Chapter : 18. AREA OF CIRCLE, SECTOR AND SEGMENT

Exercise : 18A

Question: 1

The difference be

Solution:

Given:

Difference between the circumference and the radius of circle = 37 cm

Let the radius of the circle be ‘r’.

Circumference of the circle = 2πr

So, Difference between the circumference and the radius of the circle = 2πr – r = 37

2πr – r = 37

$2 \times \frac{22}{7} \times r - r = 37$

$\frac{44}{7} \times r - r = 37$

$r\left(\frac{44}{7} - 1\right) = 37$

$\frac{37}{7} \times r = 37$

$r = 37 \times \frac{7}{37}$

r = 7 cm

∴ Circumference of circle = $2 \times \frac{22}{7} \times 7$

= 2 × 22

= 44 cm

Hence the circumference of the circle is 44 cm.

Question: 2

The circumference

Solution:

Given:

Circumference of circle = 22 cm

Let the radius of the circle be ‘r’.

∵ Circumference of circle = 2πr

∴ 22 = 2 × π × r

$\Rightarrow 22 = 2 \times \frac{22}{7} \times r$

$\Rightarrow 22 \times \frac{7}{22} \times \frac{1}{2} = r$ or $\frac{7}{2} = r$

or $r = \frac{7}{2}$

∵ Area of circle = πr^2

∴ Area of its quadrant = $\frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{8}$

Hence the area of the quadrant of the circle is $\frac{77}{8}$ cm.

Question: 3

What is the diame

Solution:

Given:

Let the two circles be C_1 and C_2 with diameters 10 cm and 24 cm respectively.

Area of circle, C = Area of C_1 + Area of C_2 ...... (i)

∵ Diameter = 2 × radius

∴ Radius of C_1, $r_1 = \frac{10}{2} = 5$ cm

and Radius of C_2, $r_2 = \frac{24}{2} = 12$ cm

∵ Area of circle = πr^2 ...... (ii)

∴ Area of $C_1 = \pi r_1^2$

$= \frac{22}{7}\ 5 \times 5$

$= \frac{22}{7} \times 25$

$= \frac{550}{7}$ cm^2

Similarly, Area of $C_2 = \pi r_2^2$

$= \frac{22}{7} \times 12 \times 12$

$= 22/7 \times 144$

$= \frac{3168}{7}$ cm^2

∴ Using equation (i), we have

Area of C $= \frac{550}{7} + \frac{3168}{7}$

$= \frac{3718}{7}$ cm^2

Now, using equation (ii), we have

$\pi r^2 = \frac{3718}{7}$

$\frac{22}{7} \times r^2 = \frac{3718}{7}$

$r^2 = \frac{3718}{7} \times \frac{7}{22}$

$r^2 = 169$

$r = \sqrt{169}$

r = 13 cm

⇒ Diameter = 2 × r

= 2 × 13

= 26 cm

Hence, the diameter of the circle is 26 cm.

Question: 4

If the area of a

Solution:

Given:

Area of circle = 2 × Circumference of circle …… (i)

Let the radius of the circle be ‘r’.

Then, the area of the circle = πr^2

and the circumference of the circle = $2\pi r$

Using (i), we have

$\pi r^2 = 2 \times 2\pi r$

$\pi r^2 = 4\pi r$

r = 4 cm

∵ Diameter = 2 × radius

∴ Diameter = 2 × 4

= 8 cm

Hence, the diameter of the circle is 8 cm.

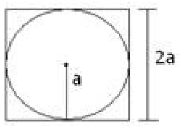
Question: 5

What is the perim

Solution:

Given:

Perimeter of square circumscribes a circle of radius ‘a’.



Side of square = Diameter of circle

Diameter of circle = 2 × radius

= 2a

So, Side of square = 2a

∵ Perimeter of square = 4 × side

∴ Perimeter of square = 4 × 2a

= 8a

Hence, the perimeter of the square is 8a.

Question: 6

Find the length o

Solution:

Given:

Diameter of circle = 42 cm

$\Rightarrow$ Radius of circle = $\frac{42}{2}$ cm = 21 cm

Angle subtended at the centre = 60°

$\because$ Length of arc = $\frac{\theta}{360} \times 2\pi r$

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$

= 22 cm

Hence, the length of the arc is 22 cm.

Question: 7

Find the diameter

Solution:

Given:

Let the two circles with radii 4 cm and 3 cm be C_1 and C_2 respectively.

$\Rightarrow r_1 = 4$ cm and $r_2 = 3$ cm

Area of circle, C = Area of C_1 + Area of C_2 ...... (i)

$\because$ Area of circle = πr^2 ...... (ii)

$\therefore$ Area of $C_1 = \pi r_1^2$

$= \frac{22}{7} \times 4 \times 4$

$= \frac{22}{7} \times 16 = \frac{352}{7}$ cm^2

Similarly, Area of $C_2 = \pi r_2^2$

$= \frac{22}{7} \times 3 \times 3$

$= \frac{22}{7} \times 9 = \frac{198}{7}$ cm^2

So, using (i), we have

Area of C = $\frac{352}{7} + \frac{198}{7} = \frac{550}{7}$ cm^2

Now, using (ii), we have

$\pi r^2 = \frac{550}{7}$

$\frac{22}{7} \times r^2 = \frac{550}{7}$

$r^2 = \frac{550}{7} \times \frac{7}{22} = 25$

$r = \sqrt{25} = 5$

r = 5 cm

$\because$ Diameter = 2 × radius

$\therefore$ Diameter = 2 × 5 = 10 cm

Hence, diameter of the circle with area equal to the sum of two circles of radii 4 cm and 3cm is 10 cm.

Question: 8

Find the area of

Solution:

Given:

Circumference of circle = 8π

∵ Circumference of a circle = 2πr

∴ 8π = 2πr

r = 4

∵ Area of circle = πr^2

∴ Area of circle = π × 4 × 4

= 16π

Hence, the area of the circle is 16π.

Question: 9

Find the perimete

Solution:

Given:

Diameter of the semicircular protractor = 14 cm

Radius of the protractor = $\frac{14}{2}$ cm = 7 cm

∵ Perimeter of semicircle = πr + d

∴ Perimeter of semicircular protractor = $\frac{22}{7}$ × 7 + 14 = 22 + 14

= 36 cm

Hence, the perimeter of the semicircular protractor is 36 cm.

Question: 10

Find the radius o

Solution:

Given:

Perimeter of circle = Area of circle …… (i)

∵ Perimeter of circle = 2πr and Area of circle = πr^2

∴ Using (i), we have

$2\pi r = \pi r^2$

$2 = \frac{\pi r2}{2\pi r}$

2 = r or r = 2

Hence, the radius of the circle is 2 cm.

Question: 11

The radii of two

Solution:

Given:

Radius of one of the circles, C_1 = 19 cm = r_1

Radius of the other circle, C_2 = 9 cm = r_2

Let the other circle be C with radius 'r'.

Circumference of C = Circumference of C_1 + Circumference of C_2 …………(i)

$\because$ Circumference of circle = $2\pi r$

$\therefore$ Circumference of $C_1 = 2\pi r_1 = 2 \times \frac{22}{7} \times 19 = \frac{836}{7}$

and Circumference of $C_2 = 2\pi r_2 = 2 \times \frac{22}{7} \times 9 = \frac{396}{7}$

Using (i), we have

$2\pi r = \frac{836}{7} + \frac{396}{7} = \frac{1232}{7}$

$2 \times \frac{22}{7} \times r = \frac{1232}{7}$

$r = \frac{1232}{7} \times \frac{7}{22} \times \frac{1}{2} = 28$

r = 28 cm

Hence, the radius of the circle is 28 cm.

Question: 12

The radii of two

Solution:

Given:

Radius of one of the circles, C_1 = 8 cm = r_1

Radius of the other circle, C_2 = 6 cm = r_2

Let the other circle be C with radius 'r'.

Area of C = Area of C_1 + Area of C_2 ...... (i)

$\because$ Area of circle = πr^2

$\therefore$ Area of $C_1 = \pi {r_1}^2 = \frac{22}{7} \times 8 \times 8 = \frac{1408}{7}$

and Area of $C_2 = \pi {r_2}^2 = \frac{22}{7} \times 6 \times 6 = \frac{792}{7}$

Using (i), we have

$\pi r^2 = \frac{1408}{7} + \frac{792}{7} = \frac{2200}{7}$

$\frac{22}{7} \times r^2 = \frac{2200}{7}$

$r^2 = \frac{2200}{7} \times \frac{7}{22} = 100$

$r^2 = 100$

$r = \sqrt{100} = 10$ or r = 10

Hence, the radius of the circle is 10 cm.

Question: 13

Find the area of

Solution:

Given:

Radius of circle = 6 cm

Angle of the sector = 30°

$\because$ Area of sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{30}{360} \times 3.14 \times 6 \times 6$

$= 3 \times 3.14 = 9.42\ cm^2$

Hence, the area of the sector is $9.42\ cm^2$.

Question: 14

In a circle of ra

Solution:

Given:

Radius of circle = 21 cm

Angle subtended by the arc = 60°

$\because$ Length of arc $= \frac{\theta}{360} \times 2\pi r$

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21 = 22$ cm

Hence, the length of the arc is 22 cm.

Question: 15

The circumference

Solution:

Given:

Ratio of circumferences of two circles = 2:3

Let the two circles be C_1 and C_2 with radii 'r_1' and 'r_2'.

$\because$ Circumference of circle = $2\pi r$

$\therefore$ Circumference of $C_1 = 2\pi r_1$

and Circumference of $C_2 = 2\pi r_2$

$\Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{2}{3}$

$\Rightarrow \frac{r_1}{r_2} = \frac{2}{3}$

Squaring both sides, we get

$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{2^2}{3^2}$

Multiplying both sides by 'π', we get

$\Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{4}{9}$

$\because$ Area of circle = πr^2

$\Rightarrow \frac{\text{Area of } C_1}{\text{Area of } C_2} = \frac{4}{9}$

Hence, the ratio between the areas of C_1 and C_2 is 4:9.

Question: 16

The areas of two

Solution:

Given:

Ratio of areas of two circles = 2:3

Let the two circles be C_1 and C_2 with radii 'r_1' and 'r_2'.

∵ Area of circle = πr^2

∴ Area of $C_1 = \pi {r_1}^2$

and Area of $C_2 = \pi {r_2}^2$

$\Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{4}{9}$

$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{4}{9}$

Taking square root on both sides, we get

$\Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{4}}{\sqrt{9}}$

$\Rightarrow \frac{r_1}{r_2} = \frac{2}{3}$

Multiplying and dividing L.H.S. by 'π', we get

$\Rightarrow \frac{\pi r_1}{\pi r_2} = \frac{2}{3}$

Multiplying and dividing L.H.S. by '2', we get

$\Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{2}{3}$

As Circumference of circle = $2\pi r$

$\Rightarrow \frac{\text{Circumference of } C_1}{\text{Circumference of } C_2} = \frac{2}{3}$

Hence, the ratio between the circumferences of C_1 and C_2 is 2:3.

Question: 17

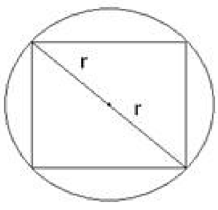
A square is inscr

Solution:

Given:

A square is inscribed in a circle.

Let the radius of circle be 'r' and the side of the square be 'x'.



⇒ The length of the diagonal = 2r

∵ Length of side of square = $\frac{\text{Length of diagonal}}{\sqrt{2}}$

∴ Length of side of square = $\frac{2r}{\sqrt{2}} = \sqrt{2}r$

Area of square = side × side = x × x = $\sqrt{2}r \times \sqrt{2}r = 2r^2$

Area of circle = πr^2

Ratio of areas of circle and square = $\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{2r^2} = \frac{\pi}{2}$

Hence, the ratio of areas of circle and square is π:2.

Question: 18

The circumference

Solution:

Given:

Circumference of circle = 8 cm

Central angle = 72°

∵ Circumference of a circle = 2πr

∴ 2πr = 8

$2 \times \frac{22}{7} \times r = 8$

$r = 8 \times \frac{7}{22} \times \frac{1}{2}$

$r = \frac{14}{11}$ cm

∵ Area of sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{72}{360} \times \pi \times \frac{14}{11} \times \frac{14}{11}$

= 1.02 cm^2

Question: 19

A pendulum swings

Solution:

Given:

Angle made by the pendulum = 30°

Length of the arc made by the pendulum = 8.8 cm

Then the length of the pendulum is equal to the radius of the sector made by the pendulum.

Let the length of the pendulum be 'r'.

∵ Length of arc = $\frac{\theta}{360} \times 2\pi r$

∴ We have,

$\frac{\theta}{360} \times 2\pi r = 8.8$

$\frac{30}{360} \times 2 \times 3.14 \times r = 8.8$

$r = 8.8 \times \frac{360}{30} \times \frac{1}{2} \times \frac{1}{3.14}$

r = 16.8 cm

Hence, the length of the pendulum is 16.8 cm.

Question: 20

The minute hand o

Solution:

Given:

Length of minute hand = 15 cm

Here, the length of the minute hand is equal to the radius of the sector formed by the minute hand.

Angle made by the minute hand in 1 minute = $\frac{360}{60}$ = 6°

Angle made by the minute hand in 20 minutes = 20 × 6 = 120°

Here, the area swept by the minute hand is equal to the area of the corresponding sector made.

$\because$ Area of sector = $\frac{\theta}{360} \times \pi r^2$

= $\frac{120}{360} \times 3.14 \times 15 \times 15 = 235.5$ cm^2

Hence, the area swept by it in 20 minutes is 235.5 cm^2.

Question: 21

A sector of 56°,

Solution:

Given:

Angle of the sector = 56°

Area of the sector = 17.6 cm^2

Let the radius of the circle be 'r'.

$\because$ Area of sector = $\frac{\theta}{360} \times \pi r^2$

$\therefore 17.6 = \frac{56}{360} \times \frac{22}{7} \times r^2$

$r^2 = \frac{360}{56} \times \frac{7}{22} \times 17.6$

$r^2 = 36$

$r = \sqrt{36}$

r = 6 cm

Hence, the radius of the circle is 6 cm.

Question: 22

The area of the s

Solution:

Given:

Radius of the circle = 10.5 cm

Area of the sector = 69.3 cm^2

$\because$ Area of the sector = $\frac{\theta}{360} \times \pi r^2$

$\therefore 69.3 = \frac{\theta}{360} \times \frac{22}{7} \times 10.5 \times 10.5$

$\theta = 69.3 \times 360 \times \frac{7}{22} \times \frac{1}{10.5} \times \frac{1}{10.5}$

$\theta = 72°$

Hence, the central angle is 72°.

Question: 23

The perimeter of

Solution:

Given:

Radius of circle = 6.5 cm

Perimeter of sector = 31 cm

Now, Perimeter of sector = 2 × radius + Length of arc

∵ Length of arc = $\frac{\theta}{360} \times 2r \times 2\pi r$

∴ Perimeter of sector = $2 \times r + \frac{\theta}{360} \times 2r \times \pi$

$= 2r \times [1 + \frac{\theta}{360} \times \pi]$

$31 = 2 \times 6.5 \times [1 + \frac{\theta}{360} \times \frac{22}{7}]$

$31 = 13 \times [1 + \frac{\theta}{360} \times \frac{22}{7}]$

$\frac{31}{13} = 1 + \frac{\theta}{360} \times \frac{22}{7}$

$\frac{31}{13} - 1 = \frac{\theta}{360} \times \frac{22}{7}$

$\frac{18}{13} = \frac{\theta}{360} \times \frac{22}{7}$

$\theta = \frac{18}{13} \times 360 \times \frac{7}{22}$ .... (i)

∵ Area of sector = $\frac{\theta}{360} \times \pi r^2$

∴ using (i), we have

Area = $\frac{18}{13} \times 360 \times \frac{7}{22} \times \frac{1}{360} \times \frac{22}{7} \times 6.5 \times 6.5$

$= 18 \times 3.25 = 58.5$ cm^2

Hence, the area of the sector is 58.5 cm^2.

Question: 24

The radius of a c

Solution:

Given:

Radius of circle = 17.5 cm

Length of arc = 44 cm

∵ Length of arc = $\frac{\theta}{360} \times 2\pi r$

∴ $44 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 17.5$

$\theta = 44 \times 360 \times \frac{1}{2} \times \frac{7}{22} \times \frac{10}{17.5}$

$\theta = \frac{2520}{17.5} = 144°$

Now, Area of sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{144}{360} \times \frac{22}{7} \times 17.5 \times 17.5 = 385$ cm^2

Hence, the area of the sector is 385 cm^2.

Question: 25

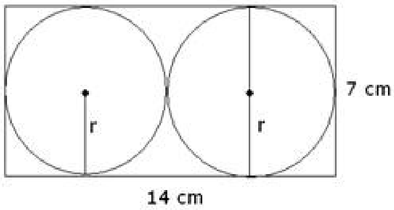
Two circular piec

Solution:

Given:

Length of the rectangular cardboard = 14 cm

Breadth of the rectangular cardboard = 7 cm



∵ Area of rectangle = length × breadth

∴ Area of cardboard = 14 × 7 = 98 cm^2

Let the two circles with equal radii and maximum area have a radius of ‘r’ cm each.

Then, 2r = 7

r = $\frac{7}{2}$ cm

∵ Area of circle = πr^2

∴ Area of two circular cut outs = 2 × πr^2

= $2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

= 11 × 7 = 77 cm^2

Thus, the area of remaining cardboard = 98 – 77 = 21 cm^2

Hence, the area of remaining cardboard is 21 cm^2.

Question: 26

In the given figu

Solution:

Given:

Side of the square = 4 cm

Radius of the quadrants at the corners = 1 cm

Radius of the circle in the centre = 1 cm

∵ 4 quadrants = 1 circle

∴ There are 2 circles of radius 1 cm

Area of square = side × side

= 4 × 4 = 16 cm^2

Area of 2 circles = 2 × πr^2

= $2 \times \frac{22}{7} \times 1 \times 1 = \frac{44}{7}$ cm^2

∵ Area of shaded region = Area of square – Area of 2 circles

= $16 - \frac{44}{7}$

= $\frac{112-44}{7} = \frac{68}{7}$ cm^2 = 9.7 cm^2

Hence, the area of shaded region is 9.72 cm^2.

Question: 27

From a rectangula

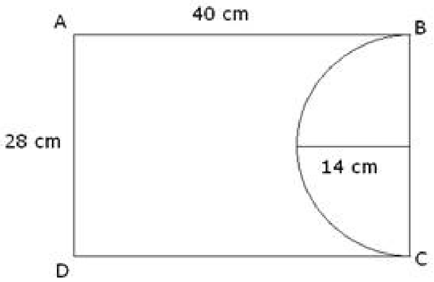
Solution:

Given:

Length of rectangular sheet of paper = 40 cm

Breadth of rectangular sheet of paper = 28 cm

Radius of the semicircular cut out = 14 cm



$\because$ Area of rectangle = length × breadth

$\therefore$ Area of rectangular sheet of paper = 40 × 28

= 1120 cm^2

$\because$ Area of semicircle = $\frac{1}{2}\pi r^2$

$\therefore$ Area of semicircular cut out = $\frac{1}{2} \times \frac{22}{7} \times 14 \times 14$

= 22 × 14 = 308 cm^2

Thus, the area of remaining sheet of paper = Area of rectangular sheet of paper – Area of semicircular cut out

= 1120 – 308 = 812 cm^2

Hence, the area of remaining sheet of paper is 812 cm^2.

Question: 28

In the given figu

Solution:

Given:

Side of square = 7 cm

Radius of the quadrant = 7 cm

Area of square = side × side

= 7 × 7 = 49 cm^2

$\because$ Area of circle = πr^2

$\therefore$ Area of a quadrant = $\frac{1}{4}\pi r^2$

= $\frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

= $\frac{77}{2}$ = 38.5 cm^2

Thus, the area of shaded region = Area of square – Area of quadrant

= 49 – 38.5 = 10.5 cm^2

Hence, the area of the shaded region is 10.5 cm^2.

Question: 29

In the given figu

Solution:

Given:

Radius of circle = 7 cm

Let the sectors with central angles 80°, 60° and 40° be S_1, S_2, and S_3 respectively.

Then, the area of shaded region = Area of S_1 + Area of S_2 + Area of S_3 .... (i)

$\because$ Area of sector $= \frac{\theta}{360} \times \pi r^2$

$\therefore$ Area of $S_1 = \frac{80}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{308}{9}$ cm^2

Similarly, Area of $S_2 = \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{154}{6}$ cm^2

and Area of $S_3 = \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{154}{9}$ cm^2

Thus, using (i), we have

Area of shaded region $= \frac{308}{9} + \frac{154}{6} + \frac{154}{9}$

$= \frac{616 + 462 + 308}{18}$

$= \frac{1386}{18} = 77$ cm^2

Hence, the area of shaded region is 77 cm^2.

Question: 30

In the given figu

Solution:

Given:

Radius of inner circle = 3.5 cm

Radius of outer circle = 7 cm

$\angle POQ = 30°$

Let the sector made by the arcs PQ and AB be S_1 and S_2 respectively.

Then, Area of shaded region = Area of S_1 – Area of S_2 ....(i)

$\because$ Area of sector $= \frac{\theta}{360} \times \pi r^2$

$\therefore$ Area of $S_1 = \frac{30}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{71}{6}$ cm^2

Similarly, Area of $S_2 = \frac{30}{360} \times \frac{22}{7} \times 3.5 \times 3.5$

$= \frac{77}{24}$ cm^2

Thus, using (i), we have

Area of shaded region $= \frac{77}{6} - \frac{77}{24}$

$= \frac{308-77}{24}$

$= \frac{231}{24} = \frac{77}{8}$ cm^2

Hence, the area of shaded region is $\frac{77}{8}$ cm^2.

Question: 31

In the given figu

Solution:

Given:

Side of square = 14 cm

Diameter of each semicircle = 14 cm

Radius of each semicircle = $\frac{14}{2}$ = 7 cm

∵ Both the semicircles have same radius.

∴ We consider one circle of radius 7 cm.

Area of shaded region = Area of square – Area of circle ……….. (i)

Area of square = side × side

= 14 × 14 = 196 cm^2

Area of circle = πr^2

= $\frac{22}{7}$ × 7 × 7 = 22 × 7 = 154 cm^2

Thus, using (i), we have

Area of shaded region = 196 – 154 = 42 cm^2

Hence, the area of shaded region is 42 cm^2.

Question: 32

In the given figu

Solution:

Give:

Radius of the circle = 42 cm

Central angle of the sector = ∠AOB = 90°

Perimeter of the top of the table = Length of the major arc AB + 2 × radius …………………. (i)

Length of major arc AB = $\frac{(360-\theta)}{360}$ × 2πr

= $\frac{(360-90)}{360}$ × 2 × $\frac{22}{7}$ × 42

= $\frac{270}{360}$ × 2 × 22 × 6

= $\frac{3}{4}$ × 264 = 3 × 66 = 198 cm

Thus, using (i), we have

Perimeter of the top of the table = 198 + 2 × 42

= 198 + 84 = 282 cm

Hence, the perimeter of the top of the table is 282 cm.

Question: 33

In the given figu

Solution:

Given:

Side of square = 7 cm

Radius of each quadrant = 7 cm

Area of square = side × side = 7 × 7 = 49 cm^2

∵ Area of quadrant = $\frac{1}{4}\pi r^2$

∴ Area of 2 quadrants = $2 \times \frac{1}{4} \times \pi r^2$

= $\frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

= 77 cm^2

Area of shaded region = Area of 2 quadrants – Area of square

= 77 – 49 = 28 cm^2

Hence, the area of shaded region is 28 cm^2.

