CHAPTER 3

Understanding Quadrilaterals

Learning Objectives

In this chapter you will learn:

- *To differentiate between different types of quadrilaterals on basis of their properties and establish relationship between them.*

3.1 Introduction :-

In our daily life, we come across various plane surfaces such as blackboard in the class, a page of a book, top of a table etc.

Figure 3.1

These are perfect samples or models for a plane surface. We know that a paper is the sample or model of the surface.

3.2 Types of Quadrilaterals :

In this section, we shall learn about some special types of quadrilaterals and their properties.

Trapezium : A quadrilateral having exactly one pair of parallel sides, is called a trapezium.

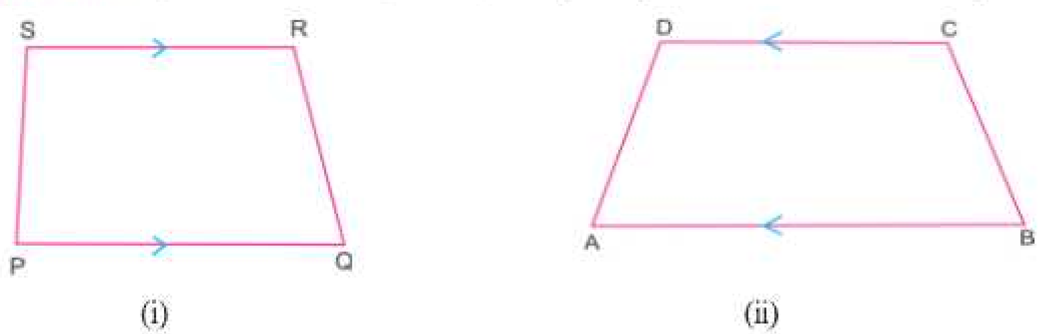


Figure 3.2

Note : The arrow marks indicate parallel lines.

Isosceles Trapezium : A trapezium is said to be an isosceles trapezium, if its non-parallel sides are equal.

Figure 3.3

Here, quadriateral ABCD is an isosceles trapezium in which AB||DC and AD = BC.

Parallelogram : As the name of this quadrilateral suggests that it has some concern with parallel lines.

"Parallelogram is a quadrilateral if both of its pair of opposite sides are parallel or equal."

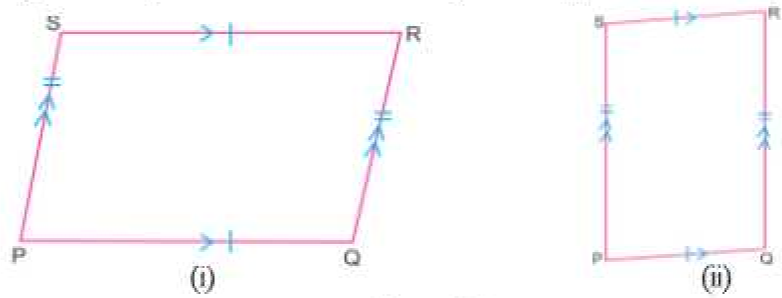


Figure 3.4

Here, quadrilateral PQRS is a parallelogram in which

PQ||RS and PS||QR or PQ=RS and PS=QR.

3.3 Properties of a Parallelogram :

Property 1 : In a parallelogram, opposite sides are equal.

Proof : Let ABCD be a parallelogram. Draw its diagonal AC.

In ΔABC and ΔCDA, We get

$\angle 3 = \angle 1$

$\angle 2 = \angle 4$ [alternate interior angles

AC = AC (Common)

$\therefore$ $\Delta ABC \cong \Delta CDA$ (ASA congruency)

$\Rightarrow$ AB = CD and BC = DA

Thus, in a parallelogram, the opposite sides are equal.

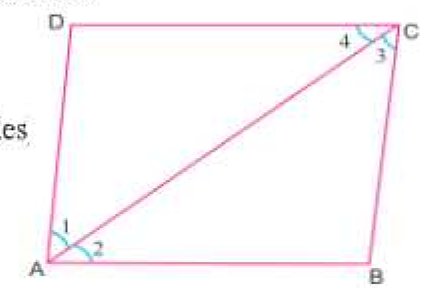


Figure 3.5

Example 3.1 : Find x and y in the following parallelograms

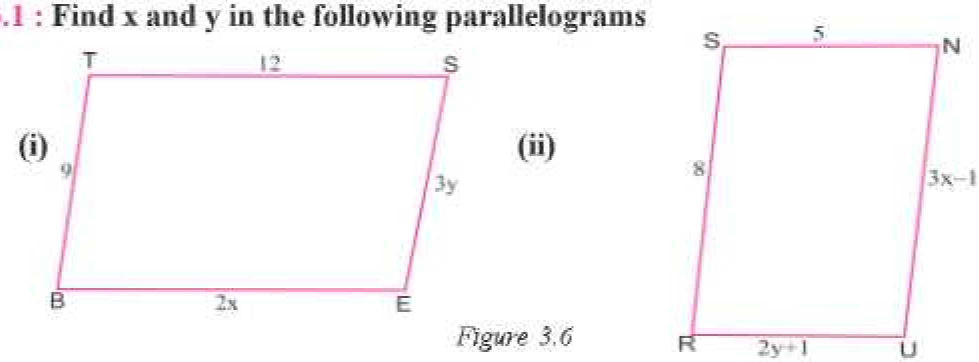


Figure 3.6

Sol. (i) Given, BEST is a parallelogram.

