Sample Question Paper - 2
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
3. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
4. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

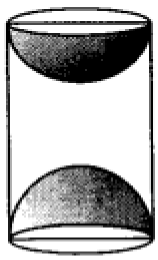
Section A

1. Find the values of k for which the equation has real and equal roots: **[2]**
$x^2 + k(2x + k - 1) + 2 = 0.$

OR

Find the roots of the quadratic equation $4x^2 - 4px + (p^2 - q^2) = 0$.

2. A wooden article was made by scooping out a hemisphere from each end of a cylinder, as shown in the figure. If the height of the cylinder is 20 cm and its base is of diameter 7 cm, find the total surface area of the article when it is ready. **[2]**



3. Given below is a cumulative frequency distribution showing the marks secured **[2]**

Marks	Number of students
Below 20	17
Below 40	22
Below 60	29
Below 80	37
Below 100	50

Form the frequency distribution table for the above data.

4. Write the first five terms of the sequence whose nth term is: $A_n = \frac{3n-2}{5}$ **[2]**

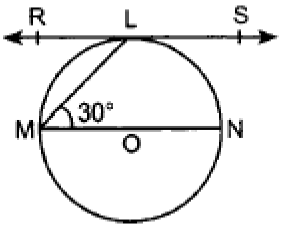
5. The marks in Science of 80 students of class X are given below. Find the mode of the marks obtained by the students in Science. **[2]**

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	5	16	12	13	20	5	4	1	1

6. Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that radius of the circle is 7 cm. **[2]**

OR

In the given figure, RS is the tangent to the circle at L and MN is a diameter. If ∠NML = 30°, determine ∠RLM.



Section B

7. If the ratio of the sums of first n terms of two A.P.'s is (7n + 1) : (4n + 27), find the ratio of their m^{th} terms. **[3]**

8. The shadow of a tower standing on a level ground is found to be 30m longer when the sun's altitude is 30°, than when it was 60°. Find the height of the tower. [Take V3 = 1.732.] **[3]**

OR

The angle of elevation of the top of a tower at a point on the level ground is 30^o. After walking a distance of 100 m towards the foot of the tower along the horizontal line through the foot of the tower on the same level ground the angle of elevation to the top of the tower is 60^o, find the height of the tower.

9. From an external point P,two tangents,PA and PB are drawn to a circle with centre O.At one point E on the circle tangent is drawn which intersects PA and PB at C and D respectively. If PA=10 cm,find the perimeter of the triangle PCD. **[3]**

10. Solve quadratic equation by factorization method: **[3]**

$4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$

Section C

11. Draw a circle of radius 3 cm. Take a point at a distance of 5.5 cm from the centre of the circle. From point P, draw two tangents to the circle. **[4]**

OR

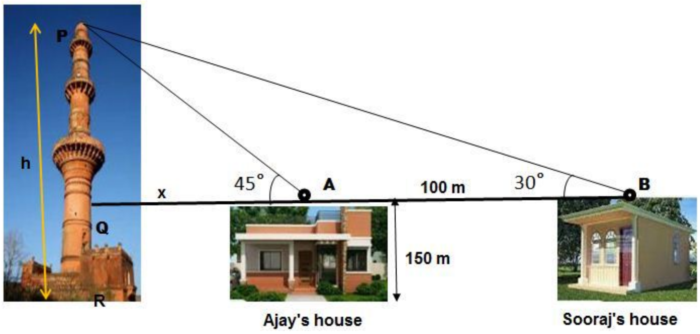
Draw a line segment of length 6 cm and divide it in the ratio 3 : 2.

12. The median of the following data is 525. Find the values of x and y, if the total frequency is 100. **[4]**

Class interval	Frequency
0-100	2
100-200	5

200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

13. The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^{o} and 30^{o} respectively as shown in the figure. **[4]**

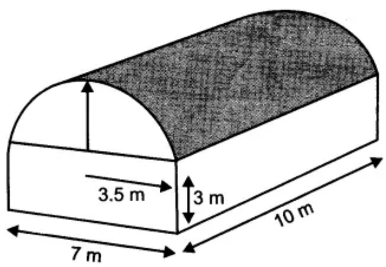


By using the above given information answer the following questions:

i. Find the height of the tower.

ii. What is the distance between the tower and the house of Sooraj?

