6. Parallel Lines

Questions Pg-102

1. Question

Draw an 8 centimetres long line and divide it in the ratio 2 : 3.

Answer

Let the ends of our 8cm line be A and B.

Method-1:

Let us divide the given line into five equal parts as below.

Here if we consider AD, DB then we can notice that AD = 2parts and DB = 3 parts

Hence

Method-2: (by using the ratios)

Then the point dividing our line from end A will be

$= \left(\frac{2}{2 + 3}\right) \times 8$

$= \left(\frac{2}{5}\right) \times 8$

$= \frac{16}{5}$

$= 3.2\text{cm}$

And it will be = 8-3.2 = 4.8cm from end B.

2. Question

Draw a rectangle of perimeter 15 centimetres and sides in the ratio 3:4.

Answer

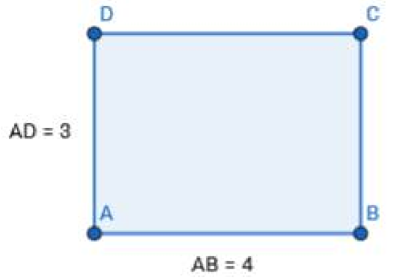
Method -1:

Given ratio of sides is 3:4.

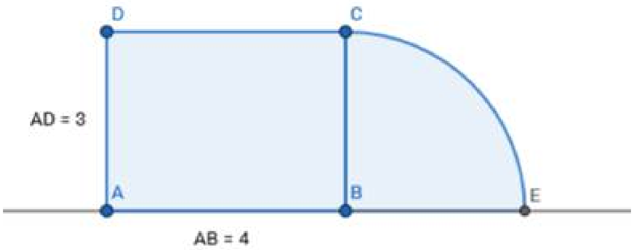
Let one side be 3 , then other side would be 4

Therefore, total perimeter = 3 + 4 + 3 + 4 = 14

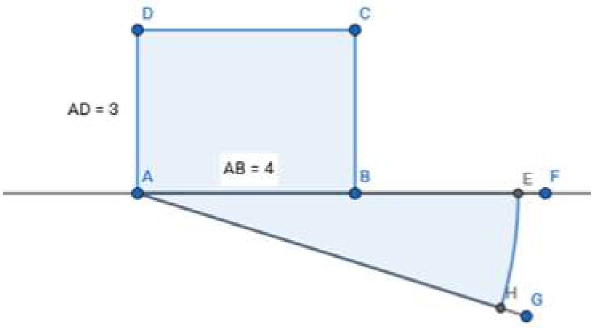
Here our perimeter is less than the required perimeter by 15-14 = 1



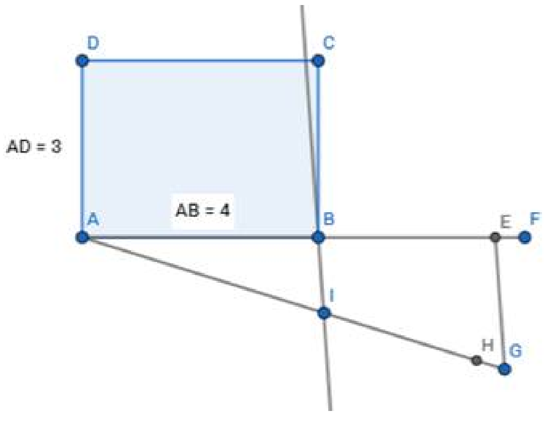
Step-1: using circle(centre B, radius BC) extend the base with side length (BC) as shown below



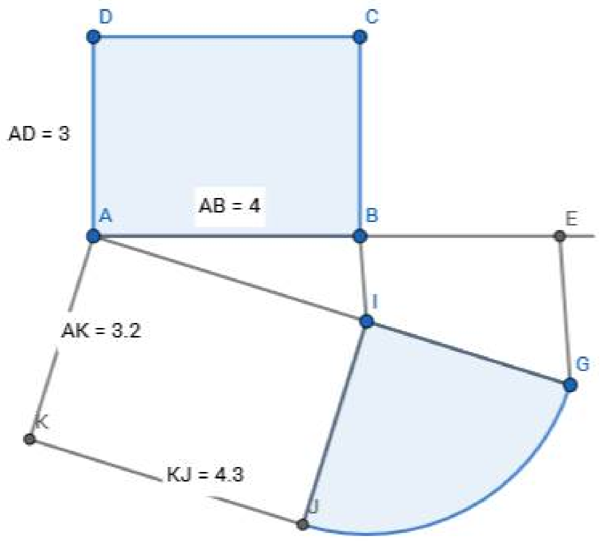
Step-2: take any slant line below base which will have a more length of 0.5cm than the base length(AE). (here the extra length is half the difference between required perimeter and present perimeter)



