Chapter – 3
Pair of Linear Equations in Two Variables

Exercise 3.7

Q. 1 The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Answer: The difference between the ages of Biju and Ani is 3 years. Either Biju is 3 years older than Ani or Ani is 3 years older than Biju. However, it is obvious that in both cases, Ani's father's age will be 30 years more than that of Cathy's age.

Let the age of Ani and Biju be x and y years respectively.

Therefore, age of Ani's father, Dharam = 2 × x = 2x years

And age of Biju's sister Cathy $=\frac{y}{2}$ years

By using the information given in the question,

Case (I) When Ani is older than Biju by 3 years, x - y = 3 (i)

4x - y = 60 (ii)

Subtracting (i) from (ii), we obtain 3x = 60 - 3 = 57

x = 57/3 = 19

Therefore, age of Ani = 19 years

And age of Biju = 19 - 3 = 16 years

Case (II) When Biju is older than Ani, y - x = 3 (i)

$$2x - \frac{y}{2} = 30$$

4x - y = 60 (ii)

Adding (i) and (ii), we obtain $3x = 63$

$x = 21$

Therefore, age of Ani = 21 years

And age of Biju = 21 + 3 = 24 years

Q. 2 One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II)

[Hint: $x + 100 = 2(y - 100)$, $y + 10 = 6(x - 10)$]

Answer: Let those friends were having Rs x and y with them. Using the information given in the question,

we obtain

From the first condition, $x + 100 = 2(y - 100)$

$x + 100 = 2y - 200$

$x - 2y = -300$ (i)

And, From the second condition$6(x - 10) = (y + 10)$

$6x - 60 = y + 10$

$6x - y = 70$ (ii)

Multiplying equation (ii) by 2, we obtain

$12x - 2y = 140$ (iii)

Subtracting equation (i) from equation (iii),

we obtain

$11x = 140 + 300$

$11x = 440$

$x = 40$

Using this in equation (i), we obtain

$40 - 2y = -300$

$40 + 300 = 2y$

$2y = 340$

$y = 170$

Therefore, those friends had Rs 40 and Rs 170 with them respectively.

Q. 3 A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Answer: Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel was d km.

We know that,

$$Speed = \frac{distance\ travelled}{time\ taken\ to\ travel\ that\ distance}$$

$$x = \frac{d}{t}$$

Or, $d = xt$ (i)

If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time

$\Rightarrow (x + 10) = \frac{d}{t-2}$

$\Rightarrow (x + 10)(t - 2) = d$

$\Rightarrow xt + 10t - 2x - 20 = d$

From (i), we have

$\Rightarrow d + 10t - 2x - 20 = d$

$\Rightarrow - 2x + 10t = 20$

$\Rightarrow x - 5t = -10 \Rightarrow x = 5t - 10$ (ii)

Also,

if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time$\Rightarrow (x - 10) = \frac{d}{t-3}$

$\Rightarrow (x - 10)(t + 3) = d$

$\Rightarrow xt - 10t + 3x - 30 = d$

By using equation (i),

$\Rightarrow d - 10t + 3x - 30 = d \Rightarrow 3x - 10t = 30$ (iii)

Substituting the value of x from eq (ii) into eq (iii), we get

$\Rightarrow 3(5t - 10) - 10t = 30$

$\Rightarrow 15t - 30 - 10t = 30$

$\Rightarrow 5t = 60 \Rightarrow t = 12$ hours

Putting this in eq(ii)$\Rightarrow x = 5t - 10$

$= 5(12) - 10$

$= 50$ km/h

From equation (i), we obtain

Distance to travel $= d = xt$

$= 50 \times 12$

$= 600$ km

Hence, the distance covered by the train is 600 km.

Q. 4 The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Answer: Let the number of rows be x and number of students in a row be y.

Total students of the class = Number of rows x Number of students in a row = xy

Using the information given in the question,

Condition 1

Total number of students = (x - 1) (y + 3)

= (x - 1) (y + 3)

= xy - y + 3x - 3

3x - y - 3 = 0

3x - y = 3 ....(i)

Condition 2

Total number of students = (x + 2) (y - 3)

= xy + 2y - 3x - 6

⇒3x - 2y = -6 ... (ii)

Subtracting equation (ii) from (i),

(3x - y) - (3x - 2y) = 3 - (-6)

⇒- y + 2y = 9

⇒3 + 6 y = 9

By using equation (i), we obtain 3x - 9 = 3,

⇒3x = 9 + 3

⇒3x = 12

⇒ x = 4

From (i),

$\Rightarrow 3(4) - y = 3$

$\Rightarrow 12 - y = 3 \Rightarrow 9 = y$

Number of rows = x = 4

Number of students in a row = y = 9

Number of total students in a class = Number of students in 1 row × Number of rows

= xy

= 4× 9 = 36.