We know, the opposite sides of a parallelogram are equal.

∴ BE = ST	and	ES = BT
$2x = 12$		$3y = 9$
$x = \frac{12}{2} = 6$		$y = \frac{9}{3} = 3$

(ii) Given, RUNS is a parallelogram.

We know, the opposite sides of a parallelogram are equal.

∴ RU = NS	and	UN = RS
⇒ $2y + 1 = 5$		$3x - 1 = 8$
$2y = 5 - 1 = 4$		$3x = 8 + 1 = 9$
$y = \frac{4}{2} = 2$		$x = \frac{9}{3} = 3$

Property 2 : In a parallelogram, opposite angles are equal.

Proof : Let ABCD be a parallelogram. Draw its diagonal AC.

In ΔABC and ΔCDA, We get

$\angle 2 = \angle 4$

$\angle 3 = \angle 1$ [alternate interior angles]

AC = AC (common)

∴ ΔABC ≅ ΔCDA (ASA congruency)

⇒ $\angle B = \angle D$ (c.p.c.t.)

Similarly, we can prove $\angle A = \angle C$ by joining diagonal BD.

Thus, In a parallelogram the opposite angles are equal.

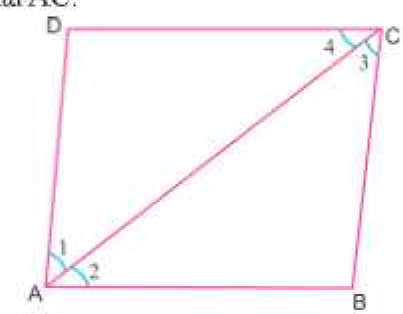


Figure 3.7

Property 3 : In a parallelogram, the sum of any two adjacent angles is 180°.

Proof : Let ABCD be a parallelogram. So AB||CD and AD is a transversal line which intersects them at A and D respectively.

As, we know that sum of interior angles on the same side of transversal between parallel lines is 180°.

∴ $\angle A + \angle D = 180^\circ$

Similarly, we can prove that

$\angle A + \angle B = 180^\circ$

and $\angle B + \angle C = 180^\circ$

and $\angle C + \angle D = 180^\circ$

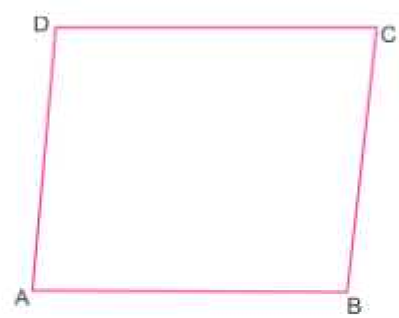


Figure 3.8

Example 3.2 : Two adjacent angles of a parallelogram are equal. What is the measure of each angle?

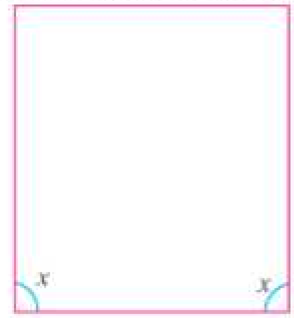


Figure 3.9

Sol. Let the measure of each of adjacent angles be x.

We know that sum of adjacent angles of a parallelogram is 180°.

$\therefore \quad x + x = 180^\circ \quad \Rightarrow \quad 2x = 180^\circ$

$x = \dfrac{180^\circ}{2} = 90^\circ$

Hence, the measure of each angle is 90°.

Example 3.3 : Two adjacent angles of a parallelogram are in 2:3. Find the measure of all the angles.

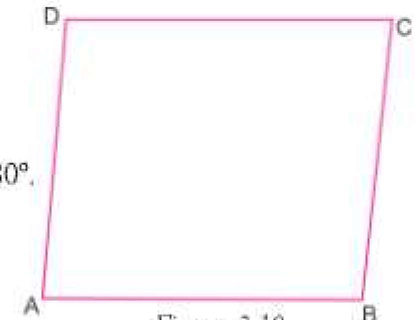


Figure 3.10

Sol. Let ABCD be a parallelogram.

Such that $\angle A : \angle B = 2 : 3$

Let $\angle A = 2x$ and $\angle B = 3x$

We know that sum of adjacent angles of a parallelogram is 180°.

$\therefore \quad \angle A + \angle B = 180^\circ$

$\Rightarrow \quad 2x + 3x = 180^\circ \quad \Rightarrow \quad 5x = 180^\circ$

$\Rightarrow \quad x = \dfrac{180^\circ}{5} = 36^\circ$

$\therefore \quad \angle A = 2x = 2 \times 36^\circ = 72^\circ, \ \angle B = 3x = 3 \times 36^\circ = 108^\circ$

Since, opposite angles of a parallelogram are equal.

$\Rightarrow \quad \angle C = \angle A = 72^\circ$ and $\angle D = \angle B = 108^\circ$

Example 3.4 : In fig, RING is a parallelogram,

If $\angle R = 70^\circ$ then find all other angles.

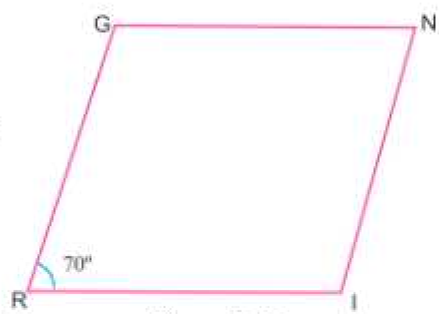


Figure 3.11

Sol. Since, sum of adjacent angles of a parallelogram is 180°

$\therefore \quad \angle R + \angle I = 180^\circ$

$\Rightarrow \quad 70^\circ + \angle I = 180^\circ \Rightarrow \angle I = 180^\circ - 70^\circ = 110^\circ$

Since, opposite angles of a parallelogram are equal.