Question: 34

In the given figu

Solution:

Given:

Radius of Circle = 3.5 cm

OD = 2 cm

∵ Area of Quadrant = $\frac{1}{4}\pi r^2$

∴ Area of Quadrant OABC = $\frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

= 9.625 cm^2

∵ Area of Triangle = $\frac{1}{2}$ × Base × Height

∴ Area of Δ COD = $\frac{1}{2} \times 3.5 \times 2$

= 3.5 cm^2

Area of Shaded Region = Area of Quadrant OABC – Area of Δ COD

= 38.5 – 3.5 = 35 cm^2

Hence, the area of shaded region is 35 cm^2.

Question: 35

Find the perimete

Solution:

Given:

Side of square = 14 cm

Diameter of semi circle = 14 cm

⇒ Radius of semi circle = $\frac{14}{2}$ = 7 cm

∵ There are 2 semi circles of same radius.

∴ We consider it as one circle with radius 7 cm.

So,

Perimeter of 2 semicircles = Perimeter of circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 7$

$= 2 \times 22 = 44$ cm

Perimeter of shaded region = Perimeter of 2 semicircles + 2 × Side of Square = 44 + 2 × 14 = 44 + 28 = 72 cm

Hence, the area of the shaded region is 72 cm.

Question: 36

In a circle of ra

Solution:

Given:

Radius of the circle = 7 cm

Diameter of the circle = 14 cm

Here, diagonal of square = 14 cm

$\because$ Side of a square = $\frac{\text{diagonal}}{\sqrt{2}}$

$\Rightarrow$ Side = $\frac{14}{\sqrt{2}} = 7\sqrt{2}$ cm

$\Rightarrow$ Area of square = side × side

$= 7\sqrt{2} \times 7\sqrt{2}$

$= 49 \times 2 = 98$ cm^2

Area of circle = πr^2

$= \frac{22}{7} \times 7 \times 7 = 22 \times 7 = 154$ cm^2

Thus, the area of the circle outside the square

= Area of circle – Area of square = 154 – 98 = 56 cm^2

Hence, the area of the required region is 56 cm^2.

Question: 37

In the given figu

Solution:

(i) Given:

Diameter of semicircles APB and CQD = 7 cm

$\Rightarrow$ Radius of semicircles APB and CQD = $\frac{7}{2}$ cm = r_1

Diameter of semicircles ARC and BSD = 14 cm

$\Rightarrow$ Radius of semicircles ARC and BSD = $\frac{14}{2}$ cm = 7 cm = r_2

Perimeter of APB = Perimeter of CQD

Area of APB = Area of CQD .... (i)

Perimeter of ARC = Perimeter of BSD

Area of ARC = Area of BSD .... (ii)

$\because$ Perimeter of semicircle = πr .... (iii)

$\therefore$ Perimeter of APB = πr_1

$= \frac{22}{7} \times \frac{7}{2} = 11$ cm

Then, using (i), we have

Perimeter of CQD = 11 cm

Now, using (iii), we have

Perimeter of ARC = πr_2

$= \frac{22}{7} \times 7 = 22$ cm

Then, using (ii), we have

Perimeter of BSD = 22 cm

Perimeter of shaded region

= (Perimeter of ARC + Perimeter of APB) + (Perimeter of BSD + Perimeter of CQD)

= (22 + 11) + (22 + 11) = 33 + 33 = 66 cm

Hence, the perimeter of the shaded region is 66 cm.

(ii) Now,

$\because$ Area of semicircle $= \frac{1}{2} \pi r^2$ .... (iv)

$\therefore$ Area of APB $= \frac{1}{2} \pi r_1^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{4}$ cm^2

Then, using (i), we have

Area of CQD $= \frac{77}{4}$ cm^2

Now, using (iv), we have

Area of ARC $= \frac{1}{2} \pi r_2^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 11 \times 7 = 77$ cm^2

Then, by using (ii), we have

Area of BSD = 77 cm^2

Area of shaded region

= (Area of ARC-Area of APB) + (Area of BSD- Area of CQD)

$= (77 - \frac{77}{4}) + (77 - \frac{77}{4})$

$= (\frac{308-77}{4}) + (\frac{308-77}{4}) = \frac{231}{4} + \frac{231}{4} = \frac{462}{4} = 115.5$ cm^2

Hence, the area of the shaded region is 115.5 cm^2.

Question: 38

In the given figu

Solution:

Given:

Diameter of semicircle PSR = 10 cm

$\Rightarrow$ Radius of semicircle PSR $= \frac{10}{2} = 5$ cm $= r_1$

Diameter of semicircle RTQ = 3 cm

⇒ Radius of semicircle RTQ = $\frac{3}{2}$ = 1.5 cm = r_2

Diameter of semicircle PAQ = 7 cm

⇒ Radius of semicircle PAQ = $\frac{7}{2}$ = 3.5 cm = r_3

∵ Perimeter of semicircle = πr

∴ Perimeter of semicircle PSR = πr_1

= 3.14 × 5 = 15.7 cm

Similarly, Perimeter of semicircle RTQ = πr_2

= 3.14 × 1.5 = 4.71 cm

and Perimeter of semicircle PAQ = πr_3

= 3.14 × 3.5 = 10.99 cm

Perimeter of shaded region = Perimeter of semicircle PSR

+ Perimeter of semicircle RTQ

+ Perimeter of semicircle PAQ

= 15.7 + 4.71 + 10.99 = 31.4 cm

Hence, the perimeter of the shaded region is 31.4 cm.

Question: 39

In the given figu

Solution:

Given:

OA = Side of square OABC = 20 cm

∵ Area of square = Side × Side

∴ Area of square OABC = 20 × 20 = 400 cm^2

Now,

∵ Length of diagonal of square = √2 × Side of Square

∴ Length of diagonal of square OABC = √2 × 20 = 20√2 cm

⇒ Radius of the quadrant = 20√2 cm

∵ Area of quadrant = $\frac{1}{4}\pi r^2$

∴ Area of quadrant OPBQ = $\frac{1}{4}$ × 3.14 × 20√2 × 20√2

= $\frac{3.14}{4}$ × 400 × 2

= 3.14 × 200 = 628 cm^2

Area of shaded region = Area of quadrant OPBQ – Area of square OABC = 628 – 400 = 228 cm^2

Hence, the area of the shaded region is 228 cm^2.

Question: 40

In the given figu

Solution:

Given:

AO = OB

Perimeter of the figure = 40 cm............. (i)

Let the diameters of semicircles AQO and APB be 'x_1' and 'x_2' respectively.

Then, using (1), we have

AO = OB

Also, AB = AO + OB = AO + AO = 2AO

$\Rightarrow x_2 = 2x_1$

So, diameter of APB = $2x_1$

and diameter of AQO = x_1

Radius of APB = x_1

and Radius of AQO = $\frac{x_1}{2}$ .... (ii)

Perimeter of shaded region = perimeter of AQO + perimeter APB + diameter of APB .... (iii)

$\because$ Perimeter of semicircle = πr

$\therefore$ Perimeter of semicircle AQO = $\frac{22}{7} \times \frac{x_1}{2} = \frac{11x_1}{7}$ cm

Perimeter of semicircle APB = $\frac{22}{7} \times x_1 = \frac{22x_1}{7}$ cm

Now, using (iii), we have

$40 = \frac{11x_1}{7} + \frac{22x_1}{7} + x_1$

$40 = \frac{11x_1 + 22x_1 + 7x_1}{7}$

$40 \times 7 = 40x_1$

$280 = 40x_1$

$x_1 = \frac{280}{40} = 7$ cm

$\therefore$ using (ii), we have

Radius of APB = 7 cm = r_1

And Radius of AQO = $\frac{7}{2}$ cm = 3.5 cm = r_2

Now,

$\because$ Area of semicircle = $\frac{1}{2}\pi r^2$

$\therefore$ Area of semicircle APB = $\frac{1}{2}\pi {r_1}^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 11 \times 7 = 77 \text{ cm}^2$

Similarly,

Area of semicircle APB = $\frac{1}{2}\pi {r_2}^2$

$= \frac{1}{2} \times \frac{22}{7} \times 3.5 \times 3.5 = 19.25 \text{ cm}^2$

Thus, Area of shaded region = Area of APB + Area of AQO

$= 77 + 19.25 = 96.25 \text{ cm}^2$

Hence, the area of the shaded region is 96.25 cm^2.

Question: 41

Find the area of

Solution:

Given:

Circumference of circle = 44 cm

Let the radius of the circle be ‘r’ cm

∵ Circumference of circle = 2πr

∴ 44 = 2πr

$\frac{44}{2} = \frac{22}{7} \times r$

$r = 22 \times \frac{7}{22} = 7$ cm

Now, Area of quadrant = $\frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

$= \frac{11 \times 7}{2} = \frac{22}{7} = 38.5$ cm^2

Hence, the area of the quadrant is 38.5 cm^2.

Question: 42

In the given figu

Solution:

Given:

Side of square = 14 cm

Let the radius of each circle be ‘r’ cm

Then, 2r + 2r = 14 cm

4r = 14 cm

$r = \frac{14}{4} = \frac{7}{2}$

Area of square = side × side = 14 × 14 = 196 cm^2

∵ Area of circle = πr^2

∴ Area of 4 circles = $4 \times \pi r^2$

$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

= 22 × 7 = 154 cm^2

Area of shaded region = Area of the square – Area of 4 circles

= 196 -154

= 42 cm^2

Hence, the area of the shaded region is 42 cm^2.

Question: 43

Find the area of

Solution:

Given:

Length of rectangle = 8 cm

Breadth of rectangle = 6 cm

Area of rectangle = length × breadth

$= 8 \times 6 = 48\ cm^2$

Consider Δ ABC,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 8^2 + 6^2 = 64 + 36 = 100$

$AC = \sqrt{100} = 10$ cm

⇒ Diameter of circle = 10 cm

Thus, radius of circle = $\frac{10}{2} = 5$ cm

Let the radius of circle be r = 5 cm

Then, Area of circle = πr^2

$= \frac{22}{7} \times 5 \times 5 = \frac{22 \times 25}{7} = \frac{550}{7} = 78.57\ cm^2$

Area of shaded region = Area of circle – Area of rectangle

= 78.57 - 48

$= 30.57\ cm^2$

Hence, the area of shaded region is $30.57\ cm^2$.

Question: 44

A wire is bent to

Solution:

Given:

Perimeter of square = Circumference of circle ……………… (i)

Area of Square = $484 m^2$

Let the side of square be ‘x’ cm.

∵ Area of Square = side × side

∴ 484 = x × x

$x^2 = 484$

$x = \sqrt{484} = 22$cm

∵ Perimeter of square = 4 × side

= 4 × 22 = 88 cm

∴ Using (i), we have

Circumference of circle = 88 cm

Also, Circumference of Circle = $2\pi r$

$2\pi r = 88$

$2 \times \frac{22}{7} \times r = 88$

$r = 88 \times \frac{1}{2} \times \frac{7}{22}$

$r = 2 \times 7 = 14$ cm

Area of Circle = $\pi r^2 = \frac{22}{7} \times 14 \times 14$

$= 22 \times 2 \times 14 = 616$ cm^2

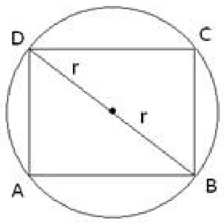
Hence, the area of Circle is 616 cm^2.

Question: 45

A square ABCD is

Solution:

Given: Radius of circle = r



Diagonal of Square = 2r

$\because$ Side of Square = $\frac{\text{length of diagonal}}{\sqrt{2}}$

$\therefore$ Side = $\frac{2r}{\sqrt{2}} = \sqrt{2}r$

Area of Square = Side × Side

$= \sqrt{2}r \times \sqrt{2}r$

$= 2r^2$

Hence, the area of square is ‘$2r^2$’ square units.

Question: 46

The cost of fenci

Solution:

Given:

Rate of fencing a circular field = Rs. 25/m

Cost of fencing a circular field = Rs. 5500

Rate of ploughing the field = 50p/m^2 = Rs. 0.5/m^2

Let the radius of circular field be ‘r’ and the length of the field fenced be ‘x’ m.

Then, $25 \times x = 5500$

$x = \frac{5500}{25} = 220$ m

$\because$ Circumference of circular field = $2\pi r$

$\therefore 220 = 2\pi r$

$220 = 2 \times \frac{22}{7} \times r$

$r = \frac{220 \times 7}{2 \times 22}$

r = 35 m

Area of the circular field = πr^2

$= \frac{22}{7} \times 35 \times 35$

$= 22 \times 5 \times 35$

$= 3850m^2$

Now, cost of ploughing the field = Rate of ploughing the field × Area of the field = 0.5 × 3850

= Rs. 1925

Hence, the cost of Ploughing the field is Rs. 1925.

Question: 47

A park is in the

Solution:

Given:

Length of the rectangular park = 120 m

Breadth of the rectangular park = 90 m

Area of the park excluding the circular lawn = $2950m^2$

Area of the rectangular park = length × breadth

= 120 × 90

$= 10800m^2$

Area of circular lawn = Area of rectangular park – Area of park excluding the lawn

= 10800 – 2950

$= 7850m^2$

∵ Area of circle = πr^2

∴ $7850 = 3.14 \times r^2$

$r^2 = \frac{7850}{3.14} = 2500$

$r = \sqrt{2500} = 50$ m

Hence, the radius of the circular lawn is 50m.

Question: 48

In the given figu

Solution:

Given:

$OP = 21 \text{ m} = r_1$

$OR = 14 \text{ m} = r_2$

Let the quadrants made by outer and inner circles be Q_1 and Q_2, with radius r_1 and r_2 respectively.

Then, Area of flower bed = Area of Q_1 – Area of Q_2

∵ Area of Quadrant = $\frac{1}{4}\pi r^2$

∴ Area of $Q_1 = \frac{1}{4}\pi r_1^2$

$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$

$= \frac{693}{2} m^2$

Similarly, Area of $Q_2 = \frac{1}{4}\pi r_2^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= \frac{308}{2}$ m^2

Thus, Area of flower bed = $\frac{693}{2} - \frac{308}{2}$

$= \frac{385}{2} = 192.5$ m^2

Hence, the area of the flower bed is 192.5 m^2.

Question: 49

In the given figu

Solution:

Given:

AC = 54 cm

BC = 10 cm

⇒ AB = AC-BC = 54-10 = 44 cm

Radius of bigger circle = $\frac{AC}{2} = \frac{54}{2} = 27$ cm = r_1

Radius of Smaller circle = $\frac{AB}{2} = \frac{44}{2} = 22$ cm = r_2

∵ Area of Circle = πr^2

∴ Area of Bigger Circle = πr_1^2

$= \frac{22}{7} \times 27 \times 27$

$= \frac{16038}{7}$ cm^2

Similarly, Area of Smaller Circle = πr_2^2

$= \frac{22}{7} \times 22 \times 22$

$= \frac{10648}{7}$ cm^2

Area of shaded region = Area of Bigger Circle – Area of Smaller Circle = $\frac{16038}{7} - \frac{10648}{7} = \frac{5390}{7} = 770$ cm^2

Hence, Area of Shaded Region is 770 cm^2.

Question: 50

From a thin metal

Solution:

Given:

AB ∥ CD

∠BCD = 90°

AB = BC = 3.5 cm = EC

DE = 2 cm

DC = DE + EC = 2 + 3.5 = 5.5 cm

Area of Trapezium = $\frac{1}{2} \times$ Sum of Parallel Sides × h

$= \frac{1}{2} \times (AB + DC) \times BC$

$= \frac{1}{2} \times (3.5 + 5.5) \times 3.5$

$= \frac{1}{2} \times 9 \times 3.5$

$= 15.75\ cm^2$

Area of Quadrant BFEC $= \frac{1}{4} \times \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

$= 9.625\ cm^2$

Thus, Area of remaining part of metal sheet

= Area of Trapezium – Area of Quadrant BFEC

$= 15.75 - 9.625 = 6.125\ cm^2$

Hence, the area of the remaining part of metal sheet is $6.125\ cm^2$.

Question: 51

Find the area of

Solution:

Given:

Radius of Circle = 35 cm

$\angle AOB = 90°$

$\because$ Area of Sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{90}{360} \times \frac{22}{7} \times 35 \times 35$

$= \frac{1925}{2}\ cm^2$

$\because$ Δ AOB is right-angled triangle.

$\therefore$ Area of Δ AOB $= \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 35 \times 35$

$= \frac{1225}{2}\ cm^2$

Now, Area of Minor Segment ACB

= Area of Sector – Area of ΔAOB

$= \frac{1925}{2} - \frac{1225}{2} = \frac{700}{2} = 350\ cm^2$

Area of Circle $= \pi r^2$

$= \frac{22}{7} \times 35 \times 35$

$= 22 \times 5 \times 35$

$= 3850\ cm^2$

Thus, Area of Major Segment = Area of Circle – Area of Minor Segment $= 3850 - 350 = 3500\ cm^2$

Hence, the area of the major segment is $3500\ cm^2$.

Exercise : 18B

Question: 1

The circumference

Solution:

In order to solve such type of questions we basically need to find the radius of the give circle and simply use it to find the area of the given circle.

Given the circumference or perimeter of the circle = 39.6 cm.

And we know, Perimeter or circumference of circle = $2\pi r$

Where, r = Radius of the circle

Therefore, $2\pi r = 39.6$

$$\Rightarrow r = \frac{39.6}{2\pi}$$

(put value of $\pi = 22/7$)

$$\Rightarrow r = \frac{39.6}{2 \times \frac{22}{7}}$$

On rearranging we get,

$$\Rightarrow r = \frac{39.6 \times 7}{2 \times 22}$$

$$\Rightarrow r = \frac{277.2}{44}$$

$\Rightarrow r = 6.3$ cm

So, the radius of the circle = 6.3 cm

And we also know, Area of the circle = πr^2

Where, r = radius of the circle

$\Rightarrow$ Area of the circle = $\pi(6.3)^2$

(putting value of r)

$$= \frac{22}{7}(6.3^2)$$

$$= \frac{22}{7}(6.3 \times 6.3)$$

$$= \frac{22}{7} \times 39.69$$

$= 22 \times 5.67$

$= 124.74\ cm^2$

The area of the circle = $124.74\ cm^2$.

Question: 2

In order to solve such type of questions we basically need to find the radius of the give circle and simply use it to find the are circumference or perimeter of the given circle.

Given the area of the circle = $98.56\ cm^2$

And we also know, Area of the circle = πr^2

Therefore, $\pi r^2 = 98.56$

$$\Rightarrow r^2 = \frac{98.56}{\pi}$$

(put value of $\pi = 22/7$)

$$\Rightarrow r^2 = \frac{98.56}{\frac{22}{7}}$$

On rearranging we get,

$$\Rightarrow r^2 = \frac{98.56 \times 7}{22}$$

$$\Rightarrow r^2 = \frac{689.92}{22}$$

$$\Rightarrow r^2 = 31.36$$

$\Rightarrow r = \sqrt{31.36}$

$\Rightarrow r = 5.6$ cm

So, the radius of the circle = 5.6 cm

And we know, Perimeter of circle = $2\pi r$

(put value of r)

⇒ Circumference or Perimeter of circle = $2\pi(5.6)$

$$= 2 \times \frac{22}{7} \times 5.6 \text{ (put } \pi = \frac{22}{7})$$

$$= \frac{2 \times 22 \times 5.6}{7}$$

$$= \frac{246.4}{7}$$

= 35.2 cm

The circumference or perimeter of the circle is 35.2 cm

Question: 3

Given, the circumference of a circle exceeds its diameter by 45 cm.

⇒ Circumference of circle = Diameter of circle + 45

Let ‘d’ = diameter of the circle

⇒ Circumference = d + 45 → eqn1

And we know, Circumference of a circle = $2\pi r$ → eqn2

Where r = radius of circle

Also, we know that the radius of the circle is half of its diameter.

$$\Rightarrow r = \frac{d}{2} \rightarrow \text{eqn3}$$

Put value of circumference in equation 1 from equation 2

⇒ $2\pi r = d + 45$ → eqn4

Put value of r in equation 4 from equation 3

$$\Rightarrow 2\pi\left(\frac{d}{2}\right) = d + 45$$

⇒ $\pi d = d + 45$

⇒ $\pi d - d = 45$

⇒ $(\pi - 1)d = 45$ (taking d common from L.H.S)

$$\Rightarrow d = \frac{45}{\pi - 1} \text{ (now put } \pi = \frac{22}{7})$$

$\Rightarrow d = \dfrac{45}{\frac{22}{7} - 1}$

$\Rightarrow d = \dfrac{45}{\frac{22-7}{7}}$ (taking 7 as LCM in denominator)

$\Rightarrow d = \dfrac{45}{\frac{15}{7}}$

On rearranging, we get

$\Rightarrow d = \dfrac{45 \times 7}{15}$

$\Rightarrow d = \dfrac{315}{15}$

$\Rightarrow d = 21$ cm

Therefore, the diameter of the circle is 21 cm.

Thus, the radius of the circle $r = \frac{d}{2}$ (from equation 3)

$\therefore r = \dfrac{21}{2}$

$\Rightarrow r = 10.5$ cm

Now put the value of r in equation 2, we get

$\Rightarrow$ Circumference or Perimeter of circle $= 2\pi(10.5)$ (put $\pi = \dfrac{22}{7}$)

$= 2 \times \dfrac{22}{7} \times 10.5$

$= \dfrac{2 \times 22 \times 10.5}{7}$

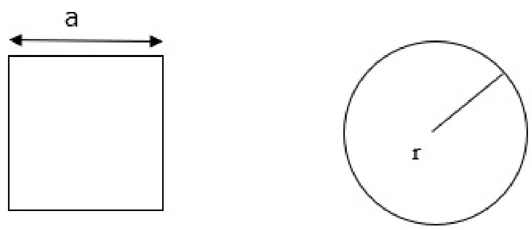
$= \dfrac{462}{7}$

$= 66$ cm

The circumference of the circle is 66 cm.

Question: 4

In this question the wire is first bent in the shape of square and then same wire is bent to form a circle. The point to be noticed is that the same wire is used both the times which implies that the perimeter of square and that of circle will be equal.



Let the square be of side ‘a’ cm and radius of the circle be ‘r’

Given the area enclosed by the square = 484 cm^2

Also, we know that Area of square = Side × Side

Area of the square = a^2

$\Rightarrow a^2 = 484$

$\Longrightarrow$ a = $\sqrt{484}$

$\Rightarrow$ a = 22 cm

Therefore, side of square, ‘a’ is 22 cm.

Also, circumference of the circle = Perimeter of square → eqn1

Perimeter of square = 4 × side

Perimeter of square = 4×22

$\Rightarrow$ Perimeter of square = 88 cm → eqn2

Also, we know, Circumference of circle = 2πr → eqn3

Put values in equation 1 from equation 2 & 3, we get

2πr = 88

$$\Longrightarrow r = \frac{88}{2\pi} \text{ (put } \pi = \frac{22}{7}\text{)}$$

$$\Longrightarrow r = \frac{88}{2 \times \frac{22}{7}}$$

On rearranging,

$$\Longrightarrow r = \frac{88 \times 7}{2 \times 22}$$

$$\Longrightarrow r = \frac{616}{44}$$

$\Rightarrow$ r = 14 cm

So, the radius ‘r’ of the circle is 14 cm.

