PRACTICE PAPER 13

MATHEMATICS

Time allowed: 45 minutes | **Maximum Marks: 200**

General Instructions: *As given in Practice Paper* – 1.

Section–A

Choose the correct option:

1. **If P and Q are symmetric matrix of same order then $PQ - QP$ is a**
   (*a*) Zero matrix
   (*b*) Identity matrix
   (*c*) Skew Symmetric matrix
   (*d*) Symmetric matrix

2. **Let A be 5×5 matrix such that det $A = -3$, then det $(-3A^{-1}) + 3$ det (A) equals**
   (*a*) –72 (*b*) 90 (*c*) 72 (*d*) –90

3. **If matrix $A = \begin{bmatrix} 2 & 3 & -1 \\ x+4 & -1 & 2 \\ 3x+1 & 2 & -1 \end{bmatrix}$ is a singular matrix, then the value of x is**
   (*a*) $\frac{-3}{16}$ (*b*) $\frac{3}{16}$ (*c*) $\frac{4}{13}$ (*d*) $\frac{8}{10}$

4. ***Read the following statements.***

   Statement 1 : If $y = x^x$ then $\frac{d^2y}{dx^2} = x^x\left\{(1+\log x)^2 + \frac{1}{x}\right\}$

   Statement 2 : If $x = at^2$, $y = at$, then $\frac{d^2y}{dx^2} = -\frac{1}{2at^3}$

   Choose the correct option:
   (*a*) Statement I is correct but statement II is not correct.
   (*b*) Statement II is correct but statement I is not correct.
   (*c*) Both statements I and II are correct.
   (*d*) None of these

5. **The tangent to the curve $y = e^{2x}$ at the point (0, 1) meets x-axis at**

(*a*) (0, 1) (*b*) (0, 2) (*c*) $\left(-\frac{1}{2}, 0\right)$ (*d*) (2, 0)

6. $\int \frac{dt}{t+\sqrt{a^2-t^2}}$ **equals**

(*a*) $\frac{1}{2}\sin^{-1}\left(\frac{t}{a}\right)+\log\left(t+\sqrt{a^2-t^2}\right)+C$ (*b*) $\frac{1}{2}\sin^{-1}\left(\frac{t}{a}\right)+\log\left(\sqrt{t+\sqrt{a^2-t^2}}\right)+C$

(*c*) $\frac{1}{2}\sin^{-1}\left(\frac{t}{a}\right)+\log\sqrt{a+\sqrt{a^2-t^2}}+C$ (*d*) none of these

7. $\int \frac{(x-1)e^x}{(x+1)^3}dx$ **equals**

(*a*) $\frac{e^x}{x+1}+C$ (*b*) $e^x\left(\frac{x}{x+1}\right)+C$ (*c*) $\frac{e^x(x-1)}{(x+1)^2}+C$ (*d*) $\frac{e^x}{(x+1)^2}+C$

8. $\int \frac{\log(x/e)}{(\log x)^2}dx$ **is equal to**

(*a*) $\frac{\log x}{x}+C$ (*b*) $\frac{x}{\log x}+C$ (*c*) $\frac{x}{(\log x)^2}+C$ (*d*) none of these

9. $\int_0^{\pi/4}\log(1+\tan\theta)\,d\theta$ **is equal to**

(*a*) $\frac{\pi}{2}\log 2$ (*b*) $-\frac{\pi}{4}\log 2$ (*c*) $\frac{\pi}{8}\log 2$ (*d*) 0

10. Using integration, the area of the region bounded by the line $2y = 5x + 7$, x-axis and the lines $x = 2$ and $x = 8$ is

(*a*) 90 sq. units (*b*) 96 sq. units (*c*) 40 sq. units (*d*) 10 sq. units

11. The Integrating Factor of the differential equation $(1-y^2)\frac{dx}{dy}+yx=ay\,(-1<y<1)$ is

(*a*) $\frac{1}{y^2-1}$ (*b*) $\frac{1}{\sqrt{y^2-1}}$ (*c*) $\frac{1}{1-y^2}$ (*d*) $\frac{1}{\sqrt{1-y^2}}$

12. The solution of differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is

(*a*) $y = \tan^{-1} x$ (*b*) $y - x = k(1 + xy)$ (*c*) $x = \tan^{-1} y$ (*d*) $\tan(xy) = k$

13. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

(*a*) $p = q$ (*b*) $p = 2q$ (*c*) $q = 2p$ (*d*) $q = 3p$

14. Let X be discrete random variable assuming values $x_1, x_2, x_3, ..., x_n$ with probability $p_1, p_2, p_3, ..., p_n$ respectively. Then the variance of X is given by

(*a*) $E(X^2)$ (*b*) $E(X^2) + E(X)$ (*c*) $E(X^2) - [E(X)]^2$ (*d*) $\sqrt{E(X^2)-(E(X))^2}$

15. A box has 100 mangoes of which 10 are rotten. What is the probability that out of a sample of 5 mangos drawn one by one with replacement at most one is rotten?