$\Rightarrow \quad \angle N = \angle R = 70^\circ$ and $\angle G = \angle I = 110^\circ$

Example 3.5 : In Fig, PQRS is a parallelogram. Find the values of x, y and z.

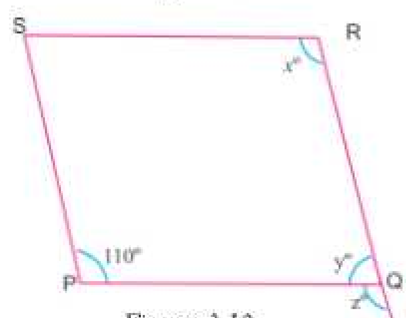


Figure 3.12

Sol. Since, opposite angles of a parallelogram are equal.

$\therefore \quad x = 110^\circ$

Since, sum of adjacent angles of a parallelogram is 180°

$\therefore \quad y + 110^\circ = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 110^\circ = 70^\circ$

Clearly $y + z = 180^\circ$ (Linear Pair)

$\Rightarrow \quad 70^\circ + z = 180^\circ \Rightarrow 180^\circ - 70^\circ = 110^\circ$

Property 4 : In a parallelogram, diagonals bisect each other.

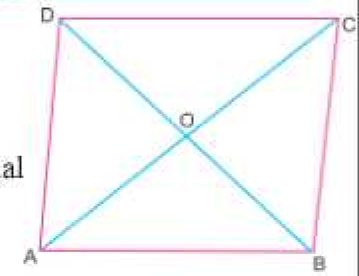


Figure 3.13

Proof : Let ABCD be a parallelogram and draw its diagonals AC and BD intersecting each other at O.

In ΔAOB and ΔCOD, We have

AB = CD (Opposite sides of a parallelogram are equal

$\angle OAB = \angle OCD$ (alternate interior angles)

$\angle OBA = \angle ODC$ (alternate interior angles)

$\therefore \Delta AOB \cong \Delta COD$ (ASA congruency criterion)

$\Rightarrow$ OA = OC and OB = OD (cpct.)

Thus, In a parallelogram the diagonals bisect each other.

Example 3.6 : In parallelogram ABCD, the diagonals AC and BD intersect at O. If OA=3cm and OB=2.5cm then find the length of AC and BD.

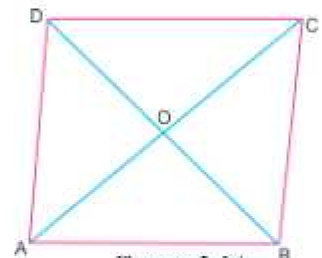


Figure 3.14

Sol. We know, the diagonals of a parallelogram bisect each other.

$\therefore$ AC = 2 (OA) = 2×3 = 6cm

and BD = 2(OB) = 2×2.5 = 5cm

Example 3.7 : In parallelogram HELP, HL = 10cm and PE = 9cm, then find the length of HO and EO.

Sol. We know, the diagonals of a parallelogram bisect each other.

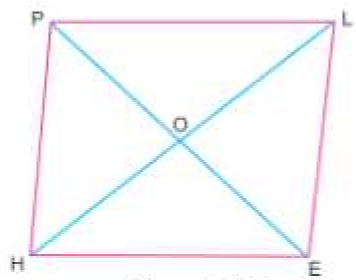


Figure 3.15

$$\therefore \quad OH = \frac{1}{2}HL = \frac{1}{2}\times 10 = 5cm$$

$$\text{and} \quad EO = \frac{1}{2}PE = \frac{1}{2}\times 9 = 4.5cm$$

Example 3.8 : Find x & y in fig, if PQRS is a paralellogram.

Sol. We know, the diagonals of a parallelogram bisect each other.

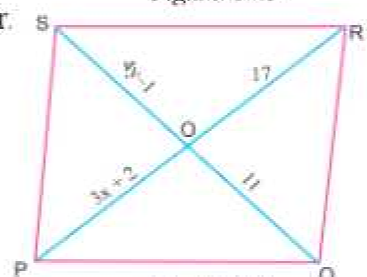


Figure 3.16

$\therefore$ OP = OR	and OQ = OS
$\Rightarrow$ 3x + 2 = 17	11 = 4y – 1
$\Rightarrow$ 3x = 17–2 = 15	4y = 11 + 1 = 12
$x = \frac{15}{3} = 5$	$y = \frac{12}{4} = 3$

Exercise 3.1

1. The two adjacent sides of a parallelogram are 6cm and 8cm. Find the perimeter of the parallelogram.
2. Given below a parallelogram PQRS, then complete the following (Using properties of paralellogram)

(i) PQ = ...

(ii) QR = ...

(iii) $\angle P$ = ...

(iv) $\angle S$ = ...

(v) $\angle P + \angle Q$ = ...

(vi) $\angle R + \angle S$ = ...