Area of circle = πr^2

Where r = radius of the circle

= $\pi(14^2)$

$$= \frac{22}{7} \times 14 \times 14 \text{ (put } \pi = \frac{22}{7}\text{)}$$

$$= \frac{22 \times 14 \times 14}{7}$$

= 4312/7

= 616 cm^2

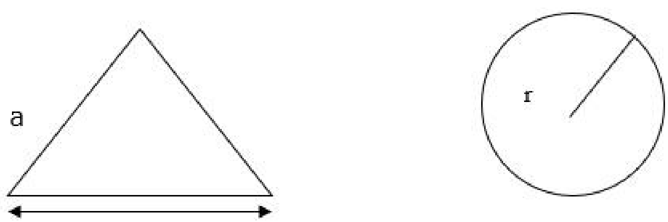
Area of the circle is 616 cm^2.

Question: 5

A wire when

Solution:

In this question the wire is first bent in the shape of equilateral triangle and then same wire is bent to form a circle. The point to be noticed is that the same wire is used both the times which implies that the **perimeter of equilateral triangle and that of circle will be equal.**



Let the equilateral triangle be of side ‘a’ cm and radius of the circle be ‘r’.

Given: Area enclosed by equilateral triangle = $123\sqrt{3}$ cm^2

Also, we know that Area of equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

Where ‘a’ = side of equilateral triangle

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 121\sqrt{3}$$

$$\Rightarrow a^2 = \frac{121\sqrt{3}}{\frac{\sqrt{3}}{4}}$$

$$\Rightarrow a^2 = \frac{121\sqrt{3} \times 4}{\sqrt{3}}$$

$$\Rightarrow a^2 = \frac{484\sqrt{3}}{\sqrt{3}}$$

$\Longrightarrow a = \sqrt{484}$

⇒ a = 22 cm

Therefore, side of equilateral triangle, ‘a’ is 22 cm.

Also, circumference of the circle = Perimeter of equilateral triangle → eqn1

Perimeter of equilateral triangle = 3 × side

= 3 × 22

= 66 cm → eqn2

Also, we know Circumference of circle = $2\pi r$ → eqn3

Put values in equation 1 from equation 2 & 3, we get

$2\pi r = 66$

$$\Rightarrow r = \frac{66}{2\pi}$$

(put $\pi = 22/7$)

$$\Rightarrow r = \frac{66}{2 \times \frac{22}{7}}$$

On rearranging,

$$\Rightarrow r = \frac{66 \times 7}{2 \times 22}$$

$$\Rightarrow r = \frac{462}{44}$$

⇒ r = 10.5 cm

So, the radius ‘r’ of the circle is 10.5 cm.

Area of circle = πr^2

Where r = radius of the circle

⇒ Area of circle = $\pi(10.5^2)$

$$\Rightarrow \text{Area of circle} = \frac{22}{7} \times 10.5 \times 10.5 \text{ (put } \pi = \frac{22}{7})$$

$= \frac{22 \times 10.5 \times 10.5}{7}$

$= \frac{2425.5}{7}$

= 346.5 cm^2

Area of the circle is 346.5 cm^2.

Question: 6

In this question the length of chain used as boundary of the semicircular park is the perimeter of the semicircular park. By using this we will first calculate the radius of the semicircular park and then area of semicircle consequently.

Length of chain = 108 m

Length of chain = Perimeter or circumference of semicircle

Therefore, Circumference or Perimeter of semicircle = 108 m

Also, Circumference or Perimeter of semicircle = πr

Where r = radius of semicircle

⇒ πr = 108

$\Rightarrow r = \frac{108}{\pi}$

(put π = 22/7)

$\Rightarrow r = \frac{108}{\frac{22}{7}}$

On rearranging,

$\Rightarrow r = \frac{108 \times 7}{22}$

$\Rightarrow r = \frac{756}{22}$

⇒ r = 34.46 m

Therefore, radius of semicircle is 34.36 m

As, Area of semicircle $= \frac{\pi r^2}{2} \rightarrow$ eqn1

Put value of ‘r’ in equation 1, we get

Area of semicircle $= \frac{\pi(34.36^2)}{2}$

(put π = 22/7)

$= \frac{\frac{22}{7} \times 34.36 \times 34.36}{2}$

On rearranging,

$= \frac{22 \times 34.3636 \times 34.3636}{7 \times 2}$

$= \frac{25973.4112}{14}$

= 1855.63 m^2

The area of the semicircular park is 1855.63 m^2.

Question: 7

Given Sum of the radius of the circles = 7 cm

the difference of their circumference = 8 cm

Let the radius one circle be 'r_1' cm and other be 'r_2' cm and circumference be 'C_1' and 'C_2' respectively.

Also, circumference of circle = $2\pi r$

Where r = radius of the circle

$C_1 = 2\pi r_1$ and $C_2 = 2\pi r_2$

$r_1 + r_2 = 7 \rightarrow$ eqn1

$C_1 - C_2 = 8 \rightarrow$ eqn2

(Note: Her it is considered that $r_1 > r_2$)

We can rewrite equation 2 as,

$2\pi r_1 - 2\pi r_2 = 8$

$\Rightarrow 2\pi(r_1 - r_2) = 8$

(taking 2π common from L.H.S)

$$\Longrightarrow r_1 - r_2 = \frac{8}{2\pi} \rightarrow \text{eqn3}$$

$$\Longrightarrow r_1 - r_2 = \frac{8}{2 \times \frac{22}{7}}$$

$$\Longrightarrow r_1 - r_2 = \frac{8 \times 7}{44}$$

$$\Longrightarrow r_1 - r_2 = \frac{56}{44}$$

$$\Longrightarrow r_1 - r_2 = \frac{14}{11}$$

$$\Longrightarrow r_1 = \frac{14}{11} + r_2 \rightarrow \text{eqn3}$$

Put the value of r_1 from equation 3 in equation 1

$$\frac{14}{11} + r_2 + r_2 = 7$$

$$\Longrightarrow \frac{14}{11} + 2r_2 = 7$$

$$\Longrightarrow 2r_2 = 7 - \frac{14}{11}$$

$$\Longrightarrow 2r_2 = \frac{77 - 14}{11}$$

(taking 11 as LCM on R.H.S)

$$\Longrightarrow 2r_2 = \frac{63}{11}$$

$$\Longrightarrow r_2 = \frac{63}{2 \times 11}$$

$$\Longrightarrow r_2 = \frac{63}{22} \text{ cm}$$

Put value of r_2 in equation 3

$\therefore\ r_1 = \frac{14}{11} + \frac{63}{22}$(from equation 3)

$\Rightarrow r_1 = \frac{28+63}{22}$ (taking 22 as LCM on R.H.S)

$\Rightarrow r_1 = \frac{91}{22}$ cm

$\therefore\ C_1 = 2\pi\left(\frac{91}{22}\right)$

(by putting value of r_1)

$\Rightarrow C_1 = 2 \times \frac{22}{7} \times \frac{91}{22}$

$= \frac{2 \times 22 \times 91}{7 \times 22}$

$= \frac{2 \times 91}{7}$

= 182/7

= 26 cm

$C_2 = 2\pi\left(\frac{63}{22}\right)$ (by putting value of r_2)

$\Rightarrow C_1 = 2 \times \frac{22}{7} \times \frac{63}{22}$

$= \frac{2 \times 22 \times 63}{7 \times 22}$

$= \frac{2 \times 63}{7}$

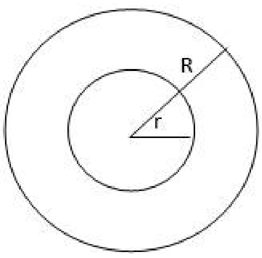
= 126/7

= 18 cm

The circumference of circles are 26 cm and 18 cm.

Question: 8

Consider the ring as shown in the figure below,



The inner radius of ring is ‘r’ and the outer radius is ‘R’.

Area of inner Circle = πr^2 and Area of outer Circle = πR^2

Where r = 12 cm and R = 23 cm

Area of ring = Area of outer circle – Area of inner circle

Area o ring = $\pi R^2 - \pi r^2$ (put values of r & R)

⇒ Area of ring = $\pi(23^2) - \pi(12^2)$

⇒ Area of ring = $\pi(23^2 - 12^2)$ (taking π common from R.H.S)

⇒ Area of ring = π(529 – 144)

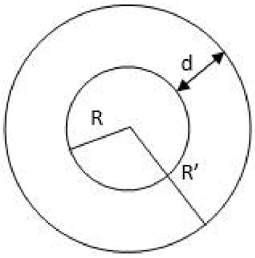
$= \dfrac{22 \times 385}{7}$

$= \dfrac{8470}{7}$

= 1210 cm^2

Area of ring is 1210 cm^2.

Question: 9



Given radius of circular park = R = 17 m

Width of the circular path outside the park = d = 8 m

Therefore, the radius of the outer circle = R’ = R + d

Outer radius = R’ = 17 + 8

R’ = 25 m

Area of inner circle = πR^2 and,

Area of outer circle = $\pi R'^2$

Area of path = Area of outer circle – Area of inner circle

= $\pi R'^2 - \pi R^2$ (put values of R’ & R)

= $\pi(25^2) - \pi(17^2)$

= $\pi(25^2 - 17^2)$ (taking π common from R.H.S)

= π(625 – 289)

$\Rightarrow$ Area of path $= \dfrac{22}{7} \times 336$

(put π = 22/7)

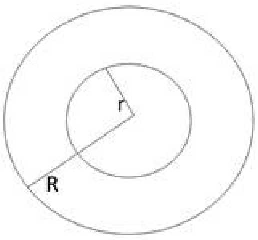
= 7392/7

= 1056 m^2

The area of the path is 1056 m^2.

Question: 10

Consider the race track as shown below,



The inner and outer radius of track is ‘r’ cm and ‘R’ cm respectively.

Let inner and outer circumference be ‘C_1’ and C_2’ respectively.

C_1 = 352 m and C_2 = 396 m.

We know,

Circumference of circle = $2\pi r$

Where r = radius of the circle

$C_1 = 2\pi r$ and $C_2 = 2\pi R$

$\Rightarrow 2\pi r = 352$ and $2\pi R = 396$

$$\Rightarrow r = \frac{352}{2\pi} \text{ and } R = \frac{396}{2\pi} \left(\text{put } \pi = \frac{22}{7}\right)$$

$$\Rightarrow r = \frac{352}{2 \times \frac{22}{7}} \text{ and } R = \frac{396}{2 \times \frac{22}{7}}$$

On rearranging,

$$\Rightarrow r = \frac{352 \times 7}{2 \times 22} \text{ and } R = \frac{396 \times 7}{2 \times 22}$$

$$\Rightarrow r = \frac{2464}{44} \text{ and } R = \frac{2772}{44}$$

$\Rightarrow r = 56$ m and $R = 63$ m

So, the width of the race track = R – r,

$\Rightarrow$ Width of the race track = 63 – 56

$\Rightarrow$ Width of the race track = 7 m

Area of race track = area of outer circle – area of inner circle

$\Rightarrow$ Area of track = $\pi R^2 - \pi r^2$ (put values of r and R)

$\Rightarrow$ Area of track = $\pi(63^2) - \pi(56^2)$

$\Rightarrow$ Area of track = $\pi(63^2 - 56^2)$ (taking π common from R.H.S)

$\Rightarrow$ Area of track = $\pi(3969 - 3136)$

$\Rightarrow$ Area of track = $\pi \times 833$

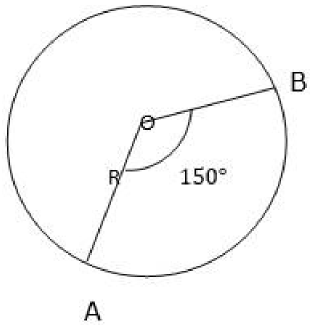
$$\Rightarrow \text{Area of track} = \frac{22}{7} \times 833 \left(\text{put } \pi = \frac{22}{7}\right)$$

= 22×119

= 2618 m^2

The width of tack is 7 m and area of track is 2618 m^2.

Question: 11



Consider the circle shown above,

Given radius of the circle = R = 21 cm → eqn1

And angle of the sector = θ = 150°→ eqn2

Length of arc of a sector $= \frac{\theta}{360} \times 2\pi R \rightarrow$ eqn3

Where 'R' = radius of sector (or circle)

θ = angle subtended by the arc on the centre of the circle

Put the values of R and θ from equation 1 and 2 in equation 3

$\Longrightarrow$ Length of arc $= \frac{150}{360} \times 2\pi(21)$ (put $\pi = \frac{22}{7}$)

$= \frac{150}{360} \times 2 \times \frac{22}{7} \times 21$

$= \frac{150 \times 2 \times 22 \times 21}{360 \times 7}$

= 138600/2520

= 55 cm

Area of a sector $= \frac{\theta}{360} \times \pi R^2 \rightarrow$ eqn4

Where 'R' = radius of sector (or circle)

θ = angle subtended by the arc on the centre of the circle

Put the values of R and θ from equation 1 and 2 in equation 3

$\Longrightarrow$ Area of sector $= \frac{150}{360} \times \pi(21^2)$ (put $\pi = \frac{22}{7}$)

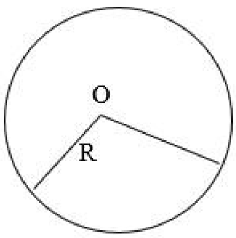
$= \frac{150 \times 22 \times 21 \times 21}{360 \times 7}$

= 1455300/2520

= 577.5 cm^2

The length of arc is 55 cm and area of sector is 577.5 cm^2.

Question: 12



Consider the circle shown above,

We know , Area of sector $= \frac{\theta}{360} \times \pi R^2 \rightarrow$ eqn1

Where R = radius of the circle and θ = central angle

Given R = 10.5 cm and Area of sector = 69.3 cm^2

Let the angle subtended at centre = θ

Put the values of R and area of sector in equation 1

$\Longrightarrow 69.3 = \frac{\theta}{360} \times \pi(10.5^2)$ (put $\pi = \frac{22}{7}$)

$\Longrightarrow 69.3 = \frac{\theta}{360} \times \frac{22}{7} \times 10.5 \times 10.5$

$\Longrightarrow 69.3 = \frac{\theta \times 22 \times 10.5 \times 10.5}{360 \times 7}$

$$\Rightarrow 69.3 = \frac{\theta \times 2425.5}{2520}$$

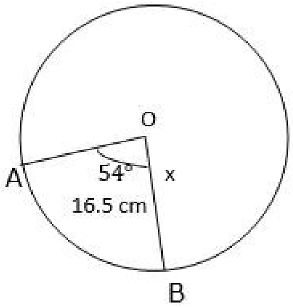
$$\Rightarrow \frac{69.3 \times 2520}{2425.5} = \theta$$

$$\Rightarrow \frac{174636}{2425.5} = \theta$$

$\Rightarrow \theta = 72^{\circ}$

The central angle of the sector is 72°.

Question: 13



Consider the Circle shown above,

We know, Length of arc of sector = $\frac{\theta}{360} \times 2\pi R \rightarrow$ eqn1

Where R = radius of circle and θ = central angle of the sector

Given, Length of arc = ℓ = 16.5 cm and θ = 54^{o}. Let the radius be x cm

Put the values of ℓ and θ in equation 1

$$\Rightarrow 16.5 = \frac{54}{360} \times 2\pi x \text{ (put } \pi = \frac{22}{7})$$

$$\Rightarrow 16.5 = \frac{54 \times 2 \times 22 \times x}{360 \times 7}$$

$$\Rightarrow 16.5 = \frac{2376 \times x}{2520}$$

On rearranging

$$\Rightarrow \frac{16.5 \times 2520}{2376} = x$$

$$\Rightarrow \frac{41580}{2376} = x$$

⇒ x = 17.5 cm

Also, we know circumference of the circle = 2πR

⇒ Circumference of the circle = 2πx (put value of x in this equation)

⇒ Circumference of the circle = 2π(17.5)

$$\Rightarrow \text{Circumference of the circle} = 2 \times \frac{22}{7} \times 17.5 \text{ (put } \pi = \frac{22}{7})$$

$$= \frac{2 \times 22 \times 17.5}{7}$$

$$= \frac{770}{7}$$

⇒ Circumference of the circle = 110 cm

Also, we know Area of the circle = πR^2

⇒ Area of the circle = πx^2

⇒ Are of the circle = $\pi(17.5^2)$

$\Longrightarrow$ Area of the circle $= \frac{22}{7} \times 17.5 \times 17.5$ (put $\pi = \frac{22}{7}$)

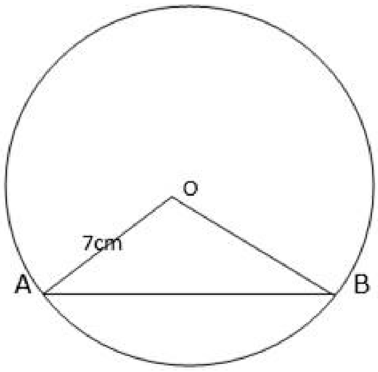
$\Longrightarrow$ Area of the circle $= \frac{22 \times 17.5 \times 17.5}{7}$

$\Longrightarrow$ Area of the circle $= \frac{6737.5}{7}$

⇒ Area of the circle = 962.5 cm^2

The radius of circle is 17.5 cm, circumference is 110 cm and area is 962.5 cm^2

Question: 14



Consider the above figure,

From here we can conclude that the portion or the segment below the chord AB is the minor segment and the segment above AB is major segment.

Also we know,

Area of minor segment = Area of sector – Area of ΔAOB → eqn1

Now, Area of sector $= \frac{\theta}{360} \times \pi R^2 \rightarrow$ eqn2

Where R = radius of the circle and θ = central angle of the sector

Given, R = 7 cm and θ = 90°

Putting these values in the equation 2, we get

Area of sector $= \frac{90}{360} \times \pi(7^2)$ (put $\pi = \frac{22}{7}$)

$= \frac{90}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{90 \times 22 \times 7 \times 7}{360 \times 7}$

$= \frac{97020}{2520}$

⇒ Area of sector = 38.5 cm^2→ eqn3

Area of △AOB = 1/2 × base × height

NOTE : In general Area of △ AOB $= \frac{1}{2} \times OA \times OB \times \sin\theta$

As triangle is isosceles therefore height and base both are 7 cm.

$\Longrightarrow$ Area of △AOB = 1/2×7×7 = $\frac{49}{2}$

$= 24.5\ cm^2 \rightarrow$ eqn4

Putting values of equation 2 and 4 in equation 1 we get

Area of minor segment = 38.5 – 24.5

⇒ Area of minor segment = 14 cm^2

Area of major segment = πR^2 – Area of minor segment → eqn5

Put the value of R, and Area of minor segment in equation 5

$= \pi(7^2) – 14$

$= 49\pi - 14$

$\Rightarrow$ Area of major segment $= \frac{22}{7} \times 49 - 14$ (put $\pi = \frac{22}{7}$)

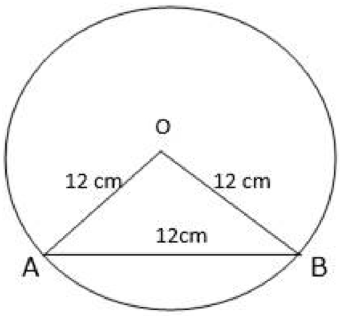
$= (22\times7) - 14$

$= 154 - 14$

$= 140\ cm^2$

Area of minor segment is 14 cm^2 and of major segment is 140 cm^2.

Question: 15



Consider the figure shown above.

In this, the triangle AOB is an equilateral triangle as all the sides are equal; therefore, it is obvious that the central angle of the sector is 60 degrees. Now by simply applying the formula of length of an arc, we can easily calculate the length of arc of the sector AOB.

Given Radius of circle = R = 12 cm,

Length of chord AB = 12 cm

∴ Central angle = θ = 60° (∵ ΔAOB is an equilateral triangle)

Length of arc $= \frac{\theta}{360} \times 2\pi(R) \rightarrow$ eqn1

Where R = radius of the circle and θ = central angle of the sector

Put the values of R and θ in equation 1

$\Rightarrow$ Length of minor arc $= \frac{60}{360} \times 2\pi(12)$ (put $\pi = 3.14$)

$= \frac{60}{360} \times 2 \times 3.14 \times 12$

$= \frac{60 \times 2 \times 3.14 \times 12}{360}$

$= \frac{2 \times 3.14 \times 12}{6}$

$= 2\times3.14\times2$

$= 12.56$ cm

Now, Length of major arc = 2πR – Length of minor arc

⇒ Length of major arc = 2π(12) – 12.56 (put π = 3.14)

⇒ Length of major arc = (2×3.14×12) – 12.56

⇒ Length of major arc = 75.36 – 12.56

⇒ Length of major arc = 62.8 cm

Now, Area of minor segment = Area of sector – Area of triangle → eqn1

$$\therefore \text{ Area pf sector } = \frac{\theta}{360} \times \pi R^2 \text{ (put the values of R and } \theta)$$

$$= \frac{60}{360} \times \pi(12^2)$$

$$= \frac{60}{360} \times 3.14 \times 144$$

= 75.36 cm^2→ eqn2

$$\text{Area of triangle } = \frac{\sqrt{3}}{4} \times a^2 \text{ (put a } = \text{ 12 cm)}$$

$$= \frac{\sqrt{3}}{4} \times (12^2)$$

$$\Longrightarrow \text{ Area of triangle } = \frac{\sqrt{3}}{4} \times 144$$

⇒ Area of triangle = 1.73×36

⇒ Area of triangle = 62.28 cm^2→ eqn3

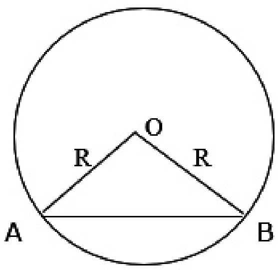
Put the values of equation 2 and 3 in equation 1,

∴ Area of minor segment = 75.36 – 62.28

= 13.08 cm^2

Length of major arc is 62.8 cm and of minor arc is 12.56 cm and area of minor segment is 13.08 cm^2.

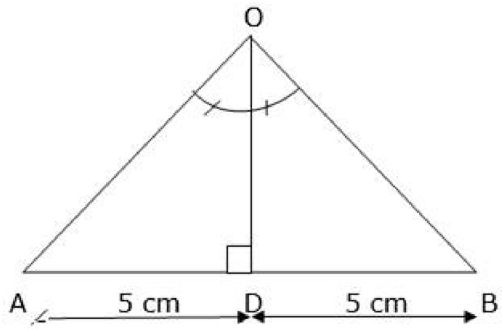
Question: 16



Consider the figure shown above.

In this, the triangle AOB is an isosceles triangle. So here we will construct a perpendicular bisector from O on AB and as this triangle is isosceles therefore this perpendicular will also act as median and angle bisector.

Therefore,



Draw a perpendicular bisector from O which meets AB at D and bisects AB, as ABO is an isosceles triangle therefore OD acts as a median.

