09

Measurement of Angle : Radian and Degree

1. **To measure an angle in degree** : The angle between two perpendicular lines is called a right angle. A right angle is equal to 90 degree, it is written as 90°.

Thus, if a right angle is divided into 90 equal parts then one part is called one degree. It is written as 1°.

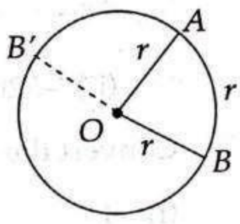
If 1° is divided into 60 equal parts, each part is called 1 minute. It is denote by 1′

$\frac{1}{60}$th part of 1′ is called one second. It is written as 1″.

Hence 1 right angle = 90°
$= 90 \times 60' = 5400' = 5400$ minute
$= 90 \times 60 \times 60'' = 324000'' = 324000$ seconds

Again, $\boxed{1^\circ = 60' = 60 \times 60'' = 3600''}$

2. **To measure an angle in radian** : Let AB be an arc of a given circle whose length is equal to radius of the circle. The angle subtended by arc AB at the centre O of the circle is measured as 1 radian i.e., $\angle AOB = 1$ radian. It is denoted by 1 or 1 rad. In the given figure $\angle AOB = 1$ rad. It is also written as 1^c.



3. **Relation between degree measure and radian measure :**

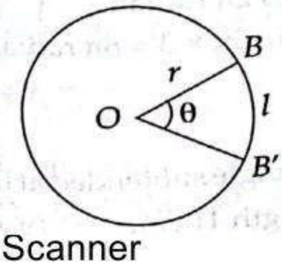
$\because\ \pi$ rad = 180° $\quad\quad \therefore\ \boxed{x^\circ = \frac{\pi x}{180}\ rad}$

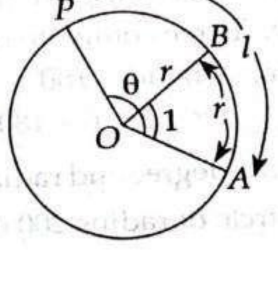
and $\boxed{x\ rad = \frac{180}{\pi}\ x^\circ}$ $\quad\quad \boxed{1 \text{rad} = \frac{180}{3.14} = 57^\circ 16' 22''}$

Thus to change degree into radian, multiply by $\frac{\pi}{180}$ and to change radian into degree multiply by $\frac{180}{\pi}$.

If mentioned, take $\pi = \frac{22}{7}$ or 3.14.

4. **Relation between length of arc (l), radius (r) and angle (θ) :**





If an arc of length l of a circle su... radius of the circle is r then $\theta = \frac{l}{r}$.

Hence, (i) when $\theta = \frac{l}{r}$ and r is constant then $\theta \propto l$ i.e., $\theta_1 : \theta_2 = l_1 : l_2$

(ii) when $\theta = \frac{l}{r}$ and θ is constant then $l \propto r$ i.e., $l_1 : l_2 = r_1 : r_2$

(iii) when $\theta = \frac{l}{r}$ and l is constant then $\theta \propto \frac{1}{r}$ or $r \propto \frac{1}{\theta}$

i.e., $\theta_1 : \theta_2 = r_2 : r_1$ *(reverse order)*

5. If θ is in radian and is very-very small then $\sin\theta = \theta = \tan\theta$ *(approximate)*

Solved Example

1. Convert the following degree measures in the radian measure.

(i) 42°30' (ii) –520°

Solution : We know that $x° = \frac{\pi x}{180}$ rad

$\therefore$ (i) $42°30' = 42\frac{1}{2}° = \frac{85°}{2}$ $\left(\because 30' = \frac{1}{2}°\right)$

$$= \frac{\pi}{180} \times \frac{85}{2} = \frac{\pi \times 17 \times 5}{5 \times 36 \times 2} = \frac{17\pi}{72}$$

(ii) $-520° = -520 \times \frac{\pi}{180} = \frac{-52\pi}{18} = \frac{-26\pi}{9}$

2. Convert the following radian measure in degree measurers $\left(\text{Use } \pi = \frac{22}{7}\right)$

(i) 4 (ii) $-\frac{5\pi}{3}$

Solution : $\because$ π radian = 180°

$\therefore$ 1 radian $= \frac{180°}{\pi} = 180° \times \frac{7}{22}$

or, 4 radian $= \frac{180° \times 7 \times 4}{22} = \frac{90° \times 7 \times 4}{11} = \frac{2520°}{11} = 229\frac{1}{11}$ degree.

(ii) $-\frac{5\pi}{3} = -\frac{5}{3} \times 180° = -5 \times 60° = -300°$

3. A wheel makes 180 revolutions in one minute. Through how many radians does it turn in one second ? Also find its degree measure.

Solution : $\because$ Wheel makes 180 revolution in 60 seconds

$\therefore$ Wheel makes $\frac{180}{60} = 3$ revolutions in 1 second.

Now, $\because$ One complete revolution measures 2π radian.