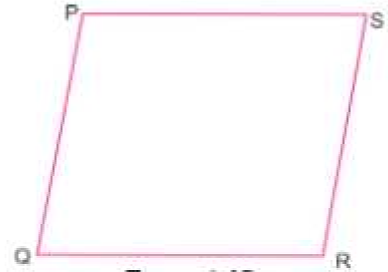
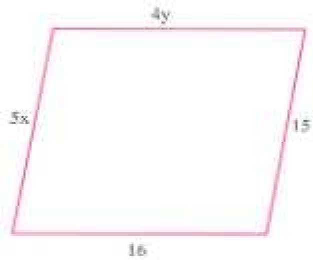


Figure 3.17

3. Find x and y in the following parallelogram.

(i)



(ii)

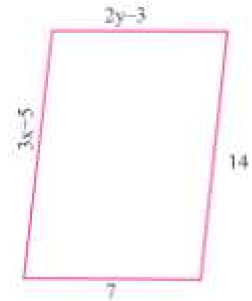


Figure 3.18

4. Two adjacent angles of a parallelogram are in ratio 4:5. Find the measure of all the angles.

5. Two adjacent angles of a parallelogram are in ratio 3:7. Find the measure of all the angles.

6. In parallelogram WXYZ, $\angle Y = 80°$. Find the measure of all the angles.

Figure 3.19

7. In parallelogram BEST, $\angle B = 105°$, Find the measure of all the angles.

Figure 3.20

8. Find x & y in the following parallelogram.

(i)

(ii)

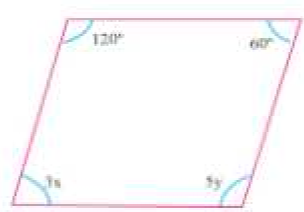


Figure 3.21

9. In parallelogram ABCD, diagonals AC and BD intersect at O. If AC = 12cm and BD = 16cm then find OA and OD.

10. In parallelogram PQRS, diagonals PR and QS intersect at O. If OP = 6cm and OS = 7cm then find PR and QS.

11. Find x and y in the following parallelogram.

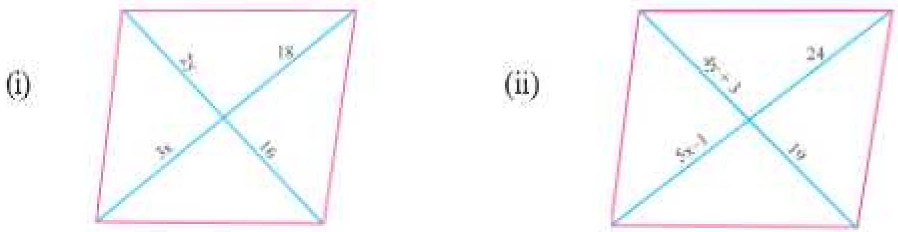


Figure 3.22

12. Find x,y and z in the following parallelogram.

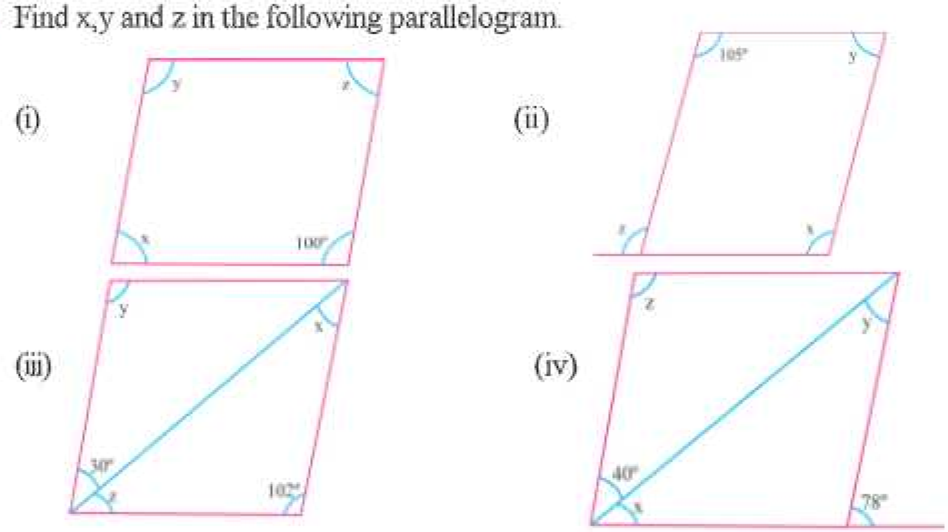


Figure 3.23

13. **Multiple Choice Questions :**

(i) If length of one diagonal of a rectangle is 6cm, what is the length of other diagonal?

(a) 3cm (b) 6cm (c) 12cm (d) 4cm

(ii) If 6x and 24 are two opposite sides of parallelogram, what is the value of x?

(a) 4 (b) 6 (c) 24 (d) 12

(iii) If 3x–2 and 7 are the two equal parts of a diagonal of parallelogram, what is the value of x ?

(a) 5 (b) 4 (c) 3 (d) 6

(iv) If 4y° and 100° are two opposite angles of a parallelogram then find the value of y?

(a) 25 (b) 20 (c) 100 (d) 10

3.4 Rhombus :

A rhombus is quadrilateral with sides of equal length.

Or A parallelogram having its adjacent sides equal is called a rhomb

ABCD is a rhombus in which AB = BC = CD = DA

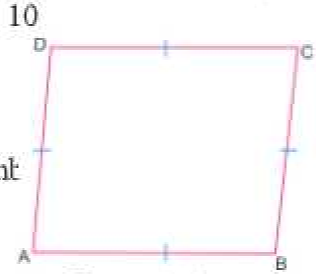


Figure 3.24

3.4.1 Properties of Rhombus:

Since rhombus is a parallelogram, so all properties of parallelogram will be contained in rhombus i.e.

