Sample Question Paper - 36
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours **Maximum Marks : 40**

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

2. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

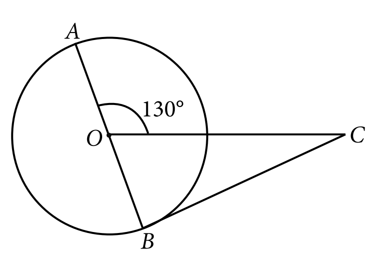
OR

In an A.P., the first term is –4, the last term is 29 and the sum of all its terms is 150. Find its common difference.

3. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. $\left[\text{Use } \pi = \frac{22}{7}\right]$

4. If the quadratic equation $px^2 - 2\sqrt{5}\,px + 15 = 0$ has two equal roots, then find the value of p.

5. In the given figure, AOB is a diameter of a circle with centre O and BC is a tangent to the circle at B. If $\angle AOC = 130°$, then find $\angle BCO$.



OR

If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

6. If median = 7 and mean = 7.5, then find the mode.

SECTION - B

7. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is 30°. If the height of the second pole is 24 m, find the height of the first pole. $\left[\text{Use } \sqrt{3} = 1.732\right]$

OR

A kite is flying at a height of 45 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.

8. For the following grouped frequency distribution find the mode.

Class-interval	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	5	10	23	21	12	3

9. Divide a line segment of length 8 cm internally in the ratio 3 : 4. Also, give justification of the construction.

10. Find the median marks for the following distribution :

Classes	0-10	10-20	20-30	30-40	40-50
Number of students	2	12	22	8	6

SECTION - C

11. Water is flowing at the rate of 15 km/hour through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm?

OR

Raman made a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 21 cm and its length is 36 cm. If each cone has a height of 9 cm, find the volume of air contained in the model that Raman made.

12. Draw a circle of radius 3 cm and draw a tangent to this circle making an angle of 30° with line passing through the centre.

Case Study - 1

13. In our daily life we use quadratic formula as for calculating areas, determining a product's profit or formulating the speed of an object and many more.

Based on the above information, answer the following questions.

(i) Find the value of k, if one root of the quadratic equation $2x^2 + kx + 1 = 0$ is $-1/2$.

(ii) Solve for x : $(x - 2)^2 - 1 = 0$.

Case Study - 2

14. There is fire incident in the house. The house door is locked so, the fireman is trying to enter the house from the window. He places the ladder against the wall such that its top reaches the window as shown in the figure.

Based on the above information, answer the following questions.

(i) If window is 6 m above the ground and angle made by the foot of ladder to the ground is 30°, then find the length of the ladder.

(ii) If fireman places the ladder 2.5 m away from the wall and angle of elevation is observed to be 60°, then find the height of the window. (Take $\sqrt{3} = 1.73$)

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Let tens place digit of the number be x and ones place digit be y.

$\therefore$ Number = $10x + y$

Now, $10x + y = 4(x + y) \Rightarrow 10x + y = 4x + 4y$

$\Rightarrow 6x = 3y \Rightarrow y = 2x$ ...(i)

Also, $10x + y = 3(x \times y)$

$\Rightarrow 10x + 2x = 3(x \times 2x)$ [Using (i)]

$\Rightarrow 12x = 6x^2 \Rightarrow 6x(x - 2) = 0$

$\Rightarrow x = 2 \quad [\because x \neq 0] \quad \therefore \quad y = 2 \times 2 = 4$

$\therefore$ Required number = 24

2. Natural numbers between 101 and 999 which are divisible by both 2 and 5 are 110, 120, ..., 990, which forms an A.P.

Here, $a = 110$, $d = 10$, $a_n = 990$

Now, $a + (n - 1)d = a_n$

$\Rightarrow 110 + (n - 1)10 = 990$

$\Rightarrow (n - 1)10 = 880$

$\Rightarrow n - 1 = 88 \Rightarrow n = 89$

Hence, there are 89 numbers which are divisible by both 2 and 5.

