Sample Paper – 5
Mathematics
Class XI Session 2022-23

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A

(Multiple Choice Questions)
Each question carries 1 mark.

1. The radius of the circle whose arc length 15π cm makes an angle of $\frac{3\pi}{4}$ radian at the centre is:
(a) 10 cm (b) 20 cm
(c) $11\frac{1}{4}$ cm (d) $22\frac{1}{2}$ cm 1

2. If $a + ib = \frac{(p^2+1)^2}{2p-i}$, then $a^2 + b^2$ is equal to:
(a) $\frac{(p^2+1)^4}{4p^2+1}$ (b) $\frac{(p+1)^2}{4p+1}$
(c) $\frac{(p-1)^2}{(4p-1)^2}$ (d) None of these 1

3. The roster form of the following set is a set of integers between -5 and 5 is:
(a) {– 4, –3, –2, –1, 0, 1, 2, 3, 4}
(b) {–5, –4 . . . 4, 5}
(c) {0, 1, 2, 3, 4, 5}
(d) None of these 1

4. If P and Q are the coefficients of a^r and a^{n-r} respectively in the expansion of $(1 + a)^n$, then:
(a) P = Q
(b) P ≠ Q
(c) P = λQ for some λ
(d) None of these 1

5. If $|3 - 6x| \geq 9$, then $x \in$
(a) $(-\infty, -1) \cup (3, \infty)$
(b) $(-\infty, 1] \cup (2, \infty)$
(c) $(-\infty, -1) \cup (0, \infty)$
(d) $(-\infty, -1] \cup [2, \infty)$ 1

6. Let A = {a, b, c}, B = {1, 3, 5}. If relation R from A to B is given by R = {(a, 3), (b, 5), (c, 3)}. Then, R^{-1} is:
(a) {(3, a), (5, b), (3, c)}
(b) {(1, a), (2, b), (3, c)}
(c) {(a, 3), (b, 2)}
(d) None of these 1

7. The value of y, if the distance between the points P(0, 0)and Q(6, y)is 10 units is:
(a) 3, –3 (b) 8, –8
(c) 0 (d) None of these 1

8. The point A(–4, –3, –2) is present in:

(a) V octant (b) VI octant

(c) VII octant (d) VIII octant 1

9. The value of $(2 + i)^4 - (2 - i)^4$ is:

(a) 48 i (b) 48

(c) –48 i (d) –48 1

10. The value of $\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$ is equal to:

(a) 1 (b) 2

(c) 3 (d) 4 1

11. If A and B are two disjoint sets, then $n(A \cap B)$ is equal to:

(a) 0

(b) $n(A) + n(B) - n(A \cap B)$

(c) $n(A) + n(B) + n(A \cap B)$

(d) $n(A)\, n(B)$ 1

12. The geometric mean of 2 and 8 is:

(a) 4 (b) 6

(c) 7 (d) 5 1

13. The value of $\lim_{x \to 0} \left(\frac{\tan^2 3x}{x^2} \right)$ is:

(a) 0 (b) 3

(c) ∞ (d) 9 1

14. If $\cos x = \frac{-1}{2}$ and $0 < x < 2\pi$, then the solutions are:

(a) $x = \frac{\pi}{3}, \frac{4\pi}{3}$ (b) $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

(c) $x = \frac{2\pi}{3}, \frac{7\pi}{3}$ (d) $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$ 1

15. Find the distance between the points P(0, 3, 4) and Q(4, 1, 0).

(a) 5 units (b) 4 units

(c) 6 units (d) 2 units 1

16. If $y = (\sin x + \tan x)$, then find $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$.

(a) $\frac{\sqrt{3}}{2} + \sqrt{3}$ (b) $\sqrt{\frac{3}{2} + \frac{4}{3}}$

(c) $\frac{\sqrt{3}}{2} + 4$ (d) $\frac{9}{2}$ 1

17. The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal, is:

(a) $\left\{-1, \frac{4}{3}\right\}$ (b) $\left\{-1, \frac{2}{3}\right\}$

(c) $\left\{-1, \frac{3}{2}\right\}$ (d) $\{-2, 2\}$ 1

18. The given inequality for the real x: $4x + 3 < 5x + 7$. The solution set is:

(a) (4, ∞) (b) (2, ∞)

(c) (–4, ∞) (d) (–2, ∞) 1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A): A slope of line $3x - 4y + 10 = 0$ is $\frac{3}{4}$.

