17

Measures of Central Tendency

17.01 Introduction :

The collection, classification, tabulation and graphical representation of initial data make the data simple and easy to understand. But when comparitive study of data is to be undertaken or any conclusion form data are to be drawn then it is necessary to make it simple and compact. So that their characterstics can be represented by a single number.

For example, if 300 students of a school are compared to 500 students of another school, it is not possible to come to any conclusion from their series showing marks obtained in different subject. But if a single number is taken from each of these series which is their repersentative, then it will be easy to compare these series. This representative number is taken in the middle of the series, where most of the observations are centred. Such a number is the representative of the whole series and it is called a **measure of** central tendency.

17.02 Measures of Central Tendency and Types of Averages :

Measures of central tendency and averages are ordinarily divided into two parts :

(1) Mathematical Average

(i) Arithmetic Mean or Average [AM]

(ii) Geometric Mean [GM]

(iii) Harmonic Mean [HM]

(2) Average of Position

(i) Median

(ii) Mode

Here at the secondary level, we shall consider only simple questions on arthimetic mean (which is usually called mean), median and mode.

17.03 Arithmetic Mean

To find arithmetic mean from initial data (individual series) : In order to find arithmetic mean from such type of data, we find the sum of all the data and divide it by the number of data. This is also called mean, i.e.

$$\text{Arithmetic Mean} = \frac{\text{Sum of data}}{\text{Number of data}}$$

For example, the marks obtained by student of class X in mathematics are 7, 8, 5, 6, 7, 8, 9, 4, 5 and 6 respectively then average of marks

$$= \frac{\text{Sum of marks (Sum of data)}}{\text{No. of students (No. of data)}}$$

$$= \frac{7+8+5+6+7+8+9+4+5+6}{10}$$

$$= \frac{65}{10} = 6 \cdot 5 \text{ Marks}$$

If values of a variate are $x_1, x_2, \ldots, x_n$ respectively, then

their A.M. $(\bar{x}) = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

$= \dfrac{\sum_{i=1}^{n} x_i}{n}$

Note : Σ is a letter of Greek alphabet and it is pronounced as 'Sigma' and it is used to denote the operation of addition in Mathematics.

$\sum_{i=1}^{n} x_i$ denotes the sum $x_1 + x_2 + x_3 + \cdots + x_n$.

Thus $\sum_{i=1}^{25} y_i = y_1 + y_2 + y_3 + \cdots + y_{25}$

Illustrative Examples

Example 1. The monthly salaries of 5 employees including Head master in a school are ₹ 8000, ₹ 5000, ₹ 4000, ₹ 2500 and ₹ 1500 respectively. Find the average monthly salary of working employees in school.

Solution : Average monthly salary

$$= \frac{8000 + 5000 + 4000 + 2500 + 1500}{5}$$

$$= \frac{21000}{5} = 4200 \text{ ₹}$$ **Answer.**

Example 2. Find the A.M. of the first ten positive odd numbers.

Solution : First ten odd positive numbers are 1, 3, 5, 7, 11, 13, 15, 17, 19 respectively.

So A.M. $(\bar{x}) = \dfrac{1+3+5+7+9+11+13+15+17+19}{10}$

$= \dfrac{100}{10} = 10$ **Answer.**

Example 3. The average of eight consecutive odd numbers is 16, find the numbers.

Solution : Let the first odd number be x, so eight odd numbers are

$x, x+2, x+4, x+6, x+8, x+10, x+12, x+14$

Average of eight odd numbers

$$= \frac{(x)+(x+2)+(x+4)+(x+6)+(x+8)+(x+10)+(x+12)+(x+14)}{8}$$

$$= \frac{8x+2+4+6+8+10+12+14}{8} = \frac{8x+56}{8}$$

Thus $\dfrac{8x+56}{8} = 16$ or $8x + 56 = 128$ or $x = 9$.

Therefore required consecutive numbers are

9, 11, 13, 15, 17, 19, 21, 23 **Answer.**

17.04 Merits, Demerits of Arithmetic Mean :

Merits :

1. It is easy to calculate.
2. It is based upon all the terms.
3. It is also used in other statistical analysis.
4. This mean is fixed and always same.
5. It is possible to test its accuracy.
6. Its value has stability.

Demerits :

1. Sometimes in its calcualtion, such value may occur which is not possible according to nature, e.g. number of members in a family is 3·8 or 5·6
2. It is not possible to calculate if any one value is missing.
3. It is affected very much by exterme values.
4. Determination of this mean is not possible by observation.

