EXPONENTIAL & LOGARITHMIC SERIES

- $\underset{x\to\infty}{Lt}\left(1+\frac{1}{x}\right)^x = e$
- $e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+.....=\sum_{r=0}^{\infty}\frac{x^r}{r!} \quad (x\in R)$

  $e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+=\sum_{r=0}^{\infty}\frac{1}{r!}\ ;\ 2<e<3$
- $e^{-x} = 1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+....=\sum_{r=0}^{\infty}(-1)^r\frac{x^r}{r!}$

  $e^{-1} = 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+....=\sum_{r=0}^{\infty}(-1)^r\frac{1}{r!}$
- $e^x+e^{-x} = 2\left[1+\frac{x^2}{2!}+\frac{x^4}{4!}+....\right]$

  $e^1+e^{-1} = 2\left[1+\frac{1}{2!}+\frac{1}{4!}+....\right]$
- $e^x-e^{-x} = 2\left[\frac{x}{1!}+\frac{x^3}{3!}+\frac{x^5}{5!}+....\right]$

  $e^1-e^{-1} = 2\left[\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+....\right]$
- $a^x = e^{\log a^x} = e^{x\log a}$

  $\therefore a^x = 1+\frac{x\log_e a}{1!}+\frac{x^2}{2!}(\log_e a)^2+\frac{x^3}{3!}(\log_e a)^3+.....(a>0)$
- In the expansion of e^x general term $T_{r+1} = \frac{x^r}{r!}$
- In the expansion of e^{-x} the general term $T_{r+1} = (-1)^r\frac{x^r}{r!}$
- The co-efficient of x^r in the expansion of a^x is $\frac{(\log_e a)^r}{r!}$

Logarithmic Series

- $\log_e(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+......\infty, -1<x\le 1$
- $\log_e 2 = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.......\infty$
- $\log_e(1-x) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}+......\infty,\ -1\le x<1$
- $\log_e(1-x^2) = \log_e(1+x)+\log_e(1-x)$

  $= -2\left[\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+......\right], -1<x<1$
- $\log_e\left(\frac{1+x}{1-x}\right) = \log_e(1+x)-\log_e(1-x)$

  $= 2\left[x+\frac{x^3}{3}+\frac{x^5}{5}+.....\right], -1<x<1$
- The general term in the expansion of log (1+x), $T_{n+1} = (-1)^n\frac{x^{n+1}}{n+1}$
- If x > 1 then

  $\log_e\left(\frac{x+1}{x-1}\right) = 2\left[\frac{1}{x}+\frac{1}{3x^3}+\frac{1}{5x^5}+.....\right]$
- If $m,n\in N$ and $m>n$ then

  $\log_e m/n = 2\left[\frac{m-n}{m+n}+\frac{1}{3}\left(\frac{m-n}{m+n}\right)^3+\frac{1}{5}\left(\frac{m-n}{m+n}\right)^5+.....\right]$
- $1+\frac{1+a}{2!}+\frac{1+a+a^2}{3!}+\frac{1+a+a^2+a^3}{4!}+$

  $......\infty = \frac{e-e^a}{1-a}$
- $\sum_1^{\infty}\frac{n}{n!} = e\ ;\ \sum_1^{\infty}\frac{n^2}{n!} = 2e\ ;\ \sum_1^{\infty}\frac{n^3}{n!} = 5e$
- The coefficient of x^n in the expansion of $\frac{(1+ax+x^2)}{e^x}$ is $\frac{(-1)^n}{(n-2)!}\left[\frac{1}{n(n-1)}-\frac{a}{n-1}+1\right]$
- Coefficient of x^n in $e^{ax} = \frac{a^n}{n!}$
- The coefficient of x^n in

  $\log_e\left(1+3x+2x^2\right)$ is $\frac{(-1)^{n-1}}{n}(1+2^n)$
- The co-efficient of x^{-r} in the expansion of $\log_{10}\left(\frac{x}{x-1}\right)$ is $\frac{1}{r}\log_{10} e$

EXPONENTIAL SERIES - CONCEPTUAL QUESTIONS

1. Sum of terms in the expansion of e^x

1. $\sum_{r=0}^{\infty}\frac{x^r}{r}$ 2. $\sum_{r=0}^{\infty}\frac{x^r}{r!}$

3. $\sum_{r=0}^{\infty}\frac{x^{r+1}}{r+1}$ 4. $\sum_{r=0}^{\infty}\frac{x^{r+1}}{(r+1)!}$

2. $1+\frac{a^2}{2!}+\frac{a^4}{4!}+\frac{a^6}{6!}+...\infty =$

1. e^a+e^{-a} 2. e^a-e^{-a}

3. $\frac{e^a+e^{-a}}{2}$ 4. $\frac{e^a-e^{-a}}{2}$

3. T_r+1 term in the expansion of e^{-x} is

1. $\frac{x^r}{r!}$ 2. $(-1)^r\frac{x^r}{r}$

3. $(-1)^r\frac{x^r}{r!}$ 4. $(-1)^r\frac{x^{r+1}}{(r+1)!}$

4. $2\left[\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+...\right]=$

1. e^1+e^{-1} 2. e^1-e^{-1}

3. $e^{-1}-e$ 4. $\frac{e^1-e^{-1}}{2}$

5. $1+\frac{x\log_e 2}{1!}+\frac{x^2}{2!}(\log_e 2)^2+\frac{x^3}{3!}(\log_e 3)^3+...\infty$, $(a>0)=$

1. e^2 2. a^2 3. 2^a 4. 2^x

KEY

1. 2 2. 3 3. 3 4. 2

EXPONENTIAL SERIES LEVEL-1

TYPE-1 (FORMULA APPLICATION)

1. $4+\frac{9}{2!}+\frac{27}{3!}+......\infty =$

1) e^2 2) e^3 3) e^4 4) $\frac{1}{e}$

2. $\left[1+\frac{4}{1!}+\frac{16}{2!}+\frac{64}{3!}+.....\right]\left[1-\frac{3}{1!}+\frac{9}{2!}-\frac{27}{3!}+.....\right]=$

1) e^3 2) e^2 3) e 4) 1

3. $\left(1+\frac{a^2x^2}{2!}+\frac{a^4x^4}{4!}+....\right)^2-\left(ax+\frac{a^3x^3}{3!}+\frac{a^5x^5}{5!}+....\right)^2$

1) e^a 2) e^{-a} 3) 2 4) 1

4. $1+(\log_e 2)\frac{x}{1!}+(\log_e 2)^2\frac{x^2}{2!}+(\log_e 2)^3\frac{x^3}{3!}+.....=$