So, consider right angle triangle AOD right angled at D

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Let ∠AOD = θ ⇒ Perpendicular = AD and Hypotenuse = AO = R

Given Radius of circle = R = 5√2 cm

Length of chord AB = 10 cm, AD = 5 cm

$\sin\theta = \frac{AD}{AO}$ (put values of AD and AO)

$\Longrightarrow \sin\theta = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Longrightarrow \sin\theta = \sin 45°$

(as $\sin 45° = \frac{1}{\sqrt{2}}$)

⇒ θ = 45°

∴ ∠AOD = 45°

Thus we can say $\angle AOB = 90°$ (As $\angle AOD = \frac{1}{2}\angle AOB$)

Area of minor segment = Area of sector – Area of right angle triangle

→ eqn1

$\text{Area of sector} = \frac{\theta}{360} \times \pi R^2$

Where R = radius of the circle and θ = central angle of the sector

$\text{Area of sector} = \frac{90}{360} \times \pi\left(\left(5\sqrt{2}\right)^2\right)$ (put $\pi = 3.14$)

$= \frac{90}{360} \times 3.14 \times 50$

$= \frac{3.14 \times 50}{4}$

∴ Area of sector = 39.25 cm^2

Area of right angle triangle = 1/2 × base × height

As this is isosceles right-angle triangle

∴ height = base = 5√2 cm

Area of right angle triangle = 1/2 ×5√2×5√2 = $\frac{50}{2}$ = 25 cm^2

Put the value of area of sector and area of right angle triangle in equation 1,

⇒ Area of minor segment = 39.25 -25

= 14.25 cm^2

Area of major segment = πR^2 – area of minor segment

Area of major segment $= \pi(\left((5\sqrt{2})^2\right) - 14.25$

$= 3.14 \times 5\sqrt{2} \times 5\sqrt{2} - 14.25$

⇒ Area of major segment = 157 – 14.25 = 142.75 cm^2

Area of major segment is 142.75 cm^2 and of minor segment is 14.25 cm^2.

Question: 17

Given R = 42 cm and central angle of sector = 120°

Area of minor segment = Area of sector – Area of triangle → eqn1

Area of sector $= \frac{\theta}{360} \times \pi R^2$

Where R = radius of the circle and θ = central angle of the sector

Area of sector $= \frac{120}{360} \times \pi(42^2)$ (put $\pi = \frac{22}{7}$)

$= \frac{120}{360} \times \frac{22}{7} \times 1764$

∴ Area of sector = 1848 cm^2

Area of right angle triangle = 1/2×base×height×sin θ

Where θ = central angle of the sector

Area of triangle $= \frac{1}{2} \times 42 \times 42 \times \sin 120°$

(put the value $\sin 120° = \frac{\sqrt{3}}{2}$)

Area of triangle = 1/2×42×42×√3/2

Area of triangle = (42×42×√3)/4

(put √3 = 1.73)

Area of triangle $= \frac{42 \times 42 \times 1.73}{4}$

= 762.93 cm^2

Put the values of area of triangle and area of sector in equation 1

⇒ Area of minor segment = 1848 – 762.93

= 1085.07 cm^2

Area of major segment = πR^2 – Area of minor segment

Put the value of area of minor segment and R in above equation

= $\pi(42^2)$ – 1085.07

⇒Area of major segment = 22/7×42×42-1085.07

(put π = 22/7)

⇒ Area of major segment = 5544 – 1085.07

∴ Area of major segment = 4458.93 cm^2

Area of major segment is 4458.93 cm^2 and of minor segment is 1085.07 cm^2.

Question: 18

Area of minor segment = Area of sector – Area of triangle → eqn1

$$\text{Area of sector} = \frac{\theta}{360} \times \pi R^2$$

Where R = radius of the circle and θ = central angle of the sector

$$\text{Area of sector} = \frac{60}{360} \times \pi(30^2)\ (\text{put } \pi = 3.14)$$

$$\text{Area of sector} = \frac{60}{360} \times 3.14 \times 900$$

$$\text{Area of sector} = \frac{3.14 \times 900}{6}$$

∴ Area of sector = 471 cm^2

$$\text{Area of right angle triangle} = \frac{\sqrt{3}}{4} \times a^2$$

Where a = side of the triangle

Area of triangle = $\sqrt{3}/4 \times 30 \times 30$

Area of triangle = $\sqrt{3}/4 \times 900$

Area of triangle = $(900 \times \sqrt{(3\ )})/4$

(put $\sqrt{3}$ = 1.732)

Area of triangle = $(1.732 \times 900)/4$

∴ Area of triangle = 389.7 cm^2

Put the values of area of triangle and area of sector in equation 1

Area of minor segment = 471 – 389.7

⇒ Area of minor segment = 81.3 cm^2

Area of major segment = πR^2 – Area of minor segment

Put the value of area of minor segment and R in above equation

⇒ Area of major segment = $\pi \times (30^2)$ – 81.3 (put π = 3.14)

⇒ Area of major segment = $3.14 \times 30 \times 30$ – 81.3

⇒ Area of major segment = 2826 – 81.3

= 2744.7 cm^2

Area of major segment is 2744.7cm^2 and of minor segment is 81.3 cm^2.

Question: 19

In a circle of ra

Solution:

Given radius of circle = R = 10.5 cm

Let the area of major sector be ‘A_1’ and that of minor sector be ‘A_2’

$$\therefore, A_2 = \frac{A_1}{5} \rightarrow \text{eqn1}$$

We know, Area of circle = Area of major sector + Area of minor sector

⇒ Area of circle = $A_1 + A_2$

$$\Rightarrow \text{Area of circle} = A_1 + \frac{A_1}{5} \rightarrow \text{eqn2 (from equation 1)}$$

We also know, Area of circle = πR^2

Where R = radius of circle, put value of area of circle in equation 2.

$$\Rightarrow \pi(10.5^2) = \frac{5A_1 + A_1}{5}$$

(taking 5 as L.C.M on R.H.S)

$$\Rightarrow \pi \times 10.5 \times 10.5 = \frac{6A_1}{5}$$

$$\Rightarrow \frac{22}{7} \times 10.5 \times 10.5 = \frac{6A_1}{5}$$

$$\Rightarrow \frac{22 \times 10.5 \times 10.5}{7} = \frac{6A_1}{5}$$

$$\Rightarrow 22 \times 10.5 \times 1.5 = \frac{6A_1}{5}$$

$$\Rightarrow 346.5 = \frac{6A_1}{5}$$

$$\Rightarrow \frac{5 \times 346.5}{6} = A_1$$

= 288.75 cm^2

The area of major sector is 288.75 cm^2.

Question: 20

In an hour the minute hand completes one rotation therefore in 24 hours the minute hand will complete 24 rotations similarly the hour hand completes one rotation in 12 hours therefore in 24 hours it will complete 2 rotations. Now we have to just calculate the perimeter of the circle traced by minute hand and hour hand and multiply it with the number of rotations of minute hand and hour hand in 2 days respectively.

Length of short/hour hand = r = 4 cm

Length of long/minute hand = R = 6 cm

∴ The perimeter of circle traced by short hand = p = 2πr → eqn1

∴ The perimeter of circle traced by Long hand = P = 2πR → eqn2

Now put the value of ‘r’ and ‘R’ in the equation 1 and 2 respectively.

⇒ p = 2π(4) & P = 2π(6) (put π = 3.14)

⇒ p = 2×3.14×4 & P = 2×3.14×6

∴ p = 25.12 cm & P = 37.68 cm

Therefore, distance covered by short hand in one rotation = 25.12 cm

Distance covered by long hand in one rotation = 37.68 cm

Number of rotation of short hand in one day = 2

Number of rotation of long hand in one day = 24

Therefore number of rotation of small hand in two days = 4

Number of rotation of long hand in two days = 48

Total distance covered by long hand in 2 days = P × no. of rotations in 2 days

⇒ Total distance covered by long hand in 2 days = 37.68×48

⇒ Total distance covered by long hand in 2 days = 1808.64 cm → eqn3

Total distance covered by short hand in 2 days = p × no. of rotations in 2 days

⇒ Total distance covered by short hand in 2 days = 25.12×24

⇒ Total distance covered by short hand in 2 days = 100.48 cm → eqn4

Now total distance covered by tip of both hands in 2 days = eqn3 + eqn4

⇒ Total distance covered by both hands in 2 days = 1808.64 + 100.48

⇒ Total distance covered by both hands in 2 days = 1909.12 cm

The distance covered by both hands tip in 2 days is 1909.12 cm

Question: 21

Quadrant is a sector in which the central angle is 90 degrees, and this is the key to solve this question. As we know the central angle of the sector so we can easily calculate the area of quadrant by first calculating the radius of the circle as the circumference of the circle is given and then applying the formula of area of sector.

So, we know Circumference of a circle = 2πR → eqn1

Where R = radius of the circle

Given Circumference of the circle = 88 cm, θ = 90°

Put the given values in equation 1

$88 = 2 \times \frac{22}{7} \times R \, (\pi = \frac{22}{7})$

$\Rightarrow 88 = \frac{2 \times 22 \times R}{7}$

$\Longrightarrow 88 = (44 \times R)/7$

$\Longrightarrow 88 = 44R/7$

$\Longrightarrow (88 \times 7)/44 = R$

$\Longrightarrow 616/44 = R$

⇒ R = 14 cm

Now we know Area of a sector $= \frac{\theta}{360} \times \pi R^2$

Put the values of R and θ in the above equation

$\Rightarrow$ Area of quadrant $= \frac{90}{360} \times \pi(14^2)$

$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14$

$= \frac{90 \times 22 \times 14 \times 14}{360 \times 7}$

$= \frac{22 \times 14 \times 14}{4 \times 7} = \frac{4312}{28}$

= 154 cm^2.

The area of quadrant is 154 cm^2.

Question: 22

Here the increase in the length of the rope simply means that there is increase in the radius of the circle within which cow can graze. Now to find the additional area available for grazing can be easily be found by simply subtracting the initial area available for grazing from the new area available.

Initial radius = r = 16 cm

Increased radius = R = 23 cm

Additional ground available = Area of new ground – Initial area → eqn1

Initial area of ground = $\pi(r^2)$

⇒ Initial area of ground = $\pi(16^2)$

⇒ Initial area of ground = 256π → eqn2

Area of new ground = πR^2

⇒ Area of new ground = $\pi(23^2)$

⇒ Area of new ground = 529π → eqn3

Now put the values of equation 2 and 3 in equation 1

⇒ Additional area of ground available = 529π – 256π

⇒ Additional area available = (529 – 256)π (Taking π common)

⇒ Additional ground available = 273π

$$\Longrightarrow \text{ Additional ground available } = 273 \times \frac{22}{7}$$

(put $\pi = \frac{22}{7}$)

= (22×273)/7

= 6006/7

= 858 cm^2

The additional ground available is 858 cm^2.

Question: 23

Here the horse is tethered to one corner implies or means that the area available for grazing is a quadrant of radius 21 m. Now we need to find the area of this quadrant to find out the area available for grazing and then subtract it from the total area of the rectangular field to obtain the area left ungrazed.

Given length of rectangular field = ℓ = 70 m

Breadth of rectangular field = b = 52 m

∴ Area of the field = $\ell \times b$

⇒ Area of the field = 70×52

⇒ Area of the field = 3640 m^2

We know in a rectangle all the angles are 90 degrees.

∴ Area available for grazing = area of quadrant

$$\Rightarrow \text{Area of quadrant/sector} = \frac{\theta}{360} \times \pi R^2$$

Where R = radius of circle & θ = central angle

Given R = 21 m and θ = 90°

$$\Longrightarrow \text{ Area available for grazing } = \frac{\theta}{360} \times \pi R^2$$

Put the given values in the above equation,

$$\Longrightarrow \text{ Area available for grazing } = \frac{90}{360} \times \pi(21^2)$$

(put $\pi = 22/7$)

$$= \frac{90}{360} \times \frac{22}{7} \times 441$$

$$= \frac{90 \times 22 \times 441}{360 \times 7}$$

$= (22 \times 63)/4$

$= 1386/4$

$\Rightarrow$ Area available for grazing = 346.5 m^2

Area left ungrazed = Area of field – Area available for grazing

$\Rightarrow$ Area left ungrazed = 3640 – 346.5

$\Rightarrow$ Area left ungrazed = 3293.5 m^2

The area available for grazing is 346.5 m^2 and area left ungrazed is 3293.5 m^2.

Question: 24

Here the horse is tethered to one corner implies or means that the area available for grazing is a sector of radius 21 m with central angle as 60 degrees as the field is in shape of equilateral triangle . Now we need to find the area of this sector to find out the area available for grazing and then subtract it from the total area of the triangular field to obtain the area left ungrazed.

Given the side of field = a = 12 m

$\therefore$ Area of field = Area of equilateral triangle

$$\Longrightarrow \text{ Area of field } = \frac{\sqrt{3}}{4} \times a^2$$

$$\Longrightarrow \text{ Area of field } = \frac{1.732}{4} \times (12^2)$$

$$\Longrightarrow \text{ Area of field } = \frac{1.732 \times 144}{4}$$

$\Rightarrow$ Area of field = 62.352 m^2

We know in an equilateral triangle all the angles are 60 degrees.

$\therefore$ Area available for grazing = Area of the sector

$$\text{Area of quadrant/sector } = \frac{\theta}{360} \times \pi R^2$$

Where R = radius of circle and θ = central angle of sector

Given R = 7 m and $\theta = 60°$

Put the given values in the above equation,

$$\Longrightarrow \text{ Area available for grazing } = \frac{60}{360} \times \pi(7^2) \left(\text{put } \pi = \frac{22}{7}\right)$$

$$\Longrightarrow \text{ Area available for grazing } = \frac{60}{360} \times \frac{22}{7} \times 49$$

$$\Longrightarrow \text{ Area available for grazing } = \frac{60 \times 22 \times 49}{360 \times 7}$$

$$\Longrightarrow \text{ Area available for grazing } = \frac{22 \times 7}{6}$$

$$\Longrightarrow \text{ Area available for grazing } = \frac{154}{6}$$

$\Rightarrow$ Area available for grazing = 25.666 m^2

Area that cannot be grazed = Area of field – Area available for grazing

⇒ Area that cannot be grazed = 62.352 – 25.666

⇒ Area that cannot be grazed = 36.686 m^2

The area that cannot be grazed is 36.656 m^2.

Question: 25

Here the 4 cows are tethered to each corner implies or means that the area available for grazing is a quadrant of radius 25 m with central angle as 60 degrees as the field is in shape of square . Now we need to find the area of this sector to find out the area available for grazing for all the cows and then subtract it from the total area of the square field to obtain the area left ungrazed.

The reason why we have taken the radius as 25 m is , basically we have considered that each cow is tethered to a rope which is equal to half of the side of the square as we had to maximize the area each cow gets to graze without sharing thus the maximum radius within which a cow can graze maximum unshared area is simply the half of the side of square.

Given the side of field which is in shape of square = a = 50 m

∴ Area of the field = Area of Square

⇒ Area of field = a^2

⇒ Area of field = (50^2)

⇒ Area of field = 2500 m^2

We know in an square all the angles are 90 degrees.

∴ Area available for grazing for one cow = area of sector/quadrant

$$\text{Area of quadrant/sector } = \frac{\theta}{360} \times \pi R^2$$

Where R = radius of circle & θ = central angle of sector

Given R = 25 m & θ = 90°

$$\Longrightarrow \text{ Area available for grazing for one cow } = \frac{\theta}{360} \times \pi R^2$$

Put the given values in the above equation,

$$\Longrightarrow \text{ Area available for grazing for one cow } = \frac{90}{360} \times \pi(25^2) \text{ (put } \pi = 3.14)$$

$$\Longrightarrow \text{ Area available for grazing for one cow } = \frac{90}{360} \times 3.14 \times 625$$

$$\Longrightarrow \text{ Area available for grazing for one cow } = \frac{90 \times 3.14 \times 625}{360}$$

$$\Longrightarrow \text{ Area available for grazing for one cow } = \frac{3.14 \times 625}{4}$$

$$\Longrightarrow \text{ Area available for grazing for one cow } = \frac{1962.5}{4}$$

⇒ Area available for grazing for one cow = 490.625 m^2

⇒ Area available for 4 cows = 4 × Area available for one cow

⇒ Area available for 4 cows = 4 × 490.625

⇒ Area available for 4 cows = 1962.5 m^2

Area left ungrazed = Area of field – Area available for grazing for 4 cows

⇒ Area that cannot be grazed = 2500 – 1962.5

$$\Longrightarrow \text{ Area that cannot be grazed } = 2500 - 1962.5$$

⇒ Area that cannot be grazed = 537.5 m^2

The area left ungrazed is 537.5 m^2.

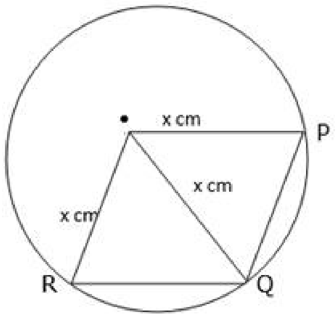
Question: 26

Here in the given figure 'O' is the centre of circle on which three vertices of rhombus lie, this implies that OP, OR are both radius of the circle. Also we know that in rhombus all the 4 sides are equal in length. Thus OP, OR, PQ, RQ, they all are radii of circle. Also OQ is equal to radius of circle. Now rhombus being a parallelogram therefore diagonal OQ will divide the rhombus into two equal halves this means that the area of triangle OQR will be equal to half of the area of rhombus. Also we can see that triangle OQR is an equilateral triangle and hence we can easily calculate its area in terms of radius of circle and equate it to half of the area of rhombus and calculate the radius of given circle.

Given Area of OPQR $= 32\sqrt{3}\ \text{cm}^2$

Let the radius of the circle = x cm

Now join OQ



Consider ΔOQR,

OQ = OR = RQ = x cm

⇒ ΔOQR is an equilateral triangle

$\therefore$ Area of ΔOQR = Area of an equilateral triangle $= \dfrac{\sqrt{3}}{4} \times a^2 \rightarrow$ eqn 1

Where a = side of equilateral triangle

Also we know OQ is a diagonal of rhombus OPQR and as in a parallelogram diagonal divides it into two equal area or halves , similarly OQ is also dividing the rhombus into two equal areas therefore,

⇒ Area of ΔOQR = Area of ΔOPQ → eqn2

Area of OPQR = Area of ΔOQR + Area of ΔOPQ

Area of OPQR = 2 × Area of ΔOQR (from eqn2) → eqn3

Put the values of area of OPQR and equation 1 in equation 3

$\Longrightarrow 32\sqrt{3} = 2 \times \dfrac{\sqrt{3}}{4} \times a^2$ (put a = x)

$\Longrightarrow 32\sqrt{3} = \dfrac{2\sqrt{3}}{4} \times x^2$

$\Longrightarrow 32\sqrt{3} = \dfrac{\sqrt{3}}{2} \times x^2$

$\Longrightarrow \dfrac{32\sqrt{3} \times 2}{\sqrt{3}} = x^2$

$\Longrightarrow 64 = x^2$

$\Longrightarrow x = \pm\sqrt{64}$

$\Rightarrow x = \pm 8$

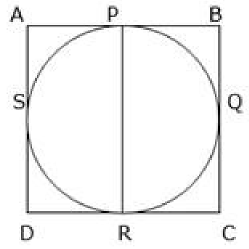
As every quadratic equation has two roots, similarly $x^2 = 64$ also have two roots i.e. x = 8 and x = -8. As we know that 'x' represents radius of circle therefore it cannot be a negative value, hence we discard the negative root.

Therefore radius of the circle = x = 8 cm.

The radius of circle is 8 cm.

Question: 27

(i)



Consider the above figure, Join PR,

Now PR = Diameter of the inscribed circle

Also, PR = BC = 10 cm.

So, PR = 10 cm

$\therefore$ radius of inscribed circle $= r = \frac{PR}{2}$

$\Rightarrow r = \frac{10}{2}$

$\Rightarrow r = 5$ cm

$\therefore$ Area of inscribed circle $= \pi r^2$ (put value of r in this equation)

$\Rightarrow$ Area of inscribed circle $= \pi(5^2)$

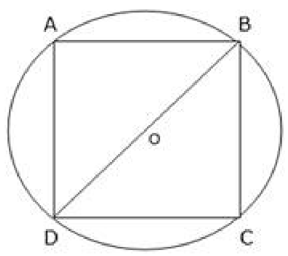
$\Rightarrow$ Area of inscribed circle $= \frac{22}{7} \times 25$ (put $\pi = \frac{22}{7}$)

$\Rightarrow$ Area of inscribed circle $= \frac{22 \times 25}{7}$

$\Rightarrow$ Area of inscribed circle = 78.57 cm^2

The area of inscribed circle is 78.57 cm^2.

(ii)



Consider the above figure, O is the centre of circle and ABCD is a square inscribed. Now OB and OD are radii of circle.

Consider ΔDBC right angled at c (as C is a vertex of square)

$\therefore$ Apply Pythagoras theorem in triangle DBC

$Hypotenuse^2 = Perpendicular^2 + Base^2$

In triangle DBC, hypotenuse = DB,

perpendicular = BC and

base = DC

$\implies BD^2 = BC^2 + DC^2$

Put the values of BC and DC i.e. 10 cm

$\implies BD^2 = 10^2 + 10^2$

$\implies BD^2 = 200$

$\implies BD = \sqrt{200}$

$\implies BD = 10\sqrt{2}$ cm

Now radius of circle = half of BD

$\therefore \text{radius of circle} = r = \frac{BD}{2}$

$\implies r = (10\sqrt{2})/2$

$\implies r = 5\sqrt{2}$ cm

Hence Area of circumscribing circle = πr^2

$\implies$ Area of circumscribing circle = $3.14 \times 5\sqrt{2} \times 5\sqrt{2}$

(put $\pi = 3.14$ and $r = 5\sqrt{2}$ cm)

$\Rightarrow$ Area of circumscribing circle = 3.14×50

$\Rightarrow$ Area of circumscribing circle = 157 cm^2

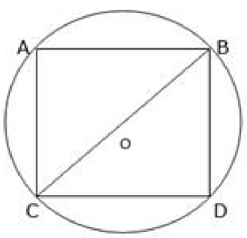
Area of circumscribing circle is 157 cm^2.

Question: 28

Consider the figure shown below where O is centre of circle, join BC which passes through O, let the side of square be 'a' and radius of circle be 'r'.

Now we know OB and OC are radius of circle

So, OB = OC = r



Consider ΔBDC right angled at D

$\therefore H^2 = P^2 + B^2$ (pythagoras theorem)

$\implies BC^2 = BD^2 + DC^2 \rightarrow \text{eqn1}$

And we know BC = OC + OB

BC = 2r and BD = DC = a (put these values in eqn1)

$\Rightarrow (2r)^2 = a^2 + a^2$

$\Rightarrow 4r^2 = 2a^2$

$\implies r^2 = \frac{2a^2}{4}$

$\implies r^2 = \frac{a^2}{2} \rightarrow \text{eqn2}$

Area of inscribed square = side × side

Areaa of inscribed square = a × a

Area of inscribed square = $a^2 \rightarrow$ eqn3

Area of circumscribing circle = πR^2 where R = radius of circle

$\Rightarrow$ Area of circumscribing circle = $\pi r^2 \rightarrow$ eqn4

Ratio of area of circumscribing cirle to that of inscribed circle

$$= \frac{\text{area of circle}}{\text{area of square}}$$

Put the values from equation 3 & 4 in above equation

$$\text{Ratio} = \frac{\pi r^2}{a^2}$$

$$\Longrightarrow \text{Ratio} = \frac{\pi \times \frac{a^2}{2}}{a^2}$$

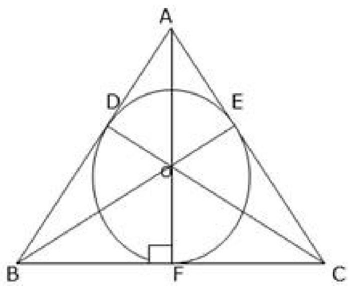
(from eqn 2)

$$\Longrightarrow \text{Ratio} = \frac{\pi \times a^2}{2 \times a^2} = \frac{\pi}{2}$$

So, Ratio is π : 2

The ratio is π:2

Question: 29



Consider the figure shown above, AF, BE and CD are perpendicular bisector.

Now we know that the point at which all three perpendiculars meet is called incentre, so O is the incentre, thus O divides all three perpendiculars in a ratio 2:1.