(i) Opposite pair of sides are parallel.

(ii) Opposite angles are equal.

(iii) Sum of adjacent angles is 180°.

(iv) Diagonals bisect each other.

Property 5 : The diagonals of a rhombus bisect each other at right angles.

Proof : Let ABCD be a rhombus and its diagonals AC and BD intersect at O.

Since every rhombus is a parallelogram.

We know, the diagonals of a parallelogram bisect each other.

Now, To prove that the diagonals of the rhombus ABCD are perpendicular to each other.

So in ΔAOB and ΔBOC, We have

AO = OC (O is mid point of AC)

OB = OB (common)

AB = BC (Sides of a rhombus)

$\Delta AOB \cong \Delta BOC$ (SSS congruency criterion)

$\Rightarrow \quad \angle AOB = \angle BOC$ [c.p.c.t.]

Since $\quad \angle AOB + \angle BOC = 180°$ (Linear Pair)

$\therefore \quad \angle AOB + \angle AOB = 180°$

$\Rightarrow \quad 2\angle AOB = 180° \Rightarrow \angle AOB = \frac{180}{2} = 90°$

Thus, the diagonals of a rhombus bisect each other at 90°.

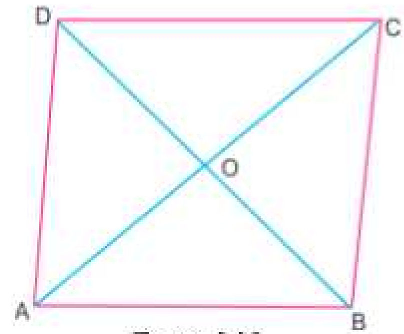


Figure 3.25

Example 3.9 : In the given figure, RICE is a rhombus. Find x, y and z.

Sol. We know, diagonals of a rhombus bisect each other.

$\therefore \quad OR = OC \Rightarrow y = 12$

and $OE = OI \Rightarrow x = 5$

Since all sides of the rhombus are equal.

IR = ER

$\therefore \quad z = 13$

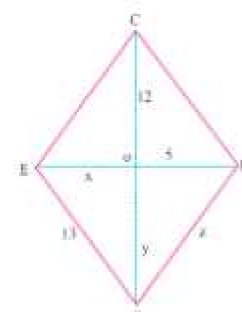


Figure 3.26

Example 3.10 : The diagonals of a rhombus are 6cm and 8cm. Find the side of the rhombus.

Sol. Let ABCD be a rhombus in which diagonals AC = 8cm and BD = 6cm.

Since diagonals of a rhombus bisect each other at 90°.

$OA = \frac{1}{2} AC = \frac{1}{2} \times 8 = 4cm$

and $OB = \frac{1}{2} BD = \frac{1}{2} \times 6 = 3cm$

In right ΔOAB.

By Pythagoras theorem,

$AB^2 = OA^2 + OB^2$

$\Rightarrow AB^2 = 4^2 + 3^2 = 16 + 9 = 25 = 5^2$

$\Rightarrow$ AB = 5cm. Thus, side of rhombus is 5cm.

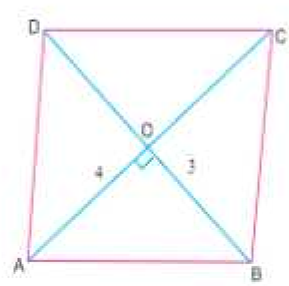


Figure 3.27

3.5 Rectangle :-

A parallelogram having all angles equal, is called rectangle.

Property 1 : Each angle of a rectangle is a right angle.

Proof: Let ABCD be a rectangle.

Since rectangle is a parallelogram having all angles equal.

$\Rightarrow \quad \angle A = \angle B = \angle C = \angle D$

We know, $\angle A + \angle B + \angle C + \angle D = 360°$

$\angle A + \angle A + \angle A + \angle A = 360°$

$\Rightarrow \quad 4\angle A = 360° \quad \Rightarrow \quad \angle A = \frac{360°}{4} = 90°$

$\therefore \quad \angle A = \angle B = \angle C = \angle D = 90°$

Thus, Each angle of the rectangle is right angle.

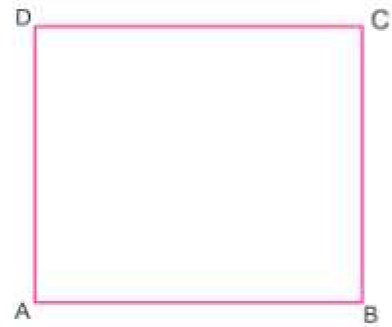


Figure 3.28

Property 2 : The diagonals of a rectangle are equal.

Proof: Let ABCD be a rectangle with diagonals AC and BD.

In ΔDAB and ΔCBA, we have

AD = BC (opposite sides of a rectangle)

AB = AB (common)

$\angle DAB = \angle CBA$ (Each 90°)

$\therefore \quad \Delta DAB \cong \Delta CBA$ (SAS congruency criterion)

$\Rightarrow \quad BD = AC$ (c.p.c.t.)

Hence, the diagonals of a rectangle are equal.