Exercise 17.1

1. If marks obtained by ten students of a class in Mathematics are 52, 75, 40, 70, 43, 40, 65, 35, 48 and 52 then find their arithmetic mean.
2. The monthly salaries (in ₹) of subordinate employees of a school are 1720, 1750, 1760 and 1710, then find the airthmetic mean.
3. If the airthmetic mean of the marks 3, 4, 8, 5, x, 3, 2, 1 is 4, then find the value of x.
4. The runs scored in 10 innings by a batsman are 60, 62, 56, 64, 0, 57, 33, 27, 9 and 71 respectively. Find the arithmetic mean of the runs scored by him in these innings.
5. Calculate arithmetic mean of the following marks obtained by 10 students in English in the monthly test :

Roll No.	:	1	2	3	4	5	6	7	8	9	10
Marks obtained	:	30	28	32	12	18	20	25	15	26	14

6. The number of books issued to the students in 10 days from the school library are given below :

   300 405 455 489 375 280 418 502 300 476

   Find the average number of books issued per day.
7. The average weight of 25 students of section A of a class is 51 kg. whereras the average weight of 35 students of section B is 54 kg. Find the average weight of 60 students of this class.
8. The mean of 5 numbers is 18, If one number is excluded, then mean becomes 16. Find the excluded number.
9. The mean of 13 numbers is 24. If 3 is added to each number, then find their new mean.
10. The monthly salary of 5 employees of a school is ₹ 3000. On the retirement of one employee, the average monthly salary of remaining employees is ₹ 3200. What was the salary of retired employeed at the time of retirement ?

17.05 Arithmetic Average from Discrete Series or Discrete Frequency Distribution

Let frequaency distribution of n values of variate x is as follows :

Value of x	:	x_1	x_2	x_3	$\cdots$	x_k
Frequency f	:	f_1	f_2	f_3	$\cdots$	f_k

It is clear form distribution that out of total n values of x, x_1, is attained f_1 times, x_2, is attained f_2 times. Thus average or arithmetic mean $(\bar{x})$ of variate x will be obtained as follows :

$$\bar{x} = \frac{\overbrace{x_1 + x_1 + \cdots + x_1}^{f_1 \text{ times}} + \overbrace{x_2 + x_2 + \cdots + x_2}^{f_2 \text{ times}} + \cdots + \overbrace{x_n + x_n + \cdots + x_n}^{f_k \text{ times}}}{f_1 + f_2 + \cdots + f_n}$$

$$= \frac{f_1 x_1 + f_2 x_2 + \cdots + f_n x_n}{f_1 + f_2 + \cdots + f_n}$$

$$= \frac{1}{n} \sum_{i=1}^{n} f_i x_i, \quad \text{Where } \sum_{i=1}^{n} f_i = n = \text{Number of total values}$$

Working steps :

Step I. First of all make frequaency table form frequency distribution in such a way that I column contains the value of x_i of x and II column contians the frequency f_i of x.

Step II. III column will contain product $f_i x_i$ of x_i and f_i .

Step III. On showing sum of II column by $\sum f_i$ and sum of III column by $\sum f_i x_i$

$$\text{Arithmetic mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

Therefore for calculation of arithmetic mean table can be prepared as follows :

Calculation of Arithmetic mean

x_i	f_i	$f_i x_i$
x_1	f_1	$f_1 x_1$
x_2	f_2	$f_2 x_2$
x_3	f_3	$f_3 x_3$
$\vdots$	$\vdots$	$\vdots$
x_n	f_n	$f_n x_n$
	$\sum f_i$	$\sum f_i x_i$

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Note : The value of x is represent by x_i and their corresponding frequencies by f_i. The average value of x is denoted by $\bar{x}$.

Example : Calculate the mean from the following frequency distribution :

x:	5	6	7	8	9	10	11
f:	5	8	9	12	6	6	4

Solution : **Calculation of Arithmetic mean**

x_i	f_i	$f_i x_i$
5	5	25
6	8	48
7	9	63
8	12	96
9	6	54
10	6	60
11	4	44
	$\sum f_i = 50$	$\sum f_i\ x_i = 390$

Thus Arithmetic mean $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

$= \dfrac{390}{50} = 7 \cdot 8$ **Answer .**

Illustrative Examples

Example 1. Find the arithmetic mean of the following frequency distribution :

x	1	2	3	4	5	6
f	2	5	6	4	2	2

Solution : **Calculation of Arithmetic mean**

x_i	f_i	$f_i x_i$
1	2	2
2	5	10
3	6	18
4	4	16
5	2	10
6	2	12
	$\sum f_i = 21$	$\sum f_i x_i = 68$

Thus Arthmetic mean $(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{68}{21} = 3 \cdot 238$ **Answer**

Example 2. The daily salaries of 50 officers of a factory are as follows :

Salary (in ₹) **x :**	**450**	**475**	**500**	**525**	**550**
No. of officers **f :**	**12**	**13**	**7**	**10**	**8**

Find arithmetic mean of their salaries.