Reason (R): x-intercepts and y-intercepts of $3x - 4y + 10 = 0$ respectively are $\frac{-10}{3}$ and $\frac{5}{2}$. 1

20. Let $\sec\theta + \tan\theta = m$, where $0 < m < 1$.

Assertion (A): $\sec\theta = \frac{m^2 + 1}{2m}$ and $\sin\theta = \frac{m^2 - 1}{m^2 + 1}$

Reason (R): θ lies in the third quadrant. 1

SECTION - B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. If X = {5, 6, 7, 8}, Y = {7, 8, 9, 10}, Z = {3, 4, 5, 6}. Find:

(A) $\{(X \cap Y) \cup Z\}$

(B) $\{(X \cup Y) \cap Z\}$

OR

Show that if $A \cup B = A \cap B$ it implies that A = B. 2

22. If $^{12}P_{x+1} > 2\ ^{12}P_x$, then find the set of values of x. 2

23. Find the radius of the circle in which a central angle of 30° intercepts an arc of length 66 cm. (Use $\pi = \frac{22}{7}$) 2

24. Solve the following inequation:
$3x + 17 \leq 2(1 - x)$.

OR

Solve $-15x > 45$, when
(A) x is a natural number.
(B) x is an integer. 2

25. Let A = {1, 3, 5, 7} and B = {2, 4, 6, 8} be two sets and let R be a relation from A to B defined by the phrase "$(x, y) \in R \Leftrightarrow x > y$". Find domain and range of R. 2

SECTION - C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Find out which one is greater $99^{50} + 100^{50}$ or 101^{50}. 3

27. Find the values of k, such that the line $x\cos\theta + y\sin\theta - k = 0$ touches the circle $x^2 + y^2 - 2ax\cos\theta - 2ay\sin\theta = 0$.

OR

If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$, then find the value of m. 3

28. One card is drawn from a pack of 52 cards. Find the probability that the card is:
(A) a face card.
(B) not a diamond.
(C) an ace. 3

29. Find real value of x and y, if

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

OR

Find the number of non-zero integral solutions of the equation $|3^{1/2} - i|^x = 4^x$. 3

30. Find the domain of the function $f(x)$ defined by $f(x) = \sqrt{4 - x} + \frac{1}{\sqrt{x^2 - 1}}$. 3

31. Find the value of $m\sin x + n\cos x$, if $\tan\frac{x}{2} = \frac{m}{n}$.

OR

Prove that:

$$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}.$$ 3

SECTION - D

(This section comprises of long answer-type questions (LA) of 5 marks each.)

32. Evaluate: $\lim_{x \to 3} \frac{x^3 - 7x^2 + 15x - 9}{x^4 - 5x^3 + 27x - 27}$. 5

33. If an integer from 1 to 1000 is chosen at random, then find the probability that the integer is a multiple of 2 or a multiple of 9. 5

34. If A is the arithmetic mean and G_1, G_2 be two geometric means between any two numbers, then prove that:

$$2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$$

OR

If p^{th}, q^{th} and r^{th} terms of both an A.P. and G.P. are a, b and c respectively, then show that:
$a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1$ 5

35. Find the equation of the ellipse whose centre is at origin and the major axis passes through the points (–3, 1) and (2, –2).