Step-3: now join EG and draw a parallel line to EG through B. this meets AG at I.



Step-4: now take AI and IG as lengths of sides of required rectangle and form a rectangle as shown below:



Hence the bottom rectangle KJIA, is our required rectangle with perimeter 15.

Method-2:

Let ‘a’ and ‘b’ be the length and width of rectangle.

Then given $a:b = 3:4$

$$a = \frac{3b}{4}$$

Then we know that perimeter will be 2(a + b)

$$2(a + b) = 15cm$$

$$2\left(\frac{3b}{4} + b\right) = 15cm$$

$$\frac{7b}{2} = 15cm$$

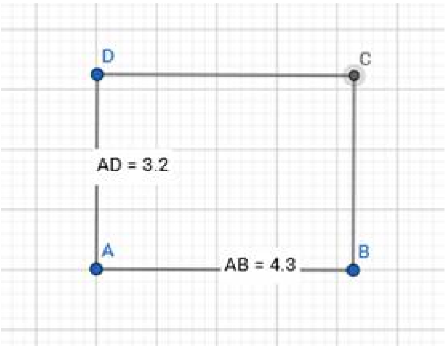
$$b = \frac{30}{7}cm$$

$$b = 4.286cm$$

Therefore, b = 4.286cm &

$$a = \frac{3b}{4} = \frac{3 \times 30}{4 \times 7}\ cm$$

$$a = 3.214cm$$



3 A. Question

Draw triangles specified below, each of perimeter 10 centimetres.

Equilateral triangle

Answer

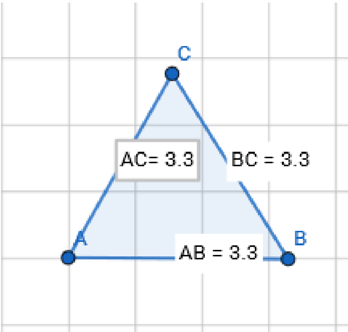
For equilateral triangle, all the sides are equal. Let ‘x’ be the length of each side.

Given perimeter = 10cm

x + x + x = 10cm

3x = 10cm

$$x = \frac{10}{3} = 3.333$$



3 B. Question

Draw triangles specified below, each of perimeter 10 centimetres.

Sides in the ratio 3 : 4 : 5

Answer

Let each part be ‘x’.

Then sides will be 3x, 4x, 5x respectively.

Given perimeter = 10cm

3x + 4x + 5x = 10cm

12x = 10cm

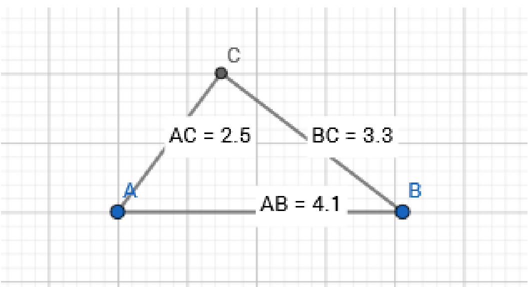
$$x = \frac{10}{12}\text{cm}$$

Then sides are

$$3\left(\frac{10}{12}\right) = 2.5 \text{ cm},$$

$$4\left(\frac{10}{12}\right) = 3.333 \text{ cm},$$

$$5\left(\frac{10}{12}\right) = 4.167\text{cm}$$



3 C. Question

Draw triangles specified below, each of perimeter 10 centimetres.

Sides in the ratio 2 : 3 : 4

Answer

Let each part be ‘x’.

Then sides will be 2x, 3x, 4x respectively.