Solution :

x_i	f_i	$f_i x_i$
450	12	5400
475	13	6175
500	7	3500
525	10	5250
550	8	4400
	$\sum f_i = 50$	$\sum f_i x_i = 24725$

Therefore the required A.M. $(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{24725}{50}$

$= ₹494 \cdot 5$ **Answer**

Exercise 17.2

Find the mean of the following frequency distribution (Q. 1-4) :

1.

x :	3	5	8	11
f :	2	4	5	3

2.

x :	2	5	7	9	11
f :	1	5	4	7	3

3.

x :	0·1	0·2	0·3	0·4	0·5	0·6
f :	30	60	20	40	10	50

4.

x :	0·1	0·3	0·5	0·7	0·89
f :	7	8	10	15	10

5. In hundred falmily, the number of childern are :

No. of children	1	2	3	4	5	6
No. of families	45	25	19	8	2	1

Find their arithmetic mean.

6. The weight of students in a class are given in the following table.

Weight (in kgs.)	20	21	22	23	24	25	26	27	28
No. of students	1	2	6	7	4	2	3	2	3

Find their arithmetic mean.

7. If mean of the following distribution is $7\cdot5$, then find the value of P.

x :	3	5	7	9	11	13
f :	6	8	15	P	8	4

8. If mean of the following frequency distribution is $1\cdot46$, then find the unknown frequencies.

x :	0	1	2	3	4	5	*Sum*
f :	46	...	...	25	10	5	200

17.06 Arithmetic mean from grouped frequency distribution

In such type of frequency distribution, the value of variable is divided in intervals. For example consider the following frequency distribution :

Marks obtained (x)	0-10	10-20	20-30	30-40	40-50
No.of students (f)	5	8	20	14	3

Here, the frequency of a class interval 10-20 is 8, i.e. the number of values of x from 10 to less than 20 is 8. When grouped frequency distribution is prepared from initial data then by seeing distribution, it is impossible to estimate about these data. For example, if values of x are 10, 11, 12, 17, 17, 18, 19, 19·5, or 11, 12, 13, 14, 15, 15, 17, 19 then in each case the class interval will be 10-20 whose frequency is 8.

Hence for the sake of convenience and simplicity it is quite logical to consider the mean of each interval as the mean of x and the frequency of each interval as the frequency x. The mean is calculated by the method used in case of ungroup frequency distribution. For example, for the interval 10-20 $x = \frac{10+20}{2} = 15$ whose frequency is 8.

Thus the ungrouped frequency distribution is obtained from above grouped frequency distribution as follow

Interval (Marks)	0-10	10-20	20-30	30-40	40-50
Marks obtained	5	15	25	35	45
Frequency	5	8	20	14	3

We obain the mean with the help of the method explained earlier as follows:

x_i	f_i	$f_i x_i$
5	5	25
15	8	120
25	20	500
35	14	490
45	3	135
Sum	$\sum f_i = 50$	$\sum f_i x_i = 1270$

Therefore the required A.M. $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

$= \dfrac{1270}{50}$

$= 25 \cdot 4$ Marks **Answer** .

Exercise 17.3

Find the arithmetic mean of the following frequency distribution : [1 to 4]

1.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	9	12	15	10	14

2.

Class	0-6	6-12	12-18	18-24	24-30
Frequency	6	8	10	9	7

3.

Marks obtained (x)	100-120	120-140	140-160	160-180	180-200
No. of students(f)	10	20	20	15	5

4.

Class	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75
Frequency	6	10	8	12	4

5. Find the mean of the following frequency distribution :

Weight (in kg.)	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of students	10	25	28	12	10	15

6. The salaries of the workers of a factory are as follows :

Monthly salary (in ₹)	1000-1200	1200-1400	1400-1600
No. of workers	10	20	20
Monthly salary (in ₹)	1600-1800	1800-2000	
No. of workers	15	5	

Find the arithmetic mean of the salaries.

17.07 Arithmetic mean using assumed mean :

If the values of x in any frequency distribution are very large, when it is difficult to calculate the arithmetic mean and much time is also consumed. In such a situated, it is conveninent to calculate the arithmetic mean form short-cut method of assumed mean.

Working Steps :

Step I. We first prepare the frequency table in such a way that first column contains x_i, and the second column contains their frequencies f_i.

Step II. In the third column we write deviation of each x_i from an appropriate value A. Here A is calld the assumed mean.

Step III. In the fourth column we write product of frequency f_i and devaiton d_i, $f_i d_i$.

Step IV. Write sum of column second, $\sum f_i$ and sum of column fourth $\sum f_i d_i$ in the corresponding last column.

Step V. We find the arithmetic mean form the formula

$$\bar{x} = A + \frac{1}{N}\left(\sum f_i d_i\right), \text{ where } N = \sum f_i \ .$$

The above procedure is clear form the following table.

x_i	f_i	$d_i = x_i - A$	$f_i d_i$
x_1	f_1	d_1	$f_1 d_1$
x_2	f_2	d_2	$f_2 d_2$
x_3	f_3	d_3	$f_3 d_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_k	f_k	d_k	$f_k d_k$
	$N = \sum f_i$		$\sum f_i d_i$

Thus Arithmetic Mean $(\bar{x}) = A + \dfrac{\sum f_i d_i}{\sum f_i}$

$$= A + \frac{1}{N}\left(\sum f_i \, d_i\right)$$

If in step II, step deviation is found from $u_i = \dfrac{x_i - A}{h}$, where h is class interval then according to step III, in column third, we write $f_i u_i$, i.e. the product of f_i and u_i . The mean is to found by the folowing formula :

$$\bar{x} = A + \frac{\sum f_i u_i}{\sum f_i} \times h$$

Important Remarks :

(i) Generally assumed mean A is taken as mid value of x or value of x having maximum frequency.

