectangular X-S/c beam can be made as beam of uniform strength by following two methods.

(i) Varying depth & keep width constant.

(ii) Varying width & keeping depth constant.

Case-1 Varying width & keeping depths constant





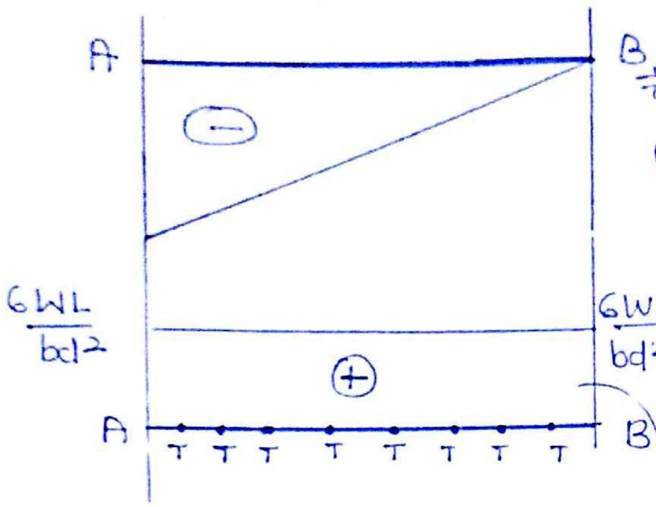
For beam uniform strength

$$(\sigma_{max})_{x-x} = \left[(\sigma_b)_{max}\right]_{I-I}$$

$$\frac{M}{(Z_{N.A.})_{x-x}} = \left(\frac{M}{Z_{N.A.}}\right)_{I-I}$$

$$\frac{Wx}{\frac{1}{6}b_x d^2} = \frac{WL}{\frac{1}{6}bd^2} \quad \because b_x = b\left[\frac{x}{L}\right]$$

$$b_x = b\left[\frac{x}{L}\right]$$



⇒ Bending moment diagram (BMD) will be a triangle.

Bending stress distribution through out X-S/c same (A ractangle)

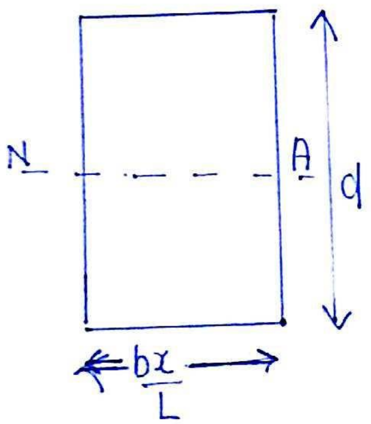
bending stress at top fiber.

Quest.

Fig:- Shape of X-S/C

Deflection of beam = ?

Find MoI at X–X = ?



$$(I)_{x-x} = \frac{1}{12} b_{x-x} (d_{x-x})^3$$

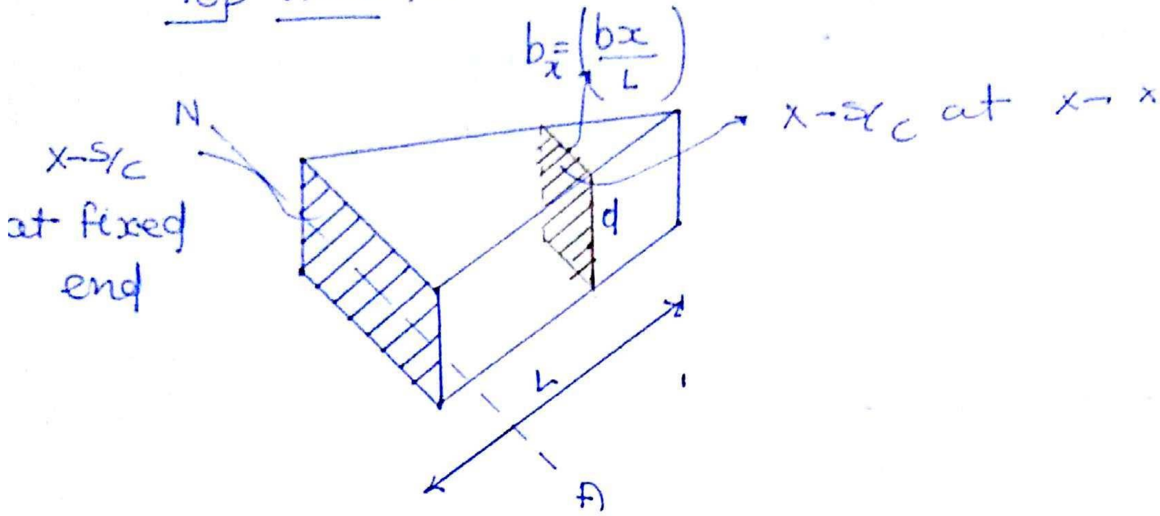
$$(I_{NA}) = \frac{1}{12}\left(\frac{bx}{L}\right) d^3$$

$$U = \int_0^L \frac{M_{x-x}^2\, dx}{2(EI_{N.A.})} = \frac{1}{2E}\int \frac{(Wx)^2\, dx}{\frac{bxd^3}{12L}}$$