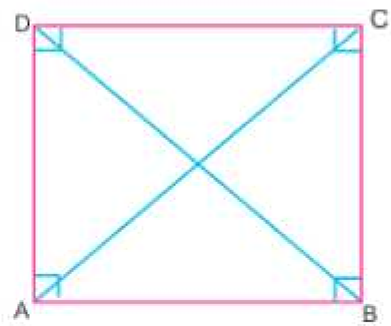


Figure 3.29

Example 3.11 : In the given figure PQRS is a rectangle in which $\angle QPR = 32°$. Find $\angle PRQ$.

Sol. We know, each angle of a rectangle is 90°.

In ΔPQR, we have

$\angle QPR + \angle PQR + \angle PRQ = 180°$

$\Rightarrow \quad 32° + 90° + \angle PRQ = 180°$

$\Rightarrow \quad 122° + \angle PRQ = 180°$

$\Rightarrow \quad \angle PRQ = 180° - 122° = 58°$

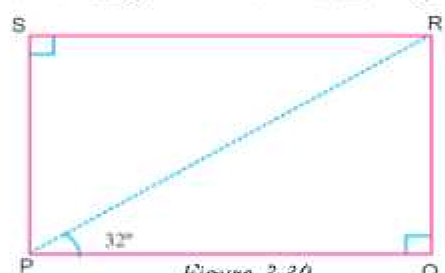


Figure 3.30

Example 3.12 : In the given figure, ABCD is a rectangle. Find x and y.

Sol. We know, opposite sides of a rectangle are equal.

$\therefore \quad BC = AD$	$AB = CD$
$\Rightarrow \quad 5x - 1 = 24$	$\Rightarrow \quad 2y - 3 = 5$
$\Rightarrow \quad 5x = 24 + 1 = 25$	$\Rightarrow \quad 2y = 5 + 3 = 8$
$\Rightarrow \quad x = \frac{25}{5} = 5$	$\Rightarrow \quad y = \frac{8}{2} = 4$

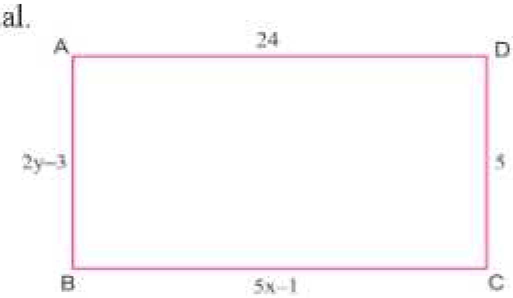


Figure 3.31

Example 3.13 : In the given figure, RENT is a rectangle and its diagonals meet at O. Find x.

Sol. Since, diagonals of a rectangle are equal and bisect each other.

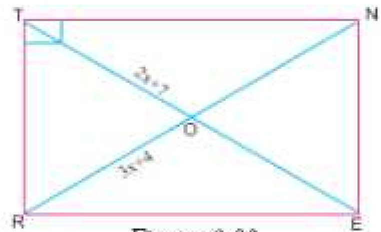


Figure 3.32

$\therefore \quad RN = TE \Rightarrow \frac{1}{2}RN = \frac{1}{2}TE$

$\Rightarrow \quad OR = OT$

$\Rightarrow \quad 3x+4 = 2x+7 \quad \Rightarrow \quad 3x - 2x = 7 - 4$

$x = 3$

3.6 Square :-

A rectangle having all sides equal is called a square.

OR

A rhombus having all angles equal is called a square.

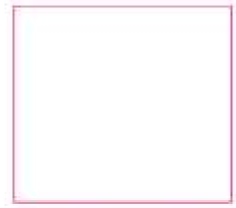


Figure 3.33

Since square is rhombus as well as rectangle. So all properties of rhombus and rectangle contained in square i.e.

(i) All the sides are equal.

(ii) Each angle is 90°.

(iii) The diagonals are of equal length.

(iv) The diagonals bisect each other at 90°.

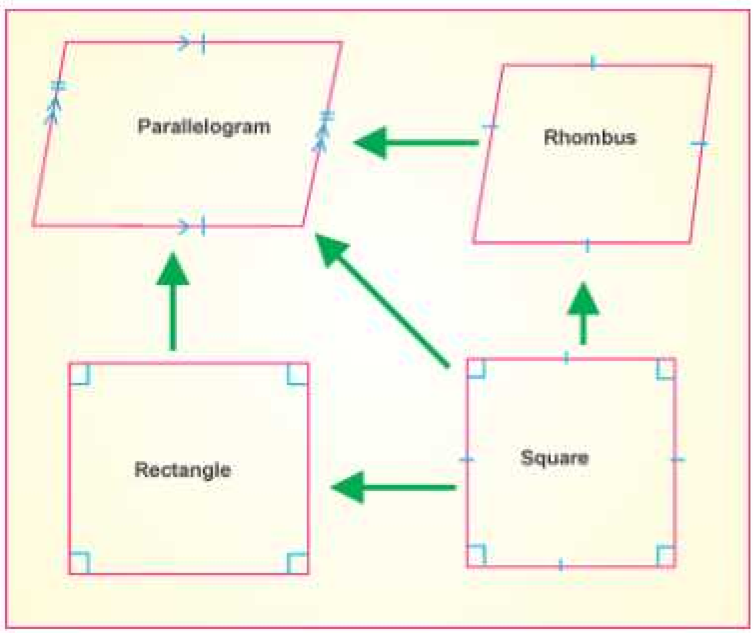


Figure 3.34

Exercise 3.2

1. Identify the quadrilateral in which
   (i) all angles are equal.
   (ii) opposite sides are equal.
2. Identify the quadrilateral in which
   (i) all the sides are equal
   (ii) each of the angle is 90°.
3. Identify the quadrilateral in which
   (i) diagonals bisect each other at 90°
   (ii) diagonals are equal in length.