$\therefore$ Three complete revolutions measure $2\pi \times 3 = 6\pi$ radian

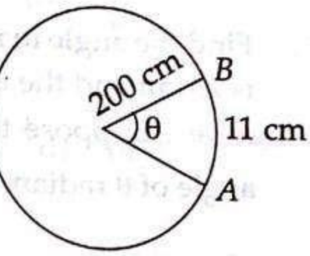
Again, $\because$ π rad = 180°

$\therefore$ 6π rad = 6 × 180° = 1080°

4. Find the degree and radian measure of the angle subtended at the centre of a circle of radius 200 cm by an arc of length 11 cm. $\left(\text{use } \pi = \frac{22}{7}\right)$

Solution : Given $r = 200$ cm, $l =$ Arc $AB = 11$cm

Suppose angle subtended at the centre of circle be θ radian



then, $\theta = \frac{l}{r} = \frac{11}{200}$ rad

$\because \pi \text{ rad} = 180°$

$\therefore 1 \text{ rad} = \frac{180°}{\pi}$

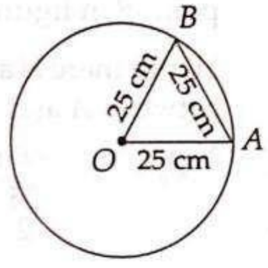
or, $1 \text{ rad} = \frac{7}{22} \times 180°$

$\therefore \frac{11}{200} \text{ rad} = \frac{11}{200} \times \frac{7}{22} \times 180° = \frac{7 \times 180°}{200 \times 2} = \frac{7 \times 45°}{100} = \frac{7 \times 9°}{20}$

$= \frac{63°}{20} = 3\frac{3°}{20} = 3 \text{ degree } \frac{3}{20} \times 60 \text{ seconds} = 3°9'$

5. In a circle of diameter 50 cm, the length of a chord is 25 cm. Find the length of minor arc and major arc of the chord.

Solution : See the figure



Given that radius of the circle $= \frac{50 \text{ cm}}{2} = 25$ cm

and chord AB of the circle $= 25$ cm

Clearly ΔOAB is an equilateral triangle,

therefore $\angle AOB = 60° = \frac{\pi}{3} = \theta$ (say)

If minor arc $AB = l$ then from $\theta = \frac{l}{r}$

$$l = r\theta = \frac{25\pi}{3}$$

and major arc $= 25\left(2\pi - \frac{\pi}{3}\right) = 25\left(\frac{5\pi}{3}\right) = \frac{125\pi}{3}$

6. If in two circles, arcs of the same length subtend angle 60° and 75° at the centre, find the ratio of their radii.

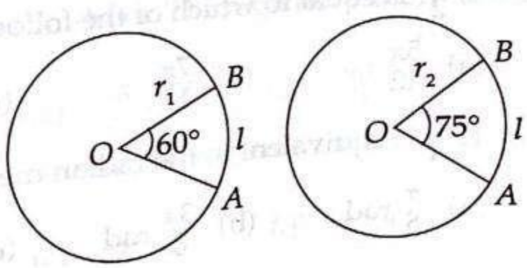
Solution : Let the radii of two circles be r_1 and r_2 respectively.

According to the question, arc $AB = l$ (say) in the two circle.

Given that $\theta_1 = 60° = 60 \times \frac{\pi}{180} = \frac{60\pi}{180}$ radian

And $\theta_2 = 75° = \frac{75\pi}{180}$ radian

$\because \theta = \frac{l}{r} \therefore \theta_1 = \frac{l}{r_1}$ and $\theta_2 = \frac{l}{r_2}$



or, $l = r_1 \theta_1 = r_2 \theta_2$

or, $\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\frac{75\pi}{180}}{\frac{60\pi}{180}} = \frac{75}{60} = \frac{5}{4}$

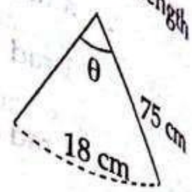
Shortcut : since l is constant,

therefore $r_1 : r_2 = \theta_2 : \theta_1 = 75° : 60° = 5 : 4$

7. Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc length 18 cm.

Solution : Suppose the pendulum swings through an angle of θ radian. then, $\theta = \frac{l}{r} = \frac{18}{75}$ rad (see figure)

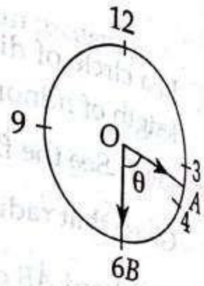
$= \frac{6}{25}$ rad



8. Find the angle in radians between the hands of a clock at half past three.

Solution: In 60 minutes the minute hand of a watch completes one revolution i.e., moves through an angle of 2π radian (360°)

Also, at three past half, the hour hand is exactly at the midway between 3 and 4, (shown by point A is figure) and minute hand is exactly at 6 (shown by point B in figure).



Hence there is a difference of $2 \times 5 + \frac{5}{2} = \frac{25}{2}$ minute between A and B.

Now, $\because$ 60 minute revolution ≡ 2π rad

$\therefore$ $\frac{25}{2}$ minute revolution $\equiv \frac{25}{2 \times 60}(2\pi) = \frac{5\pi}{12}$ rad

Hence the two hands of the clock makes an angle of $\frac{5\pi}{12}$ rad at half past three.

Shortcut : If two hands of a clock are respectively at H hour and M minute and angle between them is $A°$ then $\frac{11}{2}M = 30H \pm A$.

here, $H = 3, M = 30$

$\therefore$ $\frac{11}{2} \times 30 = 30 \times 3 \pm A$

or, $165° - 90° = \pm A \Rightarrow A = 75°$

Exercise–9A

1. In radian measure 120° equals

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{4\pi}{3}$

2. $37\frac{1}{2}^{\circ}$ is equal to which of the following radian measure ?