Given perimeter = 10cm

2x + 3x + 4x = 10cm

9x = 10cm

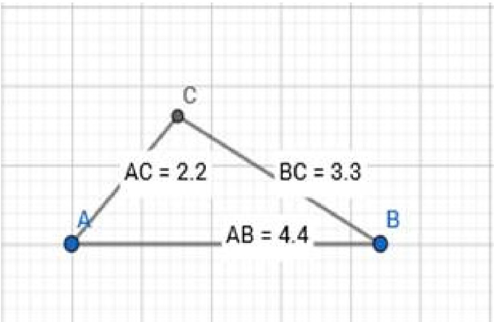
$$x = \frac{10}{9} = 1.111\text{cm}$$

Then the sides are

$$2\left(\frac{10}{9}\right) = 2.22\text{ cm},$$

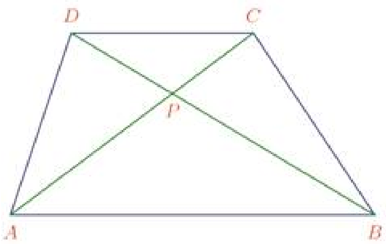
$$3\left(\frac{10}{9}\right) = 3.333\text{ cm},$$

$$4\left(\frac{10}{9}\right) = 4.44\text{cm}$$



4. Question

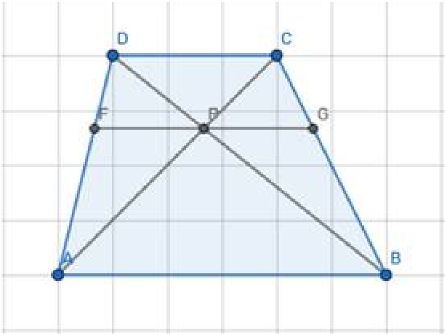
In the picture below, the diagonals of the trapezium ABCD intersect at P.



Prove that PA × PD = PB × PC

Answer

Here AB, CD are parallel, then draw a line parallel to AB or CD through P.



Then,

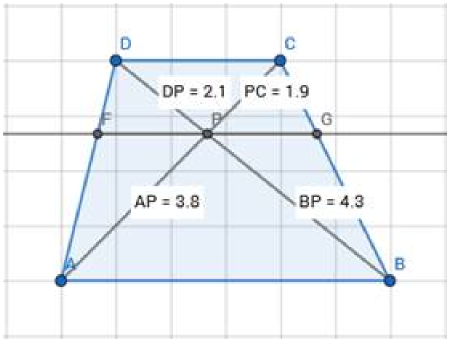
We know that ,

Any three parallel lines will cut any two lines into pieces whose lengths are in the same ratio.

Therefore, $\frac{PA}{PC} = \frac{PB}{PD}$

PA×PD = PB×PC

For verification, we can take a small trapezium and prove it as follows:

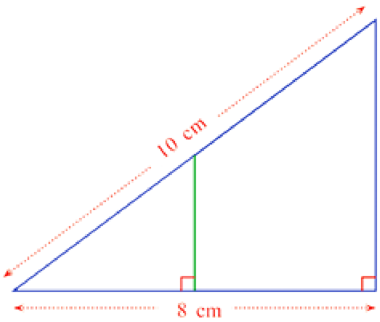


Here 3.8×2.12 = 7.98 = 1.9×4.25 approximately

Questions Pg-106

1. Question

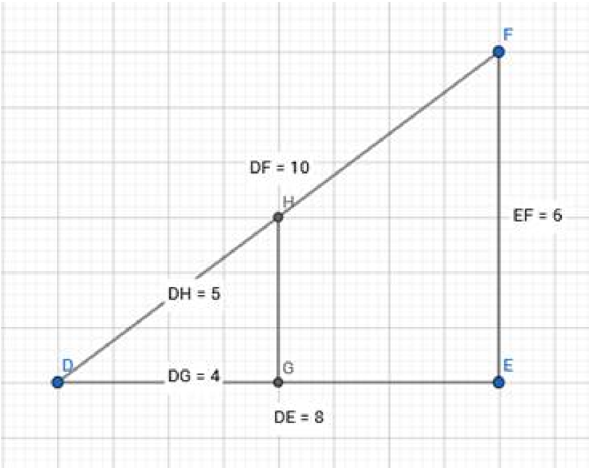
In the picture, the perpendicular is drawn from the midpoint of the hypotenuse of a right triangle to the base.