(ii) When difference in the value of x is large and the value of x are also larger or frequencies are large then it is convenient to calculate by step deviation $u_i = \dfrac{x_i - A}{h}$.

Calcualtion table for the above formula

x_i	f_i	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
x_1	f_1	u_1	$f_1 u_1$
x_2	f_2	u_2	$f_2 u_2$
⋮	⋮	⋮	⋮
x_k	f_k	u_k	$f_k u_k$
योग	$\sum f_i$		$\sum f_i u_i$

Thus arithmetic mean $(\bar{x}) = A + \frac{\sum f_i u_i}{\sum f_i} \times h$

(Here, by taking A as mid value, the values of u_i are $\cdots -3, -2, -1, \ 0, 1, 2, 3, \cdots$)

The method will be clear form the following examples.

Illustrative Examples

Example 1. Find the arithmetic mean for the following frequency distribution :

x	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
f	**20**	**43**	**75**	**67**	**72**	**45**	**39**	**9**	**8**	**6**

Solution : First of all we construct the calcualtion table by assuming 25, the corresponding value of maximum frequency 72 as assumed mean. (Here A=25 and h = 5).

Calculation for Arithmetic mean

Variate x_i	Frequency f_i	$u_i = \frac{x_i - 25}{5}$	$f_i u_i$
5	20	−4	−80
10	43	−3	−129
15	75	−2	−150
20	67	−1	−67
25	72	0	0
30	45	1	45
35	39	2	78
40	9	3	27
45	8	4	32
50	6	5	30
Total	$N = \sum f_i$ = 384		$\sum f_i u_i$ = −214

$$\text{Thus Arithmetic mean}(\bar{x}) = A + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$= 25 + \left(\frac{-214}{384}\right) \times 5$$

Answer.

$$= 25 - 2 \cdot 786 = 22 \cdot 214$$

Example 2. The following frequency distribution shows weights of 12 students.

Weight (in kg.)	**67**	**70**	**72**	**73**	**75**
No. of students	**4**	**3**	**2**	**2**	**1**

Find mean weight.

Solution : **Calculation table for Arithmetic mean**

Weight (in kg.) x_i	No. of students f_i	$d_i = x_i - 72$	$f_i d_i$
67	4	−5	−20
70	3	−2	−6
72	2	0	0
73	2	1	2
75	1	3	3
Sum	$N = \sum f_i = 12$		$\sum f_i d_i = -21$

Here by taking mid values of $x = 72$ as value of A

$$\text{Mean } (\bar{x}) = A + \frac{1}{N}\left(\sum f_i d_i\right)$$

$$= 72 + \left(\frac{-21}{12}\right)$$

$$= 72 - \frac{7}{4} = 70 \cdot 25 \text{ kg.}$$

Thus mean weight 70·25 kg. **Answer**

Example 3. In the following table, height of villages of some special region from sea level is given. Find the mean height of that region form sea level.

Height (in Metres)	**200**	**600**	**1000**	**1400**	**1800**	**2200**
No. of villages	**142**	**265**	**560**	**271**	**89**	**16**

Solution : Here we shall find the mean by taking A = 1000 and h = 400 and by calculating both types of devaition d_i as well as u_i.

Calculation table for Arithmetic mean

Height (in Metres.) x_i	No. of villages f_i	Deviation $d_i = x_i - 1000$	$f_i d_i$	Devaition $u_i = \frac{x_i - 1000}{400}$	$f_i u_i$
200	142	−800	−113600	−2	−284
600	265	−400	−106000	−1	−265
1000	560	0	0	0	0
1400	271	400	108400	1	271
1800	89	800	71200	2	178
2200	16	1200	19200	3	48
	$\sum f_i = 1343$		$\sum f_i d_i = -20800$		$\sum f_i u_i = -52$

Therefore

(i) Mean by deviation method

$$\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$$

$$= 1000 + \frac{-20800}{1343}$$

$$= 1000 - 15 \cdot 488 \text{ approx.}$$

$$= 984 \cdot 512 \quad \textbf{Answer.}$$

(ii) Mean by step deviation method

$$\bar{x} = A + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h$$

$$= 1000 + \frac{-52}{1343} \times 400$$

$$= 1000 - 15 \cdot 488 \text{ approx.}$$

$$= 984 \cdot 512 \quad \textbf{Answer.}$$

Example 4. Find the mean of the following frequency distribution by step deviation method.