OR

Let a be the first term, d be the common difference and l be the last term of the given A.P.

$\therefore \quad a = -4, l = 29$

Since, $l = a_n = a + (n - 1)d$

$\Rightarrow 29 = -4 + (n - 1)d \Rightarrow (n - 1)d = 33$ ... (i)

Now, $S_n = 150$

$$\Rightarrow \frac{n}{2}(a + l) = 150$$

$$\Rightarrow \frac{n}{2}(-4 + 29) = 150 \Rightarrow n = \frac{150 \times 2}{25} = 12$$

From (i), $(12 - 1)d = 33 \Rightarrow d = \frac{33}{11} = 3$

3. Side of cube = Diameter of sphere = 7 cm

$\therefore$ Volume of cube = $a^3 = (7)^3$

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3$

$= \frac{11}{21}(7)^3$

Hence, volume of the wood left

= Volume of cube − Volume of sphere

$$= (7)^3\left[1 - \frac{11}{21}\right] = (7)^3 \times \frac{10}{21} = 163.33 \text{ cm}^3$$

4. Given, $px^2 - 2\sqrt{5}px + 15 = 0$

Since, roots are equal

$\therefore$ Discriminant, $D = 0$

i.e., $(-2\sqrt{5}p)^2 - 4 \times p \times 15 = 0 \Rightarrow 20p^2 - 60p = 0$

$\Rightarrow p^2 - 3p = 0 \Rightarrow p(p - 3) = 0$

$\Rightarrow p = 0$ or $p = 3 \Rightarrow p = 3$ $\quad (\because p$ cannot be zero)

5. Given, $\angle AOC = 130°$

Since, BC is a tangent to the circle at B.

$\therefore \angle OBC = 90°$ [$\because$ Radius is perpendicular to the tangent at point of contact]

Now, $\angle AOC + \angle BOC = 180°$[Linear pair]

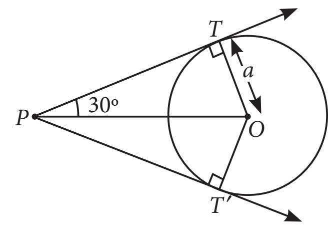
$\Rightarrow \angle BOC = 180° - 130° = 50°$

In ΔBOC, $\angle BOC + \angle BCO + \angle OBC = 180°$ [Angle sum property]

$\Rightarrow \angle BCO = 180° - 50° - 90° = 40°$

OR

Since, tangents drawn from an external point are equally inclined to the line joining centre to that point.



$\therefore \angle TPT' = 60° \Rightarrow \angle TPO = 30°$

Also, $OT \perp TP$

Now, in ΔTPO, $\sin 30° = \frac{OT}{OP}$

$$\Rightarrow \frac{1}{2} = \frac{a}{OP} \Rightarrow OP = 2a$$

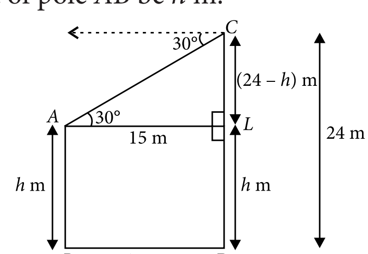
6. Given, median = 7 and mean = 7.5

We know that, Mode = 3(Median) − 2(Mean)

= 3(7) − 2(7.5) = 21 − 15 = 6

Hence, the value of mode is 6.

7. Let AB and CD be two poles, where CD = 24 m and height of pole AB be h m.



$AL = BD = 15$ m and $AB = LD = h$ m

Therefore, $CL = CD - LD = (24 - h)$ m

In ΔACL, $\tan 30° = \frac{CL}{AL} \Rightarrow \frac{1}{\sqrt{3}} = \frac{24-h}{15}$

$\Rightarrow 24 - h = \frac{15}{\sqrt{3}} \Rightarrow 24 - h = 5\sqrt{3}$

$\Rightarrow h = 24 - 5\sqrt{3} = 24 - 5 \times 1.732$

$\Rightarrow h = 15.34$

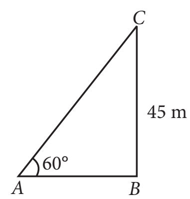
Therefore, height of the pole $AB = 15.34$ m.