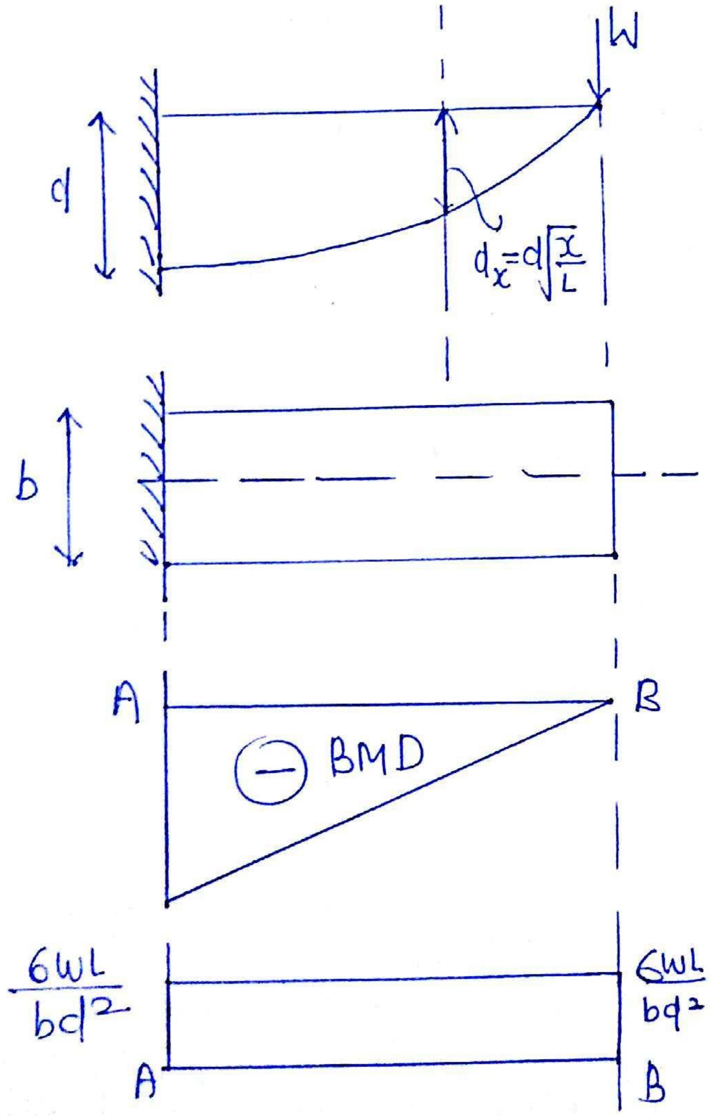
$$U = \frac{6W^2L}{Ebd^2}\int_0^x \frac{x^2}{x}\, dx = \frac{3W^2L^3}{Ebd}$$

$$y_B = \frac{du}{dw} = \frac{6WL^3}{Ebd^3}$$

Top View :-



Case-II Varying depth & keep width as Constant



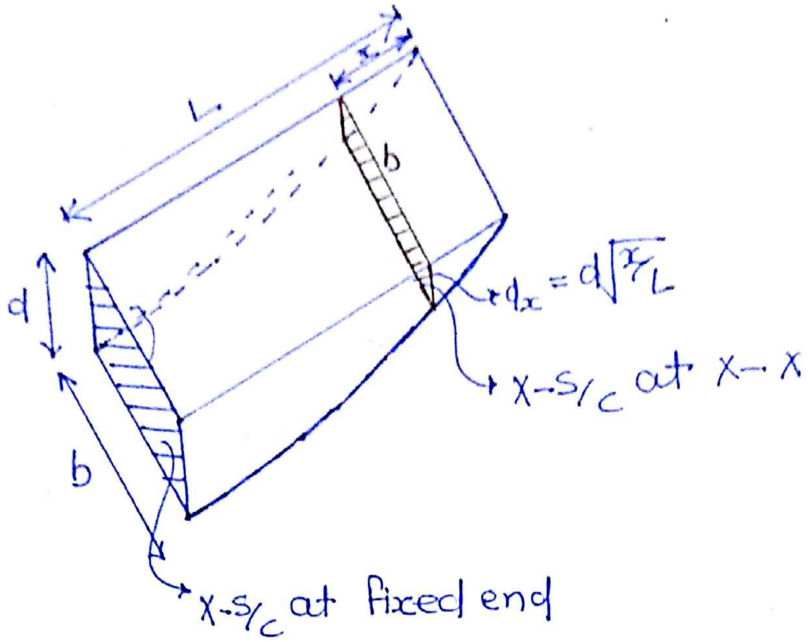
For beam uniform strength

$$\because (\sigma_{max})_{x-x} = (\sigma_{max})_{I-I}$$

$$\left(\frac{M}{Z_{NA}}\right)_{x-x} = \left(\frac{M}{Z_{N.A.}}\right)_{I-I}$$

$$\frac{Wx}{\frac{1}{6}bd_x^2} = \frac{WL}{\frac{1}{6}bd^2}$$

$$d_x = d\sqrt{x/L}$$



Safe Cond^n

$$(\sigma_{max})_{ind} \leq \sigma_{per} \text{ (or) } f$$

$$\frac{6WL}{bd^2} \leq f$$

$$\boxed{d \geq \sqrt{\frac{6WL}{bf}}}$$

For a given X-S/c Area

$$(Z)_{I} > (Z)_{T} > (Z)_{▯} > (Z)_{\square} > (Z)_{\bigcirc} > \left[(Z)_{▭} = (Z)_{\diamond}\right]$$