1) 2 2) 1/2 3) 2^{-x} 4) 2^x

5 $x+\frac{x^3}{3!}+\frac{x^5}{5!}+.....=$

1) Sinhx 2) Coshx 3) Tanhx 4) Cothx

6. $1+\frac{2^2}{2!}+\frac{2^4}{4!}+\frac{2^6}{6!}+......=$

1) e^2 2) e^{-2}

3) e^2+e^{-2} 4) $\frac{1}{2}(e^2+e^{-2})$

7. $\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}=$

1) $\frac{1}{2}(e+e^{-1})$ 2) $\frac{1}{2}(e-e^{-1})$

3) $e+e^{-1}$ 4) $e-e^{-1}$

8. $2\sum_{n=0}^{\infty}\frac{1}{(2n)!}=$

1) $\frac{1}{2}(e-e^{-1})$ 2) $\frac{1}{2}(e+e^{-1})$

3) $\frac{e^2+1}{e}$ 4) e^2

9. $\left(1+\frac{1}{2!}+\frac{1}{4!}+....\right)\div\left(1+\frac{1}{3!}+\frac{1}{5!}+.....\right)=$

1) $\frac{e-1}{e+1}$ 2) $\frac{e+1}{e-1}$ 3) $\frac{e^2-1}{e^2+1}$ 4) $\frac{e^2+1}{e^2-1}$

10. $\left(\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+....\right)\div\left(1+\frac{1}{3!}+\frac{1}{5!}+....\right)=$

1) $\frac{e-1}{e+1}$ 2) $\frac{e+1}{e-1}$ 3) $\frac{e^2-1}{e^2+1}$ 4) $\frac{e^2+1}{e^2-1}$

11. $\sum_{n=1}^{\infty}\frac{(\log_e x)^n}{n!}=$

1) $\log_e x$ 2) (x-1) 3) $\log_x e$ 4) $\log_e(x-1)$

12. $\sum_{n=0}^{\infty}\frac{2^n}{(n-1)!}e^{-2}=$

1) 2 2) 3 3) e^2 4) e^3

13. $\sum_{n=0}^{\infty}\frac{1}{(n+1)!}=$

1) e - 3 2) e - 2 3) e - 1 4) e

TYPE-2 - (GENERAL TERMS AND COEFFICIENTS)

14. The 10^{th} term in the expansion of e^x is

1) $\frac{x^{10}}{10!}$ 2) $\frac{x^9}{9!}$ 3) $\frac{x^9}{10!}$ 4) x^9

15. The 10th term in the expansion of 5^x is

1) $\frac{x^{10}}{10!}$ 2) $\frac{x^9}{9!}$

3) $\frac{x^9}{9!}(\log_e 5)^9$ 4) $\frac{x^{10}}{10!}(\log_e 5)^{10}$

16. The 4th term in the expansion of $\frac{e^{4x}-1}{2e^{2x}}$ is

1) $\frac{(2x)^7}{7!}$ 2) $\frac{(2x)^4}{4!}$ 3) $\frac{(2x)^5}{5!}$ 4) $\frac{(2x)^6}{6!}$

17. The (n+1)th term in the expansion of e^{-4} is

1) $(-1)^n\frac{4^n}{n!}$ 2) $\frac{4^n}{n!}$ 3) $(-1)^{n-1}\frac{4^n}{n!}$ 4) 4^n

18. General term in expansion of e^4 is

1) $\frac{3^n}{n!}$ 2) $\frac{4^n}{n!}$ 3) $\frac{n^4}{4!}$ 4) 3^n

19. The coefficient of x^2 in the expansion of $\frac{1-2x+3x^2}{e^x}$ is

1) $\frac{7}{2}$ 2) $\frac{9}{2}$ 3) $\frac{11}{2}$ 4) $\frac{13}{2}$

20. The coefficient of x^2 in the expansion of $\frac{1+10x+x^2}{e^x}$ is

1) -10 2) $-\frac{17}{2}$ 3) $-\frac{15}{2}$ 4) -8

21. The coefficient of x^8 in the expansion of e^{3x} is

1) $\frac{3^7}{7!}$ 2) $\frac{3^8}{8!}$ 3) $\frac{3^9}{9!}$ 4) 3^8

22. The coefficient of x^3 in the expansion of e^{-bx}

1) $\frac{-b^3}{6}$ 2) $\frac{b^3}{6}$ 3) $\frac{b^3}{3}$ 4) $\frac{-b^3}{3}$

23. The coefficient of x^2 in the expansion of e^{2x+3}

1) e^2 2) e^3 3) $2e^3$ 4) $3e^2$

24. The coefficient of x^6 in the expanssion of 6^x is

1) $\log_e 6$ 2) 6

3) $\frac{1}{6!}(\log_e 6)^6$ 4) $(\log_e 6)^6$

25. The coefficient of x^n in the series $1+\frac{(a+bx)}{1!}+\frac{(a+bx)^2}{2!}+\frac{(a+bx)^3}{3!}+.........is$

1) $\frac{b^n}{n!}$ 2) $e^a\frac{b^n}{n!}$ 3) $e^b\frac{a^n}{n!}$ 4) $\frac{e^{ab}}{n!}$

26. The coefficient of x^r in the expansion of $\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+.....\right)^2$ when 'r' is odd is

1) $\frac{2^{2r}}{(2r)!}$ 2) $\frac{r!}{2^r}$ 3) $\frac{2^r}{r!}$ 4) 0

27. Coefficient of x^n in e^{-2x^2} when n is odd

1) $(-1)^{n-1}\frac{2^n}{n!}$ 2) $(-1)^n\frac{2^n}{n!}$

3) $-\frac{2^n}{n!}$ 4) 0

28. The coefficient of x^n in the expansion of $\frac{e^{5x}+e^x}{e^{2x}}$ is

1) $\frac{3^n}{n!}$ 2) $\frac{(-1)^n}{n!}$

3) $\frac{3^n}{n!}+\frac{1}{n!}$ 4) $\frac{3^n}{n!}+\frac{(-1)^n}{n!}$

TYPE-3(SERIES)

29. $\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+....\infty=$

1) e 2) $\frac{e}{2}$ 3) 2e 4) e^2

30. $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+\frac{8}{9!}+....\infty=$

1) e^2 2) e 3) e-1 4) 1/e

31. $2-\frac{3^2}{2!}+\frac{3^3}{3!}-\frac{3^4}{4!}+......=$

1) e^{-2} 2) $-e^{-2}$ 3) e^{-3} 4) $-e^{-3}$

32. $\frac{2}{2!}+\frac{2+4}{3!}+\frac{2+4+6}{4!}+.....=$

1) e - 2 2) e - 1 3) e 4) 1/e

33. $x+\frac{x^2}{3!}+\frac{x^3}{5!}+....\infty=$

1) $2\sqrt{x}\left(e^{\sqrt{x}}-e^{-\sqrt{x}}\right)$ 2) $\frac{\sqrt{x}}{2}\left(e^{\sqrt{x}}-e^{-\sqrt{x}}\right)$