(*a*) $\left(\frac{9}{10}\right)^5$ (*b*) $\frac{1}{2}\left(\frac{9}{10}\right)^4$ (*c*) $\frac{1}{2}\left(\frac{9}{10}\right)^5$ (*d*) $\left(\frac{9}{10}\right)^5+\frac{1}{2}\left(\frac{9}{10}\right)^4$

Section–B (B1)

16. If a relation R on the set $\{1, 2, 3, 4\}$ is defined by $R = \{(1, 2), (3, 4)\}$. Then R is

(a) reflexive (b) transitive (c) symmetric (d) none of these

17. Let $A = \{x, y, z\}$ and $B = \{a, b\}$ then the number of onto function from A to B is

(a) 0 (b) 3 (c) 6 (d) 8

18. Let * be a binary operation on $\mathbb{R}$ (set of real numbers) such that $a * b = a^b$ then $(2 * 3) * 5$ equal to

(a) 2^3 (b) 3^5 (c) 8^5 (d) None of these

19. If A and B have 4 elements each then the number of one-one onto (bijective) function from A to B is

(a) 0 (b) 24 (c) 4^2 (d) None of these

20. Let $f : R \to R$ be given by $f(x) = \tan x$. Then $f^{-1}(1)$ is

(a) does not exist (b) $\frac{\pi}{4}$ (c) $n\pi + \frac{\pi}{4}; n \in Z$ (d) None of these

21. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

22. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to

(a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

23. $\sin(\tan^{-1}x), |x| < 1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

24. $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to

(a) $0, \frac{1}{2}$ (b) $1, \frac{1}{2}$ (c) 0 (d) $\frac{1}{2}$

25. If $A = \begin{bmatrix} p & q \\ 0 & 1 \end{bmatrix}$, then A^5 is

(a) $\begin{bmatrix} p^5 & q^5 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} p^5 & q\left(\frac{p^5-1}{p-1}\right) \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} p^5 & \left(\frac{p^5-1}{q-1}\right) \\ 0 & 1 \end{bmatrix}$ (d) none of these

26. The product of the matrix $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is a null matrix if $\theta - \phi$ equal to

(a) $(2n+1)\frac{\pi}{2}$ (b) $n\pi$ (c) $2n\pi$ (d) $n\frac{\pi}{2}$

27. Let $\Delta = \begin{vmatrix} 2 & \cos\theta & 2 \\ -\cos\theta & 2 & \cos\theta \\ -2 & -\cos\theta & 2 \end{vmatrix}$, where $0 \le \theta \le 2\pi$, then

(a) $\Delta = 0$ (b) $\Delta \in (0, \infty)$ (c) $\Delta \in [16, 20]$ (d) $\Delta \in [16, \infty)$

28. If A is a square matrix of order 3 such that $|adj\, A| = 36$, then $|A|$ is

(a) ± 6 (b) 6 (c) -6 (d) ± 5

MATHEMATICS

29. $f:[-2a, 2a] \to R$ is an odd function such that the left hand derivative at $x = a$ is zero and $f(x) = f(2a - x)\ \forall\ x \in (a, 2a)$. Then its left hand derivative at $x = -a$ is

(*a*) 0 (*b*) a (*c*) 1 (*d*) does not exist.

30. If $f(x) = |x| + |x - 2|$, then

(*a*) $f(x)$ is continuous at $x = 0$ but not at $x = 2$.
(*b*) $f(x)$ is continuous at $x = 0$ and at $x = 2$.
(*c*) $f(x)$ is continuous at $x = 2$ but not at $x = 0$.
(*d*) None of these.

31. The function $f(x) = \frac{1}{x-1}$ at $x = 1$

(*a*) is continuous
(*b*) has removable discontinuity.
(*c*) has jump discontinuity.
(*d*) has asymptotic discontinuity.

32. The derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x = \frac{\pi}{4}$, where $f'(1) = g'(\sqrt{2}) = 4$ is

(*a*) $\sqrt{2}$ (*b*) $\frac{1}{\sqrt{2}}$ (*c*) 1 (*d*) none of these

33. *Read the following statements.*

Statement I : $\int \frac{dx}{\sqrt{9x - 4x^2}} = \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right) + C$

Statement II : $\int e^x\left(f(x) + f'(x)\right)dx = e^x f(x) + C$

Choose the correct option:

(*a*) Statement I is correct but statement II is not correct.
(*b*) Statement II is correct but statement I is not correct.
(*c*) Both statements I and II are correct.
(*d*) None of these

34. If $f(x) = \frac{1}{4x^2 + 2x + 1}$, then its maximum value is

(*a*) 0
(*b*) $\frac{4}{3}$
(*c*) ± 5
(*d*) Maximum value does not exist.