For a given X-S/c Area & material, $(M_R \propto Z)$ {M_R same order as Z}

$$(M_R)_{I} > (M_R)_{T} > (M_R)_{▯} > (M_R)_{\square} > (M_R)_{\bigcirc} > \left[(M_R)_{▭} = (M_R)_{\diamond}\right]$$

For given X-S/c Area & B.M. $(\sigma_b)_{max} \propto \frac{1}{Z_{N.A.}}$ {Reverse order}

$$\left[\sigma_{b\,▭} = \sigma_{b\,\diamond}\right] > (\sigma_b)_{\bigcirc} > (\sigma_b)_{\square} > (\sigma_b)_{▯} > (\sigma_b)_{T} > (\sigma_b)_{I}$$

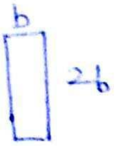
Ques Determine (a) Z (b) M_R if $\sigma_{per} = 100$ MPa

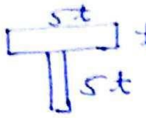
(c) $(\sigma_b)_{max}$ if M = 500 N-m

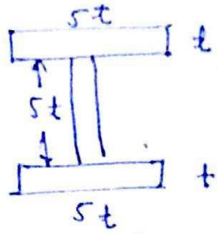
For the following X-S/c if A = 1000 mm^2

(i) S.D. (ii) 

(iii) Circular (iv) S.S.

(v) 

(vi) 

(vii) 

Shear stresses in beams :-

Let τ = shear stress developed at a fiber on the x-s/c of beam.

$$\boxed{\tau = P\left[\frac{A\bar{Y}}{I_{N.A.}\, b}\right]} \quad \text{--- (1)}$$

where P = S.F. acting on the x-s/c of beam

A = area of hatched portion which is above the fiber where τ is to be det. or below

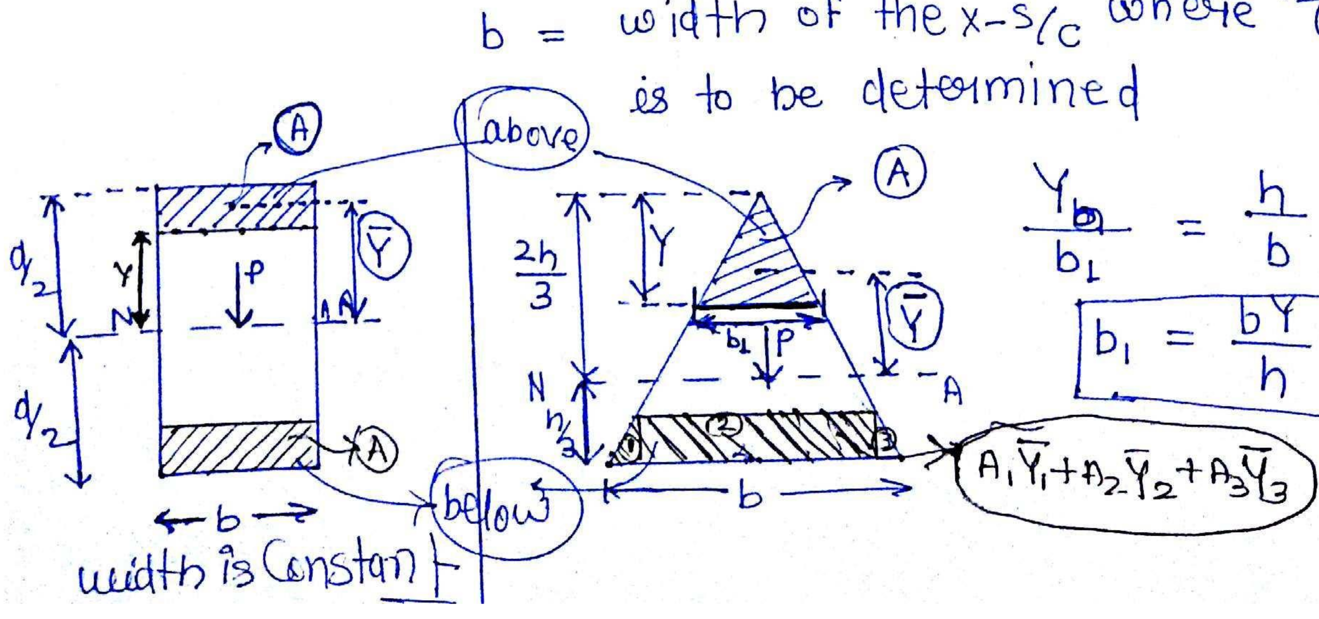
$\bar{Y}$ = distanced of centroid of hatched portion from N.A. of that x-s/c

$A\bar{Y}$ = fiber moment of area of hatched portion about N.A. of the x-s/c of beam

$I_{N.A.}$ = M.O.I. of the entire x-s/c of beam about it N.A.