3) $\frac{e^{\sqrt{x}}-e^{-\sqrt{x}}}{2}$ 4) 0

34. $\frac{1}{2!}+\frac{x}{3!}+\frac{x^2}{4!}+......\infty=$

1) $\frac{e^x}{x^2}$ 2) $\frac{e^x-1}{x^2}$

3) $\frac{e^x-1+x}{x^2}$ 4) $\frac{e^x-1-x}{x^2}$

35. $1+\frac{1+5}{2!}+\frac{1+5+5^2}{3!}+\frac{1+5+5^2+5^3}{4!}+...\infty=$

1) $\frac{1}{4}(e^5-e)$ 2) e^5-e

3) $4(e^5-e)$ 4) 4 e^5

36. $\frac{1}{1!}+\frac{1+3}{2!}+\frac{1+3+3^2}{3!}+.....\infty=$

1) $\frac{1}{2}\left(e^2-1\right)$ 2) $\frac{1}{2}\left(e^3-1\right)$

3) $\frac{1}{2}\left(e^2-e\right)$ 4) $\frac{1}{2}\left(e^3-e\right)$

37. $1+\frac{4}{2!}+\frac{13}{3!}+\frac{40}{4!}+.....=$

1) $\frac{1}{2}\left(e-e^3\right)$ 2) e - e^3

3) e^3-e 4) $\frac{1}{2}(e^3-e)$

38. 1+ $\frac{1^2+2^2}{\angle 2}+\frac{1^2+2^2+3^2}{\angle 3}+...\infty=$

1) 11e 2) 12e 3) $\frac{17e}{6}$ 4) $\frac{5e}{6}$

39. The sum of series $\frac{2}{1!}+\frac{6}{2!}+\frac{12}{3!}+\frac{20}{4!}+.....$ *is*

1) $\frac{3e}{2}$ 2) e 3) 2e 4) 3e

40. $1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+.....\infty=$

1) $\frac{e}{2}$ 2) e 3) $\frac{3}{2}e$ 4) 2e

41. $\sum_{1}^{\infty}\frac{(2n)}{n!}=$

1) e 2) 2e 3) 3e 4) e^{-1}

42. $1+\frac{3}{2!}+\frac{5}{3!}+\frac{7}{4!}+.....\infty=$

1) e 2) e -1 3) e+1 4) e+2

43. $\sum_{0}^{\infty}\frac{(\log_e x)^{2n}}{(2n)!}$

1) x +1 2) x^2 +1 3) x +1/x 4) $\frac{x^2+1}{2x}$

44. $1+\frac{2^2}{2!}+\frac{3^2}{3!}+\frac{4^2}{4!}+\frac{5^2}{5!}+......\infty=$

1) 2e 2) e^2 3) e^3 4) $\frac{e^2}{2}$

45. $1+\frac{2}{3!}+\frac{3}{5!}+\frac{4}{7!}+......=$

1) $\frac{e}{2}$ 2) $\frac{e}{3}$ 3) $\frac{1}{2e}$ 4) $\frac{1}{3e}$

46. $1+\frac{3}{2!}+\frac{7}{3!}+\frac{15}{4!}+.......=$

1) e^2-1 2) $e-e^2$ 3) e^2-e 4) $1-e^2$

47. $\frac{1}{2}-\frac{1}{3(1!)}+\frac{1}{4(2!)}-\frac{1}{5(3!)}+.....\infty=$

1) $1-\frac{1}{e}$ 2) $1-\frac{2}{e}$ 3) $1-\frac{3}{e}$ 4) $1-\frac{4}{e}$

48. $\frac{1^3}{1!}+\frac{2^3}{2!}+\frac{3^3}{3!}+\frac{4^3}{4!}+.....\infty=$

1) 4e 2) 5e 3) 6e 4) 7e

KEY

1.2	2.3	3.4	4.4	5.1
6.4	7.2	8.3	9.4	10.1
11.2	12.1	13.3	14.2	15.3
16.1	17.1	18.2	19.3	20.2
21.2	22.1	23.3	24.3	25.2
26.4	27.4	28.4	29.2	30.4
31.4	32.3	33.2	34.4	35.1
36.4	37.4	38.3	39.4	40.3
41.2	42.3	43.4	44.1	45.1
46.3	47.2	48.2		

HINTS

38. Given value = $\sum_{n=1}^{\infty}\frac{\sum n^2}{n!}$

LEVEL-II

49. If a = $\sum_{n=1}^{\infty}\frac{2n}{(2n-1)!}, b=\sum_{n=1}^{\infty}\frac{2n}{(2n+1)!}$ then ab is equal to

1) 1 2) e^2 3) $\frac{e-1}{e+1}$ 4) $\frac{e+1}{e-1}$

50. If x $\neq 0,$ then the sum of the series

$1+\frac{x}{2!}+\frac{2x^2}{3!}+\frac{3x^3}{4!}+...\infty is$

1) $\frac{e^2+1}{x}$ 2) $\frac{e^x(x-1)+(1+x)}{x}$

3) $\frac{e^x(x+1)}{x}$ 4) $\frac{e^x-1}{x}$

51. $(1+3)\log_e 3+\frac{(1+3^2)}{\angle 2}(\log_e 3)^2+.....\infty=$

1) 27 2) 28 3) 32 4) 33

52. The value of

$(x+y)(x-y)+\frac{1}{2!}(x+y)(x-y)(x^2+y^2)+$

$\frac{1}{3!}(x+y)(x-y)(x^4+y^4+x^2y^2)+...is$

1) $e^{x^2}-e^{y^2}$ 2) $e^{x^2}+e^{y^2}$

3) $e^{x^2-y^2}$ 4) $e^{x^2+y^2}$

53. If $\frac{e^x}{1-x}=B_0+B_1x+B_2x^2+...+B_nx^n ...then$

B_n-B_{n-1} is equal to

1) $\frac{1}{n!}$ 2) $\frac{1}{(n-1)!}$

3) $\frac{1}{n!}-\frac{1}{(n-1)!}$ 4) $\frac{1}{(n+2)!}$

54. $S_n=\frac{1^2.2}{1!}+\frac{2^2.3}{2!}+\frac{3^2.4}{3!}+....+\frac{n^2(n+1)}{n!}$ then $\underset{n\to\infty}{Lt}\, S_n$ is equal to

1) 3e 2) 5 e 3) 7e 4) 9e

55. $\sum_{1}^{\alpha}\frac{{}^nc_o+{}^nc_1+{}^nc_2+....{}^nc_n}{{}^nP_n}=$

1) e - 1 2) e^2 3) e^2-1 4) e^2 + 1

56. $(1+x+x^2)e^{-x}=a_0+a_1x+a_2x^2+...$

1) a_1 = -1 2) a_2 = 1/2 3) a_3 = 1/3 4) a_4 = -1/2

57. If $s=\sum_{n=2}^{\infty}\frac{\binom{n}{2}3^{n-2}}{n!}$ then 2S is equal to

1) $e^{1/2}$ 2) e^3 3) $e^{3/2}$ 4) $e^{\frac{1}{3}}$

58. The sum of the series

$\frac{4}{\angle 1}+\frac{11}{\angle 2}+\frac{22}{\angle 3}+\frac{37}{\angle 4}+\frac{56}{\angle 5}+........$

