Trigonometric Ratio

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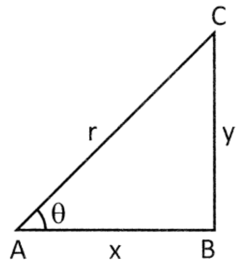


1. (i) $\sin\theta = \dfrac{Perpendicular}{Hypotenuse} = \dfrac{y}{r}$ (ii) $\cos\theta = \dfrac{Base}{Hypotenuse} = \dfrac{x}{r}$

(iii) $\tan\,\theta = \dfrac{Perpendicular}{Base} = \dfrac{y}{x}$ (iv) $\cos ec\,\theta = \dfrac{Hypotenuse}{Perpendicular} = \dfrac{r}{y}$

(v) $\sec\,\theta = \dfrac{Hypotenuse}{Base} = \dfrac{r}{x}$ (vi) $\cot\,\theta = \dfrac{Base}{Perpendicular} = \dfrac{x}{y}$

2. (i) $\cos ec\,\theta = \dfrac{1}{\sin\theta}$ (ii) $\sec\,\theta = \dfrac{1}{\cos\theta}$

(iii) $\cot\,\theta = \dfrac{1}{\tan\theta}$ (iv) $\tan\,\theta = \dfrac{\sin\theta}{\cos\theta}$

(v) $\cot\,\theta = \dfrac{\cos\theta}{\sin\theta}$

3. (i) $\sin^2 + \theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\,\theta = \sec^2\,\theta$ for $0° \le \theta < 90°$

(iii) $1 + \cot^2\theta = \cos ec^2\theta$ for $0° < \theta \le 90°$

4. (i) $sin\,(90° - \theta) = \cos\,\theta$ (ii) $\cos(90° - \theta) = \sin\theta$

(iii) $\tan(90° - \theta) = \cot\theta$ (iv) $\cot(90° - \theta) = \tan\theta$

(v) $\sec(90° - \theta) = \cos ec\,\theta$ (vi) $\cos\sec(90° - \theta) = \sec\theta$

5. The value of sin or cos never exceeds 1, whereas the value of sec or cosec is always greater or equal to 1.

6. Table for T- ratios of $0°,\ 30°,\ 45°,\ 60°,\ 90°$.

θ	sin θ	cos θ	tan θ	cosec θ	sec θ	cot θ
0°	0	1	0	not defined	1	not defined
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	not defined	1	not defined	0

Snap Test

1. $(\sec\theta - \tan\theta)^2 = ?$

(a) $\frac{1+\sin\theta}{1-\sin\theta}$ (b) $\frac{2+\sin\theta}{2-\sin\theta}$

(c) $\frac{2-\sin\theta}{2+\sin\theta}$ (d) $\frac{1-\sin\theta}{1+\sin\theta}$

(e) None of these

Ans. (d)

Explanation: $(sec\,\theta - tan\,\theta)^2 = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \frac{(1-\sin\theta)^2}{\cos^2\theta} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \frac{1-\sin\theta}{1+\sin\theta}$

2. **If $4\tan\theta = 3$, find the value of $\frac{4\sin\theta - 2\cos\theta}{4\sin\theta + 3\cos\theta}$.**

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$

(c) $\frac{1}{8}$ (d) $\frac{3}{8}$

(e) None of these

Ans. (b)

Explanation: We have $4\,tan\theta = 3$

Now, $\frac{4\sin\theta - 2\cos\theta}{4\sin\theta + 3\cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 2}{4\frac{\sin\theta}{\cos\theta} + 3}$

$$= \frac{4\tan\theta - 2}{4\tan\theta + 3}$$

$$= \frac{3-2}{3+3} = \frac{1}{6}.$$

3. **If $\tan\theta = \frac{a}{b}$, then $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = ?$**

(a) $\frac{a^2 + b^2}{a+b}$ (b) $\frac{a^2 - b^2}{a^2 - b}$

(c) $\frac{a^2 - b^2}{a^2 + b^2}$ (d) $\frac{a^2 - b^2}{a-b}$

(e) None of these

Ans. (c)

Explanation: We have $\tan\theta = \frac{a}{b}$

Now, $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a.\frac{\sin\theta}{\cos\theta} - b}{a.\frac{\sin\theta}{\cos\theta} + b}$

$= \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a.\frac{a}{b} - b}{a.\frac{a}{b} + b}$ $\left[\therefore \tan\theta = \frac{a}{b}\text{(given)}\right]$

$= \frac{a^2 - b^2}{a^2 + b^2}$

4. **Evaluate:**

$$\mathbf{Sin^2 30^\circ cos^2 45^\circ + 4tan^2 30^\circ + \frac{1}{2} sin^2 90^\circ - 2cos^2 90^\circ + \frac{1}{24}}$$