14. A godown building is in the form as shown in Fig. The vertical crosssection parallel to the width side of the building is a rectangle $7m \times 3m$, mounted by a semicircle of radius 3.5 m. The inner measurements of the cuboidal portion of the building are $10m \times 7m \times 3m$. **[4]**



i. Find the volume of the godown. (Take π = 22/7)

ii. Find the total interior surface area excluding the floor (base).

Solution

MATHEMATICS BASIC 241

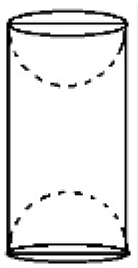
Class 10 - Mathematics

Section A

1. The given equation is $x^2 + k(2x + k - 1) + 2 = 0$
$\Rightarrow x^2 + 2kx + k(k-1) + 2 = 0$
So, a = 1, b = 2 k, c = k(k - 1) + 2
We know $D = b^2 - 4ac$
$\Rightarrow D = (2k)^2 - 4 \times 1 \times [k(k-1) + 2]$
$\Rightarrow D = 4k^2 - 4[k^2 - k + 2]$
$\Rightarrow D = 4k^2 - 4k^2 + 4k - 8$
$\Rightarrow D = 4k - 8 = 4(k - 2)$
For equal roots, D = 0
Thus, 4(k - 2) = 0
So, k = 2.

OR

we have to find the roots of the quadratic equation $4x^2 - 4px + (p^2 - q^2) = 0$.
Here, a = 4, b = - 4p, c = $(p^2 - q^2)$
The roots are given by the quadratic formula
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\therefore \quad x = \frac{4p \pm \sqrt{16p^2 - 4 \times 4 \times (p^2 - q^2)}}{2 \times 4}$
$= \frac{4p \pm \sqrt{16p^2 - 16p^2 + 16q^2}}{8}$
$= \frac{4p \pm 4q}{8}$
Therefore, the roots are $\frac{p+q}{2}, \frac{p-q}{2}$.

2. 

Given, Height of cylinder $= 20cm$
And diameter $= 7cm$ and then radius $= 3.5cm$
Total surface area of article = (curved surface area of the cylinder) + 2 (curved surface area of hemisphere)
$= \left[2\pi rh + 2 \times \left(2\pi r^2\right)\right] \; sq.\ units$
$= \left[\left(2 \times \frac{22}{7} \times \frac{7}{2} \times 20\right) + \left(4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)\right] \text{cm}^2$
= $(440 + 154)$ cm^2
$= 594$ cm^2

3.

Classes	Cumulative frequency	**Frequency**
$0 - 20$	17	17
$20 - 40$	22	22 - 17 = 5
$40 - 60$	29	29 - 22 = 7
$60 - 80$	37	37 - 29 = 8
$80 - 100$	50	50 - 37 =13
Total		**50**