1) 6e 2) 6e-1 3) 5e 4) 5e-1

59. The sum of the series

$\frac{1^2.2^2}{\angle 1}+\frac{2^2.3^2}{\angle 2}+\frac{3^2.4^2}{\angle 3}+.....$ is

1) 27e 2) 24e 3) 28e 4) e

60. $\dfrac{1+\frac{2^2}{2!}+\frac{2^4}{3!}+\frac{2^6}{4!}+......}{1+\frac{1}{2!}+\frac{2}{3!}+\frac{2^2}{4!}+........}=$

1) (e+1)(e+1) 2) $\frac{e-1}{e+1}$

3) e^2-1 4) e^2+1

61. $\frac{2}{2!}+\frac{2+4}{3!}+\frac{2+4+6}{4!}+.........=$

1) e - 2 2) e - 1 3) e 4) e^{-1}

62. If $x=1+\frac{1}{2!}+\frac{1}{4!}+....\infty$ and

$y=1+\frac{1}{3!}+\frac{1}{5!}+....\infty$ then $x^2 - y^2$=

1) 0 2) 1 3) e 4) e^2

KEY

49.1	50.2	51.2	52.1	53.1
54.3	55.3	56.2	57.2	58.2
59.1	60.3	61.3	62.2	

LEVEL - V

I. $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+...\infty$,

$e^{-x}=1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+...\infty$

$\sinh x=\frac{e^x-e^{-x}}{2}$ and $\cosh x=\frac{e^x+e^{-x}}{2}$

1. $\left(e^x+e^{-x}\right)^2-\left(e^x-e^{-x}\right)^2=$

1. 1 2. 2 3. 4 4. 3

2. $\left(\frac{e+e^{-1}}{2}\right)^2-\left(\frac{e-e^{-1}}{2}\right)^2=$

1. 1 2. 2 3. 3 4. 4

3. $Tanh(e)=$

1. $\frac{e-e^{-1}}{e+e^{-1}}$ 2. $\frac{e+e^{-1}}{e-e^{-1}}$ 3. $\frac{e^{-1}-e}{e+e^{-1}}$ 4. $\frac{e+e^{-1}}{e^{-1}-e}$

KEY

1. 3 2. 1 3. 1

HINTS

49. $a=\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}+\frac{1}{(2n-1)!}=e$

$b=\sum_{n=1}^{\infty}\frac{1}{2n!}-\frac{1}{(2n+1)!}=e^{-1}$

50. Given value

$=1+\sum_{n=1}^{\infty}\frac{n.x^n}{(n+1)!}=1+\sum_{n=1}^{\infty}\frac{x^n}{n!}-\frac{x^n}{(n+1)!}$

$=1+\left(e^x-1\right)-\frac{1}{x}\left(e^x-1-x\right)$

51. Given value

$=\left\{\log_e 3+\frac{(\log_e 3)^2}{2!}+\frac{(\log_e 3)^3}{3!}+....\right\}+$

$\left\{3.\log_e 3+\frac{(3.\log_e 3)^2}{2!}+........\right\}$

52. Given value =

$\left(x^2-y^2\right)+\frac{\left(x^4-y^4\right)}{2!}+\frac{x^6-y^6}{3!}+.....$

$=\left\{\frac{x^2}{1!}+\frac{(x^2)^2}{2!}+....\right\}-\left\{\frac{y^2}{1!}+\frac{(y^2)^2}{2!}+..\right\}$

53. $e^x(1-x)^{-1}=B_0+B_1x+B_2x^2+.....$

$B_n=1+\frac{1}{1!}+\frac{1}{2!}+......+\frac{1}{n!}$

$B_{n-1}=1+\frac{1}{1!}+\frac{1}{2!}+......+\frac{1}{(n-1)!}$

54. $\underset{n-->\infty}{Lt}S_n=\sum_{n=1}^{\infty}S_n=\sum_{n=1}^{\infty}\frac{n^2(n+1)}{n!}$

$=\sum_{n=1}^{\infty}\frac{n^3}{n!}+\frac{n^2}{n!}=5e+2e$

55. $\sum_{1}^{n}\frac{2^n}{n\ !}$

57. $S=\sum_{n=2}^{\infty}\frac{n!}{(n-2)!2!}.\frac{3^{n-2}}{n!}=\frac{e^3}{2}$

58. $T_n=2n^2+n+1$

$S_n=\sum_{n=1}^{\infty}\frac{T_n}{n!}=\sum_{n=1}^{\infty}\frac{2n^2}{n!}+\frac{n}{n!}+\frac{1}{n!}$

$=2(2e)+e+(e-1)$

59. Given value = $\sum_{n=1}^{\infty}\frac{n^2(n+1)^2}{n!}$

$\sum\frac{n^4}{n!}+\frac{2n^3}{n!}+\frac{n^2}{n!}=15e+2(5e)+2e$

60. Multiplying Nr and Dr by 2^2

61. $T_n=\frac{2+4+6+.....+2n}{(n+1)!}=\frac{n(n+1)}{(n+1)!}=\frac{1}{(n-1)!}$

$S_n=\sum_{n=1}^{\infty}\frac{1}{(n-1)!}=e$

62. $x^2-y^2=\left(\frac{e+e^{-1}}{2}\right)^2-\left(\frac{e-e^{-1}}{2}\right)^2$

LOGARTHIMIC SERIES - CONCEPTUAL QUESTIONS

1. T_n term in the expansion of $\log(1+x)=$

1. $\frac{x^{n+1}}{n+1}$ 2. $\frac{x^n}{n}$

3. $\frac{(-1)^n x^n}{n}$ 4. $\frac{(-1)^n x^{n+1}}{n+1}$

2. $\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+...\infty=$

1. $\frac{1}{2}\log\left(1-x^2\right)$ 2. $\frac{1}{2}\log\left(\frac{1}{1-x^2}\right)$

3. $2\log\left(\frac{1}{1-x^2}\right)$ 4. $2\log\left(1-x^2\right)$

3. $\frac{1}{2}+\frac{1}{12}+\frac{1}{30}+...\infty=$

1. $\log\left(\frac{1}{2}\right)$ 2. $\log 3$

3. $\log 2$ 4. $\log\left(\frac{1}{3}\right)$

4. $a+\frac{a^3}{3}+\frac{a^5}{5}+....\infty$ =

1. $\frac{1}{2}\log\left(\frac{1+a}{1-a}\right)$ 2. $\log\left(\frac{1+a}{1-a}\right)$ 3 $\frac{1}{2}\log\left(\frac{1-a}{1+a}\right)$ 4. $\log\left(\frac{1-a}{1+a}\right)$