= Second moment of area of entire x-s/c about its N.A.

b = width of the x-s/c where 'τ' is to be determined



$$\frac{Y}{b_1} = \frac{h}{b}$$

$$\boxed{b_1 = \frac{bY}{h}}$$

eq^n (1)

$$\tau = P\left[\frac{A\bar{Y}}{I_{N.A.}\, b}\right]$$

$$\tau \propto \frac{A\bar{Y}}{b} \qquad \left[\because \frac{P}{I_{N.A.}} = \text{constant}\right]$$

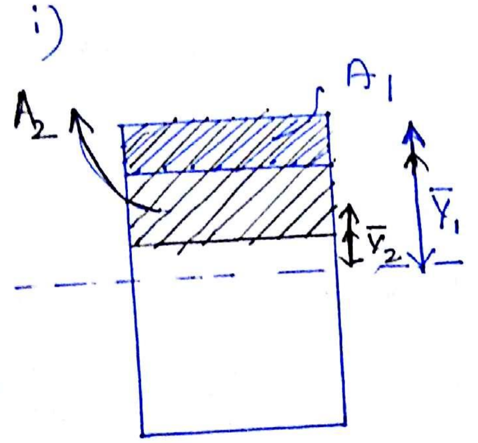
$$\tau \propto A\bar{Y} \qquad [\because \text{if } b = \text{Constant}$$

eg:- Square & Rect. X-S/C

$$\tau \propto \frac{1}{b} \qquad [A\bar{Y} = \text{Constant}]$$

if a fiber consider nearer to neutral axis

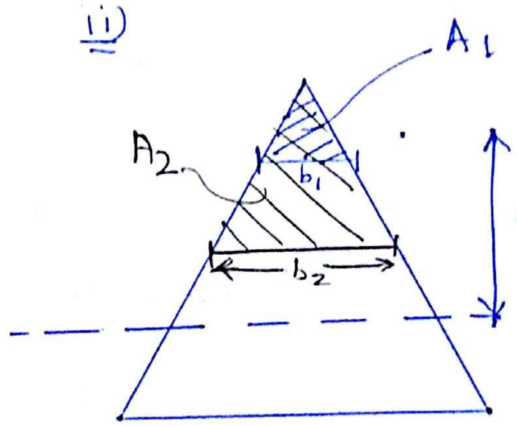
i)



$A_2 > A_1$

$\bar{Y}_2 < \bar{Y}_1$

ii)

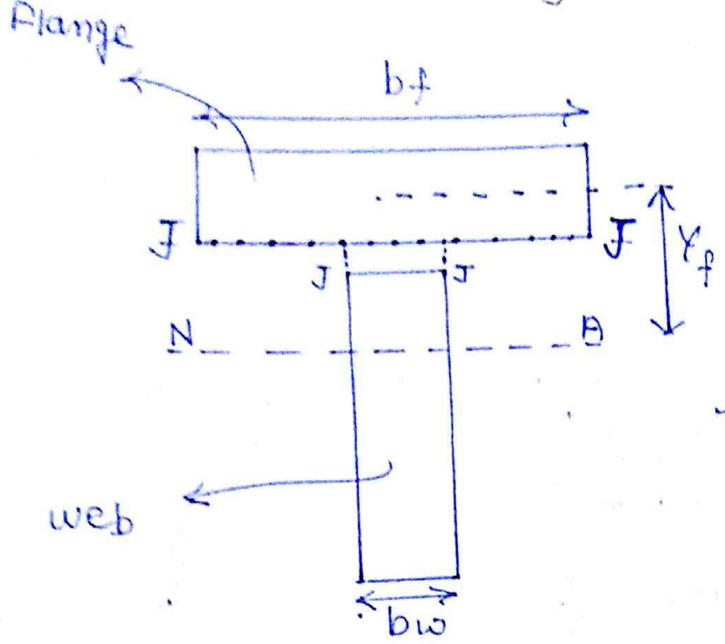


$A_2 > A_1 \Rightarrow \tau_2 > \tau_1$

$\bar{Y}_1 > \bar{Y}_2 \Rightarrow \tau_1 > \tau_2$

$\bar{b}_1 < \bar{b}_2 \Rightarrow \tau_1 > \tau_2$

Shear stress at junction of T-section



Stress at Junction Point?
(T-section)

Shear stress inversely proportional to width so Top of the web τ > ~~bottom below~~ bottom of flane.

Stress w.r.t. bottom of the flange

$$(\tau_J)_f = \frac{P}{I_{N.A.}}\left[\frac{A_f \bar{Y}_f}{b_f}\right] \quad -(1)$$