4. Here we have, $A_n = \frac{3n-2}{5}$

Put n = 1

$A_1 = \frac{3(1)-2}{5} = \frac{3-2}{5} = \frac{1}{5}$

Put n = 2

$A_2 = \frac{3(2)-2}{5} = \frac{6-2}{5} = \frac{4}{5}$

Put n = 3

$A_3 = \frac{3(3)-2}{5} = \frac{9-2}{5} = \frac{7}{5}$

Put n = 4

$A_4 = \frac{3(4)-2}{5} = \frac{12-2}{5} = \frac{10}{5} = 2$

Put n = 5

$A_5 = \frac{3(5)-2}{5} = \frac{15-2}{5} = \frac{13}{5}$

5.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	5	16	12	13	20	5	4	1	1

Here the maximum frequency is 20, so 50 - 60 is the modal class

l = 50

h = 60 - 50 = 10, f = 20, f_1 = 13, f_2 = 5

Mode = l + $\frac{f-f_1}{2f-f_1-f_2} \times$ h

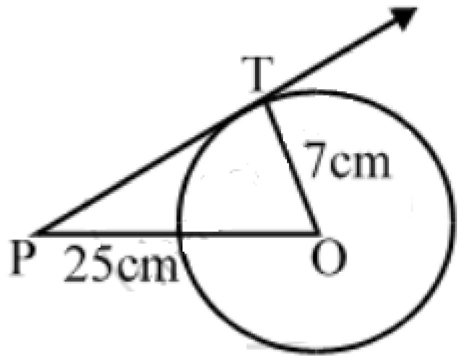
= 50 + $\frac{20-13}{2\times20-13-5} \times 10$

= 50 + $\frac{70}{22}$

= 50 + 3.18

= 53.18

6.



Let O is the centre of the circle and P is a point such that OP = 25 cm and PT is the tangent to the circle.

$OT =$radius $= 7cm$

In $\triangle$OTP, we have $\angle$T = 90°

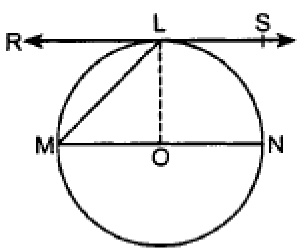
$\mathrm{OP}^2 = \mathrm{OT}^2 + \mathrm{PT}^2$ [By using Pythagorean theorem]

$\Rightarrow (25)^2 = 7^2 + \mathrm{PT}^2 \Rightarrow \mathrm{PT}^2 = 625 - 49 = 576$

$\Rightarrow PT = 24cm$

OR

Given,



Construction: Join OL

OL $\perp$ RS.

Also OL = OM

∴ ∠OML = ∠OLM

⇒. ∠OLM = 30°

⇒ ∠RLM = 90° - 30° = 60°

Section B

7. Let a, and A be the first terms and d and D be the common difference of two A.Ps

Then, according to the question,

$\frac{S_n}{S_n'} = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{7n+1}{4n+27}$

or, $\frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$

or, $\frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27}$

Putting, $\frac{n-1}{2} = m-1$

$n-1 = 2m-2$

$n = 2m-2+1$

or, $n = 2m-1$

$\frac{a+(m-1)d}{A+(m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27}$

$\frac{a+(m-1)d}{A+(m-1)D} = \frac{14m-7+1}{8m-4+27}$

$\frac{a+(m-1)d}{A+(m-1)D} = \frac{14m-6}{8m+23}$

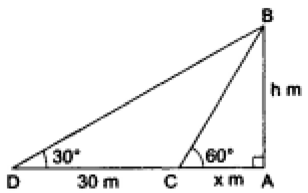
Hence, $\frac{a_m}{A_m} = \frac{14m-6}{8m+23}$

8. Let AB be the height of the tower, and AC and AD be the lengths of the shadows at the angles 60 and 30 respectively.

Clearly, $\angle ACB = 60^\circ$ and $\angle ADB = 30^\circ$.

Let AB = h m, and

AC = x m.



From right ΔCAB,we have

$\frac{AC}{AB} = \cot 60^\circ = \frac{1}{\sqrt{3}} \Rightarrow \frac{x}{h} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad x = \left(h \times \frac{1}{\sqrt{3}}\right) = \frac{h}{\sqrt{3}}$.......(i)

From right ΔDAB, we have

$\frac{AD}{AB} = \cot 30^\circ = \sqrt{3} \Rightarrow \frac{x+30}{h} = \sqrt{3}$

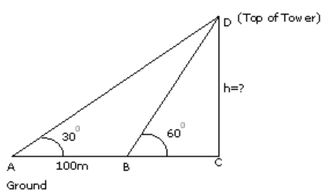
$\Rightarrow \quad x = (\sqrt{3}h - 30)$. ....(ii)