5. $2\left[\frac{1}{9}+\frac{1}{3}\left(\frac{1}{9}\right)^3+\frac{1}{5}\left(\frac{1}{9}\right)^5+...\infty\right]=$

1. $\log\left(\frac{5}{4}\right)$ 2. $\log\left(\frac{3}{2}\right)$ 3. $\log\left(\frac{6}{5}\right)$ 4. $\log\left(\frac{7}{6}\right)$

LOGARTHIMIC SERIES

(TYPE-1) LEVEL-1 GENERAL TERMS AND COEFFICIENTS

1. The nth term of $\log_e\left(\frac{6}{5}\right)$ is

1) $\frac{1}{5^n}$ 2) $\frac{(-1)^n}{n.5^n}$ 3) $\frac{(-1)^{n-1}}{n.5^n}$ 4) $\frac{(-1)n^{-1}}{n.4^n}$

2. The 4th term of $\log_e\frac{3}{2}$ is

1) $\frac{1}{16}$ 2) $-\frac{1}{16}$ 3) $-\frac{1}{64}$ 4) $\frac{1}{64}$

3. The third term in the expansion of $\log\left(\frac{1}{1-x}\right)$ is

1) $-\frac{x^3}{3}$ 2) $\frac{x^3}{3}$ 3) $\frac{x^2}{2}$ 4) $-\frac{x^2}{2}$

4. General term in the expansion of $\log_e\frac{5}{2}$ is

1) $\frac{3^n}{2^n}$ 2) $\frac{3^n}{2^n n}$ 3) $\frac{2^n}{3^n n!}$ 4) $\frac{2^n}{3^n}$

5. The coefficient of x^n in $\log_e\left(1+\frac{x}{2}\right)$ is

1) $\frac{1}{n.2^n}$ 2) $\frac{(-1)^{n-1}}{n.2^n}$ 3) $\frac{(-1)^n}{n.2^n}$ 4) $\frac{(-1)^n}{2^n}$

6. The coefficient of x^r in $\log_e(6x+1)$ is

1) $\frac{(-1)^{r-1}}{r}$ 2) $\frac{(-1)^r}{r}$ 3) $\frac{(-1)^r 6^r}{r}$ 4) $\frac{(-1)^{r-1}6^r}{r}$

7. The coefficient of x^6 in log $(1+3x+2x^2)$ is

1) $\frac{-65}{6}$ 2) $\frac{65}{6}$ 3) $\frac{31}{6}$ 4) $\frac{-31}{6}$

8. The coefficient of x^3 in the expansion of log $(1+x+x^2)$ is

1) $-\frac{2}{3}$ 2) $\frac{2}{3}$ 3) $\frac{1}{3}$ 4) $-\frac{1}{3}$

9. The coefficient of x^n in the expansion of $\log(1+x+x^2)$ when n is not a multiple of 3 is

1) $-\frac{2}{n}$ 2) $\frac{2}{n}$ 3) $\frac{1}{n}$ 4) $-\frac{1}{n}$

10. The coefficient of x^n in the expansion of $\log_e\left(\frac{1}{1+x+x^2+x^3}\right)$, when n is odd is

1) -n 2) $-\frac{1}{n}$ 3) $\frac{1}{n}$ 4) n

11. The coefficient of x^{-3} in the expansion of $\log\left(\frac{x}{x-1}\right)$ is

1) $\frac{1}{3}$ 2) $\frac{1}{3}\log_e 10$ 3) $\frac{1}{3}\log_{10} e$ 4) $\frac{1}{2}$

12. The coefficient of x^6 in the expansion of $\log\left[(1+x)^{1+x}(1-x)^{1-x}\right]$ is

1) $\frac{1}{25}$ 2) $\frac{1}{22}$ 3) $\frac{1}{19}$ 4) $\frac{1}{15}$

TYPE-2
FORMULAE APPLICATION

13. $1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+....\infty=$

1) $\log_e(1+x)$ 2) $-\log_e(1+x)$ 3) $-\frac{\log_e(1+x)}{x}$ 4) $\frac{\log_e(1+x)}{x}$

14. $\frac{x-y}{x}+\frac{1}{2}\left(\frac{x-y}{x}\right)^2+\frac{1}{3}\left(\frac{x-y}{x}\right)^3+....=$

1) $\log_e(x+y)$ 2) $\log_e\frac{x}{y}$ 3) $\log_e(x-y)$ 4) $\log_e\frac{y}{x}$

15. $\frac{1}{3}+\frac{1}{3}\left(\frac{1}{3}\right)^3+\frac{1}{5}\left(\frac{1}{3}\right)^5+....=$

1) $\frac{1}{2}\log_e 2$ 2) $2\log_e 2$

3) $\log_e 2$ 4) $\log_e 3$

16. $\frac{1}{2}+\frac{1}{3.2^3}+\frac{1}{5.2^5}+...=$

1) $\log_e 3$ 2) $\log_e 4$ 3) $\frac{1}{2}\log_e 3$ 4) $\frac{1}{2}\log_e 4$

17. If $\log_e^m = 2\left[\frac{3}{5}+\frac{1}{3}\left(\frac{3}{5}\right)^3+\frac{1}{5}\left(\frac{3}{5}\right)^5+.....\right]$ then m =

1) 2 2) 3 3) 4 4) 5

18. $\cot^2\theta-\frac{1}{2}\cot^4\theta+\frac{1}{3}\cot^6\theta-\frac{1}{4}\cot^8\theta+...=$

1) $2\log(\sec\theta)$ 2) $2\log(\cos ec\theta)$

3) $2\log(\cos\theta)$ 4) $2\log(\sin\theta)$

19. $Cos\theta+\frac{1}{3}\cos^3\theta+\frac{1}{5}\cos^5\theta+...=$

1) $\log(\tan\theta)$ 2) $\log(\cot\theta)$

3) $\log\left(\tan\frac{\theta}{2}\right)$ 4) $\log\left(\cot\frac{\theta}{2}\right)$

20. $\left(\frac{x}{x+1}\right)+\frac{1}{2}\left(\frac{x}{x+1}\right)^2+\frac{1}{3}\left(\frac{x}{x+1}\right)^3=$

1) log (1+x) 2) log (1-x)

3) $\frac{1}{2}\log(1+x)$ 4) $\frac{1}{2}\log(1-x)$

21. $(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\frac{1}{4}(x-1)^4+....=$

1) $\log_e x$ 2) 2 $\log_e x$ 3) 3 $\log_e x$ 4) 4

22. $\left(\frac{2ax}{a^2+x^2}\right)+\frac{1}{3}\left(\frac{2ax}{a^2+x^2}\right)^3+\frac{1}{5}\left(\frac{2ax}{a^2+x^2}\right)^5+....=$