Stress w.r.t

$$(\tau_J)_w = \frac{P}{I_{N.A.}}\left[\frac{A_f Y_f}{b_w}\right] \quad -(2)$$

$$\boxed{\frac{(\tau_J)_w}{(\tau_J)_f} = \frac{b_f}{b_w}} \quad -(3)$$

Data if $(\tau_J)_f = 12\text{ MPa}$ $\quad b_f = 50$

$(\tau_J)_w = ?$ $\quad b_w = 10$

So $(\tau_j)_w = \frac{50}{10} \times 12$

$(\tau_J)_w = 60\text{ MPa}$

Generalized eq^n for τ_s developed at a fiber on the rectangle X-S/C beam :-

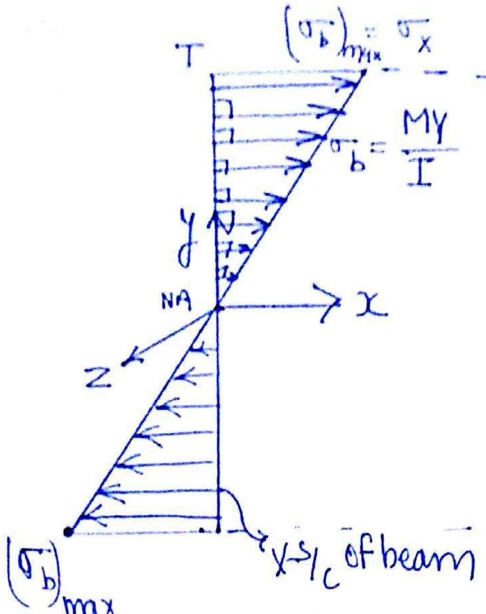


Fig :- Bending stress distribution

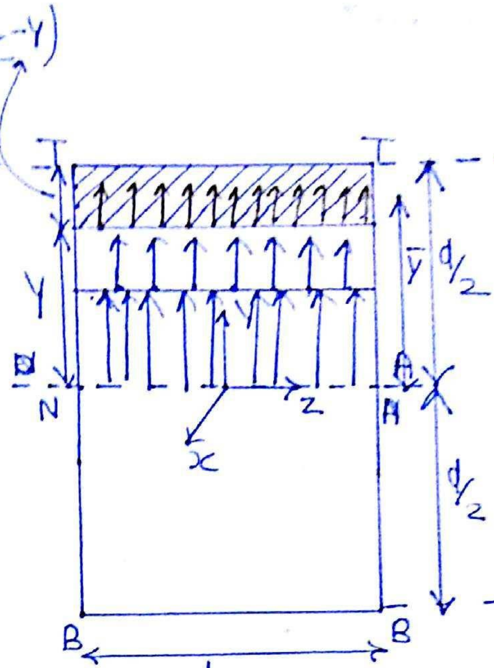


Fig X-S/C of beam.

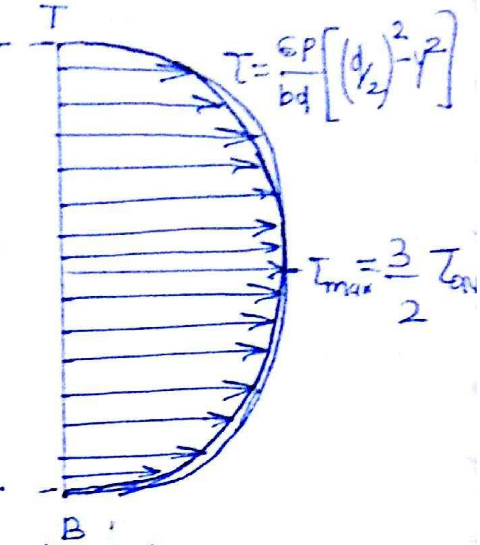


Fig (Variation of shear stress mag. on the rect. X-S/C over the depth) (only mag. No. dir^n)

$$A = b\left[\frac{d}{2} - Y\right]$$

$$\bar{Y} = Y + \frac{1}{2}\left[\frac{d}{2} - Y\right]$$

$$\bar{Y} = \frac{1}{2}\left[\frac{d}{2} + Y\right]$$

$$A\bar{Y} = \frac{b}{2}\left[\left(\frac{d}{2}\right)^2 - Y^2\right]$$

$$b = b$$

$$I_{N.A.} = \frac{bd^3}{12}$$

$$\tau = \frac{P}{I_{N.A.}}\left[\frac{A\bar{Y}}{b}\right] \quad \text{—(1)}$$

So

$$\tau = \frac{P}{\left(\frac{bd^3}{12}\right)}\left[\frac{\frac{b}{2}\left\{\left(\frac{d}{2}\right)^2 - Y^2\right\}}{b}\right]$$

$$\boxed{\tau = \frac{6P}{bd^3}\left[\left(\frac{d}{2}\right)^2 - (Y)^2\right]} \quad \text{—(11)}$$

Generalized eq^n for τ_s at a fiber on rect. X-S/C

From eq^n (11)

$$\tau \propto f[Y^2]$$

As $Y(\uparrow) \Rightarrow \tau(\downarrow)$

At extreme fiber $Y = d/2$

$$\tau_{E.F} = 0$$

To det. τ_{max} location

$$\frac{d\tau}{dY} = 0 \Rightarrow Y = 0 \text{ [i.e. N.A.]}$$

$$\tau_{max} = \tau_{y=0} = \frac{3}{2}\left[\frac{P}{bd}\right]$$

$$\tau_{max} = \frac{3}{2}\left[\frac{P}{A}\right]$$

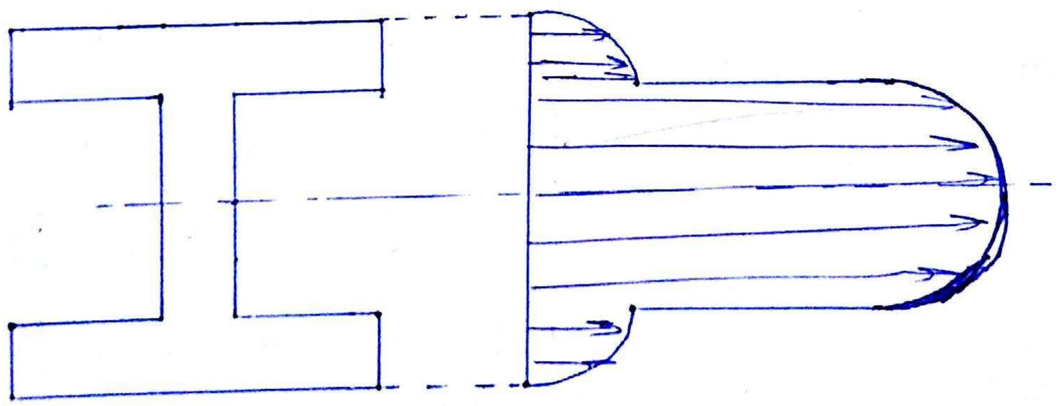
$$\boxed{\tau_{max} = \frac{3}{2}\tau_{avg}.}$$ where $\tau_{avg} = \frac{P}{A} = \frac{P}{bd}$

* Valid for any x-s/c

$(\sigma_b) \longrightarrow$ linear

$\tau_{max} \longrightarrow$ Parabolic max at N.A.