(a) $\frac{5\pi}{12}$ (b) $\frac{7\pi}{12}$ (c) $\frac{5\pi}{24}$ (d) $\frac{7\pi}{24}$

3. $11\frac{1}{4}^{\circ}$ is equivalent to the radian measure

(a) $\frac{\pi}{8}$ rad (b) $\frac{3\pi}{8}$ rad (c) $\frac{3\pi}{16}$ rad (d) $\frac{\pi}{16}$ rad

4. $\frac{5}{6}$ right angle in radian equals

(a) $\frac{5\pi}{24}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{\pi}{12}$

5. Degree equivalent of $\frac{7\pi}{12}$ radian is

(a) 105° (b) 75° (c) 135° (d) 165°

6. Radian value of 35°30' is

(a) $\frac{81\pi}{360}$ rad (b) $\frac{71\pi}{360}$ rad (c) $\frac{61\pi}{360}$ rad (d) $\frac{143\pi}{360}$ rad

7. Radian value of 560°20' is

(a) $\frac{1481\pi}{540}$ rad (b) $\frac{1481\pi}{360}$ rad (c) $\frac{1681\pi}{540}$ rad (d) $\frac{1681\pi}{360}$ rad

8. Radian measure of 72°40' is

(a) $\frac{109\pi}{270}$ (b) $\frac{109\pi}{180}$ (c) $\frac{219\pi}{540}$ (d) $\frac{219\pi}{360}$

9. One radian is equal to following degree measure,

(a) 57°14'21" (b) 57°16'22" (c) 58°14'21" (d) 58°16'22"

10. Value of $\frac{3}{5}$ rad is

(a) 34°23'19" (b) 36°23'19" (c) 36°21'49" (d) 34°21'49"

11. Measure of 6 rad is

(a) 343° 18' 11" (b) 341° 18' 11" (c) 341° 38° 11" (d) 343° 38' 11"

12. If 1 rad = 57° 16' 21" then 10 rad equals

(a) 570° 16' 21" (b) 573° 43' 10" (c) 571° 43' 40" (d) 572° 43' 30"

13. If one unit of an angle is 29° 46' 55" then five units of the angle equals

(a) 148° 54' 35" (b) 146° 54' 35" (c) 149° 34' 25" (d) 147° 44' 35"

14. If one unit of an angle is 15° 49' 50" then measure of 100 unit of the angle equals

(a) 1580° 30' 20" (b) 1582° 3' 20"

(c) 1583° 3' 20" (d) 1581° 30' 20"

15. A wheel makes 90 revolutions in half hour. Through how many degree does it turn in one minute ?

(a) 120° (b) 720° (c) 1080° (d) 540°

16. A wheel makes 720 revolutions in one hour. Through how many radian does it turn in one second ?

(a) $\frac{\pi}{5}$ rad (b) $\frac{2\pi}{5}$ rad (c) $\frac{3\pi}{5}$ rad (d) $\frac{4\pi}{5}$ rad

17. The diameter of a circle is 2 meter. The angle subtended at the centre by an arc of 22 cm is

(a) 12° 60" (b) 12° 36' (c) 24° 60' (d) 6° 36'

18. The diameter of a circle is 60 cm. The length of minor arc created by a chord of 30 cm is (take $\pi = \frac{22}{7}$)
(a) $31\frac{3}{7}$ cm (b) 34 cm (c) $32\frac{2}{7}$ cm (d) $32\frac{4}{7}$ cm

19. In a circle of radius 50 cm the length of a chord is $50\sqrt{2}$ cm. The length of major arc of the chord is (take $\pi = 3.14$)
(a) 245.5 cm (b) 235.5 cm (c) 255.5 cm (d) None of these

20. The angle in degree through which a pendulum of length 100 cm swings and the tip describes an arc length of 10 cm is
(a) 5° 43' 38" (b) 7° 43' 38° (c) 5° 34' 18" (d) 7° 34' 18"

21. The minute hand of a watch is 5 cm. How far does the tip move in 20 minutes ? (take $\pi = \frac{22}{7}$)
(a) 10 cm (b) 9.53 cm (c) 11 cm (d) 10.47 cm

22. The tip of a pendulum swings. It covers an arc of 50 cm and subtends 60° at the fixed point. The length of pendulum is (take $\pi = \frac{22}{7}$)
(a) 43.72 cm (b) 45.72 cm (c) 47.72 cm (d) 45.27 cm

23. If the arc of same length in two circle subtends angles 75° and 120° at their respective centres, then ratio of their diameter is
(a) 8 : 5 (b) 5 : 8 (c) 3 : 5 (d) 5 : 3

24. The angle between the hands of a clock at 4 hour 45 minute is
(a) $112\frac{1}{2}^\circ$ (b) $122\frac{1}{2}^\circ$ (c) 125° (d) $127\frac{1}{2}^\circ$

25. The angle between the hands of a clock at half past one is
(a) $\frac{3\pi}{4}$ rad (b) $\frac{2\pi}{3}$ rad (c) $\frac{5\pi}{12}$ rad (d) $\frac{5\pi}{6}$ rad

26. Two angles of a triangle are $\frac{3}{2}$ rad and $\frac{4}{3}$ rad. The triangle
(a) is an acute angled triangle (b) is an obtuse angled triangle
(c) is a right angled triangle (d) dosenot form

27. If two angle of a triangle are 2 rad and $\frac{1}{2}$ rad then its third angle in degree is
(a) $105\frac{3}{7}^\circ$ (b) $15\frac{3}{7}^\circ$ (c) $105\frac{5}{7}^\circ$ (d) $36\frac{9}{11}^\circ$