Calculate the length of the third side of the large right triangle and the lengths of all three sides of the small right triangle.

Answer

Let a right angled triangle DEF,



H and G are mid points of DF, DE respectively

Here by pythagorous,

$DE^2 + EF^2 = DF^2$

$8^2 + EF^2 = 10^2$

$EF^2 = 100 - 64 = 36$

$EF = 6cm$

Since H and G are mid-points of DF, DE respectively. We can have

$DH = \frac{1}{2}(DF) = \frac{10}{2} = 5cm$

$DG = \frac{1}{2}(DE) = \frac{8}{2} = 4cm$

By pythagorous theorem,

$DG^2 + GH^2 = DH^2$

$4^2 + GH^2 = 5^2$

$GH^2 = 25 - 16 = 9$

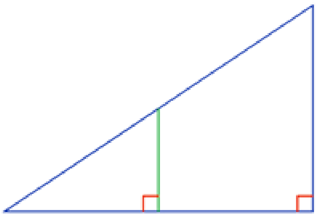
$GH = 3cm$

Hence length of third side of larger triangle is 6cm

And 3cm, 4cm, 5cm are lengths of sides of smaller triangle.

2. Question

Draw a right triangle and the perpendicular from the midpoint of the hypotenuse to the base.



i) Prove that this perpendicular is half the perpendicular side of the large triangle.

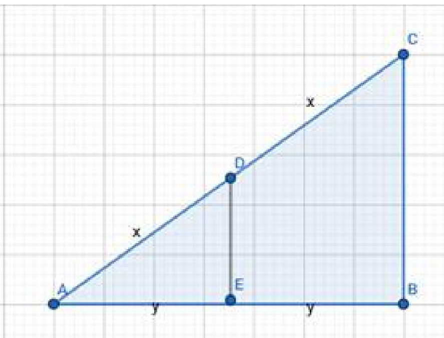
ii) Prove that in the large triangle, the distance from the midpoint of the hypotenuse to all the vertices are equal.

iii) Prove that the circumcentre of a right triangle is the midpoint of its hypotenuse.

Answer

i) We know that,

In any triangle, line drawn parallel to one side, passing through mid-point of another side will also meet the third side at its mid-point.



Therefore,

Let AD be ‘x’, then DC = x. similarly AE = y then EB = y

By pythagorus theorem for bigger triangle ABC,

$$BC = \sqrt{(AC^2 - AB^2)}$$

$$BC = \sqrt{(4x^2 - 4y^2)}$$

$$BC = 2\sqrt{(x^2 - y^2)}$$ ----1

Similarly for smaller triangle AED,

$$DE = \sqrt{(AD^2 - AE^2)}$$

$$DE = \sqrt{(x^2 - y^2)}$$

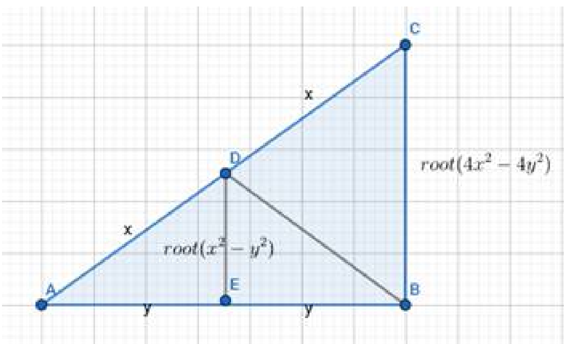
$$DE = \sqrt{(x^2 - y^2)}$$ -----2

From 1 & 2, we have

$$DE = \frac{1}{2}(BC)$$

Hence proved.

ii)



We have the above following data.