OR

Let C be the position of kite.

Now, in ΔABC,

$\sin 60° = \frac{BC}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{45}{AC}$



$\Rightarrow AC = \frac{45 \times 2}{\sqrt{3}}$

$= \frac{90}{\sqrt{3}} = 30\sqrt{3} = 51.96$ m

Thus, the length of the string is 51.96 m.

8. We observe that the class 12-15 has maximum frequency. Therefore, this is the modal class such that $l = 12, h = 3, f_1 = 23, f_0 = 10$ and $f_2 = 21$.

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

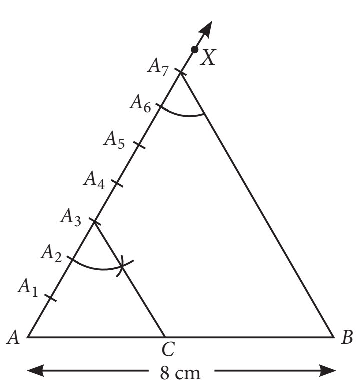
$= 12 + \left(\frac{23 - 10}{2 \times 23 - 10 - 21}\right) \times 3$

$= 12 + \frac{13}{15} \times 3 = 12 + \frac{13}{5} = 14.6$

9. Steps of construction

Step-I : Draw a line segment $AB = 8$ cm.

Step-II : Draw a ray AX making an acute angle with the line segment AB.



Step-III : Locate $(3 + 4 = 7)$ points $A_1, A_2, ..., A_7$ on AX such that $AA_1 = A_1A_2 = ... = A_6A_7$.

Step-IV : Join A_7B.

Step-V : Through A_3, draw A_3C parallel to A_7B, meeting AB at C, such that $\angle AA_7B = \angle AA_3C$.

Hence, $AC : CB = 3 : 4$

Justification : In ΔABA_7, we observe that A_3C is parallel to A_7B, therefore, by basic proportionality theorem, we have

$\frac{AA_3}{A_3A_7} = \frac{AC}{BC}$ *i.e.*, $\frac{AC}{CB} = \frac{3}{4}$

$\left[\text{By construction, we get } \frac{AA_3}{A_3A_7} = \frac{3}{4}\right]$

$\Rightarrow AC : CB = 3 : 4$

10. Cumulative frequency table for the given data is as follows:

Classes	Number of students (f_i)	Cumulative frequency ($c.f.$)
0-10	2	2
10-20	12	2 + 12 = 14
20-30	22	14 + 22 = 36
30-40	8	36 + 8 = 44
40-50	6	44 + 6 = 50
Total	$\sum f_i = 50$	

Here, $n = 50 \Rightarrow \frac{n}{2} = \frac{50}{2} = 25$

Cumulative frequency just greater than 25 is 36 and corresponding interval is 20-30.

$\therefore$ Median class is 20-30.

So, $l = 20, f = 22, c.f. = 14, h = 10$

$\therefore$ Median $= l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$

$= 20 + \left(\frac{25 - 14}{22}\right) \times 10 = 20 + \frac{11}{22} \times 10 = 20 + 5 = 25$

11. Rate of flow of water through the pipe

$= 15 \text{ km/h} = \frac{15000 \text{ m}}{3600 \text{ s}} = \frac{25}{6}$ m/s

$\therefore$ Length of standing water in the pipe in 1 sec (H) $= 25/6$

Radius of the pipe (r) $= \frac{14}{2} = 7 \text{ cm} = \frac{7}{100}$ m

Length of the cuboidal pond (l) = 50 m

Breadth of the cuboidal pond (b) = 44 m

Rise in the level of water in the pond (h) = 21 cm $= \dfrac{21}{100}$ m

Time taken by the pipe to fill the pond

$$= \frac{\text{Volume of the pond}}{\text{Volume of the water flowing through the pipe in 1 second}}$$

$$= \frac{l \times b \times h}{\pi r^2 \times H} = \frac{50 \times 44 \times \frac{21}{100}}{\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times \frac{25}{6}} = 7200 \text{ seconds}$$