OR

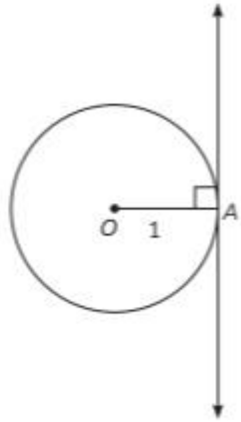
If the eccentricity of an ellipse is 5/8 and the distance between its foci is 10, then find latus rectum of the ellipse. 5

SECTION - E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (A), (B), (C) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case-Study 1:**
Consider a unit circle with centre O. Let A be any point on the circle. Consider OA as the initial side of an angle. Then the length of an arc of the circle will give the radian measure of the angle which the arc will subtend at the centre of the circle.

A circle subtends an angle at the centre whose radian measure is 2π and its degree measure is 360°.



(A) Convert 240° into radian measure. 1

(B) A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second? 1

(C) Convert $(1.2)^c$ into degree measure.

OR

Find the radius of the circle in which a central angle of 45° intercepts an arc of 132 cm. $\left(\text{Use } \pi = \frac{22}{7}\right)$ 2

37. Case-Study 2:

Sumit works at a book shop. While arranging some books on the book shelf, he observed that there are 5 History books, 3 Mathematics books and 4 Science books which are to be arranged on the shelf.

(A) In how many ways can he select either a History or a Maths book? 1

(B) If he selects 2 History books, 1 Maths book and 1 Science book to arrange them, then find the number of ways in which selection can be made. 1

(C) Find the number of ways, if the books of the same subject are put together.

OR

If we are given the number of selection of books are ${}^5P_1 \times {}^3P_2 \times {}^4P_2$, then in which manner the arrangement is? 2

38. Case-Study 3:

As Mr Anurag Thakur, Sports Minister has ordered to popularise the game of 100 m race among school students to develop their physical ability, Mr Gopi, Physical Education Teacher of a reputed CBSE school has decided to conduct an Inter School running tournament in his school premises after proper drawing of fixtures.

A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in Seconds)	0–20	20–40	40–60	60–80	80–100
No. of Students	8	10	13	6	3

(A) Find the mean time taken by a student to finish the race. 2

(B) Find the mean deviation from mean. 2

SOLUTION

SECTION - A

1. *(b) 20 cm*

Explanation: Arc length = 15π cm,

angle

$$\theta = \frac{3\pi}{4}$$

We need to calculate the radius of the circle.

Using formula of length

$$l = r \times \theta$$

Where, l = length of arc

r = radius of circle

θ = angle

Put the value into the formula

$$15\,\pi = r \times \frac{3\pi}{4}$$

$$\Rightarrow \quad r = \frac{15\,\pi \times 4}{3\,\pi}$$

$$\Rightarrow \quad r = 20 \text{ cm}$$

2. *(a)* $\dfrac{(p^2+1)^4}{4p^2+1}$

Explanation: $a + ib = \dfrac{(p^2+1)^2}{2p-i}$

Then

$$a - ib = \frac{(p^2+1)^2}{2p+i}$$

Multiplying we get,

$$a^2 + b^2 = \frac{(p^2+1)^2}{2p-i} \times \frac{(p^2+1)^2}{2p+i}$$

$$= \frac{(p^2+1)^4}{4p^2+1}$$

3. *(a) {– 4, –3, –2, –1, 0, 1, 2, 3, 4}*

Explanation: According to question, we have to write the integers that come in between –5 and 5 means –5, 5 are not included, so the correct option will be (a).

4. *(a) P = Q*

Explanation: We have,

P = coeffcient of a^r in the expansion of $(1 + a)^n$

$= {}^nC_r$

Q = coeffcient of a^{n-r} in the expansion of $(1 + a)^n$

$= {}^nC_{n-r}$

Now, ${}^nC_r = {}^nC_{n-r} \Rightarrow$ P = Q.