Class interval	**0 - 10**	**10 - 20**	**20 - 30**	**30 - 40**	**40 - 50**
Frequency	**7**	**10**	**15**	**8**	**10**

Solution : Calculation of Mean

(Here $A = 25$ and $h = 10$)

Class interval	x_i	f_i	$u_i = \frac{x_i - 25}{10}$	$f_i u_i$
0–10	5	7	–2	–14
10–20	15	10	–1	–10
20–30	25	15	0	0
30–40	35	8	1	8
40–50	45	10	2	20
		$\sum f_i = 50$		$\sum f_i u_i = 4$

$$\text{Thus Mean} = A + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$= 25 + \left(\frac{4}{50}\right) \times 10$$

$$= 25{\cdot}8 \text{ metre}$$ **Answer**

Exercise 17.4

Find the mean of following frequency distribution with the help of assumed mean (Q. 1 – 4):

1.

x	800	820	860	900	920	980	1000
f	7	14	19	25	20	10	5

2.

Weight (in kgs.)	60	61	62	63	64	65
No. of workers	5	8	14	16	10	7

3.

Expenditure (in ₹)	100-150	150-200	200-250	250-300
No. of workers	24	40	33	28
Expenditure (in ₹)	300–350	350–400	400–450	450–500
No. of workers	30	22	16	7

4.

Expenditure on water (in ₹)	15-20	20-25	25-30	30-35	35-40	40-45
No. of houses	7	5	7	8	9	11
Expenditure on water (in ₹)	45-50	50-55	55-60	60-65	65-70	
No. of houses	7	5	4	4	3	

5. Find the mean of the following distribution by taking assumed mean as 25.

Class interval	0–10	10–20	20–30	30–40	40–50
f	6	10	13	7	4

6. In the following table, the age distribution of patients of a disease in a particular year is given. Find the average age (in years) per patient.

Age (in years)	5–14	15–24	25–34	35–44	45–54	55–64
No.of patient	6	11	21	23	14	5

7. Find the mean form the following frequency distribution.

Class interval	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	10	25	28	12	10	15

17.10 Median :

If n values of a variable are arranged in ascending or descending order, then the value of the middle term of the arranged series is called the median. If the number of terms is odd then only one term i.e. $\left(\frac{n+1}{2}\right)$th term will be the middle term. But if the number of terms is even then two terms $\left(\frac{n}{2}\text{th}\right)$ and $\left(\frac{n}{2}+1\right)$th will be in the middle and mean of these two terms will be the median. For example, marks obtained by 9 students of the class A are 10, 15, 12, 18, 17, 18, 15, 16, 19 and marks obtained by 8 students of the class B are 19, 15, 18, 14, 17, 16, 15, 15. Arranging these in ascending order :

A : 10 12 15 15 16 17 18 18 19

B : 14 15 15 15 16 17 18 19

Median of A = Middle term (5^{th} term) = 16 marks

Median of B = Mean of middle terms $\left(\frac{4^{th}\text{ term} + 5^{th}\text{ term}}{2}\right)$

$= \frac{15+16}{2} = 15{\cdot}5$ Marks.

17.11 Median from ungrouped or individual series :

Working steps :

Step I. Arranging the n values of variate x in ascending or descending order like

$x_1, x_2, x_3, \ldots, x_n$

Steps II. Now obtain the median by the following formula :

$$\text{Median } (M) = \begin{cases} \frac{n+1}{2}\text{th term i.e. } x_{\frac{n+1}{2}} \text{ if } n \text{ is odd} \\ \text{Mean of the } \frac{n}{2}\text{th and } \frac{n}{2}+1\text{th terms} \\ \text{i.e. } \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}, \text{if } n \text{ is even} \end{cases}$$

Illustrative Examples

Examples 1. Find the median from following data :

25, 34, 31, 23, 22, 26, 35, 28, 20, 32

Solution : Arranging the given data in ascending order :

S. No.	1	2	3	4	5	6	7	8	9	10
Value of variate (x)	20	22	23	25	26	28	31	32	34	35

Here, number of term $(n) = 10$ (even number)

$$\text{Therefore Median } (M) = \frac{\frac{10}{2}\text{th term} + \left(\frac{10}{2}+1\right)\text{th term}}{2}$$

$$= \frac{x_5 + x_6}{2} = \frac{26+28}{2} = 27 \quad \textbf{Answer}$$

Example 2. Find the median of the following values of Variate :

37, 31, 42, 43, 46, 25, 39, 45, 32

Solution : Arranging the given data in ascending order

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
25	31	32	37	39	42	43	45	46

$\because$ 9 values of x in ascending order are $x_1, x_2, \ldots, x_9$

$$\text{Therefore Median } (M) = \left(\frac{9+1}{2}\right)\text{ th term} = x_5 = 39$$

Example 3. The values of (x) in ascending order are as follows :

8 11 12 16 $16+x$ 20 25 30

if the median is 18, then find the value of x.

