Polynomials

An algebraic expression f(x) of the form $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$, where $a_0, a_1, a_2, ..., a_n$ are real numbers and all the index of x are non-negative integers is called polynomials in x and the highest Index n is called the degree of the polynomial.

Quadratic polynomial

$f(x) = ax^2 + bx + c$

Relationship b/w zeros & coefficients

Sum of zeros $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{b}{a}$

Product of zeros $= \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{c}{a}$

If zeros of quadratic polynomial is α and β then polynomial is $f(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ where k is any real number.

Cubic polynomial

$f(x) = ax^3 + bx^2 + cx + d$

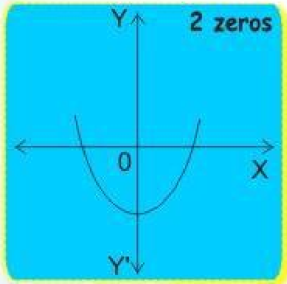
Relationship b/w zeros & coefficients

Sum of zeros $= \frac{-\text{coefficient of } x^2}{\text{coefficient of } x^3} = -\frac{b}{a}$

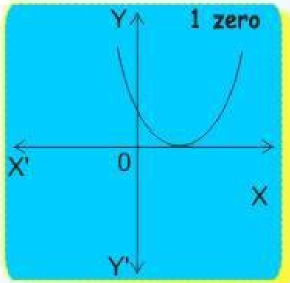
Sum of product of zeros taken two at a time $= \frac{\text{coefficient of } x}{\text{coefficient of } x^3} = \frac{c}{a}$

Product of zeroes $= \frac{-\text{constant term}}{\text{coefficient of } x^2} = -\frac{d}{a}$

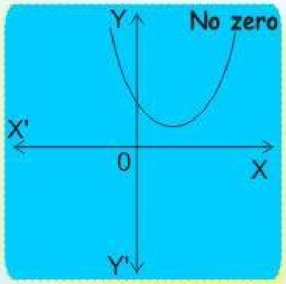
If zeros of cubic polynomial is α, β and γ then polynomial is $f(x) = k\{x^3 - (\alpha + \beta + \gamma)x + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$ where k is any real number.



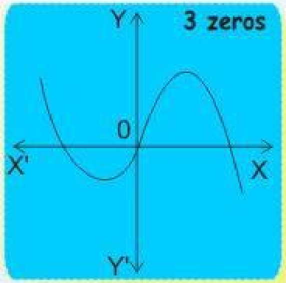
It cuts x axis twice.



It touches x axis.



Doesn't cuts x axis.



It cuts x axis 3 times.

Value of polynomial

The value of a polynomial f(x) at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$.

e.g. If $f(x) = 2x^3 - 13x^2 + 17x + 12$ then its value at x = 1 is

$f(1) = 2(1)^3 - 13(1)^2 + 17(1) + 12$

$= 2 - 13 + 17 + 12 = 18.$

Factor theorem

If p(x) is a polynomial and 'a' be a real number, such that p(a) = 0, then (x-a) is a factor of p(x).

e.g. Factors of $f(x) = x^2 - 3x + 2$ is $(x-2)(x-1)$

$\because f(1) = 1^2 - 3(1) + 2 = 1 - 3 + 2 = 0$

$\& f(2) = 2^2 - 3(2) + 2 = 4 - 6 + 2 = 0$

Division Algorithm

Dividend = Divisior × Quotient + Remainder

OR

$f(x) = g(x) \times q(x) + r(x)$

Degree of q(x) = deg. f(x) – deg. g(x)

Degree of r(x) < deg. g(x)

Remainder theorem

If f(x) is a polynomial and 'a' be a real number, then if f(x) is divided by (x–a), then the remainder is equal to f(a).

e.g Find the remainder when $f(x) = x^3 + 6x^2 - 3x + 5$ is divided by $g(x) = x + 2$.

Sol. $x + 2 = 0$

$\Rightarrow x = -2$

Remainder = f(-2)

$= (-2)^3 + 6(-2)^2 - 3(-2) + 5$

$= -8 + 24 + 6 + 5$

$= 27$