And from pythagorous theorem,

We have

$$DB = \sqrt{(DE^2 + EB^2)}$$

$$DB = \sqrt{\left(\sqrt{(x^2 - y^2)}^2 + y^2\right)}$$

$$DB = \sqrt{(x^2 - y^2 + y^2)}$$

$$DB = \sqrt{(x^2)} = x$$

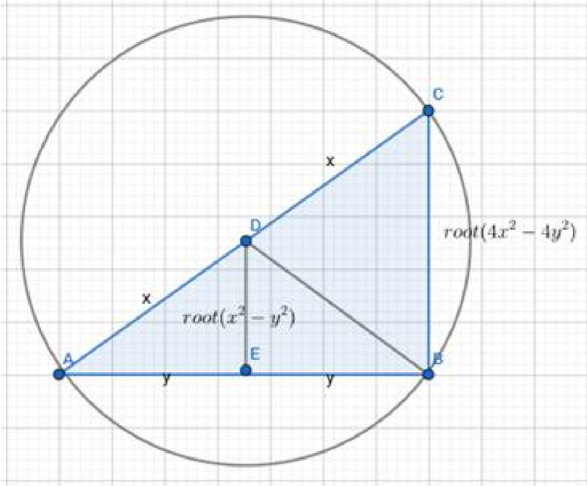
Hence the distance between mid-point of hypotenuse to vertex B (or) A (or) C is x

Hence proved.

iii) We know that

circumcentre is a point from which all the vertices are at equal distances.

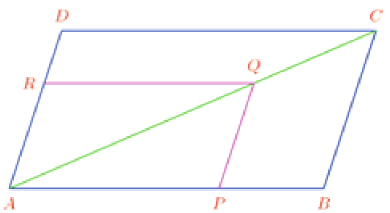
In the above case, we have proved the mid-point of hypotenuse is equidistant from all the three vertices. Therefore mid-point of hypotenuse is the circumcentre of the given triangle.



Hence proved.

3. Question

In the parallelogram ABCD, the line drawn through a point P on AB, parallel to BC, meets AC at Q. The line through Q, parallel to AB meets AD at R.

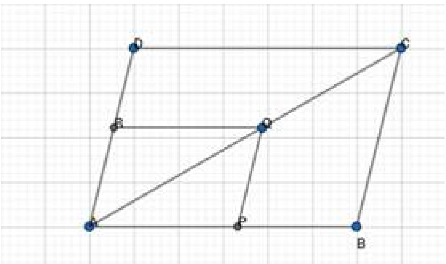


i) Prove that $\frac{AP}{PB} = \frac{AR}{RD}$

ii) Prove that $\frac{AP}{AB} = \frac{AR}{AD}$

Answer

i) Here we have AD, PQ, BC as three parallel lines.



We know that

Any three parallel lines will cut any two lines into pieces whose lengths are in the same ratio.

For triangle ABC,

Therefore, $\frac{AP}{PB} = \frac{AQ}{QC}$ --------1

Similarly, for triangle ACD,

$\frac{AQ}{QC} = \frac{AR}{RD}$ --------2

From 1 & 2,

$$\frac{AP}{PB} = \frac{AR}{RD}$$

Hence proved

ii) From previous part we have,

$$\frac{AP}{PB} = \frac{AR}{RD}$$

On inverting the numerators and denominators,

$$\frac{PB}{AP} = \frac{RD}{AR}$$

On adding one on both sides of equation,

$$\frac{PB}{AP} + 1 = \frac{RD}{AR} + 1$$

From figure, on simplifying,

$$\frac{AB}{AP} = \frac{AD}{AR}$$

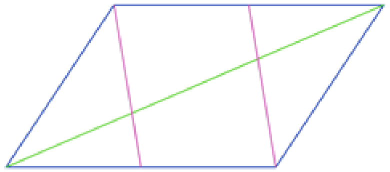
On inverting the numerators and denominators,

$$\frac{AP}{AB} = \frac{AR}{AD}$$

Hence proved.

4. Question

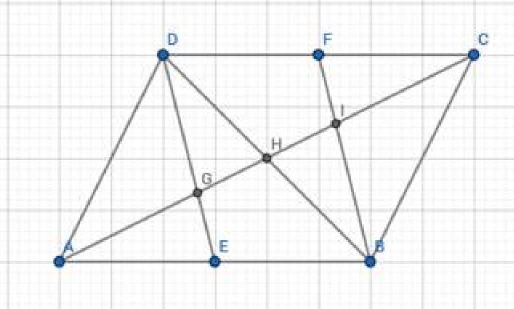
In the picture below, two vertices of a parallelogram are joined to the midpoints of two sides.