Solution : Here total number of variate is 8 therefore the two terms are 16 and $16+x$ respectively.

$$\text{Therefore median } = \frac{(16)+(16+x)}{2} = 18 \text{ (Given)}$$

$$\Rightarrow \quad 32+x = 36 \quad \Rightarrow x = 4$$

Therefore value of $x = 4$ **Answer**

17.12 Median from ungrouped frequency distribution :

The working rule to find the median form ungrouped frequency distribution is as follows :

Working steps :

Steps I. Preparation of cumulative frequency table.

Steps II. Obtain the value of $N/2$, where $N = \sum f_i$

Steps III. Variate value of the cumulative frequency just greater than or equal to $N/2$ will be the median.

Illustrative Examples

Example : Find the median form the following frequency distribution :

x :	1	2	3	4	5	6	7	8	9
f :	8	10	11	16	20	25	15	9	6

Solution : Calculation of median

x_i	f_i	$c.f.$
1	8	8
2	10	18
3	11	29
4	16	45
5	20	65
6	25	90
7	15	105
8	9	114
9	6	120

$N = 120$

Here $\frac{N}{2} = 60.$

The term whose cumulative frequency is just greater than 60, i.e. the term for which the value cumulative frequency 65 is 5. **Answer**

Therefore median = 5

Exercise 17.5

1. Find the median of the following variates :

   25, 34, 33, 13, 20, 26, 36, 28, 19, 34

2. Find median of the following data.

   19, 25, 59, 48, 35, 31, 30, 32, 51.

   If 25 is replaced by 52, then find new median.

3. The marks obtained by students of a class are given below. Find their median.

Marks Obtained	15	20	25	30	35	40	45	50
No. of students	2	8	16	26	20	16	7	4

4. The number of children in 100 families are as follows, find their median :

No. of children	0	1	2	3	4	5	6
No. of families	10	35	27	17	6	3	2

5. Find the median of the following frequency distribution.

x	20	25	30	35	40	45	50	55
f	14	28	33	30	20	15	13	7

17.13 Median from grouped frequency distribution :

Working steps for finding median from grouped frequency distribution are as follows :

Step I. To prepare the cumulative frequency table.

Step II. After calcuating $N/2$, find the class interval of cumulative frequency just greater than it.

Step III. Now to find the median for this class interval with the help of following formula :

$$\text{Median } (M) = \ell + \left(\frac{\frac{N}{2} - C}{f_i} \right) \times h$$

Where ℓ = lower limit of the median class

N = Total frequency $\left(\sum f_i\right)$

C = Cumulative frequency of class preceding the median class

h = interval of the median class

f = frequency of the median class

This method will be clear form the following example.

Example 1. Find median of following frequency distribution :

Class interval	**10 - 25**	**25 - 40**	**40 - 55**	**55 - 70**	**70 - 85**	**85 - 100**
f_i	**6**	**20**	**44**	**26**	**3**	**1**

Solution : Cumulative frequency table

Class interval	f_i	Cummulative frequency (c)
10-25	6	6
25-40	20	26
40-55	44	70
55-70	26	96
70-85	3	99
85-100	1	100

$N = 100$

Here, $\frac{N}{2} = 50 \Rightarrow$ class interval of median is "$40 - 55$"

Here, $\ell = 40,\ C = 26,\ h = 15$ and $f = 44.$

$\therefore$ The median $(M) = \ell + \frac{\left(\frac{N}{2} - c\right)}{f} \times h$

$$= 40 + \frac{(50 - 26)}{44} \times h$$

$$= 40 + \frac{24}{44} \times 15$$

$$= 48 \cdot 18$$

Therefore the median is 48·18, **Answer.**

17.14 Merits and Demerits of Median :

Merits of Median :

(i) It is the best in study of qualitative characterstics.

(ii) Finding median is easy and convenient. Sometimes it can be known by inspection.

(iii) It does not need all data for its calculation.

(iv) Median is always fixed and clear.

(v) Extreme values have no affect on it, while they have more affect on mean.

Demerits of Median :

(i) When values are distruibuted irregularly, median does not represent the representative number and gives absurd conclusion. For example a student obtained 40, 30, 5, 3, 2 marks in 5 subjects. Here the median is 5 which does not properly represent the data.

(ii) When equal importance is to be given to extreme values then the measure of central tendency is not suitable.

(iii) It can not be use in mathematical operations.

Exercise 17.6

1. The marks obtained by 100 students are given in following table. Find median from these.

Marks obtained	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	6	20	44	26	3	1

2. The marks of students of a class are given in following frequency distribution. Find the median form these :

Marks obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of students	4	28	42	20	6

Find the median form following frequency distribution : [Q. 3 to 4]

3.

Class interval	0 – 10	10 – 20	20 – 30	30 – 40
f_i	2	6	10	17
Class interval	40 – 50	50 – 60	60 – 70	70 – 80
f_i	30	15	10	10

4.

Class interval	0 – 8	8-16	16-24	24-32	32-40	40-48
f_i	42	30	50	22	8	5

17.15 Mode :

The value of a series whose frequency is maximum is called mode. The terms of the series have the maximum tendency to accumulate around the mode.