28. A wheel revolves 24 times in 10 seconds. How many time does it take in revolving an angle of 110 rad ?
(a) 5 sec (b) 7.5 sec (c) 10 sec (d) None of these

29. Radian measure of 40° 20' 50" is
(a) $\frac{481}{1196}\pi$ rad (b) $\frac{681}{1296}\pi$ rad (c) $\frac{581}{2592}\pi$ rad (d) $\frac{581}{1296}\pi$ rad

30. A pendulum of length 60 cm swings and creates an arc of 18 cm. The angle at the fixed point of the pendulum is

(a) 15° (b) $17\frac{1}{2}^\circ$ (c) 20° (d) $22\frac{1}{2}^\circ$

31. Radius of a circle is 54 cm. If an arc of circle subtends an angle of 20° at centre then length of the arc is (take $\pi = \frac{22}{7}$)

(a) $19\frac{1}{7}$ cm (b) $17\frac{4}{7}$ cm (c) $18\frac{6}{7}$ cm (d) None of these

32. An arc of length 40 cm subtends $22\frac{1}{2}^\circ$ at the centre of the circle. Radius of the circle is

(a) 92 cm (b) 102 cm (c) 96 cm (d) 108 cm

33. Ananta's (A) and Shailvi's (S) house are situated at a circular road and subtends 90° at a fixed point. If fixed point is at a distance of 100 meter from each house, the distance travelled between the both house on the road is

(a) 628 meter (b) 314 meter (c) 157 meter (d) 235.5 meter

34. The angle covered by minute hand of a watch during 1 hour 15 minutes noon to half past three noon is

(a) 4.5π (b) 5π (c) 4.25π (d) None of these

35. The angle covered by hour hand of a clock from half past six in the morning to three o'clock in the noon is

(a) 270° (b) 245° (c) 255° (d) 265°

36. Assuming that the Moon's diameter subtends an angle $\left(\frac{1}{2}\right)^\circ$ at the eye of an observer, find how far from the eye of a coin of 1 cm diameter must be held so as just to hide Moon ? (take $\pi = \frac{22}{7}$)

(a) $112\frac{5}{11}$ cm (b) $110\frac{6}{11}$ cm (c) $116\frac{5}{11}$ cn (d) $114\frac{6}{11}$ cm

37. The earth revolves in its axis in 24 hours. How much angle does it move in 4 hours and 12 minutes ?

(a) 63° (b) 64° (c) 65° (d) 70°

38. If angle of a triangle are in AP, then the middle one is

(a) always 60° (b) less than 60°

(c) move than 60° (d) less than 90°

Answer–9A

1. (b)	2. (c)	3. (d)	4. (b)	5. (a)	6. (b)	7. (c)	8. (a)
9. (b)	10. (c)	11. (d)	12. (d)	13. (a)	14. (c)	15. (c)	16. (b)
17. (b)	18. (a)	19. (b)	20. (a)	21. (d)	22. (c)	23. (a)	24. (d)
25. (a)	26. (a)	27. (d)	28. (b)	29. (c)	30. (b)	31. (c)	32. (b)
33. (c)	34. (a)	35. (c)	36. (d)	37. (a)	38. (a)		

1. (b) $120^\circ = \frac{120}{180} \times \pi \text{ rad} = \frac{2\pi}{3} \text{ rad}$

2. (c) $37\frac{1}{2}^\circ = \left(\frac{75}{2}\right)^\circ$

$$= \frac{75}{2\times180} \times \pi = \frac{5\pi}{2\times12} = \frac{5\pi}{24} \text{ rad}$$

3. (d) $11\frac{1}{4}^\circ = \left(\frac{45}{4}\right)^\circ$

$$= \frac{\left(\frac{45}{4}\right)}{180} \times \pi \text{ rad} = \frac{\pi}{16} \text{ rad}$$

4. (b) $\frac{5}{6}$ right angle $= \frac{5}{6} \times \frac{\pi}{2} \text{ rad} = \frac{5\pi}{12} \text{ rad}$

5. (a) $\frac{7\pi}{12} = \frac{7\times180^\circ}{12} = 7\times 15^\circ = 105^\circ$

6. (b) $35^\circ30' = 35\frac{1}{2}^\circ = \left(\frac{71}{2}\right)^\circ$

$$= \frac{71}{2} \times \frac{\pi}{180} = \frac{71\pi}{360} \text{ rad}$$

7. (c) $560^\circ20' = \left(560\frac{20}{60}\right)^\circ = \left(560\frac{1}{3}\right)^\circ$

$$= \left(\frac{1681}{3}\right)^\circ = \frac{1681}{3} \times \frac{\pi}{180} = \frac{1681}{540} \pi \text{ rad}$$

8. (a) $72^\circ40' = \left(72\frac{40}{60}\right)^\circ = \left(72\frac{2}{3}\right)^\circ$

$$= \left(\frac{218}{3}\right)^\circ = \left(\frac{218}{3} \times \frac{\pi}{180}\right)$$

$$= \left(\frac{109\,\pi}{3\times90}\right) = \frac{109\,\pi}{270} \text{ rad}$$

9. (b) $\because \pi \text{ rad} = 180^\circ$

$$\therefore 1 \text{ rad} = \frac{180^\circ}{\pi} \text{ rad} = \left(\frac{180}{22} \times 7\right)^\circ$$

$$= \left(\frac{90\times7}{11}\right)^\circ = \left(\frac{630}{11}\right)^\circ = \left(57\frac{3}{11}\right)^\circ$$

$$= 57^\circ \left(\frac{3}{11} \times 60'\right) = 57^\circ \left(\frac{180}{11}\right)'$$

$$= 57^\circ \left(16\frac{4}{11}\right)' = 57^\circ16'\left(\frac{4}{11} \times 60''\right)$$

$$= 57^\circ16'\frac{240''}{11} = 57^\circ16'22'' \text{ (approximate)}$$

10. (c) $\because \pi \text{ rad} = 180^\circ$

$\therefore \frac{3}{5} \text{ rad} = \frac{3}{5} \times \frac{180^\circ}{\pi}$

$= \frac{3}{5} \times \frac{7}{22} \times 180^\circ = \frac{7 \times 3 \times 18^\circ}{11}$