4. In the given figure, RACE is a rectangle find x, y and z.

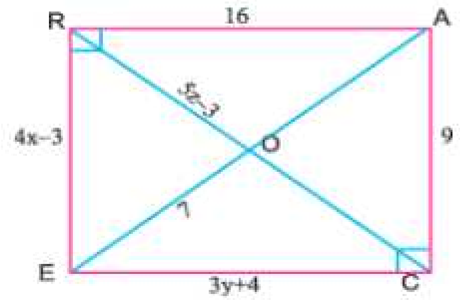


Figure 3.35

5. In the given figure, PQRS is a rhombus find x and y.

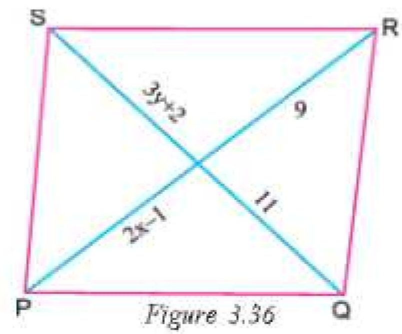


Figure 3.36

6. In the given figure, ABCD is a rectangle $\angle BAC = 36^\circ$, find $\angle ACB$.

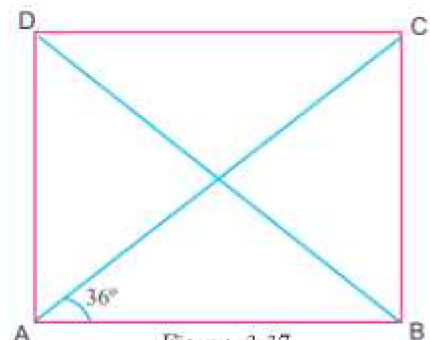


Figure 3.37

7. **Multiple Choice Questions :**

(i) Sum of adjacent angles in a parallelogram is :

(a) 90° (b) 180° (c) 360° (d) None of them

(ii) If adjacent angles of a parallelogram are equal then which polygon it will become :

(a) Rectangle (b) Rhombus (c) Square (d) Trapezium

(iii) If adjacent angles of a rhombus are equal then which polygon it will become :

(a) Rectangle (b) Square (c) Trapezium (d) Parallelogram

(iv) If 3y° and 120° are the adjacent angles of rhombus then find the value of y.

(a) 15° (b) 90° (c) 20° (d) 60°

Activities

Activity 1: Prove that the sum of interior angles of a quadrilateral is 360°.

Required Material : chart paper, geometry box, coloured pen or pencil.

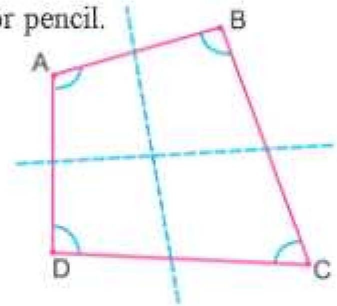
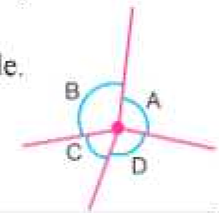


Figure 3.38

Procedure :

1. Take a chart paper and draw a quadrilateral ABCD.
2. Cut the quadrilateral from the chart as shown.
3. Cut all the four angles from the quadrilateral.
4. Draw a dot on another chart paper.
5. Paste all the angles ∠A, ∠B, ∠C and ∠D along their vertices on the dot.



Observation: All four angles after pasting along a dot form a complete circle.

∴ $\angle A + \angle B + \angle C + \angle D = 360°$

i.e. Sum of all interior angles of a quadrilateral is 360°

VIVA VOCE

Q. 1. How many diagonals does a quadrilateral have?

Ans: 2

Q. 2. What is the sum of interior angles of a quadrilateral?

Ans: 360°.

Q. 3. What is the sum of exterior angles of a quadrilateral?

Ans: 360°

Activity 2 : Prove by cutting and pasting the paper that sum of exterior angles, taken in order, of any polygon is 360°.

Material Required : chart paper, geometry box, colour pen Or pencil.

Procedure:

Triangle

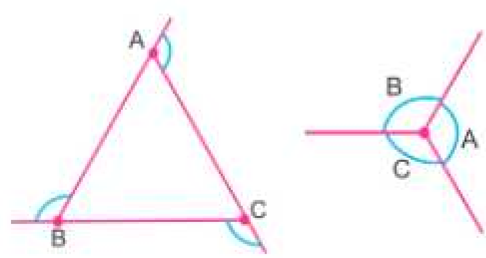


Figure 3.39

1. Take a chart paper and draw a triangle ABC.
2. Cut ΔABC from the chart paper along its exterior angles as shown.
3. Cut the exterior angles A, B, C from the triangle.
4. Draw a dot on another chart paper.
5. Paste all exterior angles ∠A, ∠B, ∠C along their vertices on dot as shown

Quadrilateral

1. Take a chart paper and draw a quadrilateral PQRS.
2. Cut PQRS from the chart paper along its exterior angles as shown.
3. Cut the exterior angles P, Q, R, S from the quadrilateral.
4. Draw a dot on another chart paper.
5. Paste all exterior angles ∠P, ∠Q, ∠R, ∠S along their vertices on a dot.