Shear stress distribution on I-section



18 July 2016

Bending stress

① Bending stress act ⊥r of beam

② Bending stress mag. varies linearly over depth of beam.

③ At extreme fibers Bending stress is max.

④ At Neutral axis Bending stress zero

⑤ $\sigma_b = \frac{MY}{I_{N.A.}}$ (or) $(\sigma_b)_{max}\left[\frac{Y}{Y_{max}}\right]$ or $\frac{E}{R}$

⑥ $(\sigma_b)_{max} = \frac{M}{Z_{N.A.}}$ (or) $\frac{E Y_{max}}{R}$

⑦ Bending stress distribution consists of 2 similar Δ for all the X-S/c of beam.

Shear stress

① Shear act ||el to X-S/c of beam

② shear stress mag varies parabolically over depth of beam.

③ At extreme fiber shear stress is zero

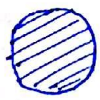
④ At Neutral axis shear stress non zero but becomes to maximum in case of circular, square, ractangular, T-Section & I-Section

⑤ $\tau_s = \frac{P}{I_{N.A.}}\left[\frac{A\bar{Y}}{b}\right]$

⑥ $\tau_{max} = K\,\tau_{avg}$ where $\tau_{avg} = \frac{P}{A}$

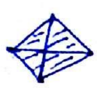
$K = \frac{3}{2}$

$k = 4/3$



$\tau_{max} = \frac{4}{3}\left(\frac{P}{A}\right)$

$k = 9/8$



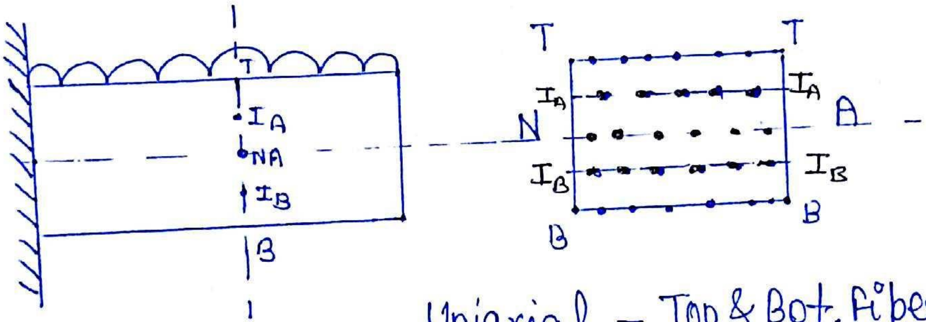
$\tau_{max} = \frac{9}{8}\left(\frac{P}{A}\right)$

⑦ Shape ~~stress~~ shear stress variation varies from X-S/c to X-S/c

* Uniaxial state of stress is develope at extreme fibe of beam (i.e. $\sigma_x = (\sigma_b)_{max}$, $\sigma_y = 0$, $\tau_{xy} = 0$

* Biaxial combined state of stress is develope on inner feber of beam. (i.e. $\sigma_x = \sigma_b$, $\sigma_y = 0$, $\tau_{xy} = \tau$

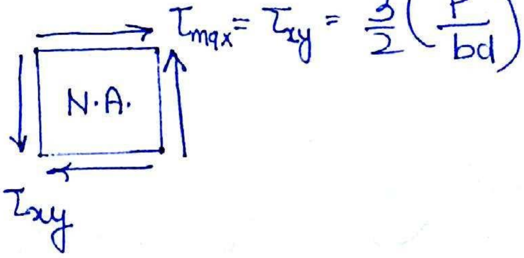
* Pure shear state of stress is develope at any point on the Neutral axis of beam (i.e. $\sigma_x = \sigma_y = 0$, $\tau_{xy} = (\tau)_{max}$) except in Δ & ◇ (S.D.)

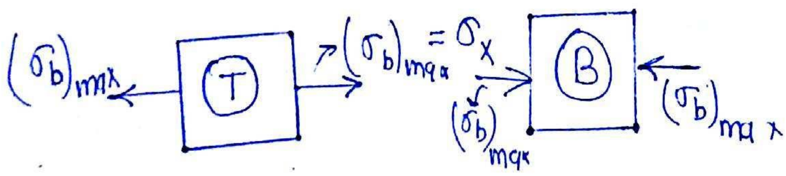


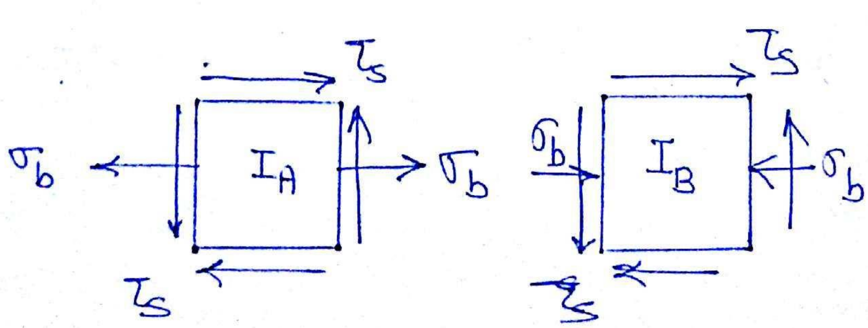
Uniaxial – Top & Bot. fibers i.e. extreme fiber

Biaxial - I_A, I_B i.e. inner febers

Pure shear - N.A.