Let AB = BC = CA = a cm

Therefore let AF = h cm

$\Longrightarrow \angle AFC = 90°$ and OF = 1/3 × AF

$\Longrightarrow$ OF = h/3 cm (putting value of OF)

$\Rightarrow$ h = 3×OF → eqn1

And we can see from figure that OF = radius of circle

Now let radius of circle be = r cm

$\therefore$ Area of circle = πR^2

where R = radius of circle

Given area of circle = 154 cm^2

$\Rightarrow \pi r^2 = 154$

$$\Longrightarrow \frac{22}{7} \times r^2 = 154 \left(\text{put } \pi = \frac{22}{7}\right)$$

$$\Longrightarrow r^2 = \frac{154 \times 7}{22}$$

$\Longrightarrow r^2 = 49$

$\Rightarrow r = 7$ cm

Therefore OF = 7 cm

$\Rightarrow h = 3\times7$ (from eqn 1)

$\Rightarrow h = 21$ cm

we know area of an equilateral triangle = $\frac{\sqrt{3}}{4}\times a^2$

where a = side of triangle

Also, Area of triangle = 1/2 ×base×height

Equating both the areas we get,

$$\frac{\sqrt{3}}{4}\times a^2 = \frac{1}{2}\times BC\times AF$$

Put the values of BC and AF

$$\Longrightarrow \frac{\sqrt{3}}{4}\times a^2 = \frac{1}{2}\times a\times h$$

$$\Longrightarrow \frac{\sqrt{3}}{4}\times a^2 = \frac{1}{2}\times a\times 21$$

(putting value of h = 21 cm)

$$\Longrightarrow \frac{\sqrt{3}}{4}\times a = \frac{21}{2}$$

$$\Longrightarrow a = \frac{21\times4}{2\times\sqrt{3}}$$

(rationalize it)

$$\Longrightarrow a = \frac{21\times4\times\sqrt{3}}{2\times\sqrt{3}\times\sqrt{3}}$$

$$\Longrightarrow a = \frac{42\times\sqrt{3}}{3}$$

$\Longrightarrow a = 14\sqrt{3}$ cm

∴ Perimeter of equilateral triangle = 3×side of triangle

$\Longrightarrow$ Perimeter of Δ ABC = $3\times14\sqrt{3}$ (put $\sqrt{3} = 1.73$)

$\Rightarrow$ Perimeter of $\Delta ABC = 42\times1.73$

$\Rightarrow$ Perimeter of $\Delta ABC = 72.66$ cm

The perimeter of triangle is 72.66 cm

Question: 30

In one revolution a wheel will cover a distance equal to its circumference, so in order to find the number of revolutions we have to first calculate the circumference of the wheel and then divide it with the total distance covered to find out the total number of revolutions

Given radius of wheel = r = 42 cm

Circumference of wheel = $2\pi R$ where R = radius of the wheel

= $2\pi(42)$ (putting value of r)

$$\text{Circumference of wheel} = \frac{2\times22\times42}{7} = 264 \text{ cm}$$

Therefore distance covered in one revolution = 264 cm

Total distance covered = 19.8 km = 1980000 cm

Total number of revolutions = n

Distance covered on 1 revolution ×no. of revolutions = Total distance

264×n = 1980000

$\Rightarrow n = \frac{1980000}{264}$

⇒ n = 7500

Total number of revolutions is 7500.

Question: 31

Given radius of wheel = R = 2.1m

Number of revolutions in one minute = 75

Number of revolutions in 1 hour = 75×60

Number of revolutions in 1 hour = 45000

Distance covered in one revolution = Circumference of wheel

Distance covered in 1 revolution = 2πR (where R = radius of wheel)

Distance covered n 1 revolution = 2π(2.1)

Distance covered in one revolution $= 2 \times \frac{22}{7} \times 2.1$ (put $\pi = \frac{22}{7}$)

= 13.2 m

So, distance covered in 4500 revolutions = 4500×distance covered in 1

Distance covered in 4500 revolution = 4500× 13.2

Distance covered in 4500 revolutions = 59400 m = 59.4 km

∴ Distance covered in 1 hour = 59.4 km

Hence speed of the locomotive = 59.4 km/hr

The speed of locomotive is 59.4 km/hr

Question: 32

Let the diameter of the wheel be ‘d’ cm

Total distance covered in 250 revolutions = 49.5 km = 495000 m

distance covered in one revolution $= \frac{495000}{2500}$

⇒ Distance covered in one revolution = 198 cm → eqn1

Also, Distance covered in one revolution = circumference of wheel

∴ Distance covered in one revolution = πD where d = diameter of wheel

Distance covered in one revolution $= \frac{22 \times d}{7} \rightarrow$ eqn2 $\left(\text{put } \pi = \frac{22}{7}\right)$

Equate equation 1 and 2 we get,

$\frac{22 \times d}{7} = 198$

$\Rightarrow d = \frac{198 \times 7}{22}$

⟹ d = 9×7

$\Longrightarrow$ d = 63 cm

The diameter of the wheel is 63 cm.

Question: 33

Given diameter of wheel = d = 60 cm

Number of revolutions in one minute = 140

Number of revolutions in one hour = 140×60

Number of revolutions in one hour = 8400

Distance covered in one revolution = circumference of wheel

⇒ Distance covered in one revolution = πd

Distance covered in one revolution $= \frac{22}{7} \times 60$ (put $\pi = \frac{22}{7}$ and value of d)

= 188.57 cm

Distance covered in one hour = Distance in 1 revolution × no. of revolutions

⇒ Total distance covered in one hour = 188.57× 8400

⇒ Total distance covered in one hour = 1583988 cm = 15.839 km

∴ speed with which boy is cycling = 15.839 km/hr

The speed with which boy is cycling is 15.839 km/hr

Question: 34

Given diameter of wheel of bus = d = 140 cm

So radius of wheel $= R = \frac{d}{2} = \frac{140}{2} = 70$ cm

Speed of bus = 72.6 km/hr

∴ Distance covered by bus in one hour = 72.6 km = 7260000 cm

So distance covered by wheels in one minute $= \frac{7260000}{60}$

Distance covered in one minute = 121000 cm → eqn1

Let the number of revolutions made by wheel per minute = x

Distance covered by wheel in one revolution = circumference of wheel = 2πR

Distance covered by wheel in one revolution = 2π(70)

(putting value of R)

$= 2 \times \frac{22}{7} \times 70 \left(\text{putting value of R and } \pi = \frac{22}{7}\right)$

$= \frac{2 \times 22 \times 70}{7}$

= 2×22×10 = 440 cm

∴ Total distance = No. of revolution× Distance covered in1 revolution

On putting the required values we get,

121000 = 440×(x)

$\Longrightarrow x = \frac{121000}{440}$

⇒ x = 275

Number of revolutions made per minute is 275.

Question: 35

Given diameter of front wheel = d = 80 cm

so, Radius of front wheel = r = d/2 = 80/2 = 40 cm

Diameter of rear wheel = D = 2 m = 200 cm

so, Radius of front wheel $= R = \frac{D}{2} = \frac{200}{2} = 100$ cm

Distance covered by wheel in 1 revolution = Circumference of wheel

⇒ Distance covered by front wheel = 2πr = 2π(40)

⇒ Distance covered by front wheel = 80π

∴ Distance covered by front wheel in 800 revolutions = 80π×800

⇒ Distance covered by front wheel in 800 revolutions = 6400π → eqn1

Similarly

⇒ Distance covered by rear wheel = 2πR = 2π(100)

⇒ Distance covered by rear wheel = 200π → eqn2

Let the number of revolutions made by rear wheel to cover 6400π cm be "x"

∴ (x)×200π = 6400π (from eqn1 and eqn2)

$\Longrightarrow x = \frac{64000\pi}{200\pi}$

⟹ x = 64000/200

⇒ x = 320

Number of revolution made by rear wheel to cover the distance covered by front wheel in 800 revolutions is 320.

Question: 36

Here the distance between the center of circles touching each other is equal to the side of the square. Therefore, we can say that the radius of ach circle is equal to the half of the side of the square. Now by simply calculating the area of the 4 quadrants and then subtracting it from the area of the square we can easily calculate the area of the shaded region.

Given side of square = a = 14 cm

Central angle of each sector formed at corner = θ = 90°

So, radius of 4 equal circles = r = a/2 = 14/2

∴ Radius of 4 circles = r = 7 cm

Area of quadrant formed at each corner $= \frac{\theta}{360} \times \pi R^2$

where R = radius of circle

$\Longrightarrow$ Area of one quadrant $= \frac{90}{360} \times \pi(7^2)$

$= \frac{49\pi}{4}\ \text{cm}^2 \rightarrow$ eqn1

Area of all the 4 quadrant = 4 × Area of one quadrant

$= 4 \times \frac{49\pi}{4}$ (from eqn 1)

⇒ Area of all 4 quadrants = 49π → eqn2

Also, Area of square = side×side = a×a = a^2 = 14^2 (putting value of side of square)

⇒ Area of square = 196 cm^2→ eqn3

∴ Area of shaded region = Area of square – Area of all 4 quadrants

⇒ Area of shaded region = 196 – 49π (fromeqn3 and eqn2)

$$\Rightarrow \text{ Area of shaded region } = 196 - \left(49 \times \frac{22}{7}\right) \text{ (put } \pi = \frac{22}{7}\text{)}$$

= 196 – (7×22)

= 196 – 154

= 42 cm^2

The area of shaded region is 42 cm^2.

Question: 37

Here, first we join the center of all adjacent circles then the distance between the center of circles touching each other is equal to the side of the square formed by joining the center of adjacent circles. Therefore, we can say that the side of the square equal to the twice of the radius of circle. Now by simply calculating the area of the 4 quadrants and then subtracting it from the area of the square we can easily calculate the area of the shaded region.

Given radius of each circle = r = 5 cm

Central angle of each sector formed at corner = θ = 90°

Side of square ABCD = a = 2×r = 2×5 = 10 cm

$$\text{Area of quadrant formed at each corner } = \frac{\theta}{360} \times \pi R^2$$

where R = radius of circle

$$\Rightarrow \text{ Area of one quadrant } = \frac{90}{360} \times \pi(5^2)$$

(putting value of r and θ)

$$= \frac{25\pi}{4} \text{ cm}^2 \rightarrow \text{ eqn1}$$

Area of all 4 quadrants = 4×Area of one quadrant

$$= 4 \times \frac{25\pi}{4} \text{ (from eqn 1)}$$

⇒ Area of all 4 quadrants = 25π → eqn2

Area of square = side×side = a×a = a^2

⇒ Area of square = 10^2 (putting value of side of square)

⇒ Area of square = 100 cm^2→ eqn3

Area of shaded region = Area of square – Area of all 4 quadrants

Area of shaded region = 100 – 25π (from eqn3 and eqn2)

= 100 – (25×3.14) (put π = 3.14)

= 100 – 78.5

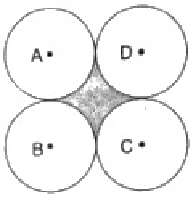
= 21.5 cm^2

The area of shaded region is 21.5 cm^2.

Question: 38

Four equal circle

Solution:



Here, first we join the centre of all adjacent circles then the distance between the centre of circles touching each other is equal to the side of the square formed by joining the centre of adjacent circles. Therefore, we can say that the side of the square equal to the twice of the radius of circle. Now by simply calculating the area of the 4 quadrants and then subtracting it from the area of the square we can easily calculate the area of the shaded region.

Given radius of each circle = "a" units

Central angle of each sector formed at corner = θ = 90°

Side of square ABCD = 2×a units

Area of quadrant formed at each corner $= \frac{\theta}{360} \times \pi R^2$

where R = radius of circle

$\Rightarrow$ Area of one quadrant $= \frac{90}{360} \times \pi(a^2)$

$\Rightarrow$ Area of one quadrant $= \frac{\pi a^2}{4}$ sq. units $\rightarrow$ eqn1

∴ Area all 4 quadrants = 4×Area of one quadrant

$\Rightarrow$ Area of all the 4 quadrant $= 4 \times \frac{\pi a^2}{4}$ (from eqn 1)

$= \pi a^2$ sq. units → eqn2

Area of square = side×side = 2a×2a = $4a^2$

⇒ Area of square = $4a^2$ sq. units → eqn3

Area of shaded region = Area of square – Area of all 4 quadrants

⇒ Area of shaded region = $4a^2 - \pi a^2$ (from eqn3 and eqn2)

$\Rightarrow$ Area of shaded region $= 4a^2 - \left(a^2 \times \frac{22}{7}\right)$ (put $\pi = \frac{22}{7}$)

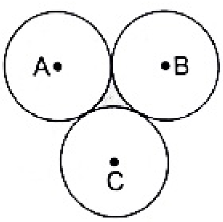
$\Rightarrow$ Area of the shaded region $= 4a^2 - \frac{22a^2}{7}$

$\Rightarrow$ Area of shaded region $= \frac{28a^2 - 22a^2}{7}$

$\Rightarrow$ Area of shaded region $= \frac{6a^2}{7}$ sq. units

Area of shaded region is $\frac{6a^2}{7}$ sq. units

Question: 39



Consider the above figure,

Here, first we join the center of all adjacent circles then the distance between the center of circles touching each other is equal to the side of an equilateral triangle formed by joining the center of adjacent circles. Therefore, we can say that the side of the equilateral triangle is equal to the twice of the radius of circle. Now by simply calculating the area of the 3 sectors and then subtracting it from the area of the equilateral triangle we can easily calculate the area of the enclosed region.

Given radius of each circle = r = 6 cm

Central angle of each sector = θ = 60° (∵ ΔABC is equilateral)

Side of equilateral ΔABC = a = 2×r = 2×6

∴ Side of equilateral ΔABC = a = 12 cm

$$\text{Area of sector formed at each corner} = \frac{\theta}{360} \times \pi R^2 \text{ where R} = \text{radius of circle}$$

$$\Longrightarrow \text{Area of one sector} = \frac{60}{360} \times \pi(6^2)$$

$$\Longrightarrow \text{Area of one sector} = \frac{36\pi}{6}\ \text{cm}^2$$

⇒ Area of one sector = 6π cm^2→ eqn1

Area of all the 3 sector = 3×Area of one sector

= 3×6π (from eqn1)

= 18π cm^2→ eqn2

$$\text{Area of equilateral } \Delta ABC = \frac{\sqrt{3}}{4} \times a^2 = \frac{\sqrt{3}}{4} \times (12^2)$$

$$\Longrightarrow \text{Area of equilateral } \Delta ABC = \frac{\sqrt{3} \times 144}{4}$$

$$\Longrightarrow \text{Area of equilateral } \Delta ABC = 36\sqrt{3}\ \text{cm}^2 \rightarrow \text{eqn3}$$

Area of enclosed region = Area of equilateral ΔABC – Area of all 3 sectors

⟹ Area of enclosed region = 36√3-18π (from eqn 3 and eqn 2)

⟹ Area of enclosed region = (36×1.732)-(18×3.14)

(put π = 3.14 &√3 = 1.732)

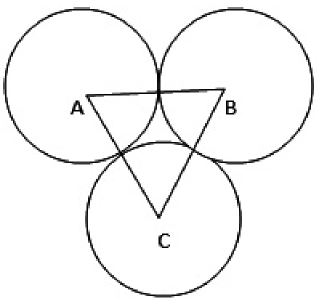
= 62.352 - 56.52

= 5.832 cm^2

The area of enclosed region is 5.832 cm^2.

Question: 40

Consider the figure shown below



Here, first we join the center of all adjacent circles then the distance between the center of circles touching each other is equal to the side of an equilateral triangle formed by joining the center of adjacent circles. Therefore, we can say that the side of the equilateral triangle is equal to the twice of the radius of circle. Now by simply calculating the area of the 3 sectors and then subtracting it from the area of the equilateral triangle we can easily calculate the area of the enclosed region.

Given radius of each circle = “a” units

Central angle of each sector = θ = 60° (∵ ΔABC is equilateral)

Side of equilateral ΔABC = 2×a units

$$\text{Area of sector formed at each corner } = \frac{\theta}{360} \times \pi R^2$$

$$\Longrightarrow \text{ Area of one sector } = \frac{60}{360} \times \pi(a^2)$$

$$\Longrightarrow \text{ Area of one sector } = \frac{\pi a^2}{6} \text{ sq. units } \rightarrow \text{ eqn1}$$

∴ Area of all 3sectors = 3×Area of one sector

$$\Longrightarrow \text{ Area of all the 3 sector } = 3 \times \frac{\pi a^2}{6} \text{ (from eqn 1)}$$

$$= \frac{\pi a^2}{2} \text{ sq. units } \rightarrow \text{ eqn2}$$

$$\text{Area of equilateral } \Delta ABC = \frac{\sqrt{3}}{4} \times (2a)^2$$

$$= \frac{\sqrt{3} \times 4a^2}{4}$$

$$= a^2\sqrt{3} \text{ sq.units } \rightarrow \text{ eqn3}$$

Area of enclosed region = Area of equilateral ΔABC – Area of all 3 sectors

$$\Longrightarrow \text{Area of enclosed region} = a^2\sqrt{3} - \frac{\pi a^2}{2} \text{ (from eqn 3 and eqn 2)}$$

$$= a^2 \times 1.73 - \frac{3.14 \times a^2}{2}$$

$$= \frac{a^2 \times 1.73 \times 2 - 3.14 \times a^2}{2}$$

$$= \frac{(3.46 - 3.14)a^2}{2}$$

(taking a^2 common)

$$\Longrightarrow \text{ Area of the enclosed region } = \frac{0.32a^2}{2}$$

$$= \frac{32a^2}{200}$$

$$= \frac{4a^2}{25} \text{ sq. units}$$

$$\text{Area of the enclosed region is } \frac{4a^2}{25} \text{ sq. units}$$

Question: 41

In the give

Solution:

Here in order to find the area of the shaded region we have to calculate the area, or the quadrant shown and subtract it from the area of the trapezium. And in order to find the area of the quadrant we have to calculate the radius of the sector EAB by the area of trapezium.

Given Area of trapezium ABCD = 24.5 cm^2 → eqn1

AD ∥ BC, AD = 10 cm, BC = 4 cm, ∠DAB = 90°

We also now Area of trapezium $= \frac{1}{2} \times$ (sum of parallel sides) × height

Area of trapezium $= \frac{1}{2} \times (AD + BC) \times AB \rightarrow$ eqn2

Putting the values in equation 2, we get,

$$24.5 = \frac{1}{2} \times (10 + 4) \times AB$$

$$\Rightarrow 24.5 = \frac{14 \times AB}{2}$$

$$\Rightarrow 24.5 = 7AB$$

$$\Rightarrow AB = \frac{24.5}{7}$$

⇒ AB = 3.5 cm

Therefore radius of the sector EAB = r = 3.5 cm

Area of quadrant EAB $= \frac{\theta}{360} \times \pi R^2$ where R = radius of the sector

$$\Rightarrow \text{Area of quadrant EAB} = \frac{90}{360} \times \pi(3.5^2) \left(\text{put } \pi = \frac{22}{7}\right)$$

$$\Rightarrow \text{Area of the quadrant} = \frac{90}{360} \times \frac{22}{7} \times 3.5 \times 3.5$$

$$\Rightarrow \text{Area of the quadrant EAB} = \frac{22 \times 3.5 \times 3.5}{4 \times 7}$$

$$\Rightarrow \text{Area of the quadrant EAB} = \frac{269.5}{28}$$

⇒ Area of the quadrant EAB = 9.625 cm^2 → eqn3

∴ Area of shaded region = Area of trapezium – Area of quadrant EAB

⇒ Area of shaded region = 24.5 – 9.625 (putting values from eqn1 and eqn3)

⇒ Area of shaded region = 14.875 cm^2

Question: 42

Here in order to find the area of the shaded region we have to calculate the area, or the quadrant shown and subtract it from the area of the trapezium. And in order to find the area of the quadrant we have to calculate the radius of the sector EAB by the area of trapezium.

Given AB = 30 m, AD = 55 m, BC = 45 m

θ_A = 90°, θ_B = 90°, θ_C = 120°, θ_D = 60°

Radius of each sector = r = 14 m

(i) total area of 4 sectors

Area of sector $= \frac{\theta_i}{360} \times \pi R^2 \rightarrow$ eqn1

$$= \frac{\theta_A}{360} \times \pi R^2$$

Area of sector at corner A $= \frac{90}{360} \times \pi \times 14^2$ (putting values in eqn 1)

Area of sector at corner A $= \frac{196\pi}{4}$

Area of sector at corner A = 49π m^2→ eqn2

As we know that central angle at A and B are both 90 degrees and radius is also same i.e. 14 m therefore the area of the sector at B will be exactly same as that of sector at A.

∴ Area of sector at corner B = Area of sector at corner A

⇒ Area of sector at corner B = 49π → eqn3

Similarly,

Area of sector $= \frac{\theta_C}{360} \times \pi R^2$

Area of sector at corner C $= \frac{120}{360} \times \pi \times 14^2$ (putting values in eqn 1)

Area of sector at corner C $= \frac{196\pi}{3}$

Area of sector at corner C = 65.33π m^2→ eqn4

Similarly,

Area of sector $= \frac{\theta_D}{360} \times \pi R^2$

Area of sector at corner D $= \frac{60}{360} \times \pi \times 14^2$ (putting values in eqn 1)

Area of sector at corner D $= \frac{196\pi}{6}$

Area of sector at corner D = 32.67π → eqn5

Total area of 4 sectors = eqn2 + eqn3 + eqn4 + eqn5

⇒ Total area of 4 sectors = 49π + 49π + 65.33π + 32.67π

⇒ Total area of 4 sectors = 196π

Total area of 4 sectors $= 196\pi$

Total area of 4 sectors $= 196 \times \frac{22}{7} \left(\text{put } \pi = \frac{22}{7}\right)$

∴ Total area of 4 sectors = 616 m^2

Total area of 4 sectors is 616 m^2.

(ii) Area of the remaining portion

Here in order to find the area of the remaining portion of the trapezium we have to subtract the area of the 4 sectors from the area of the trapezium.

Area of trapezium $= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

Area of trapezium $= \frac{1}{2} \times (AD + BC) \times AB$

On putting the values,

Area of trapezium $= \frac{1}{2} \times (55 + 45) \times 30$

$= \frac{100 \times 30}{2}$

= 50×30

Area of trapezium = 1500 m^2→ eqn1

Area of remaining portion = Area of trapezium – Area of the 4 sectors

⇒ Area of remaining portion = 1500 – 616 (from eqn1 and part (i))

∴ Area of remaining portion = 884 m^2

The area of the remaining portion is 884 m^2.

Question: 43

Area of shaded region can be calculated by subtracting the area of minor sector at vertex B from the sum of areas of the major sector at O and area of equilateral triangle.

Given Radius of circle at O = r = 6 cm

Side of equilateral triangle = a = 12 cm

Central angle at O = 360 -60 = 300°

Central angle at B = 60°

Area of the equilateral triangle = $\frac{\sqrt{3}}{4} \times a^2$

where a = side of equilateral triangle

Area of the equilateral triangle = $\sqrt{3}/4 \times (12)^2$ (putting the value of a)

Area of the equilateral triangle = (144×√3)/4

Area of the equilateral triangle = 36√3 cm^2 → eqn1

Area of sector = $\theta/360 \times \pi R^2$ where r = radius of the sector

Area of minor sector at B = $60/360 \times \pi \times (6^2)$ (given)

∴ Area of minor sector at B = 6π cm^2→ eqn2

Similarly,

Area of major sector at O $= \frac{300}{360} \times \pi(6^2)$

∴ Area of major sector at O = 30π cm^2→ eqn3

Area of shaded region = eqn1 + eqn3 – eqn2

On putting values

⇒ Area of shaded region = 36√3 + 30π-6π

Area of shaded region = 36√3 + 24π

(put π = 3.14 and √3 = 1.73

∴ Area of shaded region = (36×1.73) + (24×3.14)

⇒ Area of shaded region = 62.28 + 75.36

∴ Area of shaded region = 137.64 cm^2

Area of the shaded region is 137.64 cm^2.