1) log (a+x) 2) log (a-x) 3) $\log\left(\frac{a+x}{a-x}\right)$ 4) e

23. If m= log $\left(1+x+x^2+...\infty\right)$ and

$n=x+\frac{x^2}{2}+\frac{x^3}{3}+...\infty$ then

1) m = 2n 2) 2m = n 3) m =n 4) m = 4n

24. If x, y, z are three consecutive positive integers, then

$\frac{1}{2}\log_e x+\frac{1}{2}\log_e z+\left(\frac{1}{2xz+1}\right)+\frac{1}{3}\left(\frac{1}{2xz+1}\right)^3+...=$

1) $\log_e x$ 2) $\log_e y$ 3) $\log_e z$ 4) $-\log_e x$

25. $\frac{a-b}{a}+\frac{1}{2}\left(\frac{a-b}{a}\right)^2+\frac{1}{3}\left(\frac{a-b}{a}\right)^3+=...$

1) $\log_e\left(\frac{a}{b}\right)$ 2) $\log_e\left(b/a\right)$

3) $\log_e(a+b)$ 4) $\log_e(a-b)$

26. $\frac{1}{2}\left(\frac{1}{2}\right)^2+\frac{1}{4}\left(\frac{1}{2}\right)^4+\frac{1}{6}\left(\frac{1}{2}\right)^6+......=$

1) $\log_e\frac{4}{3}$ 2) $-\frac{1}{2}\log_e\frac{4}{3}$

3) $\log_e\frac{3}{4}$ 4) $-\frac{1}{2}\log_e\frac{3}{4}$

27. $1+\frac{1}{3.2^2}+\frac{1}{5.2^4}+\frac{1}{7.2^6}+........=$

1) $\log_e 6$ 2) $\log_e 5$ 3) $\log_e 4$ 4) $\log_e 3$

28. $\frac{1}{2}-\frac{1}{2.2^2}+\frac{1}{3.2^3}+......=$

1) $\log_e\left(\frac{3}{2}\right)$ 2) $\frac{1}{2}\log_e\left(\frac{3}{2}\right)$

3) $\log_e\left(\frac{2}{3}\right)$ 4) $\frac{1}{2}\log_e\left(\frac{2}{3}\right)$

29. $\frac{x-1}{x+1}+\frac{1}{2}\frac{(x^2-1)}{(x+1)^2}+\frac{1}{3}\frac{(x^3-1)}{(x+1)^3}+.......\infty=$

1) log x 2) 2 log x 3) 3 log x 4) 4 log x

TYPE-3
SERIES-PROBLEMS

30. $\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+....\infty=$

1) $\log_e 2$ 2) $\log_e 3$ 3) $\log_e 4$ 4) $\log_e^{\frac{1}{2}}$

31. $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\frac{1}{4.5}+....=$

1) $\log_e 2$ 2) $\log_e 3$ 3) $\log_e^{\left(\frac{4}{e}\right)}$ 4) $\log_e^{\left(\frac{3}{e}\right)}$

32. $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+......\infty=$

1) $1+\log_e 2$ 2) $1-\log_e 2$

3) $2+\log_e 2$ 4) $2-\log_e 2$

33. $\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+...=$

1) $\frac{1-x}{x}\log(1-x)$ 2) $\frac{x-1}{x}\log(1-x)$

3) $\frac{x-1}{x}\log(1+x)$ 4) $1+\left(\frac{1-x}{x}\right)\log(1-x)$

KEY

1.3	2.3	3.3	4.2	5.1
6.4	7.1	8.1	9.3	10.2
11.3	12.4	13.4	14.2	15.1
16.3	17.3	18.2	19.4	20.1
21.1	22.3	23.3	24.2	25.1
26.4	27.4	28.1	29.1	30.1
31.3	32.2	33.4		

HINTS

9. In $\log\left(1+x+x^2\right)$, coefficient of $x^n = \frac{-2}{n}$, if n = 3 m, $x^n = \frac{1}{n}$, if $n \neq 3m$

24. Given value $= \frac{1}{2}\log(xz) + \frac{1}{2}\log\left[\frac{1+\frac{1}{2xz+1}}{1-\frac{1}{2xz+1}}\right]$

$= \frac{1}{2}\log(xz+1) = \frac{1}{2}\log\left(y^2\right)$

29. $\frac{x}{x+1} - \frac{1}{x+1} + \frac{1}{2}\frac{x^2}{(x+1)^2} - \frac{1}{2(x+1)^2} + ---$

$= \left(\frac{x}{x+1} + \frac{1}{2}\frac{x^2}{(x+1)^2} + ...\right) - \left(\frac{1}{x+1} + \frac{1}{2(x+1)^2} + ...\right)$

LEVEL- II

34. If $\log\ (1-x+x^2) = a_1x + a_2x^2 + a_3x^3 + ...$ then $a_3 + a_6 + a_9 + ...$

1) log 2 2) $\frac{2}{3}\log 2$ 3) $\frac{1}{3}\log 2$ 4) 2 log 2

35. $\sum_{n=1}^{\infty} \frac{x^{n+\frac{1}{2}}}{n+1}$ =

1) $\frac{1}{\sqrt{x}}[x+\log(1-x)]$ 2) $\frac{-1}{\sqrt{x}}[x+\log(1-x)]$

3) $\frac{1}{x}[x+\log(1-x)]$ 4) $\frac{-1}{x}[x+\log(1-x)]$

36. $-\frac{1}{2} - \frac{x}{3} - \frac{x^2}{4} - \frac{x^3}{5} - ....\infty =$

1) $\frac{\log_e(1-x)+x}{x^2}$ 2) $\frac{\log_e(1-x)-x}{x^2}$

3) $\frac{\log_e(1+x)-x}{x^2}$ 4) $\frac{\log_e(1+x)+x}{x^2}$

37. $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \frac{4}{5}x^5 + ... =$

1) $\frac{x}{1-x} + \log(1-x)$ 2) $\frac{x}{1+x} + \log(1-x)$

3) $\frac{x}{1-x} + \log(1+x)$ 4) $\frac{x}{1+x} + \log(1+x)$

38. $5x - \frac{13}{2}x^2 + \frac{35}{3}x^3 - \frac{97}{4}x^4 + ... =$

1) $\log\left(1+4x+3x^2\right)$ 2) $1+4x+3x^2$

3) $1+5x+6x^2$ 4) $\log\left(1+5x+6x^2\right)$

39. If $y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\infty$ then

$y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 + ...\infty =$

1) 4x 2) 3x 3) 2x 4) x

40. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\infty$ then

$y + \frac{y^2}{2!} + \frac{y^3}{3!} + ... =$

1) $\frac{x}{3}$ 2) $\frac{x}{2}$ 3) x 4) 2x

41. If $y = x + \frac{x^2}{2} + \frac{x^3}{3} + ...\infty\ then\ 1-e^{-y}$ =

1) 2x 2) x 3) $\frac{x}{2}$ 4) $\frac{x}{3}$

42. If a=1, b = $\log_e 2$, c = $\log_e 3$ then

1) a < b < c 2) a > b > c

3) a < c < b 4) b < a < c

43. If $e^x = 1+y, then\ x =$

1) $y - \frac{y^2}{2!} + \frac{y^3}{3!} - ....$ 2) $y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + ...$