17.16 Calculation of Mode :

(i) Mode form Individual Series or Discrete Series

From this series we first prepare frequency distribution table. The value with maximum frequency is called mode. This can be easily understood by the following example.

Marks obtained	0	1	2	3	4	5
No. of students	5	8	13	5	3	2

Here it is clear form frequency distribution that frequency of 2 is maximum i.e. 13, thus mode of distribution is 2 marks.

If distribution of frequencies are irregular or value of maximum frequencies are more than one, then it becomes difficult to find mode. In such situation mode is determined by the method of grouping. Here we shall study only regular distributed frequency distribution.

(ii) Mode from ungrouped frequency distribution :

Here in regular frequency distribution, the value whose frequency is maximum, the value of that term is mode.

Example : The marks obtained by some students are as follows, find their mode.

Marks obtained	30	31	32	33	34	35	36	37	38	39	40
No. of students	1	5	15	16	20	19	15	8	7	3	2

Solution : Here ferquency of marks 34 is maximum, i.e. 20.

Therefore Mode = 34 Marks

(iii) Mode from grouped frequency distribution :

Working rule to find mode from gruoped frequency distribution is as follows :

Step I. The class of grouped frequency distribution having maximum frequency is called the modal class interval. First we find modal class interval.

Step II. With the help of modal class we find the mode by using the followig formula :

$$\text{Mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where ℓ = lower limit of modal class

f_1 = frequency of modal class

f_0 = frequency of the class interval just preceeding the modal class interval

f_2 = frequency of the class interval just after the modal class interval

h = width of the modal class

Illustrative Examples

Example 1. Find mode from the following frequency distribution.

Class	10-25	25-40	40-55	55-70	70-85	85-100
f_i	6	20	44	26	3	1

Solution : Here maximum frequency is 44 of class '40-55'

So Mode class = 40 – 55

Again $\ell = 40$, $f_1 = 44$, $f_0 = 20$, $f_2 = 26$ and $h = 15$

$$\text{Mode according to formula} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 40 + \left(\frac{44 - 20}{88 - 20 - 26} \right) \times 15$$

$$= 48 \cdot 57$$

Therefore required Mode = 48·57 **Answer**

Exercise 17.7

1. Find the mode of the following distribution :

(i) 2 5 7 5 3 1 5 8 7 5

(ii) 2 4 6 2 6 6 7 8

(iii) 2·5 2·5 2·1 2·5 2·7 2·8 2·5

2. Find the Mode of following frequency distribution :

(i)

x	3	4	5	6	7	8
f	2	4	6	3	2	1

(ii)

x	1·1	1·2	1·3	1·4	1·5	1·6
f	20	50	80	60	15	8

3. The number of members of 30 families of a village is given in the following table. Find their Mode.

No. of members	2	3	4	5	6	7	8
No. of families	1	2	4	6	10	3	5

4. The Ages (in years) of 20 students of a class are as follows :

15 16 13 14 14 13 15 14 13 13
14 12 15 14 16 13 14 14 13 15

Find mode by representing these in frequency distribution.

5. The marks obtained by some students are given below, find the mode of marks obtained :

Marks obtained	10	20	30	40	50	60	70	80
No. of students	2	8	16	26	20	16	7	4

Find the mode form following frequency distribution :

6.

Class	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	3	7	16	12	9	5	3

7.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	12	14	10	8	6

8.

Marks obtained	20-30	30-40	40-50	50-60	60-70
No. of students	4	28	42	20	6

9.

Height (in cm)	52-55	55-58	58-61	61-64
No. of students	10	20	25	10

Important Points

1. Arithmetic mean $(\bar{x})$:

(i) Individual series : $(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n} x_i$

(ii) Ungrouped distribution : $(\bar{x}) = \frac{\sum f_i u_i}{\sum f_i}$

(iii) By assumed mean : $\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$ or $\bar{x} = A + \frac{\sum f_i u_i}{\sum f_i} \times h$

where assumed mean A, $d_i = x_i - A$ and $u_i = \frac{x_i - A}{h}$

2. Median (M)

(i) *Individual series :* Arranging values in ascending or descending order as $x_1, x_2, x_3, \ldots, x_n$

$$\text{Median } (M) = \begin{cases} x_{\frac{n+1}{2}}, & \text{if } n \text{ is odd number} \\ \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}, & \text{if } n \text{ is even number} \end{cases}$$

(ii) *Ungrouped frequency distribution :* The value from cumulative frequency table cumulative frequency is just greater than $N/2$.

(iii) *Grouped frequency distribution :* The class interval whose frequency is just greater than $N/2$ is median class interval and

$$\text{Median } (M) = \ell + \left(\frac{\frac{N}{2} - C}{f}\right) \times h$$

where ℓ = lower limit of median class interval

$N = \sum f_i$ i.e. Total frequencies

C = C.F. of class interval preceding the median class

h = width of the median class

f = frequency of the median class

3. Mode :

(i) *Individual series* : The term whose frequency is maximum.