Prove that these line divide the diagonal in the picture into three equal parts.

Answer

Given ,



Here AC and BD are diagonals and they bisect each other as we know.

Therefore we can say that AC is the median and passes through mid-poin of DB.

We know that

In any triangle, all the medians intersect at a single point and that point divides each median in the ratio 2:1 measured from vertex.

Therefore from triangle ABD,

We have

$\frac{AG}{GH} = \frac{2}{1}$ -------1

Similarly, from triangle BCD,

We have

$\frac{CI}{IH} = \frac{2}{1}$; $\frac{HI}{IC} = \frac{1}{2}$ ----2

From 1 & 2

AG:GI:IC = AG:(GH + HI):IC

AG:GI:IC = 2:2:2 = 1:1:1

Hence the lines from vertices to mid-points of opposite side will divide the diagonal into three equal parts as shown above.

Hence proved.

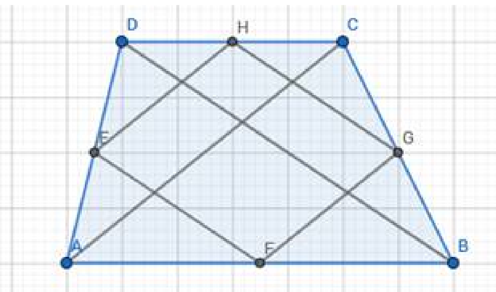
5. Question

Prove that the quadrilateral formed by joining the midpoints of a quadrilateral is a parallelogram. What if the original quadrilateral is a rectangle? What if it is a square?

Answer

Given ABCD is a quadrilateral. And EFGH be the mid-points of the above quadrilateral.

Then,



We know that,

If the mid-points of adjacent sides of a triangle are joined, then the line joining those mid-points will be parallel to the third side and half of its length.

Therefore, from triangle’s ABD and BCD,

EF//DB and HG//DB

⇒ EF//HG

$$\mathrm{EF} = \frac{1}{2}\mathrm{DB} = \mathrm{HG}$$

Similarly, from triangle ACD and triangle ABC,

FG//AC and EH//AC

⇒ FG//EH

$$\mathrm{FG} = \frac{1}{2}\mathrm{AC} = \mathrm{EH}$$

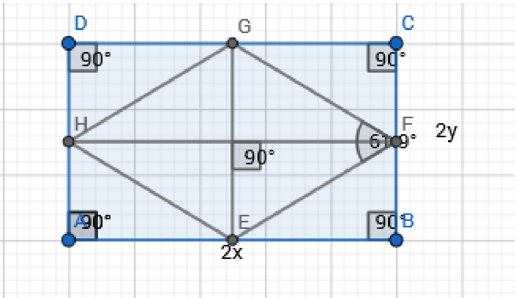
Since EF//HG , $\mathrm{EF} = \mathrm{HG}$

And FG//EH , $\mathrm{FG} = \mathrm{EH}$

Therefore EFGH forms a parallelogram for any type of quadrilateral taken.

Hence proved.

If original quadrilateral was a rectangle then,



Then the new quadrilateral formed would be a rhombus as shown in figure, due to

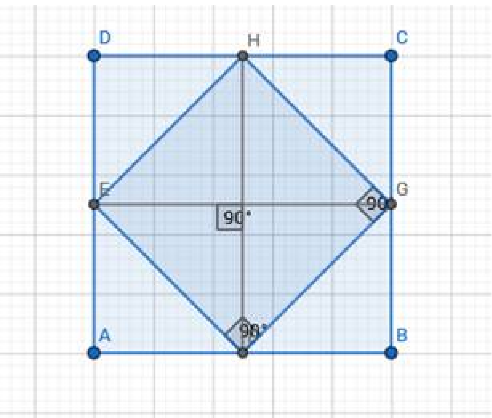
Equal sides are generated because of equal opposite lengths of original quadrilateral.