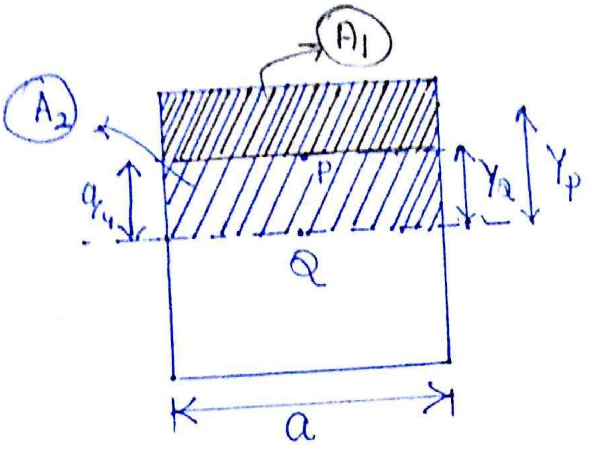






Quest

For the square x-s/c as shown in fig determine ratio of shear stress at point P & Q.



$$\tau = \frac{6P}{bd^3}\left(\left(\frac{d}{2}\right)^2 - (y)^2\right) \qquad b = d = a$$

$$\tau_P = \frac{6P}{a^4}\left(\frac{a^2}{4} - \frac{a^2}{16}\right) = \frac{6P}{a^2}\left(\frac{3}{16}\right)$$

$$\tau_Q = \frac{6P}{a^4}\left(\frac{a^2}{4}\right)$$

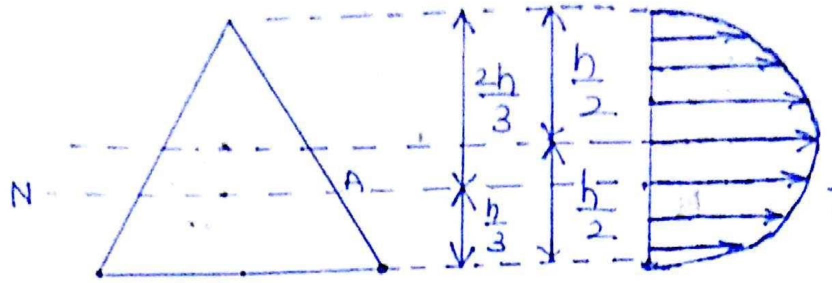
$$\frac{\tau_P}{\tau_Q} = \frac{3/16}{1/4} = \frac{3}{4}$$

OR

$$\frac{\tau_P}{\tau_Q} = \frac{A_P \bar{Y}_P}{A_P Y_Q} = \frac{(a \times a/4)(a/4 + a/8)}{a \times a/2\,(a/4)} = \frac{3}{4}$$

$$\frac{\tau_P}{\tau_Q} = \frac{3}{4}$$

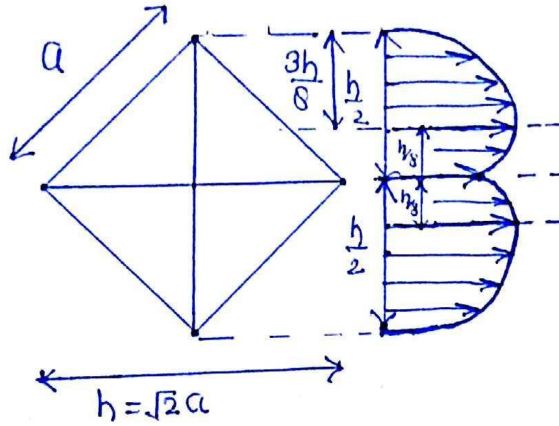
Shear ~~[illegible]~~ stress distribution of Δ triangle.



$\tau_{max} = \frac{3}{2}\tau_{avg}$

$\tau_{N.A.} = \frac{4}{3}\tau_{avg}$

shear stress variation of S.D.