Equating the values of x from (i) and (ii), we get

$\frac{h}{\sqrt{3}} = (\sqrt{3}h - 30) \Rightarrow h = 3h - 30\sqrt{3}$

$\Rightarrow 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3} = (15 \times 1.732) = 25.98$

OR

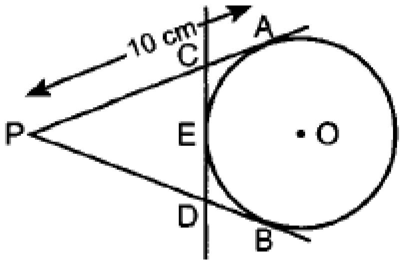


In $\triangle BCD$, $\frac{h}{x} = \tan 60^\circ = \sqrt{3}$ (BC=x)

$h = \sqrt{3x}$..............(i)

In $\triangle ACD, \frac{h}{100+x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$
$\Rightarrow h\sqrt{3} = 100 + x$
$\Rightarrow h\sqrt{3} = 100 + \frac{h}{\sqrt{3}}$
$\Rightarrow h\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = 100$
$\Rightarrow h\left[\frac{3-1}{\sqrt{3}}\right] = 100$
$\Rightarrow h = \frac{100\sqrt{3}}{2} = 50\sqrt{3} = 50 \times 1.732 = 86.6\text{m}$

9. Given,

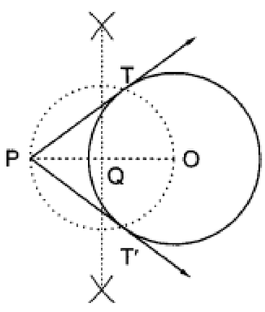


PA = 10 cm.
PA = PB [If P is external point] ...(i)[From an external point tangents drawn to a circle are equal in length]
If C is external point, then CA = CE
If D is external point, then
DB = DE ...(ii)
Perimeter of triangle $\triangle$PCD
= PC + CD + PD
= PC + CE + ED + PD
= PC + CA + DB + PD
= PA + PB
= PA + PA
=2 PA
$= 2 \times 10 = 20\text{cm}$ [From (i)]

10. We have,
$4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$
$\Rightarrow 4x^2 - (2a^2 + 2b^2)x + a^2b^2 = 0$
$\Rightarrow 4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$
$\Rightarrow 2x(2x - a^2) - b^2(2x - a^2) = 0$
$\Rightarrow (2x - a^2)(2x - b^2) = 0$
$\Rightarrow (2x - a^2) = 0$ or, $(2x - b^2) = 0$
$\Rightarrow \quad x = \frac{a^2}{2}$ or, $x = \frac{b^2}{2}$

Section C

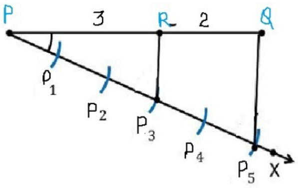
11. In order to construct the required tangents, we follow the following steps:
STEP I Take a point O in the plane of the paper and draw a circle of radius 3 cm.
STEP II Mark a point P at a distance of 5.5 cm from the centre O and join OP.
STEP III Draw the right bisector of OP, intersecting OP at Q.



STEP IV Taking Q as centre and OQ - PQ as radius, draw a circle to intersect the given circle at T and T'.
STEP V Join PT and PT' to get the required tangents.

OR

Now we will divide the line segment PQ in the ratio 3 : 2
Step 1: At first we will draw a line segment of length 6cm i.e. PQ
Step 2: Draw any other ray PX, making an angle less than 90° (acute angle)
Step 3: Mark the 5 (because 3 + 2 = 5) arcs in a line PX, named P_1, P_2, P_3, P_4, P_5
such that,
$PP_1 = P_2 = P_3 = P_4 = P_5$
Step 4: Join QP_5
Step 5: Draw a parallel line QP_5 from P_3. That intersect the PQ line at point R