3) $1 - \frac{y^2}{2} + \frac{y^4}{4}.- .....$ 4) $1 + \frac{y^2}{2!} + \frac{y^4}{4!} + ...$

44. $\log_e(1+x+x^2+x^3) = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$ then

1) $a_1 = -1, a_2 = \frac{1}{2}$ 2) $a_1 = 1, a_2 = \frac{1}{2}$

3) $a_0 = 1, a_1 = -1$ 4) $a_0 = 1, a_1 = 1$

45. *If* α, β are the roots of $x^2 - px + q = 0$ then

$(\alpha+\beta)x - \left(\alpha^2+\beta^2\right)\frac{x^2}{2} + (\alpha^3+\beta^3)\frac{x^3}{3} - ...\infty =$

1) log $(1-px+qx^2)$ 2) log $(1-qx+px^2)$

3) log $(1+qx+px^2)$ 4) $\log(1+px+qx^2)$

46. $2\log n-\log(n+1)-\log(n-1)$ is equal to

1) $\frac{1}{n^2}+\frac{1}{2n^4}+\frac{1}{3n^6}+.....$

2) $\frac{1}{n^2}-\frac{1}{2n^4}+\frac{1}{3n^6}-.....$

3) $\frac{1}{n^2}+\frac{1}{n^4}+\frac{1}{n^6}+.....$ 4) $\frac{1}{n^2}-\frac{1}{n^4}+\frac{1}{n^6}-.....$

47. If $f(x)=1+x^2+x^4+x^6+......\infty$ then

$\int_0^x f(x)dx =$

1) $\log(1-x)$ 2) $\log(1+x)$

3) $\log\frac{(1+x)}{(1-x)}$ 4) $\frac{1}{2}\log\frac{(1+x)}{(1-x)}$

KEY

34.2	35.2	36.1	37.1	38.4
39.4	40.3	41.2	42.4	43.2
44.2	45.4	46.1	47.4	

HINTS

34. $\log(1-x+x^2)=$

$$\begin{cases}\frac{(-1)^n}{n} & \text{If } n \text{ is not multiple of } 3\\ \frac{2(-1)^{n-1}}{n} & \text{If } n \text{ is multiple of } 3\end{cases}$$

$$\therefore a_3+a_6+a_9+....=2\left(\frac{1}{3}-\frac{1}{6}+\frac{1}{9}-\frac{1}{12}+....\right)=\frac{2}{3}\log 2$$

35. $\frac{1}{\sqrt{x}}\sum_{n=1}^{\infty}\frac{x^{n+1}}{n+1}=\frac{1}{\sqrt{x}}\left[\frac{x^2}{2}+\frac{x^3}{3}+.......\right]$

$=\frac{1}{\sqrt{x}}[-\log(1-x)-x]$

37. $\sum_{n=1}^{\infty}\frac{n}{n+1}\bullet x^{n+1}=\sum_{n=1}^{\infty}\left(1-\frac{1}{n+1}\right)x^{n+1}$

$=\sum_{n=1}^{\infty}x^{n+1}-\sum_{n=1}^{\infty}\frac{x^{n+1}}{n+1}$

38. $(2x+3x)-\left(\frac{(2x)^2+(3x)^2}{2}\right)+\left(\frac{(2x)^3+(3x)^3}{3}\right)+....=$

$\left(2x-\frac{(2x)^2}{2}+\frac{(2x)^3}{3}-....\right)+\left(3x-\frac{(3x)^2}{2}+\frac{(3x)^3}{3}-....\right)$

42. $2<e<3$

$\log_e 2<\log_e e<\log_e 3$

43 $x=\log_e(1+y)$

44. $\log(1+x+x^2+x^3)=\log(1+x^2)+\log(1+x)$

45. Given value = $\log(1+\alpha x)+\log(1+\beta x)$

$=\log(1+(\alpha+\beta)x+\alpha\beta x^2)$

46. $\log\left(\frac{n^2}{(n+1)(n-1)}\right)=\log\left(1-\frac{1}{n^2}\right)$

47. $f(x)=(1-x^2)^{-1}$

NPQ

1 Observe the following lists

List – I	List – II
(1) $\sum_{n=0}^{\infty}\frac{x^n(\log_e a)^n}{n!}=$	1) $\frac{e^x-e^{-x}}{2}$
(2) $\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}=$	2) e^{-ax}
(3) $\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)}=$	3) a^x
(4) $\sum_{n=0}^{\infty}\frac{(-1)^n.(ax)^n}{n!}=$	4) $\frac{a^x-a^{-x}}{2}$
	5) $\frac{e^x+e^{-x}}{2}$

The correct match for List - I from List - II is

	A	B	C	D
(1)	3	5	3	5
(2)	2	4	3	5
(3)	3	5	1	2
(4)	2	4	3	1

2. Assertion (A) **:**

$x\in R,\ e^x=1+\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+....to\ \infty$

Reason (R) **:**

$x\in R^+,\ 1+\log_e x+\frac{(\log_e x)^2}{2!}+\frac{(\log_e x)^3}{3!}+....to\ \infty=x$

1) Both A and R are true and R is a consequence of A
2) Both A and R are true and R is a not consequence of A
3) A is true but R is false
4) A is false but R is true

3 $A=\frac{2}{1!}+\frac{4}{3!}+\frac{6}{5!}+.....to\infty$

$B=\sum_{n=1}^{\infty}\frac{1}{(n+1)!}$

$C=1-\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}........to\infty;\ e^{-1}$

$D=1-\frac{2}{\lfloor 1}+\frac{4}{\lfloor 2}-\frac{8}{\lfloor 3}+........to\infty;\ e^{-2}$

Arrange A, B, C, D in descending order

1) B < A < D < C 2) A < B < C < D
3) A < B < D < C 4) D < C < B < A

4 $I:\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+........to\infty=\frac{e}{3}$

$II:\sum_{n=1}^{\infty}\frac{n_{C_0}+n_{C_1}+n_{C_2}+......+n_{C_n}}{n_{P_n}}=e^2-1$

Which of the above statements is (are) true ?

1) only I 2) only II
3) both I and II 4) neither I or II

5. I: Coeff. of x^n

$\frac{1+x}{1!}+\frac{(1+x)^2}{2!}+\frac{(1+x)^3}{3!}+........to\infty=\frac{1}{n!}$

I: Coeff. of x^5 $\sum_{n=1}^{\infty}(-1)^n.\frac{(2x+3x)^n}{n!}=\frac{3^n}{n!}$

Which of the above statements is (are) true ?