Question: 44

Here in order to find the area of the shaded region we have to subtract the area of the semicircle and the triangle from the area of the rectangle.

Given AB = 80 cm, BC = 70 cm, DE = 42 cm, ∠AED = 90°

Here we see that the triangle AED is right angle triangle, therefore, we can apply Pythagoras theorem i.e.

$H^2 = P^2 + B^2$ (pythagoras theorem)

$AD^2 = DE^2 + AE^2$

$\Rightarrow 70^2 = 42^2 + AE^2$ (putting the given values)

$\Rightarrow 4900 = 1764 + AE^2$

$\Rightarrow 4900 - 1764 = AE^2$

$\Rightarrow 3136 = AE^2$

$AE = \sqrt{3136}$

$\therefore$ AE = 56 cm

Area of $\Delta AED = 1/2 \times AE \times DE$

(Area of triangle = $1/2 \times$ base $\times$ height)

On putting values we get,

Area of $\Delta AED = 1/2 \times 56 \times 42$

$\Rightarrow$ Area of $\Delta AED = 28 \times 42$

$\therefore$ Area of $\Delta AED = 1176\ cm^2 \rightarrow$ eqn1

$$\text{Area of semicircle} = \frac{\pi R^2}{2}$$

$$\text{Here radius of semicirle} = \frac{BC}{2} = \frac{70}{2}$$

R = 35 cm

$$\therefore \text{ Area of semicircle} = \frac{\pi \times 35^2}{2}$$

$$\text{Area of semicircle} = \frac{22 \times 1225}{2 \times 7} \text{ (putting } \pi = \frac{22}{7}\text{)}$$

$\Rightarrow$ Area of semicircle = 11×175

$\therefore$ Area of semicircle = $1925\ cm^2 \rightarrow$ eqn2

Area of rectangle = $\ell \times b$ (ℓ = length of rectangle, b = breadth of rectangle)

$\Rightarrow$ Area of rectangle = $80 \times 70 = 5600\ cm^2 \rightarrow$ eqn3

Area of shaded region = Area of rectangle – Area of semicircle – Area of Δ

$\Rightarrow$ Area of shaded region = 5600 -1925 – 1176 (fromeqn1, eqn2 and eqn3)

$\therefore$ Area of shaded region = $2499\ cm^2$

Area of the shaded region is $2499\ cm^2$.

Question: 45

Here in order to find the area of the shaded region (region excluding the triangle) we have to subtract the area of the triangle from the area of the rectangle and then add the area of the semicircle.

Given AB = 20 cm, DE = 12 cm, AE = 9 cm and $\angle AED = 90°$

Here we see that the triangle AED is right angle triangle, therefore, we can apply Pythagoras theorem i.e.

$H^2 = P^3 + B^2$ (pythagoras theorem)

$AD^2 = DE^2 + AE^2$

$AD^2 = DE^2 + AE^2$

$AD^2 = 12^2 + 9^2$ (putting given values)

$\Rightarrow AD^2 = 144 + 81$

$\Rightarrow AD^2 = 225$

$\Rightarrow AD = \sqrt{225}$

$\therefore AD = 15$ cm

Area of $\Delta AED = 1/2 \times AE \times DE$

(Area of triangle = 1/2×base×height)

On putting values we get,

$$\text{Area of } \Delta AED = \frac{1}{2} \times 9 \times 12$$

$\Rightarrow$ Area of $\Delta AED = 9 \times 6$

$\therefore$ Area of $\Delta AED = 54\ cm^2 \rightarrow$ eqn1

$$\text{Area of semicircle} = \frac{\pi R^2}{2}$$

Here radius of semicircle = BC/2 = 15/2

$\Rightarrow R = 7.5$ cm

$$\therefore \text{ Area of semicircle} = \frac{\pi \times 7.5^2}{2}$$

$$\text{Area of semicircle} = \frac{3.14 \times 56.25}{2} \text{ (putting } \pi = 3.14)$$

$\Rightarrow$ Area of semicircle = 1.07×56.25

$\therefore$ Area of semicircle = $88.3125\ cm^2 \rightarrow$ eqn2

Area of rectangle = $\ell \times b$

(ℓ = length of rectangle, b = breadth of rectangle)

$\Rightarrow$ Area of rectangle = 20×15

(putting the values of ℓ & b)

$\therefore$ Area of rectangle = $300\ cm^2 \rightarrow$ eqn3

Area of shaded region = Area of rectangle + Area of semicircle – Area of Δ

$\Rightarrow$ Area of shaded region = 300 + 88.3125 – 53 (from eqn1, eqn2, eqn3)

$\therefore$ Area of shaded region = $334.3125\ cm^2$

Area of shaded region is $334.3125\ cm^2$.

Question: 46

Here in order to find the area of the shaded region (region excluding the area of segment AC and quadrant OCD) can be calculated by subtracting the area of triangle and quadrant OBD from the area of the circle.

Given AC = 24 cm, AB = 7 cm and $\angle BOD = 90°$

Here we see that the triangle ACB is right angle triangle, therefore, we can apply Pythagoras theorem i.e.

$H^2 = P^2 + B^2$ (pythagoras theorem)

$BC^2 = AC^2 + AB^2$

$\Rightarrow BC^2 = 24^2 + 7^2$ (putting the given values)

$\Rightarrow BC^2 = 576 + 49$

$\Rightarrow BC^2 = 625$

$BC = \sqrt{625}$

$\therefore BC = 25$ cm

Area of $\Delta ACB = 1/2\times AB\times AC$ (Area of triangle = $1/2\times$base$\times$height)

On putting values we get,

$$\text{Area of } \Delta ACB = \frac{1}{2}\times 7\times 24$$

$\Rightarrow$ Area of $\Delta AED = 7\times 12$

$\therefore$ Area of $\Delta AED = 84\ cm^2 \rightarrow$ eqn1

Area of circle $= \pi R^2$ (R = radius of circle)

$$\text{Here radius of cicrle} = \frac{BC}{2} = \frac{25}{2} \text{ (because ABCD is a rectangle)}$$

$\Rightarrow R = 12.5$ cm

$\therefore$ Area of circle $= \pi\times 12.5^2$

$\Rightarrow$ Area of circle $= 156.25\times 3.14$ (put $\pi = 3.14$)

$\therefore$ Are of circle $= 490.625\ cm^2 \rightarrow$ eqn2

$$\text{Are of quadrant OBD} = \frac{\theta}{360}\times \pi R^2$$

$$\text{Area of quadrant OBD} = \frac{90}{360}\times \pi \times 12.5^2 \text{ (put } \pi = 3.14)$$

$$\text{Area of quadrant} = \frac{3.14\times 156.25}{4}$$

$\Rightarrow$ Area of quadrant OBD $= 122.65625\ cm^2 \rightarrow$ eqn3

Area of shaded region = Area of circle – Area of quadrant – Area of Δ

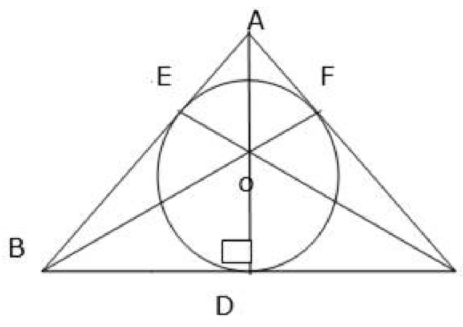
$\Rightarrow$ Area of shaded region = 490.625 – 84 – 122.65625 (from eqn1, 2 and 3)

$\Rightarrow$ Area of shaded region $= 283.96875\ cm^2$

Area of shaded region is $283.96875\ cm^2$.

Question: 47

Here we will draw median from all the vertices of the equilateral triangle and the point at which they intersect will be the incircle of the triangle and that will be the centre of the circle. Thenwith the help of which we will find out the height of the triangle and subsequently the radius of the circle and ultimately the area of the shaded region (region of equilateral triangle excluding the area of circle inscribed).



As AD = BF = CE = h

Consider ΔADB, $\angle ADB = 90°$, BD = 6 cm

$AB^2 = AD^2 + BD^2$ (Phythagoras theorem)

$12^2 = AD^2 + 6^2$ (putting the given values)

$144 = AD^2 + 36$

$144 - 36 = AD^2$

$AD^2 = 108$

$AD = \sqrt{108}$

$AD = \sqrt{9 \times 3 \times 4}$

$AD = 6\sqrt{3}$ cm

so, $h = 6\sqrt{3}$ cm

We also know that a point O will divide each median in a ratio of 2:1

$$\text{So, } OD = \frac{h}{3}$$

$$OD = \frac{6\sqrt{3}}{3}$$

$OD = 2\sqrt{3}$ cm

$\therefore$ radius of the circle $= r = 2\sqrt{3}$ cm

Area of the circle $= \pi r^2$

Area of the circle $= \pi \times \left(2\sqrt{3}\right)^2$ (putting the value of r)

$\therefore$ Area of the circle = 12π cm$^2 \rightarrow$ eqn1

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4} \times a^2 \text{ where } a = \text{ side of equilateral triangle}$$

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4} \times 12^2$$

$$\text{Area of } \Delta ABC = \frac{144 \times \sqrt{3}}{4}$$

Area of $\Delta ABC = 36\sqrt{3}$ cm$^2 \rightarrow$ eqn2

Area of shaded region = area of triangle – area of circle

Area of the shaded regiom $= 36\sqrt{3} - 12\pi$ $\left(\text{put } \pi = 3.14 \text{ \& } \sqrt{3} = 1.73\right)$

$\Rightarrow$ Area of the shaded region = (36×1.73) – (12×3.14)

$\Rightarrow$ Area of the shaded region = 62.28 -37.68

$\therefore$ Area of the shaded region = 24.6 cm^2

The radius of the circle is $2\sqrt{3}$ cm and area of shaded region is 24.6 cm^2.

Question: 48

Here we will first find the sides of equilateral triangle ant then subtract the area of the triangle from the area of the circle.

Given radius of circle = r = 42 cm

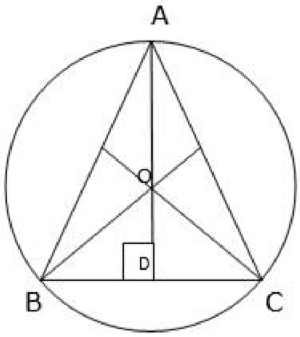
$\therefore$ Area of the circle = πR^2, where R = radius of the circle

$\Rightarrow$ Area of the circle = $\pi(42^2)$

$\therefore$ Area of circle $= \frac{22}{7} \times 1764 \left(\text{putting } \pi = \frac{22}{7}\right)$

$\Rightarrow$ Area of the circle = 22×252

$\therefore$ Area of the circle = 5544 $cm^2 \rightarrow$ eqn1



Consider the figure shown,

In ΔABD, ∠ADB = 90°

$\therefore\ AB^2 = AD^2 + BD^2 \rightarrow$ eqn2 (Pythagoras theorem)

Let the sides of the equilateral triangle = a cm

And as we know AD is a median therefore it will bisect the side BC into two equal parts i.e.

BD = DC → eqn3

Also, BC = BD + DC

⇒ BC = BD + BD (from eqn3)

⇒ a = 2BD (BC = a)

$BD = \frac{a}{2}$ cm

So, $a^2 = AD^2 + \left(\frac{a}{2}\right)^2$ (putting values of AC and BD in eqn2)

$\Longrightarrow a^2 = AD^2 + \frac{a^2}{4}$

$\Longrightarrow a^2 - \frac{a^2}{4} = AD^2$

$\Longrightarrow \frac{4a^2 - a^2}{4} = AD^2$

$\Longrightarrow \frac{3a^2}{4} = AD^2$

$\Longrightarrow AD = \sqrt{\frac{3a^2}{4}}$

$\Longrightarrow AD = \frac{a\sqrt{3}}{2}$ cm $\rightarrow$ eqn4

Now, we also know that the point 'O' which is the intersection of all the three medians i.e. centroid of the triangle. Also we know that the centroid divides the median in the ratio 2:1.

So, we can say that $AO = \frac{2AD}{3}$

Also, we know AO = radius = r = 42 cm

$\therefore\ 42 = \frac{2AD}{3}$

$\Rightarrow \frac{42 \times 3}{2} = AD$

$\Rightarrow AD = 63$ cm

Putting the value in equation 4,

$63 = \frac{a\sqrt{3}}{2}$

$\Rightarrow \frac{63 \times 2}{\sqrt{3}} = a$

$\Rightarrow \frac{126}{\sqrt{3}} = a$

$\Rightarrow \frac{126 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = a$ (rationalizing L.H.S)

$\Rightarrow \frac{126\sqrt{3}}{3} = a$

$\Rightarrow a = 42\sqrt{3}$ cm

o, area of equilateral triangle ABC $= \frac{\sqrt{3}}{4} \times a^2$ (where a = side of triangle)

$\Rightarrow$ Area of triangle ABC $= \frac{\sqrt{3}}{4} \times (42\sqrt{3})^2$ (putting the value of a)

$\Rightarrow$ Area of triangle ABC $= \frac{\sqrt{3}}{4} \times (1764 \times 3)$

$\Rightarrow$ Area of triangle ABC $= \frac{\sqrt{3}}{4} \times 5292$

$\Rightarrow$ Area of triangle ABC $= 1323\sqrt{3}\ cm^2 \rightarrow$ eqn5

Area of covered by design = Area of circle – Area of triangle ABC

Area covered by design $= 5544 - 1323\sqrt{3}$ (from eqn1 and eqn5)

$\Rightarrow$ Area covered by design $= 5544 - (1323 \times 1.73)$ (putting $\sqrt{3} = 1.73$)

$\Rightarrow$ Area covered by design = 5544 – 2288.79

$\therefore$ Area covered by design = 3255.21 cm^2

Area covered by design is 3255.21 cm^2.

Question: 49

We know perimeter of a sector = Length of its arc + 2R → eqn1

Where R = radius of the sector.

Perimeter = 25 cm

Also, length of arc of sector $= \frac{\theta}{360} \times 2\pi R$

$\theta = 90°$

$\therefore 25 = \frac{90}{360} \times 2\pi R + 2R$ (putting the values in eqn1)

$\Rightarrow 25 = \frac{2\pi R}{4} + 2R$

$$\Rightarrow 25 = \frac{\pi R}{2} + 2R$$

$$\Rightarrow 25 \frac{\pi R + 4R}{2} \text{ (taking 2 as L. C. M on R. H. S)}$$

$$\Rightarrow 25 \times 2 = (\pi + 4)R \text{ (taking R common)}$$

$$\Rightarrow 50 = \left(\frac{22}{7} + 4\right) R \left(\text{putting } \pi = \frac{22}{7}\right)$$

$$\Rightarrow 50 = \left(\frac{22 + 28}{7}\right) R \text{ (taking 7 as L. C. M onR. H. S)}$$

$$\Rightarrow 50 \times 7 = 50R$$

$$\Rightarrow \frac{50 \times 7}{50} = R$$

$\Rightarrow R = 7$ cm $\rightarrow$ eqn2

$$\text{Area odf a secotor} = \frac{\theta}{360} \times \pi R^2$$

$$\Rightarrow \text{Area of quadrant} = \frac{90}{360} \times \pi(7^2) \text{ (putting the value of } \theta \text{ amd R)}$$

$$\Rightarrow \text{Area of the quadrant} = \frac{49\pi}{4} \left(\text{put } \pi = \frac{22}{7}\right)$$

$$\Rightarrow \text{Area of the quadrant} = \frac{49 \times 22}{4 \times 7}$$

$$\Rightarrow \text{Area of the quadrant} = \frac{7 \times 11}{2}$$

$\therefore$ Area of the quadrant = 38.5 cm^2

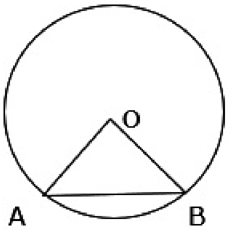
Area of the quadrant is 38.5 cm ^{2}.

Question: 50

Given the radius of the circle = 42 cm

Central angle of the sector = θ = 90°

Area of the minor segment = Area of sector – area of the right angle triangle



$$\text{Area of the sector} = \frac{\theta}{360} \times \pi R^2$$

$$\text{Area of the sector} = \frac{90}{360} \times \pi(10^2) \text{ (putting the values of } \theta \text{ and R)}$$

$$\Rightarrow \text{Area of the sector} = \frac{100\pi}{4} \text{ (put } \pi = 3.14)$$

$$\Rightarrow \text{Area of the sector} = \frac{100 \times 3.14}{4}$$

$\Rightarrow$ Area of the sector = 25×3.14

$\therefore$ Area of the sector = 78.5 cm^2 $\rightarrow$ eqn1

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\Rightarrow \text{Area of triangle} = \frac{1}{2} \times 10 \times 10$$

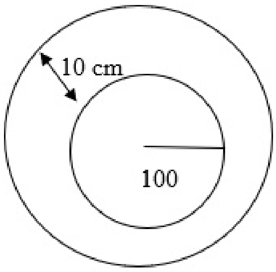
∴ Area of triangle = 50 cm^2→ eqn2

Area of the minor segment = 78.5 – 50 (from eqn1, eqn2)

∴ Area of the minor segment = 28.5 cm^2

Area of the minor segment is 28.5 cm^2.

Question: 51



Here we will first find out the area of the road running around the circular garden and then multiplying it with rate per square meter to calculate the cost of leveling.

Here we see in the figure there are two concentric circles so,

Area of road = Area of outer circle- Area of circular garden

Area of circle = πR^2 (where R = radius of circle) → eqn1

Let the radius of inner circle = r = 100 m

Also, radius of outer circle = R = 110 m (R = r + 10)

Area of outer circle = $\pi(110)^2$→ eqn2 (putting R in eqn1)

Area of inner circle = $\pi(100)^2$→ eqn2 (putting r in eqn1)

∴ Area of road = $\pi(110)^2 - \pi(100)^2$ (from eqn2 and 3)

⇒ Area of road = $\pi(12100 - 10000)$

⇒ Area of road = 2100 π (put π = 3.14)

⇒ Area of road = 2100×3.14

∴ Area of road = 6594 m^2

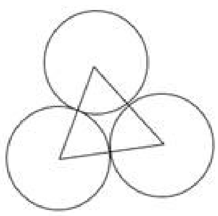
Cost of leveling = Rate of leveling × Area of road

⇒ Cost of leveling = 20×6594

∴ Cost of leveling = Rs.131880

Area of road is 6594 m^2 and cost of leveling is Rs.131880.

Question: 52



$$\text{Area of equilateral triangle} = 49\sqrt{3}\ \text{cm}^2$$

Each angle of triangle = θ = 60°

Area of triangle not included in circles = Area of triangle – Area of all sectors

Area of all 3 sectors area equal as all the three circles are having same radius which is equal to the half of the side of the equilateral triangle.

Let the side of equilateral triangle be = a cm

$\text{also, area of equilateral triangle} = \frac{\sqrt{3}}{4} \times a^2 \text{ (a = side of triangle)}$

$49\sqrt{3} = \frac{\sqrt{3}}{4} \times a^2 \text{ (equating the value of area to the above equation)}$

$\Rightarrow \frac{49\sqrt{3} \times 4}{\sqrt{3}} = a^2$

$\Rightarrow a^2 = 49 \times 4$

$\Rightarrow a = \sqrt{49 \times 4}$

⇒ a = 7×2

⇒ a = 14 cm

So radius of the circles = 7 cm

$\text{Area of sector} = \frac{\theta}{360} \times \pi R^2$

$\text{Area of sector} = \frac{60}{360} \times \pi(7)^2$

$\text{Area of sector} = \frac{22 \times 49}{6 \times 7} \left(\text{put } \pi = \frac{22}{7}\right)$

$\text{Area of sector} = \frac{11 \times 7}{3}$

$\text{Area of sector} = \frac{77}{3} \text{ cm}^2$

$\text{Area of 3 sector} = 3 \times \text{area of one sector}$

$\text{Area of 3 sector} = 3 \times \frac{77}{3}$

∴ Area of all 3 sectors = 77 cm^2→ eqn1

Area of triangle not included in circles

$= 49\sqrt{3} - 77 \text{ (putting value of area of triangle and sector)}$

Area of triangle not included = (49×1.73) - 77

⇒ Area of triangle not included = 84.77 – 77

∴ Area of triangle not included = 7.77 cm^2

Area of triangle not included in circles is 7.77 cm^2.

Question: 53

Area of whole figure = ar || ABCD + ar || FGHI + ar DCIF + ar ΔDEF + area semicircle

CD = 8 cm, BP = HQ = 4 cm, DE = EF = 5 cm, CI = 8 cm

ar ∥ ABCD = ar ∥ FGHI = base × height

ar ∥ ABCD = ar ∥ FGHI = BP×DC

ar ∥ ABCD = ar ∥ FGHI = 4×8

ar ∥ ABCD = ar ∥ FGHI = 32 cm^2→ eqn1 and eqn2 ar DCIF = area of square = side×side

ar DCIF = DC×CI

ar DCIF = 8×8

ar DCIF = 64 cm^2→ eqn3

Consider ΔDEF, EF⊥DF and ΔDEF is isosceles

So, FL = LD

$$FL = LD = \frac{DF}{2}$$

$$FL = LD = \frac{8}{2}$$

FL = LD = 4 cm

In ΔDEL, ∠DLE = 90°

$ED^2 = EL^2 + LD^2$ (Pythagoras theorem)

$5^2 = EL^2 + 4^2$ (putting the values)

$25 = EL^2 + 16$

$25 - 16 = EL^2$

⇒ $EL^2 = 9$

$EL = \sqrt{9}$

∴ EL = 3 cm

$$\text{Area of } \Delta\text{ DEF} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area of } \Delta\text{DEF} = \frac{1}{2} \times DF \times EL$$

$$\text{Area of } \Delta\text{DEF} = \frac{1}{2} \times 8 \times 3$$

⇒ Area of ΔDEF = 4×3

∴ Area of ΔDEF = 12 cm^2→ eqn4

$$\text{Area of semicrcle} = \frac{\pi R^2}{2} \text{ where R = radius of the semicircle}$$

R = 4 cm

$$\text{Area of semicircle} = \frac{\pi(4)^2}{2} \text{ (put } \pi = 3.14)$$

$$\text{Area of semicircle} = \frac{3.14 \times 16}{2}$$

⇒ Area of semicircle = 3.14×8

∴ Area of semicircle = 25.12 cm^2→ eqn5

Area of whole figure = eqn1 + eqn2 + eqn3 + eqn4 + eqn5

⇒ Area of whole figure = 32 + 32 + 64 + 12 + 25.12

∴ Area of whole figure = 165.12 cm^2

Area of the whole figure is 165.12 cm^2.