(a) 3 (b) 6

(c) 4 (d) 2

(e) None of these

Ans. (d)

Explanation: $sin^2 30^\circ\ cos + \ 45^\circ \ + \ 4\ tan^2 30^\circ \ + \frac{1}{2}\ sin^2 90^\circ \ - \ 2\ cos^2 90^\circ + \frac{1}{24}$

$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4 \times \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{2}\right) \times 1^2 - 2 \times (0)^2 + \frac{1}{24}$

$= \left(\frac{1}{4} \times \frac{1}{2}\right) + \left(4 \times \frac{1}{3}\right) + \left(\frac{1}{2} + \frac{1}{24}\right) = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$

$= \frac{3 + 32 + 12 + 1}{24} = \frac{48}{24} = 2$

5. **In a $\triangle$ABC, if $\angle$A = 30°, $\angle$B = 90° and AC = 10 cm, then find AB and BC.**

(a) $5\sqrt{3}\,cm,\ 5\ cm$ (b) $5\sqrt{2}\,cm,\ 5\ cm$

(c) $5\sqrt{2}\,cm,\ 3\ cm$ (d) $5\sqrt{2}\,cm,\ 8\ cm$

(e) None of these

Ans. (a)

Explanation: It is given that in ΔABC $\angle A = 30°$, and $AC = 10\ cm$.

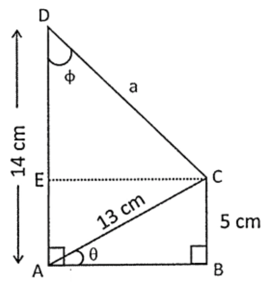
We have:

(i) $\frac{AB}{AC} = cos\ A = cos\ 30°$

$$\Rightarrow \frac{AB}{10} = \frac{\sqrt{3}}{2} \Rightarrow AB = \left(10 \times \frac{\sqrt{3}}{2}\right) cm\ 5\sqrt{3}\ cm.$$

(ii) $\frac{BC}{AC} = \sin A = \sin 30° \quad \Rightarrow \frac{BC}{10} = \frac{1}{2} \quad \Rightarrow BC = \frac{10}{2}\ cm = 5\ cm$

6. **In the given figure, $\angle ABC = 90°$, $\angle BAC = \theta$, $\angle ADC = \phi$. BC = 5 cm, AC = 13 cm and AD = 14 cm. Also, $\angle BAD = 90°$. Find the values of cosec ϕ and tan ϕ.**



(a) $\frac{12}{13}, \frac{4}{3}$ (b) $\frac{5}{4}, \frac{4}{3}$

(c) $\frac{4}{3}, \frac{4}{5}$ (d) $\frac{4}{3}, \frac{7}{5}$

(*e*) None of these

Ans. (b)

Explanation: In right Δ ABC we have: $AC^2 = AB^2 + BC^2$

$\Rightarrow AB = \sqrt{AC^2 - BC^2} = \sqrt{(13)^2 - (5)^2}$

$= \sqrt{169 - 25} = \sqrt{144} = 12\ cm.$

Now, $EC = AB = 12\ cm.$

and $DE = AD - EA = AD - BC$

$= (14 - 5)\ cm = 9\ cm.$

In right ΔCDE we have:

$CD^2 = DE^2 + EC^2 = 9^2 + 12^2 = 81 + 144 = 225$

$\Rightarrow CD = \sqrt{225} = 15\ cm.$

(i) $cosec\,\phi = \frac{CD}{EC} = \frac{15}{12} = \frac{5}{4}.$

(ii) $\tan\phi = \frac{EC}{DE} = \frac{12}{9} = \frac{4}{3}.$

7. $\left(\textbf{cosec } \theta - \textbf{cot } \theta\right)^2$ **is equal to**:

(a) $\frac{2-\cos\theta}{1+\cos\theta}$ (b) $\frac{1}{1-\cos\theta}$

(c) $\frac{1-\cos\theta}{1+\cos\theta}$ (d) $\frac{2+\cos\theta}{1+\cos\theta}$

(e) None of these

Ans. (c)

Explanation: $\left(cosec\ \theta - cot\ \theta\right)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)^2(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

8. $\frac{\textbf{sin } \theta}{\textbf{1+cos } \theta} + \frac{\textbf{1+cos } \theta}{\textbf{sin } \theta} =$

(a) $2\ cosec\,\theta$ (b) $2\ cos\,\theta$

(c) $2\ sec\,\theta$ (d) $2\ sin\,\theta$

(e) None of these

Ans. (a)

Explanation: $\frac{\sin^2\theta}{1+\cos\theta} + \frac{1+\cos\theta}{sin\theta} = \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \frac{2}{\sin\theta} = 2\ \text{cos}ec\,\theta$$