$= \frac{378^\circ}{11} = \left(34\frac{4}{11}\right)^\circ$

$= 34^\circ\left(\frac{4}{11} \times 60'\right) = 34^\circ\left(\frac{240'}{11}\right)$

$= 34^\circ\left(21\frac{9}{11}\right)' = 34^\circ 21'\left(\frac{9}{11} \times 60'\right)$

$= 34^\circ 21'\left(\frac{540}{11}\right)'' = 34^\circ 21' 49''$

11. (d) $\because \pi \text{ rad} = 180^\circ$

$\therefore 1 \text{ rad} = \frac{180^\circ}{\pi}$

$6 \text{ rad} = \frac{6 \times 180^\circ}{\pi} = \frac{6 \times 180^\circ \times 7}{22} = \frac{21 \times 180^\circ}{11} = \frac{3780^\circ}{11}$

$\left(343\frac{7}{11}\right)^\circ = 343^\circ\left(\frac{420}{11}\right)' = 343^\circ\left(38\frac{2}{11}\right)'$

$= 343^\circ 38'\left(\frac{120}{11}\right)'' = 343^\circ 38' 11''$ (approximate)

Second Method,

$\because 1 \text{ rad} = 57^\circ 16' 22''$

$\therefore 6 \text{ rad} = 57^\circ \times 6 + 16' \times 6 + 22'' \times 6$

$= 342^\circ + 96' + 132''$

$= 342^\circ + (1^\circ + 36') + (2' + 12'')$

$= (\because 1^\circ = 60' \text{ and } 1' = 60'') = 343^\circ 38' 12''$ (approximate)

12. (d) $10 \text{ rad} = (57^\circ 16' 21'') \times 10$

$= 570^\circ + 160' + 210'' = 570^\circ + 2^\circ 40' + 3' 30'' = 572^\circ 43' 30''$

13. (a) $5 \text{ unit} = (29^\circ 46' 55'') \times 5$

$= 145^\circ 230' 275'' = 145^\circ + 3^\circ 50' + 4' 35'' = 148^\circ 54' 35''$

14. (c) $100 \text{ unit} = (15^\circ 49' 50'') \times 100$

$= 1500^\circ 4900' 5000'' = 1500^\circ + (81^\circ 40') + 83' 20''$

$\left(\because \frac{4900}{60} = 81\frac{40}{60}, \frac{5000}{60} = 83\frac{20}{60}\right)$

$= 1500^\circ + 81^\circ 40' + 1^\circ 23' 20''$

$= 1582^\circ 63' 20'' = 1583^\circ 3' 20''$

15. (c) Wheel revolves $= \frac{90}{30} = 3$ turn in one minute

$\because 1 \text{ turn} = 360^\circ$

$\therefore 3 \text{ turn} = 1080^\circ$

16. (b) Wheel covers $\frac{720}{3600} = \frac{1}{5}$ turn in 1 second

$\because$ 1 turn = 2π rad

$\therefore \frac{1}{5}$ turn = $\frac{2\pi}{5}$ rad

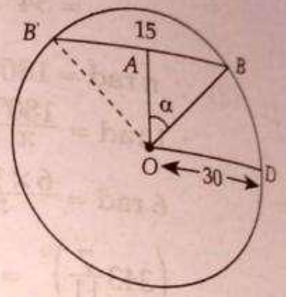
17. (b) Radius = 1 meter = 100 cm = r

$\because \theta = \frac{l}{r}$, here $l = 22$ cm.

$\therefore \theta = \frac{22}{100}$ rad $= \frac{22}{100} \times \frac{180}{\pi}$ degree $= \frac{22}{100} \times \frac{180}{22} \times 7$ degree

$= \frac{180 \times 7}{100} = \frac{126}{10}$ degree $= \left(12\frac{6}{10}\right)^\circ = 12^\circ\left(\frac{6}{10} \times 60\right)' = 12^\circ\, 36'$

18. (a) In $\Delta\, OAB$,



$\text{Sin}\alpha = \frac{15}{30} = \frac{1}{2} \Rightarrow \alpha = 30^\circ = \frac{\pi}{6}$

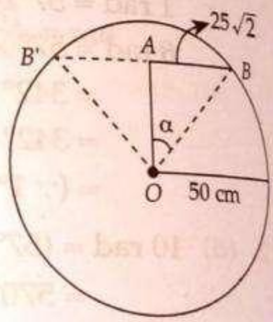
In figure chord BB' is 30 cm that subtends

$2\theta = 2 \times \frac{\pi}{6} = \frac{\pi}{3}$ at the centre.