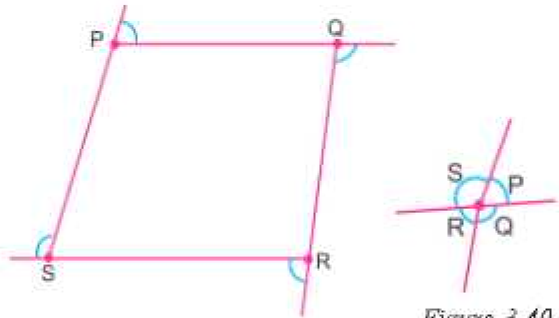


Figure 3.40

Observation: In both cases, exterior angles form a complete circle:

∴ $\angle A + \angle B + \angle C = 360^\circ$

and $\angle P + \angle Q + \angle R + \angle S = 360^\circ$

Similary, students can verify the result for other polygons.

i.e. Sum of exterior angles of a polygon is 360°.

VIVA VOCE

Q. 1. What is the sum of interior angles of a triangle?

Ans: 180°

Q. 2. What is the sum of interior angles of a pentagon?

Ans: 540°

Q. 3. What is the sum of exterior angles of hexagon?

Ans: 360°

Activity 3: Verify (i) The diagonals of a rectangle are equal in length.

(ii) The diagonals of a square are equal in length.

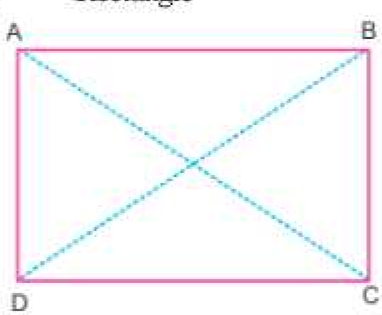
(iii) The diagonals of a rhombus and parallelogram are not equal in length.

Required Material: Chart paper, Geometry Box, Coloured Pen or Pencil.

Procedure :

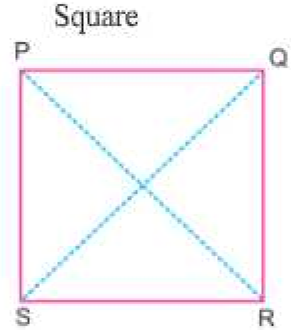
1. Take a chart paper and draw a rectangle, square, parallelogram and rhombus as shown.
2. Join the diagonals of all quadrilaterals.
3. Measure the lengths of diagonals.

(i) Rectangle



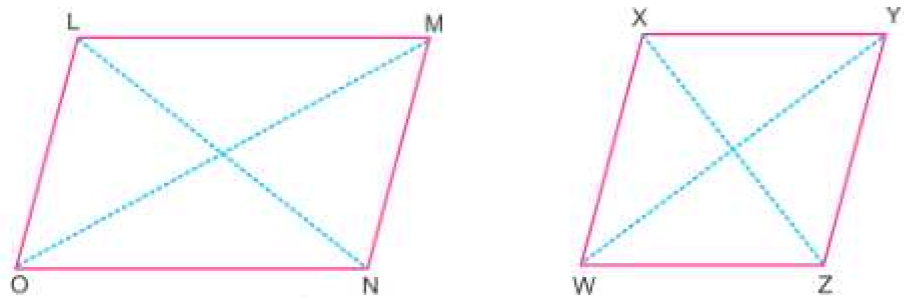
AC = ______, BD = ______

(ii) Square



PR = ______, QS = ______

(iii) Parallelogram (iv) Rhombus



NL = ______, OM = ______ XZ = ______, YW = ______

Figure 3.41

Observation:

(i) The diagonals of rectangle are equal in length.

(ii) The diagonals of the square are equal in length.

(iii) The diagonals of the parallelogram are not equal in length.

(iv) The diagonals of the rhombus are not equal in length.

VIVA VOCE

1. If one diagonal of rectangle is 6cm then what is the length of other diagonal?

Ans: 6cm.

2. In rhombus PQRS, diagonals PR=6cm and QS=8cm intersect at O then OP = ... & OS = ...

Ans: 3cm and 4cm

3. The diagonals of a square bisect each other at............ .

Ans: 90°

Learning Outcomes

After completion of the chapter, students are now able to

- *Differentiate between different types of quadrilaterals on the basis of their properties and establish relationship between them.*

Answers

Exercise 3.1

1. 28cm
2. (i) SR (ii) PS (iii) ∠R (iv) ∠Q (v) 180° (vi) 180°
3. (i) x = 3, y=4 (ii) x=3, y = 5

4. 80°, 100°, 80°, 100°

5. 54°, 126°, 54°, 126°

6. ∠X=100°, ∠W=80°, ∠Z=100°

7. ∠E = 75°, ∠S = 105°, ∠T =75°

8. (i) x=20°, y = 20° (ii) x = 20°, y = 24°

9. OA = 6cm, OD = 8cm

10. PR = 12cm, QS = 14cm

11. (i) x = 6, y = 8 (ii) x = 5, y = 4

12. (i) x = 80°, y = 100°, z = 80° (ii) x = 105°, y = 75°, z = 105°
(iii) x = 30°, y = 102°, z = 48° (iv) x = 38°, y = 40°, z = 102°

13. (i) b (ii) a (iii) c (iv) a

Exercise 3.2

1. Rectangle
2. Square
3. Square
4. x = 3, y = 4, z= 2
5. x = 5, y = 3
6. 54°
7. (i) b (ii) a (iii) b (iv) c