1) only I 2) only II
3) both I and II 4) neither I or II

6. Assertion (A) : If |x| < 1, $\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+........to\infty$

Reason (R): $\frac{1}{2}\log\frac{1+x}{1-x}=\sum_{n=1}^{\infty}\frac{x^{2n-1}}{2n-1}$

1) Both A and R are true and R is a consequence of A

2) Both A and R are true and R is a not consequence of A

3) A is true but R is false

4) A is false but R is true

7 **I :** $\left(\frac{a-b}{a}\right)+\frac{1}{2}\left(\frac{a-b}{a}\right)^2+\frac{1}{3}\left(\frac{a-b}{a}\right)^3+.....=\log_e a-\log_e b$

II : $(a-1)-\frac{(a-1)^2}{2}+\frac{(a-1)^3}{3}-\frac{(a-1)^4}{4}+...=\log_e a$

Which of the above statements is (are) true ?

1) only I 2) only II
3) both I and II 4) neither I or II

8. Observe the following lists

List – I	List – II
1) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+....to\ \infty$	(1) $\log_e 3$
2) $\frac{1}{5}+\frac{1}{2.5^2}+\frac{1}{3.5^3}+\frac{1}{4.5^4}....to\ \infty$	(2) $\log_e 2$
(3) $\frac{1}{n+1}+\frac{1}{2(n+1)^2}+\frac{1}{3(n+1)^3}....to\ \infty$	(3) $\log_e\left(\frac{4}{5}\right)$
(4) $1+\frac{1}{3.2^2}+\frac{1}{5.2^4}+\frac{1}{7.2^6}....to\ \infty$	(4) $\log_e\left(\frac{5}{4}\right)$
	(5) $\log_e\left(1+\frac{1}{n}\right)$

The correct match for List - I from List - II is

	A	B	C	D
1)	2	4	1	5
2)	4	2	5	3
3)	3	2	5	4
4)	2	4	5	1

9. I : $0<y<2^{1/3}$ and $x(y^3-1)=1$, then

$\frac{2}{x}+\frac{2}{3x^3}+\frac{2}{5x^5}+....to\ \infty=\log\left(\frac{y^3}{2-y^3}\right)$

II : $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+....to\ \infty=1-\log_e 2$

Which of the above statements is (are) true ?

1) only I 2) only II
3) both I and II 4) neither I or II

NPQ KEY

1.3	2.1	3.2	4.2	5.4
6.1	7.3	8.4	9.3	

LEVEL - V

I. $\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-...\infty$

$\log(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-\frac{x^5}{5}-...\infty$

1. $\frac{1}{2}-\frac{1}{2^2.2}+\frac{1}{2^3.3}-\frac{1}{2^4.4}+...\infty=$

1. log 2 2. log 3 3. $\log\frac{3}{2}$ 4. $\log\frac{2}{3}$

2. $Tan\theta-\frac{Tan^2\theta}{2}+\frac{Tan^3\theta}{3}-\frac{Tan^4\theta}{4}+...\infty$

1. $2\log(\sec\theta)$ 2. $2\log\cos\theta$
3. $2\log\tan\theta$ 4. $2\log\cot\theta$

3. $-\left(\frac{4}{5}+\frac{4^2}{5^2.2}+\frac{4^3}{5^3.3}+\frac{4^4}{5^4.4}+...\infty\right)=$

1. $\log\frac{9}{5}$ 2. $\log\frac{4}{5}$ 3. $\log\frac{5}{4}$ 4. $-\log 4$

KEY

1. 3 2. 1 3. 4

PREVIOUS EAMCET QUESTIONS

EAMCET - 2005

1. $\sum_{n=1}^{\infty}\frac{2n^2+n+1}{n!}$

1) $2e-1$ 2) $2e+1$ 3) $6e-1$ 4) $6e+1$

2. $|a|<1, b=\sum_{k=1}^{\infty}\frac{a^k}{k}\Rightarrow a=$

1) $\sum_{k=1}^{\infty}\frac{(-1)^k b^k}{k}$ 2) $\sum_{k=1}^{\infty}\frac{(-1)^{k-1} b^k}{k!}$

3) $\sum_{k=1}^{\infty}\frac{(-1)^k b^k}{(k-1)!}$ 4) $\sum_{k=1}^{\infty}\frac{(-1)^{k-1} b^k}{(k+1)!}$

EAMCET - 2003

3. $\frac{1}{\lfloor 2}+\frac{1+2}{\lfloor 3}+\frac{1+2+3}{\lfloor 3}+..........\infty=$

1) $\frac{e}{2}$ 2) $\frac{e}{3}$ 3) $\frac{e}{4}$ 4) $\frac{e}{5}$

EAMCET - 2002

4. For a>0, $x\in R$,

$1+x\log_e a+\frac{x^2}{2!}(x\log_e a)^2+\frac{x^3}{3!}.(\log_e a)^3+.....=$

1) a 2) a^x 3) $\log_e a$ 4) x

5. $1+\frac{1+2}{2!}+\frac{1+2+2^2}{3!}+\frac{1+2+2^2+2^3}{4!}....=$

1) e^2+e 2) e^2 3) e^2-e 4) e^3-e

EAMCET - 2001

6. $\frac{2}{2!}+\frac{2+4}{3!}+\frac{2+4+6}{4!}+......=$

1) e - 2 2) e-1 3) e 4) e^{-1}

7. If $|x|<1$, the coefficient of x^3 in the expansioin of $\log(1+x+x^2)$ in ascending powers of x, is

1) 2/3 2) 4/3 3) -2/3 4) -4/3

EAMCET - 2007

8. If $S_n=1^3+2^3+........+n^3$ and $T_n=1+2+........+n$ then **(E-2007)**

1) $S_n=T_{n^3}$ 2) $S_n=T_{n^2}$

3) $S_n=T_n^2$ 4) $S_n=T_n^3$

9. The sum of the series $\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}-......$ **(E-2007)**

1) $\sqrt{\frac{3}{2}}-\frac{3}{4}$ 2) $\sqrt{\frac{2}{3}}-\frac{3}{4}$ 3) $\sqrt{\frac{3}{2}}-\frac{1}{4}$ 4) $\sqrt{\frac{2}{3}}-\frac{1}{4}$

10. $\frac{1}{2}-\frac{1}{2.2^2}+\frac{1}{3.2^3}-\frac{1}{4.2^4}.....=$
(E-2007)

1) $\frac{1}{4}$ 2) $\log_e\left(\frac{3}{4}\right)$

3) $\log_e\left(\frac{3}{2}\right)$ 4) $\log_e\left(\frac{2}{3}\right)$

KEY

1. 3	2.2	3. 1	4. 2	5. 3
6. 3	7. 3	8. 3	9. 2	10. 3