35. $\int \cos\sqrt{x}\, dx$ equals

(*a*) $-\frac{\sin\sqrt{x}}{2\sqrt{x}} + C$
(*b*) $\sqrt{x}\,.\sin\sqrt{x} + \cos\sqrt{x} + C$
(*c*) $2(\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}) + C$
(*d*) $2(\sqrt{x}\sin\sqrt{x} - \cos\sqrt{x}) + C$

36. *Read the following statements.*

Statement I : If f and g are continuous functions on [0, 1] satisfying $f(x) = f(a - x)$ and $g(x) + g(a - x) = a$, then $\int_0^a f(x)g(x)dx = \frac{a}{2}\int_0^a f(x)dx$

Statement II : $\int_{-\pi/4}^{\pi/4} \sin x\, dx = 0$

Choose the correct option:

(*a*) Statement I is correct but statement II is not correct.
(*b*) Statement II is correct but statement I is not correct.
(*c*) Both statements I and II are correct.
(*d*) None of these

37. **The area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ and $x = a$ is**

(*a*) $\frac{\pi a^2}{4}$ sq. units (*b*) $\frac{a^2}{4}$ sq. units (*c*) πa^2 sq. units (*d*) 4π sq. units

38. **Which of the following is a homogeneous differential equation?**

(*a*) $(4x + 6y + 5)\, dy - (3y + 2x + 4)\, dx = 0$ (*b*) $(xy)\, dx - (x^3 + y^3)\, dy = 0$

(*c*) $(x^3 + 2y^3)\, dx + 2xy\, dy = 0$ (*d*) $y^2\, dx + (x^2 - xy - y^2)\, dy = 0$

39. **The degree and order of the differential equation of the family of all parabola whose axis is x-axis, are respectively**

(*a*) 1, 2 (*b*) 2, 3 (*c*) 3, 2 (*d*) 2, 1

40. **If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then value of $\vec{a} \cdot \vec{b}$ is**

(*a*) $6\sqrt{3}$ (*b*) $8\sqrt{3}$ (*c*) $12\sqrt{3}$ (*d*) None of these

41. **The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of ΔABC. The length of the median through A is**

(*a*) $\frac{\sqrt{34}}{2}$ (*b*) $\frac{\sqrt{48}}{2}$ (*c*) $\sqrt{18}$ (*d*) None of these

42. **The projection of vector $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ along $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is**

(*a*) $\frac{2}{3}$ (*b*) $\frac{1}{3}$ (*c*) 2 (*d*) $\sqrt{2}$

43. **If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 1$ then the angle between $\vec{a}$ and $\vec{b}$ is**

(*a*) $\cos^{-1}\left(\frac{1}{6}\right)$ (*b*) $\cos^{-1}\left(\frac{1}{2}\right)$ (*c*) $\cos^{-1}\left(\frac{1}{3}\right)$ (*d*) None of these

44. **The DRs of normal to the plane passing through (1, 0, 0), (0, 1, 0) which makes an angle $\frac{\pi}{4}$ with plane $x + y = 3$ are**

(*a*) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (*b*) $-1, 1, \sqrt{2}$ (*c*) $-1, -1, \sqrt{2}$ (*d*) $1, 1, \sqrt{2}$

45. **A line makes angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals**

(*a*) $\frac{3}{5}$ (*b*) $\frac{1}{5}$ (*c*) 0 (*d*) $\frac{2}{3}$

46. **$A(3, 2, 0), B(5, 3, 2), C(-9, 6, -3)$ are three points forming a triangle. If AD, the bisector of $\angle BAC$ meets BC in D, then coordinates of D are**

(*a*) $\left(\frac{1}{13}, \frac{2}{13}, \frac{20}{3}\right)$ (*b*) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$ (*c*) $\left(\frac{11}{3}, \frac{12}{5}, \frac{21}{5}\right)$ (*d*) $\left(\frac{17}{3}, \frac{2}{13}, \frac{2}{3}\right)$

47. **If a line makes angle with positive direction of x-axis and y-axis 120° and 60° respectively, then the angle made by the line with z-axis is**

(*a*) 60° (*b*) 45° (*c*) 135° (*d*) (*b*) and (*c*)

48. **Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is**

(*a*) $\frac{1}{18}$ (*b*) $\frac{5}{18}$ (*c*) $\frac{1}{5}$ (*d*) $\frac{2}{5}$

49. **In a college, 30% students play cricket, 25% play volleyball and 10% play both. One student is chosen at random. The probability that he plays cricket if he is player of volleyball is**

(*a*) $\frac{1}{10}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{9}{20}$ (*d*) $\frac{1}{3}$

50. For two events A and B, if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then

(*a*) A and B are dependent events

(*b*) A and B are independent events

(*c*) $P(A) = P(B)$

(*d*) none of these