Hence, from $\theta = \frac{l}{r}$, $\frac{\pi}{3} = \frac{l}{30} \Rightarrow l = 10\pi$

$= \frac{10 \times 22}{7} = \frac{220}{7}$ cm $= 31\frac{3}{7}$ cm

19. (b) $\sin\alpha = \frac{25\sqrt{2}}{50} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$



$\therefore \quad 2\alpha = 90^\circ$

Hence major arc of chord BB' subtends

$360^\circ - 90^\circ = 270^\circ = \frac{3\pi}{2}$ at centre.

$\therefore$ using $\theta = \frac{l}{r}$

major arc $l = \theta \times r = \frac{3\pi}{2} \times 50$

$= 75\pi = 75 \times 3.14$ cm $= 235.50$ cm

20. (a) From $\theta = \frac{l}{r}$, $\theta = \frac{10}{100} = \frac{1}{10}$ rad

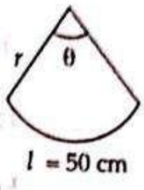
$= \frac{180^\circ}{\pi \times 10} = \left(\frac{18 \times 7}{22}\right)^\circ$

$= \left(\frac{63}{11}\right)^\circ = \left(5\frac{8}{11}\right)^\circ = 5^\circ\left(\frac{8}{11} \times 60'\right) = 5^\circ\left(\frac{480'}{11}\right)$

$= 5^\circ\left(43\frac{7}{11}\right)' = 5^\circ\, 43'\left(\frac{7}{11} \times 60''\right)$

$= 5^\circ\, 43'\, 38''$ (approximate)

21. (d) In 20 minute, hand covers $\frac{20}{60} \times 2\pi = \frac{2\pi}{3}$ rad distance.



from $\theta = \frac{l}{r}$, $l = \theta r$

$$= \frac{2\pi}{3} \times 5 = \frac{10\pi}{3} = \frac{10}{3} \times \frac{22}{7} = \frac{220}{21} = 10.47 \text{ cm}$$

22. (c) $\theta = 60^\circ = \frac{\pi}{3}$ and $l = 50$ cm

$\therefore$ using $\theta = \frac{l}{r}$,

$$r = \frac{l}{\theta} = \frac{50 \text{ cm}}{\frac{\pi}{3}} = \frac{150}{\pi} \text{ cm} = \frac{150}{\frac{22}{7}} = \frac{150 \times 7}{22} = 47.72 \text{ cm}$$

23. (a) $\theta = \frac{l}{r} \Rightarrow r = \frac{l}{\theta} \Rightarrow \frac{r_1}{r_2} = \frac{\frac{l}{75^\circ}}{\frac{l}{120^\circ}} = \frac{120^\circ}{75^\circ} = 8:5$

24. (d) Using, $\frac{11}{2} M = 30H \pm A$

here, $H = 4, M = 45$

$\because \frac{11}{2} \times 45 = 30 \times 4 \pm A$

$247.5 = 120^\circ \pm A$

$\therefore A = 247.5^\circ - 120^\circ = 127.5^\circ$

25. (a) using $\frac{11}{2}$ M = 30 H ± A

$\frac{11}{2} \times 30 = 30 \times 1 \pm A$

$\Rightarrow A = 11 \times 15 - 30 = 135^\circ = \frac{135}{180} \times \pi \text{ rad} = \frac{3\pi}{4}$ rad

26. (a) 1 right angle = $\frac{\pi}{2}$ rad = 1.57 rad (approximate)

$\frac{3}{2} = 1.5$ rad, which is an acute angle.

$\frac{4}{3} = 1.33$ rad, which is an acute angle

Third angle = π rad – 1.5 rad – 1.33 rad

= (3.14 – 1.5 – 1.33)

= 0.31 rad which is also an acute angle.

27. (d) Third angle = $\pi \text{ rad} - 2 \text{ rad} - \frac{1}{2}$ rad

$$= \left(\frac{22}{7} - \frac{5}{2}\right) \text{rad} = \frac{9}{14} \text{ rad}$$

$$= \frac{9}{14} \times \frac{180^\circ}{\pi} = \frac{9}{14} \times \frac{180}{22} \times 7 = \frac{45 \times 9}{11} = \frac{405}{11} = 36\frac{9^\circ}{11}$$

28. (b) The wheel makes $\frac{[illegible]}{10}$ rotation in one second.

$\therefore$ It covers $\frac{24}{10} \times 2\pi$ rad angle in 1 second.

Hence in covering 110 rad, wheel takes

$$\frac{110}{\frac{24}{10}\times 2\pi} = \frac{110\times 10\times 7}{24\times 2\times 22} = 7.3 \text{ second.}$$

29. (c) $40^\circ\, 20'\, 50'' = 40^\circ \left(20\frac{50}{60}\right)'$

$$= 40^\circ \left(\frac{125}{6}\right)' = 40^\circ \left(\frac{125}{6\times 60}\right)^\circ$$

$$= \left(40\frac{25}{72}\right)^\circ = \left(\frac{2905}{72}\right)^\circ$$

$$= \frac{2905}{72}\times\frac{\pi}{180} \text{ rad}$$

$$= \frac{581\pi}{72\times 36} = \frac{581\pi}{2592} \text{ rad}$$

30. (b) From $\theta = \frac{l}{r}$, $\theta = \frac{18}{60} = \frac{3}{10}$ rad $= 0.3$ rad

$\because$ 1 rad $= 57^\circ\, 16''\, 22'$ (approximate)

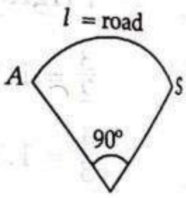
$\therefore$ 0.3 rad $= 5.7^\circ \times 3$ = more than 17° and less than 18°