Question: 54

A $\theta_1 = 90°$ $\theta_2 = 120°$, $\theta_3 = 150°$

Radius of circle = r = 6 cm

$$\text{Area of sector} = \frac{\theta}{360} \times \pi R^2 \rightarrow \text{eqn1}$$

Area of circle = πR^2

⇒ Area of circle = $\pi \times 6^2$

⇒ Area of circle = 36π → eqn2

$$\text{Area of sector}(\theta_3) = \frac{150}{360} \pi \times 6^2 \text{ (from eqn1)}$$

$$\text{Area of sector}(\theta_3) = \frac{15\pi}{36} \times 36$$

$$\text{Area of sector } (\theta_3) = \frac{15}{36} \times 36\pi$$

$$\text{Area of sector } (\theta_3) = \frac{15}{36} \times \text{Area of circle (from eqn2)}$$

$$\text{Area of sector}(\theta_3) = \frac{5}{12} \times \text{Area of circle}$$

Also, Area of sector (θ_3) = 15π cm^2

$$\text{Area of sector}(\theta_2) = \frac{120}{360} \times \pi 6^2 \text{ (putting values in eqn 1)}$$

$$\text{Area of sector}(\theta_2) = \frac{12}{36} \times 36\pi$$

Area of sector (θ_2) = 12π cm^2→ eqn3

$$\text{Area of sector}(\theta_1) = \frac{90}{360} \times \pi 6^2 \text{ (putting values in eqn 1)}$$

$$\text{Area of sector}(\theta_1) = \frac{9}{36} \times 36\pi$$

Area of sector (θ_1) = 9π cm^2→ eqn4

Ratio of three sectors ∷ $9\pi:12\pi:15\pi$

Ratio of three sectors ∷ 3:4:5

Area of sector(θ_3) is $\frac{5}{12} \times$ Area of circle and Ratio of three sectors ∷ 3 : 4 : 5

Question: 55

Total area of design = Area of all the minor segments

Here we will find out the area of one segment and then multiply it with 6 to get the total area of design. And as the figure inscribed in the circle is a regular hexagon this implies that it will be having all edges of same length. Therefore we can say that the angle subtended by each chord which are actually the edges of regular hexagon are equal(theorem).

Let angle subtended by chord AB on centre O be θ

$$\text{So, angle subtended} = \theta = \frac{360}{6}$$

∴ Angle subtended = $\theta = 60°$

Radius of circle = 35 cm

$$\text{Area of sector} = \frac{\theta}{360} \times \pi R^2 \rightarrow \text{eqn1}$$

$$\text{Area of one sector} = \frac{60}{360} \times \pi(35)^2 \text{ (putting values in eqn1)}$$

$$\text{Area of one sector} = \frac{1225\pi}{6} \text{ cm}^2$$

$$\text{Areaof } \Delta OAB = \frac{1}{2} \times OA \times OB \times \sin\theta \text{ (where } \theta = \text{ central angle of sector)}$$

$$\text{Area of } \Delta OAB = \frac{1}{2} \times 35 \times 35 \times \sin 60$$

$$\text{Area of } \Delta OAB = \frac{1}{2} \times 1225 \times \frac{\sqrt{3}}{2} \left(\sin 60 = \frac{\sqrt{3}}{2}\right)$$

$$\text{Area of } \Delta OAB = \frac{1225\sqrt{3}}{4} \text{ cm}^2$$

Area of minor segment OAB = Area of sector – Area of ΔOAB

$$\text{Area of minor segment OAB} = \frac{1225\pi}{6} - \frac{1225\sqrt{3}}{4} \left(\text{put } \pi = 3.14 \text{ \& } \sqrt{3} = 1.732\right)$$

$$\text{Area of minor segment OAB} = \frac{1225 \times 3.14}{6} - \frac{1225 \times 1.732}{4}$$

$$\text{Area of minor segment OAB} = \frac{3846.5}{6} - \frac{2121.7}{4}$$

Area of minor segment OAB = 641.0833333 – 530.425

∴ Area of minor segment OAB = 110.6583333 $\text{cm}^2 \rightarrow$ eqn2

Total area of design = 6 × Area of minor segment OAB

⇒ Total area of design = 6×110.6583333 (from eqn2)

∴ Total area of design = 663.95 cm^2

Total area of design is 663.95 cm^2.

Question: 56

Here we will subtract the area of right angle triangle PQR and semicircle from the area of entire circle.

Given PQ = 24 cm, PR = 7 cm

Consider ΔPQR, ∠QPR = 90°

$$RQ^2 = PQ^2 + PR^2 \text{ (Pyhtagoras theorem)}$$

$RQ^2 = 24^2 + 7^2$

⇒ $RQ^2 = 576 + 49$

⇒ $RQ^2 = 625$

$$\Rightarrow RQ = \sqrt{625}$$

$$\therefore RQ = 25 \text{ cm}$$

Therefore Radius of the circle = half of RQ

Let radius be ‘r’

$$r = \frac{25}{2}$$

∴ r = 12.5 cm

$$\text{Area } \Delta PQR = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area of } \Delta PQR = \frac{1}{2} \times PR \times PQ$$

$$\text{Area of } \Delta PQR = \frac{1}{2} \times 7 \times 24$$

Area of ΔPQR = 7×12

∴ Area of ΔPQR = 84 cm^2→ eqn1

$$\text{Area of semicircle} = \frac{\pi r^2}{2}$$

$$\text{Area of semicircle} = \frac{3.14 \times (12.5^2)}{2} \text{ (putting } \pi = 3.14)$$

$$\text{Area of semicircle} = \frac{3.14 \times 156.25}{2}$$

∴ Area of semicircle = 245.3125 cm^2→ eqn2

Area of circle = πr^2

Area of circle = $\pi(12.5^2)$

⇒ Area of circle = 3.14×156.25 (putting π = 3.14)

∴ Area of circle = 490.625 cm^2→ eqn3

Area of shaded region = eqn3 – eqn2 – eqn1

⇒ Area of shaded region = 490.625 – 245.3125 – 84

∴ Area of shaded region = 161.3125 cm^2

Area of shaded region is 161.3125 cm^2.

Question: 57

Given AB = 6 cm, BC = 10 cm

Consider ΔABC, ∠BAC = 90°

$BC^2 = AB^2 + AC^2$ (Pythagoras theorem)

⇒ $10^2 = 6^2 + AC^2$ (putting given values)

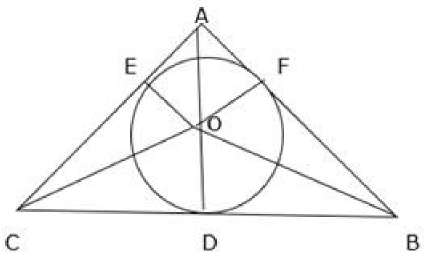
⇒ $100 = 36 + AC^2$

⇒ $100 – 36 = AC^2$

⇒ $AC^2 = 64$

$AC = \sqrt{64}$

∴ AC = 8 cm



Join OB, OA, OC, OE, OF, OD

Here OE = OF = OD = radius of circle = r cm

$\angle OEC = \angle ODB = \angle OFB = 90°$ (angle at the point of contact of radius & tangent)

Area ΔABC = Area of ΔOAC + Area of ΔOCB + Area of ΔOAB → eqn1

$$\text{Area of } \Delta OAC = \frac{1}{2} \times OE \times AC$$

$$\text{Area of } \Delta OAC = \frac{1}{2} \times r \times 8$$

∴ Area of ΔOAC = 4r → eqn2

$$\text{Area of } \Delta OCB = \frac{1}{2} \times OD \times BC$$

$$\text{Area of } \Delta OCB = \frac{1}{2} \times r \times 10$$

∴ Area of ΔOCB = 5r → eqn3

$$\text{Area of } \Delta OAB = \frac{1}{2} \times OF \times AB$$

$$\text{Area of } \Delta OAB = \frac{1}{2} \times r \times 6$$

∴ Area of ΔOAB = 3r → eqn4

$$\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times AC$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 6 \times 8$$

⇒ Area of ΔABC = 3×8

∴ Area of ΔABC = 24 cm^2→ eqn5

Putting all the values in equation we get;

⇒ 24 = 4r + 5r + 3r

⇒ 24 = 12r

$$r = \frac{24}{12}$$

∴ r = 2 cm

Area of circle = πr^2

Put the value of r, we get,

⇒ Area of circle = $\pi \times 2^2$

⇒ Area of circle = 3.14×4 (putting π = 3.14)

∴ Area of circle = 12.56 cm^2→ eqn6

Area of shaded region = Area of triangle – Area of circle

⇒ Area of shaded region = 24 – 12.56 (from eqn5 and eqn6)

∴ Area of shaded region = 11.44 cm^2

Area of shaded region is 11.44 cm^2.

Question: 58

Here we will first find out the area of semicircle whose diameter is BC and then subtract the area of right angle triangle ABC from it and then we will subtract this result from the area of semicircles whose diameters are AB and AC.

Consider ΔABC, $\angle BAC = 90°$

$BC^2 = AC^2 + AB^2$ (Pythagoras theorem)

$\Rightarrow BC^2 = 4^2 + 3^2$

$\Rightarrow BC^2 = 16 + 9$

$\Rightarrow BC^2 = 25$

$BC = \sqrt{25}$

$\therefore BC = 5$ cm

Area of semicircle $= \frac{\pi r^2}{2} \rightarrow$ eqn1 where r = radius of the semicircle

Area of semicircle whose diameter is AC

Radius $= \frac{AC}{2}$

Radius $= \frac{4}{2}$

$\therefore$ Radius = 2 cm

Area of semicircle $= \frac{\pi(2)^2}{2}$ (from eqn1)

$\therefore$ Area of semicircle = 2π cm^2 –eqn2

Area of semicircle whose diameter is AB

Radius $= \frac{AB}{2}$

Radius $= \frac{3}{2}$

$\therefore$ Radius = 1.5 cm

Area of semicircle $= \frac{\pi(1.5)^2}{2}$ (from eqn1)

$\therefore$ Area of semicircle = 1.125π $cm^2 \rightarrow$ eqn3

Area of semicircle whose diameter is BC

Radius $= \frac{BC}{2}$

Radius $= \frac{5}{2}$

$\therefore$ Radius = 2.5 cm

Area of semicircle $= \frac{\pi(2.5)^2}{2}$ (from eqn1)

$\therefore$ Area of semicircle = 3.125π $cm^2 \rightarrow$ eqn4

Area of triangle PQR $= \frac{1}{2} \times AB \times AC$

Area of triangle PQR $= \frac{1}{2} \times 3 \times 4$

$\Rightarrow$ Area of triangle PQR = 3×2

$\therefore$ Area of triangle PQR = 6 $cm^2 \rightarrow$ eqn5

Now subtract equation 5 from equation 4,

⇒ Area of semicircle excluding ΔABC = eqn4 – eqn5

⇒ Area of semicircle excluding ΔABC = 3.125π – 6

$$\Rightarrow \text{ Area of semicircle excluding } \Delta ABC = 3.125 \times \frac{22}{7} - 6 \text{ cm}^2$$

$$\Rightarrow \text{ Area of semicircle excluding } \Delta ABC = \frac{3.125 \times 22}{7} - 6$$

$$\Rightarrow \text{ Area of semicircle excluding } \Delta ABC = \frac{68.75 - 42}{7}$$

$$\Rightarrow \text{ Area of semicircle excluding } \Delta ABC = \frac{26.75}{7}$$

∴ Area of semicircle excluding ΔABC = 3.8214 cm^2→ eqn6

Area of shaded region = eqn3 + eqn2 – eqn6

⇒ Area of shaded region = 2π + 1.125π – 3.8214

⇒ Area of shaded region = 3.125π – 3.8214

$$\text{Area of shaded region} = 3.125 \times \frac{22}{7} - 3.8214$$

$$\text{Area of shaded region} = \frac{68.75}{7} - 3.8214$$

⇒ Area of shaded region = 9.8214 – 3.8214

∴ Area of shaded region = 6 cm^2

Area of shaded region 6 cm^2.

Question: 59

Here we will subtract the area of semicircle whose diameter is QS from the area of the semicircle whose diameter PS and add the area of semicircle whose diameter is PQ so as to find out the area of the shaded region.

Given PS = 12 cm

Radius of the circle = 6 cm

PQ = QR = RS

So let PQ = QR = RS = k cm

Also, PQ + QR + RS = PS

⇒ k + k + k = 12

⇒ 3k = 12

$$k = \frac{12}{3}$$

∴ k = 4 cm

So, PQ = QR = RS = 4 cm

$$\text{Area of semicircle} = \frac{\pi r^2}{2} \rightarrow \text{ eqn1 where r = radius of the semicircle}$$

Area and perimeter of semicircle whose diameter is PS

$$\text{Radius} = \frac{PS}{2}$$

$$\text{Radius} = \frac{12}{2}$$

∴ Radius = 6 cm

Area of semicircle $= \dfrac{\pi(6)^2}{2}$ (from eqn1)

∴ Area of semicircle = 18π cm^2→ eqn2

Perimeter of semicircle = πr

⇒ Perimeter of semicircle = π×6

∴ Perimeter of semicircle = 6π cm → eqn3

Area of semicircle whose diameter is QS

Radius $= \dfrac{QS}{2}$

Radius $= \dfrac{8}{2}$

⇒ Radius = 4 cm

Area of semicircle $= \dfrac{\pi(4)^2}{2}$ (from eqn1)

∴ Area of semicircle = 8π cm^2→ eqn4

Perimeter of semicircle = πr

⇒ Perimeter of semicircle = π×4

∴ Perimeter of semicircle = 4π cm → eqn5

Area of semicircle whose diameter is PQ

Radius $= \dfrac{PQ}{2}$

Radius $= \dfrac{4}{2}$

∴ Radius = 2 cm

Area of semicircle $= \dfrac{\pi(2)^2}{2}$ (from eqn1)

∴ Area of semicircle = 2π cm^2→ eqn6

Perimeter of semicircle = πr

⇒ Perimeter of semicircle = π×2

∴ Perimeter of semicircle = 2π cm → eqn7

Area of the shaded region = eqn2 – eqn4 + eqn6

Area of shaded region = 18π – 8π + 2π

⇒ Area of shaded region = 12π

⇒ Area of shaded region = 12×3.14 (putting π = 3.14)

∴ Area of shaded region = 37.68 cm^2

Perimeter of shaded region = eqn3 – eqn5 + eqn7

⇒ Perimeter of shaded region = 6π -4π + 2π

⇒ Perimeter of shaded region = 4π

⇒ Perimeter of shaded region = 4×3.14 (put π = 3.14)

∴ Perimeter of shaded region = 12.56 cm

Perimeter of the shaded region is 12.56 cm and Area of shaded region is 37.68 cm^2.

Question: 60

Consider the figure as a combination of two semicircles on the ends of the rectangle

Let the length of rectangle be ‘L’ m and breadth be ‘B’ cm

Given L = 90 m, W = 14 m

Perimeter of running track = 400 m

Perimeter of inside of running track = 2L + Arc of two semicircles → eqn1

Arc length of a semicircle = πr where r = radius

⇒ 400 = (2×90) + (2×πr) (putting values in eqn1)

⇒ 400 = 180 + 2πr

⇒ 400 – 180 = 2πr

⇒ 220 = 2πr

$$220 = 2 \times \frac{22}{7} \times r \left(\pi = \frac{22}{7}\right)$$

$$220 = \frac{44}{7} \times r$$

$$\frac{220 \times 7}{44} = r$$

∴ r = 35 m

Area of inner of running track = Area of rectangle + 2×area of semicircles → eqn2

Area of rectangle = L×B

Here B = 2r

B = 70 m

⇒ Area of inner rectangle = 90×70

∴ Area of inner rectangle = 6300 m^2→ eqn3

$$\text{Area of inner semicircles} = 2 \times \frac{\pi r^2}{2}$$

$$\text{Area of inner semicircles} = 2 \times \frac{\pi (35)^2}{2}$$

⇒ Area of inner semicircles = 1225π

$$\text{Area of inner semicircles} = 1225 \times \frac{22}{7}$$

⇒ Area of inner semicircle = 175×22

∴ Area of inner semicircle = 3850 m^2→ eqn4

Area of inner of running track = 6300 + 3850 (from3&4)

∴ Area of inner of running track = 10150 m^2

Now radius of semicircles of outer of the running track = R = r + W

⇒ R = 35 + 14

∴ R = 49 m

$$\text{Area of outer semicircles} = 2 \times \frac{\pi R^2}{2}$$

$$\text{Area of outer semicircles} = 2 \times \frac{\pi(49)^2}{2}$$

⇒ Area of outer semicircles = 2401π

$$\Rightarrow \text{Area of outer semicircles} = 2401 \times \frac{22}{7}$$

⇒ Area of outer semicircles = 343×22

∴ Area of outer semicircle = 7546 m^2→ eqn5

Breadth of outer running track = B’ = 2R

⇒ B’ = 2×49

∴ B’ = 98 m

Area of outer rectangle = L×B’

⇒ Area of outer rectangle = 90×98

∴ Area of outer rectangle = 8820 m^2→ eqn6

Area of entire ground = 8820 + 7546 (from 5&6)

∴ Area of entire ground = 16366 m^2

Area of running track = Area of entire ground – Area of inner ground

⇒ Area of running track = 16366 – 10150

∴ Area of running track = 6216 m^2

Perimeter of outer boundary = 2L + Arc of outer semicircles

Arc length of an outer semicircle = πR, where R = outer radius

$$\text{Perimeter of outer boundary} = (2 \times 90) + \left(2 \times \frac{22}{7} \times 49\right) \text{ (putting values)}$$

⇒ Perimeter of outer boundary = 180 + (2×22×7)

⇒ Perimeter of outer boundary = 180 + 308

∴ Perimeter of outer boundary = 488 m

Area of running track is 6216 m^2 and perimeter of outer boundary is 488 m.

Exercise : MULTIPLE CHOICE QUESTIONS (MCQ)

Question: 1

Let the radius if circle be r

Given, Area of circle = 38.5 cm^2

Area of circle = πr^2

⇒ πr^2 = 38.5

Since π = 22/7

⇒ 22/7 × r^2= 38.5

⇒ r^2 = 38.5 × (7/22)

⇒ r^2 = 12.25

⇒ r = √12.25 = 3.5 cm

∴ Radius of circle = 3.5 cm

Circumference of circle = $2\pi r$

= $2 \times 22/7 \times 3.5$ cm

= 22 cm

∴ Circumference of the circle is 22 cm

Let the radius if circle be r

Given, Area of circle = 38.5 cm^2

Area of circle = πr^2

$\pi r^2 = 38.5$

Since = 22/7

∴ $\pi r^2 = 38.5$

⇒ $22/7 \times r^2 = 38.5$

⇒ $r^2 = 12.25$

⇒ $r = \sqrt{12.25} = 3.50$cm

∴ Radius of circle = 3.5 cm

Circumference of circle = $2\pi r$

= $2 \times 22/7 \times 3.5$

= 22 cm

∴ Circumference of the circle is 22 cm

Question: 2

The area of a cir

Solution:

Let the radius if circle be r

Given, Area of circle = 49π cm^2

Area of circle = πr^2

$\pi r^2 = 49\pi$

Since = 22/7

∴ $\pi r^2 = 49\pi$

⇒ $r^2 = 49$

⇒ $r = \sqrt{49} = 7$ cm

∴ Radius of circle = 7 cm

Circumference of circle = $2\pi r$

= $2 \times \pi \times 7$ cm

= 14π cm

∴ Circumference of the circle is 14π cm

Question: 3

The difference be

Solution:

Let the radius if circle be r

Circumference of circle = $2\pi r$

Difference between the circumference and radius of a circle = 37 cm

$\Rightarrow 2\pi r - r = 37$ cm

$\Rightarrow 2 \times 22/7 \times r - r = 37$ cm

$\Rightarrow 44/7 \times r - r = 37$ cm

$\Rightarrow (44/7 - 1) \times r = 37$ cm

$\Rightarrow 37/7 \times r = 37$ cm

$\Rightarrow r = 37 \times 7/37$

$\Rightarrow r = 7$ cm

Area of circle = πr^2

$= 22/7 \times 7 \times 7$ cm^2

$= 22/7 \times 49$ cm^2 $= 22 \times 7$ cm^2 $= 154$ cm^2

$\therefore$ Area of the circle is 154 cm^2

Question: 4

The perimeter of

Solution:

Let the radius if circular field be r

Perimeter of circular field = $2\pi r$

Perimeter of circular field = 242 m

$\Rightarrow 2\pi r = 242$ m

$\Rightarrow 2 \times 22/7 \times r = 242$ m

$\Rightarrow r = 242 \times 1/2 \times 7/22 = 38.5$ m

$\therefore$ Radius of circular field = 38.5 m

Area of the field = πr^2

$= 22/7 \times 38.5^2$ m^2

$= 22/7 \times 1482.5$ m^2 $= 4658.5$ m^2

$\therefore$ Area of the field = 4658.5 m^2

Question: 5

On increasing the

Solution:

Let the radius if circle be r

Area of circle = $A = \pi r^2$

Radius increases by 40%

So, New Radius $r' = r + 40/100 \times r = 1.4r$

New Area of circle = $A' = \pi r'^2 = \pi \times (1.4r)^2$

$= 1.96\pi r^2$

Percentage increase in area = $\frac{A' - A}{A} \times 100$

$= \frac{1.96\pi r^2 - \pi r^2}{\pi r^2} \times 100$ = .96 × 100 = 96

∴ Increase in area = 96%

Question: 6

On decreasing the

Solution:

Let the radius if circle be r

Area of circle = A = πr^2

Radius decreases by 30%

So, New Radius r' = r - 30/100 × r = 0.7r

New Area of circle = A' = $\pi r'^2 = \pi \times (0.7r)^2$

= $.49\pi r^2$

Percentage decrease in area = $\frac{A - A'}{A} \times 100$

$= \frac{\pi r^2 - .49\pi r^2}{\pi r^2} \times 100$ = .51 × 100 = 51

∴ Decrease in area = 51%

Question: 7

The area of a squ

Solution:

Let the length of the side of the square be a

Let the radius if circle be r

Area of a square = a^2

Area of circle = πr^2

Area of a square = Area of a circle

$a^2 = \pi r^2$

a = √π × r

Perimeter of circle = 2πr

Perimeter of square = 4a

= 4√π r

$$\frac{\text{Perimeter of square}}{\text{Perimeter of circle}} = \frac{4\sqrt{\pi}\, r}{2\pi r}$$

$$\frac{\text{Perimeter of circle}}{\text{Perimeter of square}} = \frac{\sqrt{\pi}}{2}$$

Ratio of perimeter of circle and square = √π : 2

Question: 8

The circumference

Solution:

Let the bigger circle be C_1 and other circles be C_2 and C_3

Radius of circle $C_1 = r_1$

Diameter of circle C_2 = 36 cm

Radius of circle $C_2 = r_2$ = 36/2 cm = 18cm

Diameter of circle C_3 = 20 cm

Radius of circle $C_3 = r_3$ = 20/2 cm = 10 cm

Circumference of circle $C_2 = 2\pi r_2$

$= 2 \times \pi \times 18$ cm = 36π cm

Circumference of circle $C_3 = 2\pi r_3$

$= 2 \times \pi \times 10$ cm = 20π cm

Circumference of circle C_1 = Circumference of circle C_2 + Circumference of circle C_3

$\Rightarrow 2\pi r_1 = 2\pi r_2 + 2\pi r_3$

$\Rightarrow 2\pi r_1 = 36\pi + 20\pi$

$\Rightarrow 2\pi r_1 = 56\pi$

$\Rightarrow r_1 = 28$ cm

Radius of circle $C_1 = r_1 = 28$ cm

Question: 9

The area of a cir

Solution:

Let the bigger circle be C_1 and other circles be C_2 and C_3

Radius of circle $C_1 = r_1$

Radius of circle $C_2 = r_2 = 24$ cm

Radius of circle $C_3 = r_3 = 7$ cm

Area of circle $C_2 = \pi r_2^2$

$= \pi \times 24^2 = 576\pi$

Area of circle $C_3 = \pi r_3^2$

$= \pi \times 7^2 = 49\pi$

Area of circle C_1 = Area of circle C_2 + Area of circle C_3

$\pi r_1^2 = \pi r_2^2 + \pi r_3^2$

$\pi r_1^2 = 576\pi + 49\pi$

$\pi r_1^2 = 625\pi$

$r_1 = 25$ cm

Diameter of new circle = 25×2 cm = 50cm

Question: 10

If the perimeter

Solution:

Let the length of the side of the square be a

Let the radius if circle be r

Perimeter of circle = $2\pi r$

Perimeter of square = 4a

Perimeter of circle = Perimeter of square

$\Rightarrow 2\pi r = 4a$

$a = \pi \times r/2$

Area of a square = a^2

Area of circle = πr^2

$$\frac{\text{Area of square}}{\text{Area of circle}} = \frac{a^2}{\pi r^2}$$

$$\frac{\text{Area of square}}{\text{Area of circle}} = \frac{(\frac{\pi r}{4})^2}{\pi r^2}$$

$$\frac{\text{Area of square}}{\text{Area of circle}} = \frac{\pi^2 r^2}{4\pi r^2}$$

$$\frac{\text{Area of square}}{\text{Area of circle}} = \frac{\pi}{4}$$

Ratio of area of square to circle = π: 4

Question: 11

If the sum of the

Solution:

Let three circles be C_1, C_2 and C

Area of circle C = Area of circle C_1 + Area of circle C_2

$$\pi R^2 = \pi R_1^2 + \pi R_2^2$$

$$R_1^2 + R_2^2 = R^2$$

Question: 12

Let three circles be C_1, C_2 and C

Circumference of circle C = Circumference of circle C_1 + Circumference of circle C_2

$\Rightarrow 2\pi R = 2\pi R_1 + 2\pi R_2$

$R = R_1 + R_2$

Question: 13

If the circumfere

Solution:

Let the length of the side of the square be a

Let the radius if circle be r

Perimeter of circle = $2\pi r$

Perimeter of square = 4a

Perimeter of circle = Perimeter of square

$2\pi r = 4a$

$a = \pi \times r/2$

Area of a square = a^2

$= (\pi \times r/2)^2 = \pi/4 \times \pi r^2$

Area of circle = πr^2

Seeing the co-efficient of πr^2

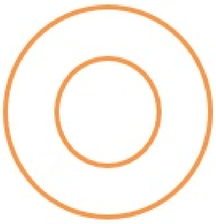
$1 > \pi/4 \therefore \pi r^2 > \pi/4 \times \pi r^2$

So, (area of the circle) > (area of the square)

Question: 14

The radii of two

Solution:



Radius of circle 1 = r_1 = 19 cm

Radius of circle 2 = r_2 = 16 cm

Area of Ring = $\pi(r_1^2 - r_2^2)$

= π (19^2 – 16^2) cm^2

= 22/7 × 105

= 330 cm^2

Question: 15

The areas of two

Solution:

Let the radius of circle 1 & 2 be R_1 and R_2 respectively

Area of circle 1 = 1386 cm^2

πR_1^2 = 1386 cm^2

22/7 × R_1^2 = 1386 cm^2

R_1^2 = 1386 × 7/22 cm^2 = 441 cm^2

R_1 = 21 cm

Area of circle 2 = 962.5 cm^2

πR_2^2 = 962.5 cm^2

22/7 × R_2^2 = 962.5 cm^2

R_2^2 = 962.5 × 7/22 cm^2 = 306.25 cm^2

R_2 = 17.5 cm

Width of the ring = R_1 – R_2 = 21 -17.5 = 3.5 cm

Question: 16

The circumference

Solution:

Circumference of circle C_1 = $2\pi r_1$

Circumference of circle $C_2 = 2\pi r_2$

$$\frac{\text{Circumference of circle C1}}{\text{Circumference of circle C2}} = \frac{2\pi r_1}{2\pi r_2} = \frac{3}{4}$$

$$\frac{r_1}{r_2} = \frac{3}{4}$$

$$\frac{\text{Area of circle C1}}{\text{Area of circle C2}} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = (\frac{3}{4})^2 = \frac{9}{16}$$

∴ Ratio of two circles = 9: 16

Question: 17

The areas of two

Solution:

Area of circle $C_1 = \pi r_1^2$

Area of circle $C_2 = \pi r_2^2$

$$\frac{\text{Area of circle C1}}{\text{Area of circle C2}} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{9}{4}$$

$$\frac{r_1}{r_2} = \frac{3}{2}$$

$$\frac{\text{Circumference of circle C1}}{\text{Circumference of circle C2}} = \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{3}{2}$$

Ratio of their circumferences = 3: 2

Question: 18

The radius of a w

Solution:

Radius of wheel = r = 0.25 m

Distance the wheel travels = 11 km = 11000 m

In 1 revolution wheel travels $2\pi r$ distance

No. of revolutions a wheel makes = $\frac{\text{distance travelled by the wheel}}{2\pi r}$

$$= \frac{11000}{2\pi \times 0.25} = \frac{11000}{2 \times \frac{22}{7} \times 0.25} = \frac{11000 \times 7}{2 \times 22 \times 0.25}$$

= 7000 revolutions

Question: 19

The diameter of a

Solution:

Diameter of wheel = 40 cm

Radius of wheel = r = 40/2 cm = 20 cm

Distance the wheel travels = 176 m = 17600 cm

In 1 revolution wheel travels $2\pi r$ distance

No. of revolutions a wheel makes = $\frac{\text{distance travelled by the wheel}}{2\pi r}$

$$= \frac{17600}{2\pi \times 20} = \frac{17600}{2 \times \frac{22}{7} \times 20} = \frac{17600 \times 7}{2 \times 22 \times 20}$$

= 140 revolutions

Question: 20

In making 1000 re

Solution:

Distance the wheel travels = 88 km = 88000 m

In 1 revolution wheel travels 2πr distance

No. of revolutions a wheel makes = $\frac{\text{distance travelled by the wheel}}{2\pi r}$

No. of revolutions a wheel makes = 1000

$$r = \frac{\text{distance travelled by the wheel}}{2\pi \times \text{No. of revolutions a wheel makes}} = \frac{88000}{2 \times \frac{22}{7} \times 1000}$$

$$= \frac{88000 \times 7}{2 \times 22 \times 1000}$$

r = 14 m

Radius of wheel = 14 m

Diameter of wheel = 2 × 14 m = 28 m

Question: 21

The area of a sec

Solution:

Area of a sector of angle $\theta°$ of a circle with radius R = area of circle × $\frac{\theta}{360}$

$$= \frac{\pi R^2 \theta}{360}$$

Question: 22

The length of an

Solution:

Length of an arc of a sector of angle $\theta°$ of a circle with radius R

= Circumference of circle × $\frac{\theta}{360}$

$$= \frac{2\pi R \theta}{360}$$

Question: 23

The length of the

Solution:

Length of the minute hand of a clock = 21 cm

∴ Radius = R = 21 cm

In 1 minute, minute hand sweeps 6°

So, in 10 minutes, minute hand will sweep 10 × 6° = 60°

Area swept by minute hand in 10 minutes = Area of a sector of angle $\theta°$ of a circle with radius R

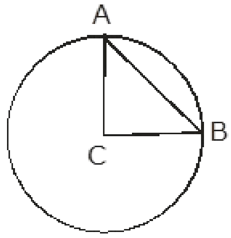
$= \frac{\pi R^2 \theta}{360}$ = 22/7 × 21 × 21 × 60/360 = 231 cm^2

Question: 24

A chord of a circ

Solution:

Radius of Circle = R = 10 cm



Area of minor Segment = Area of sector subtending 90° – Area of triangle ABC

Area of sector subtending 90° = $\frac{\pi R^2 \theta}{360}$ = 3.14 × 10 × 10 × 90/360 cm^2

= 78.5 cm^2

Area of triangle ABC = 1/2 × AC × BC

= 1/2 × 10 × 10 cm^2 = 50 cm^2

Area of Minor segment = 78.5 cm^2 – 50 cm^2

= 28.5 cm^2

Question: 25

In a circle of ra

Solution:

Radius of Circle = R = 21 cm

Angle Subtended by the arc = 60°

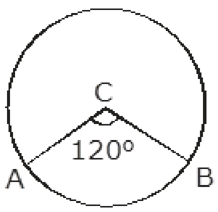
Length of an arc of a sector of angle $\theta°$ of a circle with radius R = $\frac{2\pi R\theta}{360}$

Length of arc = 2 × 22/7 × 21 × 60/360 cm = 22 cm

Question: 26

In a circle of ra

Solution:



Radius of Circle = R = 14 cm

Angle Subtended by the arc = θ = 120°

Area of sector subtending 120° = $\frac{\pi R^2 \theta}{360}$ = 22/7 × 14 × 14 × 120/360 cm^2

= 205.33 cm^2

In Triangle ABC

AC = BC = 14 cm = R

Area of triangle ABC = 1/2 × base × height

$= 2 \times 1/2 \times R \sin \theta/2 \times R \times \cos \theta/2$

$= 2 \times 1/2 \times 14 \times 14 \times \sin 60° \times \cos 60°$

$= 84.77\ cm^2$

Area of Segment = Area of sector subtending 120° - Area of triangle ABC

$= 205.33 – 84.77\ cm^2 = 120.56\ cm^2$

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

In the given figu

Solution:

Length of side of square = OA = 20 cm

Radius of Quadrant = OB = $\sqrt{20^2 + 20^2}$ cm = $20\sqrt{2}$ cm

Area of Quadrant = $\pi R^2 \times \theta/360 = 3.14 \times 20\sqrt{2} \times 20\sqrt{2} \times 90/360 = 628\ cm^2$

Area of Square = $a^2 = 20^2\ cm^2 = 400\ cm^2$

Area of Shaded region = Area of Quadrant - Area of Square

$= 628\ cm^2 – 400\ cm^2$

$= 228\ cm^2$

Question: 2

The diameter of a

Solution:

Diameter of wheel = 84 cm

Radius of wheel = r = 84/2 cm = 42 cm

Distance the wheel travels = 792 m = 79200 cm

In 1 revolution wheel travels 2πr distance

No. of revolutions a wheel makes = $\frac{\text{distance travelled by the wheel}}{2\pi r}$

$= \frac{79200}{2\pi \times 42} = \frac{79200}{2 \times \frac{22}{7} \times 42} = \frac{79200 \times 7}{2 \times 22 \times 42}$

= 300 revolutions

Question: 3

The area of a sec

Solution:

Area of a sector of angle $\theta°$ of a circle with radius R = area of circle × $\frac{\theta}{360}$

$= \pi r^2 \times \frac{x°}{360°}$

Question: 4

In the given figu

Solution:

Given:

Length of rectangle = 8 cm

Breadth of rectangle = 6 cm

Area of rectangle = length × breadth

$= 8 \times 6 = 48 \text{ cm}^2$

Consider Δ ABC,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 8^2 + 6^2 = 64 + 36 = 100$

$AC = \sqrt{100} = 10$ cm

⇒ Diameter of circle = 10 cm

Thus, radius of circle = $\frac{10}{2}$ = 5 cm

Let the radius of circle be r = 5 cm

Then, Area of circle = πr^2

$= \frac{22}{7} \times 5 \times 5 = \frac{22 \times 25}{7} = \frac{550}{7} = 78.57 \text{ cm}^2$

Area of shaded region = Area of circle – Area of rectangle

= 78.57 - 48

$= 30.57 \text{ cm}^2$

Hence, the area of shaded region is 30.57 cm^2.[None of the option is correct]

Question: 5

The circumference

Solution:

Let the radius if circle be r

Circumference of circle = 22 cm

$2\pi r = 22$ cm

$2 \times 22/7 \times r = 22$ cm

$r = 22 \times 1/2 \times 7/22$ cm

r = 3.5 cm

Area of Circle = πr^2

$= 22/7 \times 3.5 \times 3.5 \text{ cm}^2$

$= 38.5 \text{ cm}^2$

∴ Area of Circle = 38.5 cm^2

Question: 6

In a circle of ra

Solution:

Radius of circle = R = 21 cm

Angle subtended by arc = 60°

Length of an arc of a sector of angle $\theta°$ of a circle with radius R

= Circumference of circle × θ/360

$= \frac{2\pi R\theta}{360}$

Length of arc = 2 × 22/7 × 21 × θ/360 cm = 22 cm

Length of arc = 22 cm

Question: 7

The minute hand o

Solution:

Length of the minute hand of a clock = 12 cm

∴ Radius = R = 12 cm

In 1 minute, minute hand sweeps 6°

So, in 35 minutes, minute hand will sweep 35 × 6° = 210°

Area swept by minute hand in 35 minutes = Area of a sector of angle $\theta°$ of a circle with radius R

$= \frac{\pi R^2\theta}{360}$ = 22/7 × 12 × 12 × 60°/360° = 264 cm^2

Area swept by minute hand in 35 minutes = 264 cm^2

Question: 8

The perimeter of

Solution:

Radius of circle = 5.6 cm

Perimeter of a sector of a circle = 2R + Circumference of circle × θ/360

$= 2R + \frac{2\pi R\theta}{360}$

Perimeter of a sector of a circle = 2 × 5.6 + 2 × 22/7 × 5.6 × θ/360 cm

= 27.2 cm

⇒ 2 × 22/7 × 5.6 × θ/360 = 27.2 – 11.2 cm = 16 cm

⇒ θ = 16 × 1/2 × 1/5.6 × 7/22

⇒ θ = 163.63°

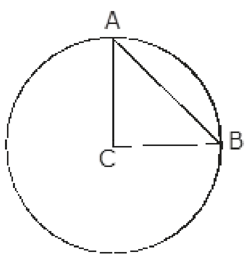
Area of Sector = πr^2 × θ\360 = 22/7 × 5.6 × 5.6 × 163.63/360 = 44.8cm^2

∴ Area of Sector = 44.8 cm^2

Question: 9

A chord of a circ

Solution:



Chord AB subtends an angle of 90° at the centre of the circle

Radius of Circle = R = 14 cm

Area of sector of circle of radius R = $\frac{\pi R^2 \theta}{360}$

= 22/7 × 14 × 14 × 90/360 cm^2 = 154 cm^2

Question: 10

In the give figur

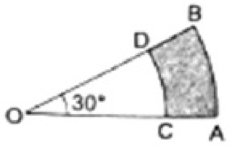
Solution:

Given,

Radius of smaller circle = R_1 = 3.5 cm

Radius of bigger circle = R_2 = 7 cm

Angle subtended = 30°



Area of Shaded region = $\frac{\pi R_2^2 \theta}{360} - \frac{\pi R_1^2 \theta}{360} = \pi\ (\ R_2^2 - \ R_1^2\ )\frac{\theta}{360}$

= 22/7 × (7^2 – 3.5^2) × 30/360 cm^2

= 22/7 × (49 – 12.22) × θ/360 cm^2

= 9.625 cm^2

∴ Area of shaded region = 9.625 cm^2

Question: 11

A wire when bent

Solution:

Let the sides of equilateral triangle be a cm

Area of equilateral triangle = 121√3 cm^2

Area of equilateral triangle = √3/4 × a^2

⇒ √3/4 a^2 = 121√3

⇒ a^2 = 121√3 × 4/√3 = 121 × 4 cm^2

⇒ a^2 = 484 cm^2

⇒ a = 22 cm

Perimeter of equilateral triangle = 3a

= 3 × 22 cm = 66 cm

Perimeter of equilateral triangle = Circumference of circle

Circumference of circle = 66 cm

Let the radius of circle be r

Circumference of circle = 2πr

⇒ 2πr = 66 cm

⇒ 2 × 22/7 × r = 66 cm

⇒ r = 66 × 1/2 × 7/22 cm

⇒ r = 10.5 cm

Area of circle = πr^2 = 22/7 × 22/7 × 10.5 × 10.5 cm2

= 346.5 cm^2

Question: 12

Diameter of the wheel = 84 cm

Let the radius of the wheel be R cm

Radius of the wheel = 84/2 cm = 42 cm

No. of revolutions wheel makes = 5 rev/sec

Since, 1 revolution = 2πR

Speed of the wheel = 5 × 2πR rev/sec

= 5 × 2 × 22/7 × 42 = 1320 cm/sec

= 13.20 m/sec

= 13.20 × 3600/1000 km/h

= 47.52 km/h

Since, 1 m/sec = 3600/1000 km/h

Question: 13

OACB is a quadran

Solution:

Radius of circle = R = 3.5 cm

OD = 2 cm

OA = OB = R = 3.5 cm

Since, OACB is a quadrant of a circle ∴ angle subtended by it at the centre = 90°

(i) Area of quadrant = $\frac{\pi R^2 \theta}{360}$

= 22/7 × 3.5 × 3.5 × 90°/360° cm^2

= 9.625 cm^2

(ii) Area of shaded region = Area of quadrant – Area of triangle OAD

Area of triangle OAD = 1/2 × base × height

= 1/2 × OA × OD

= 1/2 × 3.5 × 2 cm^2

= 3.5 cm^2

Area of shaded region = 9.625 cm^2 – 3.5cm^2

= 6.125 cm^2

Question: 14

In the given figu

Solution:

Length of the sides of square = 28 cm

Area of square = $a^2 = 28^2$ cm^2

= 784 cm^2

Since, all the circles are identical so, they have same radius

Let the radius of circle be R cm

From the figure 2R = 28 cm

R = 28/2 cm

R = 14 cm

Quadrant of a circle subtends 90° at the centre.

Area of quadrant of circle = $\frac{\pi R^2 \theta}{360}$

= 22/7 × 14 × 14 × 90°/360° cm^2 = 154 cm^2

Area of 4 quadrants of circle = 154 × 4 cm^2 = 616 cm^2

Area of shaded region = Area of square – Area of 4 quadrants of circle

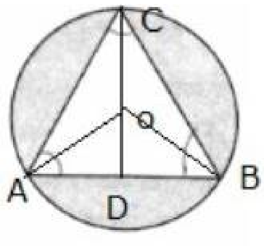
= 784 cm^2 – 616 cm^2

= 168 cm^2

Question: 15

In the given figu

Solution:



Radius of circle = R = 4 cm

OD perpendicular to AB is drawn

ΔABC is equilateral triangle,

∠A = ∠B = ∠C = 60°

∠OAD = 30°

OD/AO = sin 30°

AO = 4 cm

$\frac{OD}{AO} = \frac{1}{2}$

OD = 1/2 × 4 cm

OD = 2 cm

$AD^2 = OA^2 - OD^2$

= $4^2 - 2^2$ = 16 – 4 = 12 cm^2

AD = 2√3 cm

AB = 2 × AD

= 2 × 2√3 cm = 4√3 cm

Area of triangle ABC = √3/4 × AB^2

= √3/4 × 4√3 × 4√3

$= 20.71\ cm^2$

Area of circle $= \pi R^2$

$= 3.14 \times 4 \times 4\ cm^2$

$= 50.24\ cm^2$

Area of shaded region $= 29.53\ cm^2$

Question: 16

The minute hand o

Solution:

Length of minute hand = 7.5 cm

In a clock, length of minute hand = radius

Radius = R = 7.5 cm

In 1 minute, minute hand moves 6°

So, in 56 minutes, minute hand moves 56 × 6° = 336°

Area described by minute hand $= \frac{\pi R^2 \theta}{360°}$

$= 22/7 \times 7.5 \times 7.5 \times 336°/360°\ cm^2$

$= 165\ cm^2$

Question: 17

A racetrack is in

Solution:

Let the inner radius be R_1 and outer radius be R_2

Inner circumference $= 2\pi R_1 = 352$ m

$\Rightarrow 2 \times 22/7 \times R_1 = 352$ m

$\Rightarrow R_1 = 352 \times 1/2 \times 7/22$

$\Rightarrow R_1 = 56$ m

Outer Circumference $= 2\pi R_2 = 396$ m

$\Rightarrow 2 \times 22/7 \times R_2 = 396$ m

$\Rightarrow R_2 = 396 \times 1/2 \times 7/22$ m

$\Rightarrow R_2 = 63$ m

Width of the track $= R_2 – R_1 = 63$ m – 56 m = 7 m

Area of track $= \pi(R_2^2 - R_1^2) = 22/7 \times (63^2 - 56^2)$

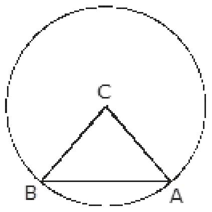
$= 22/7 \times (3969 – 3136)\ m^2$

$= 22/7 \times 833\ m^2 = 2618\ m^2$

Question: 18

A chord of a circ

Solution:



$\angle ACB = 60°$

Chord AB subtends an angle of 60° at the centre

Radius = 30 cm

Let Radius be R

In triangle ABC, AC = BC

So, $\angle CAB = \angle CBA$

$\angle ACB + \angle CAB + \angle CBA = 180°$

$60° + 2\angle CAB = 180°$

$2\angle CAB = 180° - 60° = 120°$

$\angle CAB = 120°/2 = 60°$

$\angle CAB = \angle CBA = 60°$

$\therefore$ ΔABC is a equilateral triangle

Length of side of an equilateral triangle = radius of circle = 30 cm

Area of equilateral triangle = $\sqrt{3}/4 \times \text{side}^2 = 1.732/4 \times 30 \times 30\ cm^2$

$= 389.7\ cm^2$

Area of sector ACB = $\frac{\pi R^2 \theta}{360} = 3.14 \times 30 \times 30 \times 60°/360° = 471.45\ cm^2$

Area of minor Segment = Area of sector ACB – Area of ΔABC

$= 471.45\ cm^2 – 389.7\ cm^2 = 81.75\ cm^2$

Area of circle = $\pi R^2 = 3.14 \times 30 \times 30\ cm^2 = 2828.57\ cm^2$

Area of major segment = Area of circle – Area of minor segment

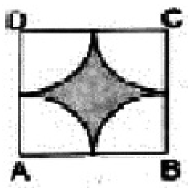
$= 2826\ cm^2 – 81.75\ cm^2$

$= 2744.25\ cm^2$

Question: 19

Four cows are tet

Solution:



From the figure we see that cows are tethered at the corners of the square so while grazing they form four quadrants as shown in the figure

Length of side of square = 50 m

Length of side of square = 2 × Radius of quadrant

Radius of quadrant = R = 50/2 m

= 25 m

Area of square = side^2

= 50^2 m^2 = 2500 m^2

Area of quadrant = 1/4 π R^2 = 1/4 × 3.14 × 25 × 25 m^2

= 490.625 m^2

Area of 4 quadrants = 4 × 490.625 m^2

= 1962.5 m^2

Area left ungrazed = Area of shaded part

= Area of square – Area of 4 quadrants

= 2500 m^2 – 1962.5 m^2

= 537.5 m^2

Question: 20

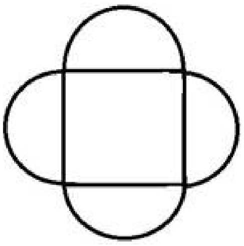
A square tank has

Solution:

Let the length of side of the square tank be a

Area of square tank = a^2 = 1600 m^2

⇒ a = √1600 m = 40 m



Let the radius of semicircle be R

From the figure we can see that

Length of the side of the square = Diameter of semicircle

40 m = 2 × R

R = 40/2 m

R = 20 m

Area of semi-circle = 1/2 πR^2 = 1/2 × 3.14 × 20 × 20 m^2

= 628 m^2

Area of 4 semi-circles = 4 × 628 m^2

= 2512 m^2

Cost of turfing the plots = Rs. 12.50 per m^2

Cost of Turfing = Cost of turfing per m^2 × Area of 4 semicircle

= Rs. 12.50 × 2512

= Rs. 31400