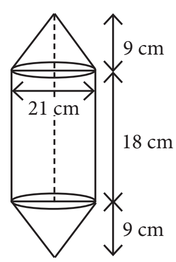
$$= \frac{7200}{3600} \text{ hours} = 2 \text{ hours}$$

Hence, level of water in the pond rises by 21 cm. in 2 hours.

OR

Radius of each cone (r) = Radius of cylinder (r)

$$= \frac{21}{2} = 10.5 \text{ cm}$$



Height of each cone (h) = 9 cm

$\therefore$ Height of cylindrical portion (h_1) = 36 – 9 – 9 = 18 cm

$\therefore$ Volume of the air contained in model = Volume of cylindrical part + 2 × Volume of cone

$$= \pi r^2 h_1 + 2 \times \frac{1}{3}\pi r^2 h = \pi r^2 \left[h_1 + \frac{2}{3}h\right]$$

$$= \frac{22}{7} \times (10.5)^2 \left[18 + 2 \times \frac{1}{3} \times 9\right]$$

$$= \frac{22}{7} \times (10.5)^2 [18 + 6] = \frac{22}{7} \times 110.25 \times 24 = 8316 \text{ cm}^3$$

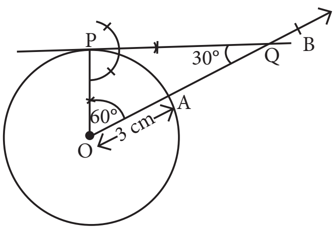
12. Steps of construction :

Step-I : Draw a circle with centre O and radius 3 cm.

Step-II : Draw a radius OA and produce it to B.

Step-III : Construct an $\angle AOP$ equal to the complement of 30° *i.e.*, 60°.

Step-IV : Draw a perpendicular to OP at P which intersects OB at Q.



Hence, PQ is the required tangent such that $\angle OQP = 30°$.

13. (i) We have, $2x^2 + kx + 1 = 0$

Since, –1/2 is the root of the equation, so it will satisfy the given equation.

$$\therefore \quad 2\left(-\frac{1}{2}\right)^2 + k\left(-\frac{1}{2}\right) + 1 = 0 \Rightarrow 1 - k + 2 = 0 \Rightarrow k = 3$$

(ii) The given equation is $(x - 2)^2 - 1 = 0$

$\Rightarrow x^2 - 4x + 4 - 1 = 0 \Rightarrow x^2 - 4x - 3 = 0$

Compare the quadratic equation $x^2 - 4x - 3 = 0$ with $ax^2 + bx + c = 0$.

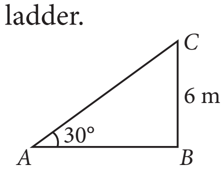
$\Rightarrow a = 1, b = -4, c = -3$

$$x = \frac{-(-4) \pm \sqrt{16 + 12}}{2}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

$$= 2 \pm \sqrt{7}$$

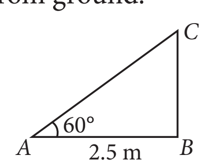
14. (i) Let AC be the length of the ladder.



In ΔABC, $\dfrac{BC}{AC} = \sin 30°$

$$\Rightarrow \frac{6}{AC} = \frac{1}{2} \Rightarrow AC = 12 \text{ m}$$

(ii) Let BC be the height of window from ground.



In ΔABC, $\dfrac{BC}{AB} = \tan 60°$

$$\Rightarrow \frac{BC}{2.5} = \sqrt{3}$$

$\Rightarrow BC = 2.5 \times 1.73 = 4.325$ m