31. (c) $20^\circ = \frac{20^\circ}{180^\circ}\pi = \frac{\pi}{9}$ rad

From, $\theta = \frac{l}{r}$ $\qquad l = \theta r = \frac{\pi}{9}\times 54 = \frac{22}{7\times 9}\times 54$ cm

$$= \frac{22\times 6}{7} = \frac{132}{7} = 18\frac{6}{7} \text{ cm}$$

32. (b) $22\frac{1^\circ}{2} = \frac{\left(\frac{45^\circ}{2}\right)}{180}\times\pi \text{ rad} = \frac{\pi}{8}$ rad

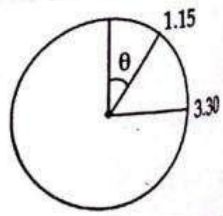


$\because\ \theta = \frac{l}{r} \Rightarrow r = \frac{l}{\theta} = \frac{(40)}{\left(\frac{\pi}{8}\right)} = \frac{320}{\pi} = \frac{320\times 7}{22} = 101.8$ cm

33. (c) $90^\circ = \frac{\pi}{2}$

From, $\theta = \frac{l}{r}$ $l = \theta r = \frac{\pi}{2}\times 100$ meter

$$= \frac{3.14}{2}\times 100 = 157 \text{ meter}$$



34. (a) From 1 hour 15 minutes to half past three, minute hand covers 2 hours 15 minutes i.e., $2\frac{1}{4}$ rotations.

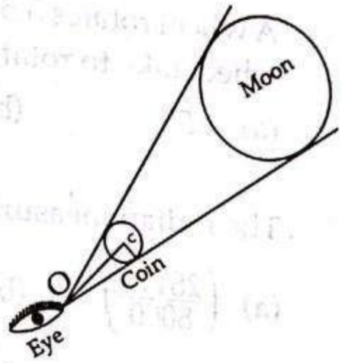
$\therefore$ It covers $2\frac{1}{4}\times 2\pi = (4.5)\,\pi$ rad distance.

(d) From half past six in the morning to 3 o'clock at noon, time elapsed is 8 hours 30 minutes.

Since hour hand covers 30° in 5 minute therefore it covers $30° \times 8\frac{1}{2}$ $= 255°$ in $8\frac{1}{2}$ hours.

(d) See the figure, let coin is placed at a distance l cm from the eye of the observer

i.e. $OC = l$ cm



In ΔOCF, $\angle COF = \frac{1}{2} \times 30' = 15' = \left(\frac{1}{4}\right)^\circ$

$$\sin(\angle COF) = \frac{CF}{OC} = \frac{\left(\frac{1}{2}\right)}{l}$$

$(\because$ diamter = 1 cm $\therefore$ CF = $\frac{1}{2}$ cm)

$$\Rightarrow \sin\left(\frac{1}{4}\right)^\circ = \frac{1}{2l}$$

When θ is very-very small then $\sin\theta = \theta$

(Learn that it is true when θ is in radian)

$$\therefore \left(\frac{1}{4}\right)^\circ = \frac{1}{2l}$$

$$\Rightarrow \left(\frac{1}{4} \times \frac{180}{\pi}\right) \text{rad} = \frac{1}{2l}$$

$$\Rightarrow l = \frac{4 \times 180}{2 \times \pi} = \frac{360}{\pi}$$

$$= \frac{360 \times 7}{22} = \frac{1260}{11} = 114\frac{6}{11} \text{ cm}$$

37. (a) Revolution in 24 hours = 360°

$\therefore$ Revolution in 1 hours = $\frac{360°}{24} = 15°$

Revolution in 4 hours = $15° \times 4 = 60°$

$\because$ Revolution in 60 minutes = 15°

Revolution in 12 minutes = $\frac{15° \times 12°}{60°} = 3°$

$\therefore$ Revolution in 4 hours 12 minutes = $60° + 3° = 63°$

Exercise–9B

1. The angle formed by the hour-hand and the minute-hand of a clock at 2 : 15 p.m. is

(a) $22\frac{1}{2}°$ (b) 30° (c) $27\frac{1}{2}°$ (d) 45°

[SSC Tier-I 2012]

2. Two angles of triangle are $\frac{1}{2}$ and $\frac{1}{3}$ radian. The measure of the third angle in degree $\left(\text{taking } \pi = \frac{22}{7}\right)$ is

(a) $132\frac{1}{11}^\circ$ (b) $132\frac{2}{11}^\circ$ (c) $132\frac{3}{11}^\circ$ (d) 132°

3. A wheel rotates 3.5 times in one second. What time (in second) does the wheel take to rotate 55 radian of angle ? [SSC Tier-I 2012]

(a) 1.5 (b) 2.5 (c) 3.5 (d) 4.5

4. The radian measure of 63°14'51" is [SSC Tier-I 2012]

(a) $\left(\frac{2811\pi}{8000}\right)^c$ (b) $\left(\frac{3811\pi}{8000}\right)^c$ (c) $\left(\frac{4811\pi}{8000}\right)^c$ (d) $\left(\frac{5811\pi}{8000}\right)^c$

[SSC Tier-I 2012]

5. When a pendulum of length 50 cm oscillates, it produce an arc of 16 cm. The angle so formed in degree measure is (approx)

(a) 18°25' (b) 18°35' (c) 18°20' (d) 18°08'

[SSC Tier-I 2012]

6. A rail road curve is to be laid out on a circle. What radius should be used if the track is to change direction by 25° in a distance of 40 metres?