Therefore,

$$\mathrm{HE} = \mathrm{EF} = \mathrm{FG} = \mathrm{GH} = \sqrt{x^2 + y^2}$$

And diagonals are perpendicular to each other. Hence EFGH forms a rhombus.

If original quadrilateral was a square then,



Then the new quadrilateral formed would be a square as shown in figure, due to

Equal and perpendicular adjacent sides are generated because of equal lengths of original quadrilateral. Even new diagonals are equal and perpendicular.

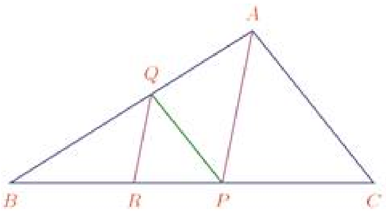
Therefore,

$HE = EF = FG = GH = \sqrt{x^2 + x^2} = x\sqrt{2}$

And diagonals are equal and perpendicular to each other. Hence EFGH forms a square.

6. Question

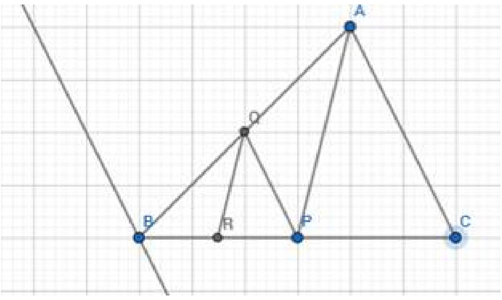
In □ABC, the line through a point P on BC, parallel to AC meets AB at Q. The line through Q, parallel to AP, meets BC at R.



Prove that $\frac{BP}{PC} = \frac{BR}{RP}$

Answer

Here AB, CD are parallel, then draw a line parallel to AB or CD through P.



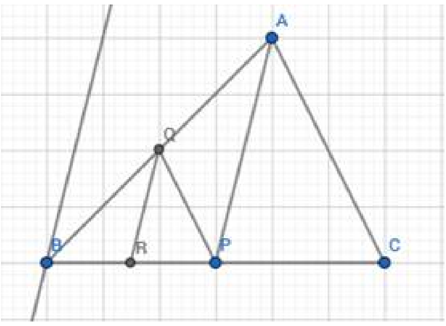
Then,

We know that ,

Any three parallel lines will cut any two lines into pieces whose lengths are in the same ratio.

Therefore, if we consider QR, AP, line through B parallel to QR, then it divides the BA and BP in the same ratio.

$\frac{BP}{PC} = \frac{BQ}{QA}$----1



Similarly, if we consider QR,AP, line through B parallel to QR, then it divides the BA and BP in the same ratio.

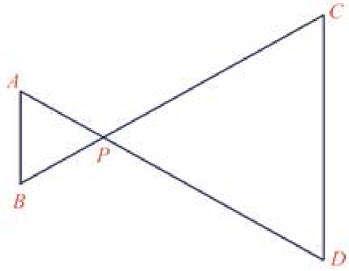
$\frac{BQ}{QA} = \frac{BR}{RP}$--------2

From 1 & 2, we have,

$\frac{BP}{PC} = \frac{BR}{RP}$

7. Question

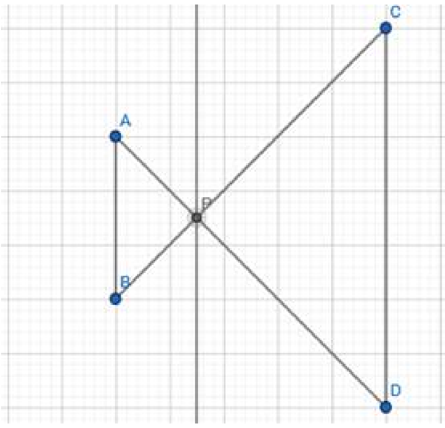
AB and CD are parallel lines in the picture.



Prove that AP × PC = BP × PD

Answer

Here AB, CD are parallel, then draw a line parallel to AB or CD through P.



Then,

We know that ,

Any three parallel lines will cut any two lines into pieces whose lengths are in the same ratio.

Therefore, $\frac{AP}{PD} = \frac{BP}{PC}$

$AP \times PC = BP \times PD$