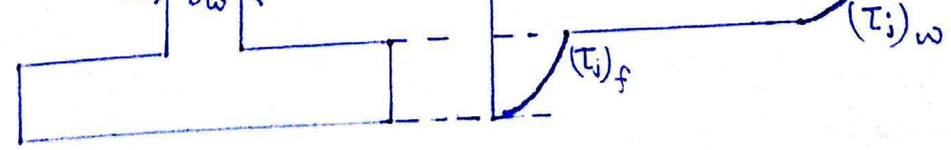
$\tau_{max} = \frac{9}{8}\tau_{avg}$

$\tau_{N.A.} = \tau_{avg}.$

$\tau_{max} = \frac{9}{8}\tau_{avg}$

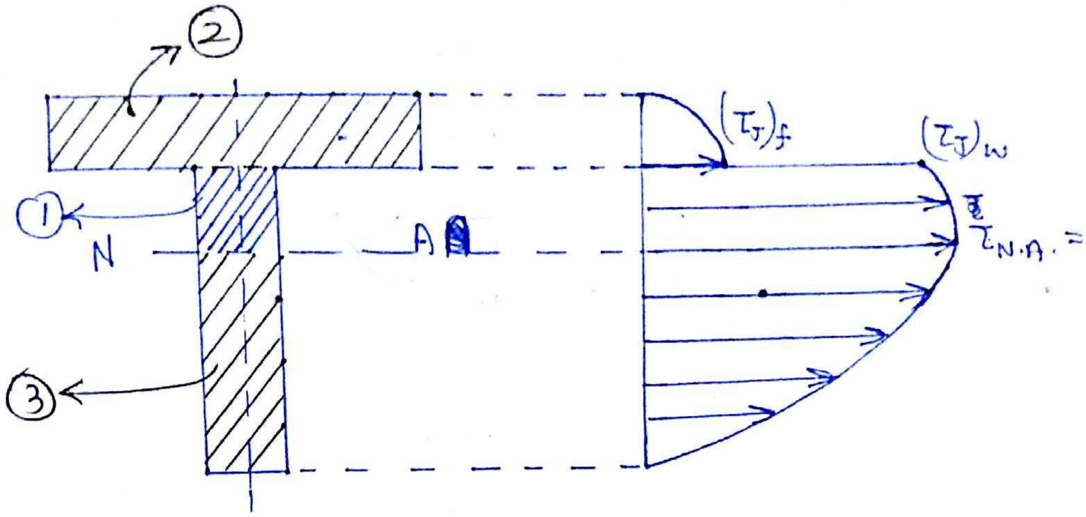
$$\tau_{avg} = \frac{P}{a^2} \text{ (or) } \frac{2P}{h^2} \; ; \; \tau_{max} = \frac{9}{8}\left(\frac{P}{a^2}\right) \Rightarrow \frac{\tau_{max}}{\tau_{avg}} = \frac{9}{8}$$

Shear stress variation of I-section.



$$(\tau_j)_w = (\tau_j)_f\left[\frac{b_f}{b_w}\right]$$

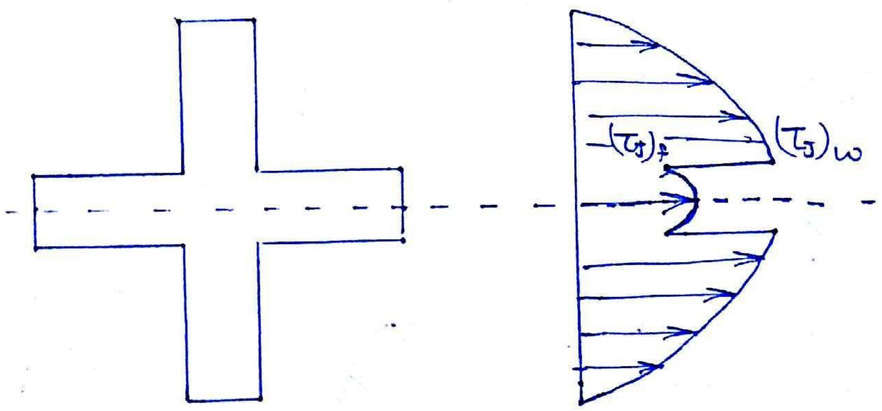
Shear ~~Force~~ stress distribution of T - Section.



$$\tau_{N.A.} = \frac{P}{I_{N.A.}} \left[\frac{A_2 \bar{Y}_2 + A_1 \bar{Y}_1}{b_w} \right]$$

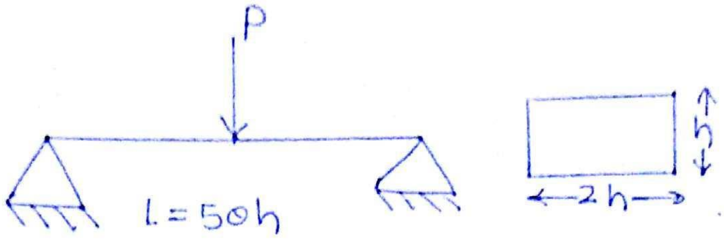
$$\tau_{N.A.} = \frac{P}{I_{N.A}} \left[\frac{A_3 \bar{Y}_3}{b_w} \right]$$

Shear stress distribution of perticular (+) section



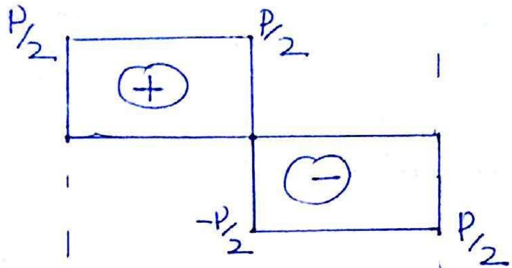
Q.25

workbook



$$\frac{\tau_{max}}{(\sigma_b)_{max}} = \frac{\frac{3}{2}\left[\frac{P_1}{A}\right]}{\frac{PL/4}{\frac{1}{6}(2h)(h^2)}} = \frac{\frac{3}{2}\left(\frac{P/2}{2\times h^2}\right)}{\frac{PL/4}{\frac{1}{6}(2h)(h^2)}} = 0.01$$

SFD



$$\boxed{P_1 = \frac{P}{2}}$$

$$\boxed{M_b = \frac{PL}{4}}$$

BMD