(a) 91.64 metres (b) 90.46 metres (c) 89.64 metres (d) 93.64 metres

[SSC Tier-I 2012]

7. An arc of a circle of radius 42 cm subtends an angle of 15° at the centre. Taking $\pi = \frac{22}{7}$, the length of the arc is :

(a) $\frac{88}{5}$ cm (b) 11 cm (c) 12 cm (d) $\frac{44}{5}$ cm

[SSC Tier-I 2012]

Answer–9B

1. (a) 2. (c) 3. (b) 4. (a) 5. (b) 6. (a) 7. (b)

Explanation

1. (a) From Trick,

using, $\frac{11}{2} M = 30H \pm A$

Here, $M = 15, H = 2$

Hence, $\frac{11}{2} \times 15 = 30 \times 2 \pm A$

or, $A = \frac{165}{2} - 60 = 82\frac{1}{2} - 60 = 22\frac{1}{2}$.

Therefore, angle will be $22\frac{1}{2}^\circ$

Second Method : Minute hand forms an angle of 30° when moves from 2 to 3. During this time, hour hand forms an angle of $\frac{15}{60} \times 30° = 7\frac{1}{2}°$

Therefore, angle will be $= 30° - 7\frac{1}{2}° = 22\frac{1}{2}°$

(c) Sum of two angle $= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ rad

$\frac{22}{7}$ rad $= 180°$ $\quad (\because \pi = \frac{22}{7})$

$\therefore \frac{5}{6}$ rad $= \frac{180°}{22} \times 7 \times \frac{5}{6} = \frac{30° \times 35}{22} = \frac{15° \times 35}{11}$

Remain angle $= 180° - \frac{15° \times 35}{11} = 180° - \frac{525°}{11}$

$= 180° - 47\frac{8°}{11} = 132\frac{3°}{11}$

(b) 1 rotation $\equiv 2\pi$ radian

$\therefore$ 3.5 rotation $= 3.5 \times 2\pi$ radian

$= 3.5 \times 2 \times \frac{22}{7} = 22$ radian

$\therefore$ Wheel rotation in one second is 22 radian.

$\therefore$ Wheel rotation in 55 radian $\frac{55}{22} = 2.5$ second.

(a) $63° 14' 51'' = 63\left(14\frac{51}{60}\right)' = 63\left(14\frac{17}{20}\right)'$

$= 63\left(\frac{297}{20}\right)' = \left(63\frac{297}{20 \times 60}\right)°$

$= \left(63\frac{99}{20 \times 20}\right)° = \left(63\frac{99}{400}\right)°$

$= \left(\frac{25299}{400}\right)° = \left(\frac{25299}{400} \times \frac{\pi}{180}\right)$ rad

$= \left(\frac{2811\pi}{400 \times 20}\right) = \left(\frac{2811\pi}{8000}\right)$

Trick, Value of 63°14'51'' is 60° (approximate)

$\therefore$ Value of 63°14'51'' should be more than $\frac{\pi}{3} = 0{\cdot}33\pi$

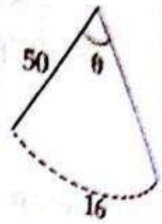
From option (a),

$\left(\frac{2811\pi}{8000}\right) = \left(\frac{2800\pi}{8000}\right)$ (approximate)

$= \left(\frac{28}{80}\pi\right) = 0{\cdot}35\pi$ (approximate)

$\frac{38}{80}\pi = 0{\cdot}47\pi$ (approximate)

So, option (a) is correct.

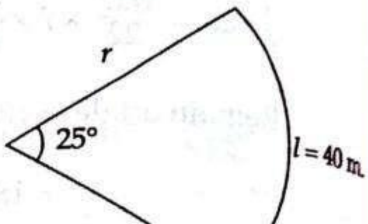


5. (b) $\theta = \frac{l}{r} \Rightarrow \theta = \frac{16}{50}$ rad

$$= \left(\frac{16}{50} \times \frac{180}{\pi}\right)^{\circ} = \left(\frac{16 \times 18}{5} \times \frac{7}{22}\right)^{\circ}$$

$$\left(\frac{16 \times 9 \times 7}{5 \times 11}\right)^{\circ} = \left(\frac{1008}{55}\right)^{\circ}$$

$$= \left(18\frac{18}{55}\right)^{\circ} = 18^{\circ}35' \text{ (approximate)}$$

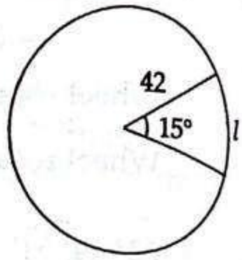


6. (a) $25^{\circ} = 25 \times \frac{\pi}{180}$ rad $= \frac{5\pi}{36}$ rad

From $\theta = \frac{l}{r}$, $\frac{5\pi}{36} = \frac{40}{r}$

$$\therefore \; r = \frac{40 \times 36}{5\pi}$$

$$= \frac{8 \times 36 \times 7}{22} = \frac{2016}{22} = 91{\cdot}64 \text{ meter}$$



7. (b) $15^{\circ} = \frac{15}{180} \times \pi = \frac{\pi}{12}$

From $\theta = \frac{l}{r}$, $l = \theta r$

or, $l = \frac{\pi}{12} \times 42 = \frac{22}{7} \times \frac{1}{12} \times 42 = 11$ cm

★★★