(ii) *Ungrouped frequency distribution* : Value of the term with maximum frequency.

(iii) *Grouped frequency distribution* : The class of maximum frequency is called modal class.

and mode $(z) = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

where ℓ = lower limit of modal class

f_1 = frequency of modal class

f_0 =	frequency of the class interval just preceding the modal class interval
f_2 =	frequency of the class interval just after the modal class interval
h =	width of middle class

Miscellaneous Exercise 17

Four possible options of the following questions are given. Choose the correct option : [1-10]

1. Positional mean is :
   (A) Arthmetic mean (B) Geometric mean
   (C) Harmonic mean (D) Median
2. Mode value of any series is :
   (A) Middle value (B) Value whose frequency is maximum
   (C) Minimum frequency value (D) Limit value
3. The median of following series is :
   520, 20, 340, 190, 35, 800, 1210, 50, 80
   (A) 1210 (B) 520 (C) 190 (D) 35
4. The marks obtained by four students in statics are 53, 75, 42, 70. Mean of their marks is :
   (A) 42 (B) 64 (C) 60 (D) 56
5. A student secured 85, 87 and 83 marks in Mathematics, Physics and Chemistry respectively. The average marks of these subject is :
   (A) 86 (B) 84 (C) 85 (D) 85·5
6. If the arithmetic mean of 5, 7, 9, x is 9, then the value of x is :
   (A) 11 (B) 15 (C) 18 (D) 16
7. The median of the distribution 2, 3, 4, 7, 5, 1 is :
   (A) 3 (B) 4 (C) 2 (D) 20
8. The median of the distribution 1, 3, 2, 5,. 9 is :
   (A) 3 (B) 4 (C) 2 (D) 20
9. The mode of the distribution 3, 5, 7, 4, 2, 1, 4, 3, 4
   (A) 7 (B) 4 (C) 3 (D) 1
10. The no. of students of a school according to their ages are as follows :

Age (in years)	8	9	10	11	12	13	14	15	16	17
No. of students	15	25	40	36	41	37	20	13	5	3

Their mode will be :
(A) 41 (B) 12 (C) 3 (D) 17

Find the airthmetic mean of the following distribution [Q. 11-14]

11.

x	5	6	7	8	9
f	4	8	14	11	3

12.

x	10	15	17	20	22	30	35
f	5	10	2	8	3	6	6

13.

x	19	21	23	25	27	29	31
f	13	15	16	18	16	15	13

14.

x	1	2	3	4	5	6
f	45	25	19	8	2	1

15. Find the arithmetic mean from following frequency distribution :

Weight (in kg.)	40-44	44-48	48-52	52-56	56-60	60-64
No. of persons	5	6	5	9	3	2

Find median of the following distribution : [Q. 16-17]

16.

x	0·1	0·2	0·3	0·4	0·5	0·6	0·7
f	30	60	20	40	10	50	35

17.

Measure of shoes	4·5	5·0	5·5	6·0	6·5	7·0	7·5	8·0	8·5	9·0
No. of shoes	1	2	4	5	15	30	60	95	82	75

18. The run scored by players of a cricket tean is as follows :

57, 17, 26, 91, 115, 26, 83, 41, 57, 0, 26.

Find their A.M. median and mode.

Find mode of following distributions : [Q. 19-20]

19.

20.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	3	15	24	8	5

21. Define Arithmetic mean and given their two demerits.

22. Give main uses of median.

23. Give difference between arithmetic mean and weighted arithmetic mean.

Answers

Exercise 17.1

1. 52 Marks 2. ₹1735 3. 6 4. 43.9 run 5. 22 Marks 6. 400 Books
7. 52.75 kg. 8. 26 9. Mean 24 +3 10. ₹ 2200

Exercise 17.2

1. 7·07 2. 7·55 3. 0·34 4. 0·55 5. 2 6. 23·9 7. 3 8. 76 व 38

Exercise 17.3

1. 26·33 approx 2. 15·45 3. 145·71 4. 49·5 5. 68·2 6. 1457·14

Exercise 17.4

1. 891·2 2. 62·65 3. 266·25 4. 39·57 5. 23·25 6. 34·87 7. 68·2

Exercise 17.5

1. 27 2. 32 *and* 35 3. 30 4. 2 5. 35

Exercise 17.6

1. 45·45 2. 24·29 3. 45 4. 17·04

Exercise 17.7

1. (i) 5 (ii) 6 (iii) 2·5 2. (i) 5 (ii) 1·3 3. 6 4. 14 5. 40
6. 23·46 7. 23·33 8. 43.89 9. 58·75

Miscellaneous Exercise 17

1. (D) 2. (B) 3. (C) 4. (C) 5. (C) 6. (B) 7. (D)
8. (A) 9. (B) 10. (B) 11. 7·025 12. 21·25 13. 25 14. 2
15. 50·67 16. 0.4 17. 8 18. 49, 41 and 26 19. 26 20. 47·2
21. 22. 23. 24. 25. 61.5 and 62.4

❑