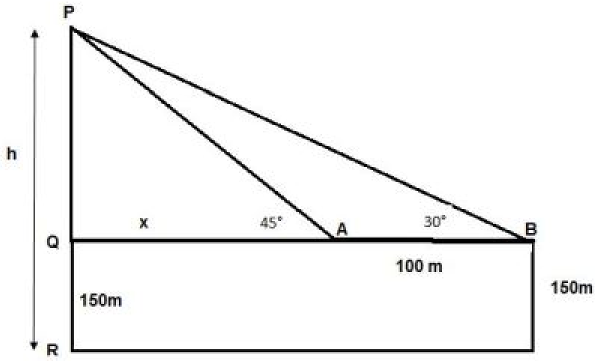
Thus, PR : RQ = 3 : 2

12.

Class intervals	Frequency (f)	Cumulative frequency (cf/F)
0-100	2	2
100-200	5	7
200-300	x	7 + x
300-400	12	19 + x
400-500	17	36 + x
500-600	20	56 + x
600-700	y	56 + x + y
700-800	9	65 + x + y
800-900	7	72 + x + y
900-1000	4	76 + x + y
		Total = 76 + x + y

We have,
$N = \Sigma f_i = 100$
$\Rightarrow$ 76 + x + y = 100
$\Rightarrow$ x + y = 24
It is given that the median is 525. Clearly, it lies in the class 500 - 600
$\therefore$ l = 500, h = 100, f = 20, F = 36 + x and N = 100
Now, Median = $l + \frac{\frac{N}{2} - F}{f} \times h$
$\Rightarrow 525 = 500 + \frac{50-(36+x)}{20} \times 100$
$\Rightarrow$ 525 - 500 = (14 - x)5
$\Rightarrow$ 25 = 70 - 5x
$\Rightarrow$ 5x = 45
$\Rightarrow$ x = 9
Putting x = 9 in x + y = 24, we get y = 15
Hence, x = 9 and y = 15

13. The above figure can be redrawn as shown below:



i. Let PQ = y

In ΔPQA,

tan 45 = $\frac{PQ}{QA} = \frac{y}{x}$

1 = $\frac{y}{x}$

x = y .....(i)

In ΔPQB,

$\tan 30 = \frac{PQ}{QB} = \frac{PQ}{x+100} = \frac{y}{x+100} = \frac{x}{x+100}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+100}$

$x\sqrt{3} = x + 100$

$x = \frac{100}{\sqrt{3}-1} = 136.61 \text{ m}$

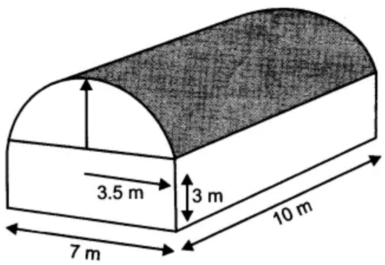
From the figure, Height of tower h = PQ + QR

= x + 150 = 136.61 + 150 = 286.61 m

ii. Distance of Sooraj's house from tower = QA + AB

= x + 100 = 136.61 + 100 = 236.61 m

14.



Since the top of the building is in the form of half of the cylinder of radius 3.5 m and height 10 m., split along the diameter.

Dimensions of the cuboidal portion of the building are $10m \times 7m \times 3m$.

Let us suppose that V be the volume of the godown.

So, V = Volume of the cuboid + $\frac{1}{2}$(Volume of the cylinder)

$\Rightarrow V = \left\{10 \times 7 \times 3 + \frac{1}{2}\left(\frac{22}{7} \times 3.5 \times 3.5 \times 10\right)\right\} \text{m}^3$

= 210 + 192.5

= 402.5 m^3

Let S be the total interior surface area excluding the base floor.

So, S = Area of four walls + $\frac{1}{2}$(Curved surface area of cylinder) + 2(Area of the semi-circles)

= $2(10 + 7) \times 3 + \frac{1}{2}(2 \times \frac{22}{7} \times 3.5 \times 10) + 2(\frac{1}{2} \times \frac{22}{7} \times 3.5^2) = 250.5 \text{ m}^2$