Q. 5 In a Δ ABC, <C=3<B=2 (<A+<B) Find the three angles.

Answer: Given that,

$\angle C = 3\angle B = 2(\angle A + \angle B)$

Let's take $3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2\angle A + 2\angle B$

$\angle B = 2\angle A$

$2\angle A - \angle B = 0$ … (i)

We know that the sum of the measures of all angles of a triangle is 180°. Therefore,

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + 3\angle B = 180°$

$\angle A + 4\angle B = 180°$ … (ii)

Multiplying equation (i) by 4, we obtain

$8\angle A - 4\angle B = 0$ … (iii)

Adding equations (ii) and (iii), we obtain

$9\angle A = 180°$

$\angle A = 20°$

From equation (ii), we obtain

$20° + 4 \angle B = 180°$

$4 \angle B = 160°$

$\angle B = 40°$

and

$\angle C = 3 \angle B$

$= 3 \times 40° = 120°$

Therefore, $\angle A$, $\angle B$, $\angle C$ are 20°, 40°, and 120° respectively.

Q. 6 Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

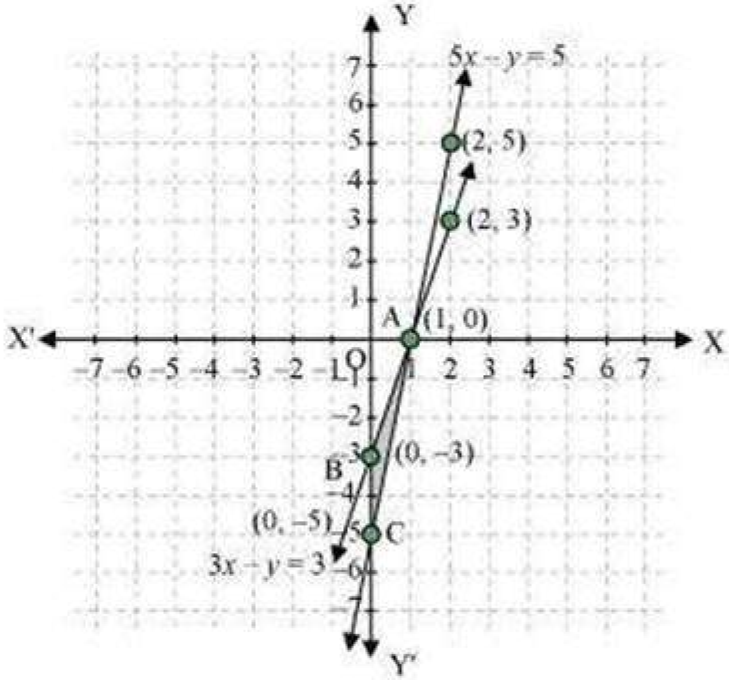
Answer: $5x - y = 5$ Or,

$y = 5x - 5$

The solution table will be as follows.

x	0	1	2
y	-5	0	5
3x-y =3 or, y =3x - 3			
The solution table will be as follows.			
x	0	1	2
y	-3	0	3

The graphical representation of these lines will be as follows.



It can be observed that the required triangle is ΔABC formed by these lines and y-axis. The coordinates of vertices are A (1, 0), B (0, - 3), C (0, - 5).

Q. 7 Solve the following pair of linear equations.

(i) px + qy = p - q

qx - py = p + q

(ii) ax + by = c

bx + ay = 1 + c

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

ax + by = $a^2 + b^2$

(iv) (a - b) x + (a + b) y = a^2 - 2ab - b^2

(a + b) (x + y) = $a^2 + b^2$

(v) 152 x - 378 y = - 74

-378 x + 152 y = - 604

Answer:

(i) $p x + q y = p - q$ … (1)

$q x - p y = p + q$ … (2)

Multiplying equation (1) by p and equation (2) by q,

we obtain $p^2 x + pq y = p^2 - pq$ … (3)

$q^2 x - pq y = pq + q^2$ … (4)

Adding equations (3) and (4),

we obtain $p^2x + q^2 x = p^2 + q^2$

$(p^2 + q^2) x = p^2 + q^2$

$$x = \frac{p^2+q^2}{p^2+q^2} = 1$$

From equation putting the value of x (1),

we obtain $p (1) + qy = p - q$

$qy = - q$

$y = - 1$

(ii) $ax + by = c$ … (1)

$bx + ay = 1 + c$ … (2)

Multiplying equation (1) by a and equation (2) by b,

we obtain $a^2 x + ab y = ac$ … (3)

$b^2 x + ab y = b + bc$ … (4)

Subtracting equation (4) from equation (3),

$(a^2 - b^2) x = ac - bc – b$

$$x = \frac{c(a-b)-b}{a^2-b^2}$$

From equation (1), we obtain ax + by = c, now putting the value of x in the equation

$$a\left\{\frac{c(a-b)-b}{a^2-b^2}\right\}+by=c$$

$$\frac{ac(a-b)-ab}{a^2-b^2}+by=c$$

$$by=c-\frac{ac(a-b)-ab}{a^2-b^2}$$

$$by=\frac{a^2c-b^2c-a^2c+abc+ab}{a^2-b^2}$$

$$by=\frac{abc-b^2c+ab}{a^2-b^2}$$

$$by=\frac{bc(a-b)+ab}{a^2-b^2}$$

$$y=\frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a}-\frac{y}{b}=0$

or, bx - ay = 0 … (1)

ax + by = $a^2 + b^2$ … (2)

Multiplying equation (1) and (2) by b and a respectively, we obtain b2 x - ab y = 0 … (3)

a^2 x + ab y = $a^3 + ab^2$ … (4)

Adding equations (3) and (4), we obtain $b^2x + a^2x = a^3 + ab^2$

x ($b^2 + a^2$) = a ($a^2 + b^2$) x

Thus, x = a

By using (1), and putting the value of x in the equation we obtain b (a) - ay = 0

ab - ay = 0

$ay = ab$

$y = b$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ … (1)

$(a + b)(x + y) = a^2 + b^2$

$(a + b)x + (a + b)y = a^2 + b^2$… (2)

Subtracting equation (2) from (1),

we obtain

$(a - b)x - (a + b)x = (a^2 - 2ab - b^2) - (a^2 + b^2)$ $(a - b - a - b)x = - 2ab - 2b^2$

$- 2bx = - 2b(a + b)$

$x = a + b$

Using equation (1), and putting the value of x in the equation we obtain

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2 a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

$(a + b)y = - 2ab$

$$y = \frac{-2ab}{a+b}$$

(v) $152x - 378y = - 74$ ------------ (1)

$-378x + 152y = - 604$ -------- (2)

Multiply eq (2) by 152 and equation (1) by 378

$378 \times 152x - 3782y = -74 \times 378$

$-378 \times 152x + 1522y = -604 \times 152$

Adding both the questions we get

$(152^2 - 378^2)y = -119780$

$-119780y = -119780$

$y = 1$

put the value in eq 1,

152 x - 378 x 1 = - 74 152 x = 378 - 74152 x = 304x = 2we get x = 2.

Q. 8 ABCD is a cyclic quadrilateral (see Fig. 3.7). Find the angles of the cyclic quadrilateral.

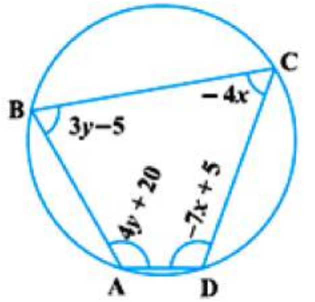


Fig. 3.7

Answer: We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°. Therefore, ∠ A + ∠ C = 180

⇒4y + 20 - 4x = 180

⇒- 4x + 4y = 160

⇒x - y = - 40 ....(i)

Also, ∠ B + ∠ D = 180

⇒3y - 5 - 7x + 5 = 180

⇒- 7x+ 3y = 180 .... (ii)

Multiplying equation (i) by 3, we obtain 3x - 3y = - 120.... (iii)

Adding equations (ii) and (iii), we obtain

- 7x + 3x = 180 - 120

- 4x = 60

x = -15

By using equation (i), we obtain x - y = - 40

-15 - y = - 40

y = -15 + 40

= 25

$\angle A = 4y + 20$

$= 4(25) + 20$ $\angle A = 120°$

$\angle B = 3y - 5$

$= 3(25) - 5$ $\angle B = 70°$

$\angle C = -4x$

$= -4(-15)$

$\angle C = 60°$

$\angle D = -7x + 5$ $\quad = -7(-15) + 5$

$= 105 + 5$

$\angle D = 110°$.