5. *(d)* $(-\infty, -1] \cup [2, \infty)$

Explanation: We have, $|3 - 6x| \geq 9$

$\Rightarrow \quad 3 - 6x \leq -9$ or $3 - 6x \geq 9$

$\Rightarrow \quad -6x \leq -12$ or $-6x \geq 6$

$\Rightarrow \quad x \geq 2$ or $x \leq -1$

$\Rightarrow \quad (-\infty, -1] \cup [2, \infty)$

⚠ **Caution**

➥ *Whenever there is an inequality involving '≤' or '≥', then brackets should be closed.*

6. *(a) {(3, a), (5, b), (3, c)}*

Explanation: If R = {(a, 3) (b, 5) (c, 3)},

Then

R^{-1} = {(3, a), (5, b), (3, c) }

7. *(b) 8, –8*

Explanation: P(0, 0) and Q(6, y)

PQ = 10

$$\Rightarrow \sqrt{(6-0)^2 + (y-0)^2} = 10$$

$$\Rightarrow \quad 36 + y^2 = 100$$

$$\Rightarrow \quad y^2 = 64$$

$$\Rightarrow \quad y = 8, -8$$

8. *(c) VII octant*

Explanation:

1st octant → (+ + +)

2nd octant → (- + +)

3rd octant → (- - +)

4th octant → (+ - +)

5th octant → (+ + -)

6^{th} octant → (- + -)
7^{th} octant → (- - -)
8^{th} octant → (+ - -)

Therefore, (–4, –3, –2) lies in 7^{th} octant.

Caution

➥ *Remember the sign convention in different octants.*

9. *(a) 48i*

Explanation: $(2+i)^4 - (2-i)^4$

$= ((2+i)^2)^2 - ((2-i)^2)^2$

$= (i^2 + 4 + 4i)^2 - (4 + i^2 - 4i)^2$

$= (3 + 4i)^2 - (3 - 4i)^2$

$= (3 + 4i + 3 - 4i)\ (3 + 4i - 3 + 4i)^2$

$= 6 \times 8i = 48i$

10. *(b) 2*

Explanation: We have,

$$\lim_{x \to \frac{1}{2}} \frac{4x^2-1}{2x-1} = \lim_{x \to \frac{1}{2}} \frac{(2x)^2-1^2}{2x-1}$$

$$= \lim_{x \to \frac{1}{2}} \frac{(2x-1)(2x+1)}{2x-1}$$

$$= \lim_{x \to \frac{1}{2}} 2x + 1$$

$$= 2 \times \frac{1}{2} + 1$$

$$= 2$$

11. *(a) 0*

Explanation: If A and B are two disjoint sets, then there will be no common elements in the sets A and B.

Thus, $A \cap B = \phi$

Hence, $n(A \cap B) = 0.$

12. *(a) 4*

Explanation: The geometric mean of 2 and 8 is $\sqrt{16}$ *i.e.*, 4.

13. *(d) 9*

Explanation: Given, $\lim_{x \to 0} \left(\frac{\tan^2 3x}{x^2}\right)$

We know that, $\lim_{x \to 0} \frac{\tan ax}{ax} = 1$

So, $\lim_{x \to 0} \frac{\tan^2 3x}{x^2} = \lim_{x \to 0} 9 \frac{\tan^2 3x}{9x^2}$

$$= \lim_{x \to 0} 9\left(\frac{\tan 3x}{3x}\right)^2$$

$$= 9 \times 1$$

$$= 9$$

14. *(b)* $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Explanation: If $\cos x = -\frac{1}{2}$

Then, $\cos x = -\cos\frac{\pi}{3}$

$\Rightarrow \cos x = \cos\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow \cos x = \cos\frac{2\pi}{3}$

$\Rightarrow x = \frac{2\pi}{3}$

and $\cos x = \cos\left(\pi + \frac{\pi}{3}\right)$

$\Rightarrow \cos x = \cos\frac{4\pi}{3}$

$\Rightarrow x = \frac{4\pi}{3}$

Hence, $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

15. *(c) 6 units*

Explanation: The distance between the points P(0, 3, 4) and Q(4, 1, 0) is

$$PQ = \sqrt{(4-0)^2 + (1-3)^2 + (0-4)^2}$$

$$= \sqrt{16+4+16} = \sqrt{36} = 6 \text{ units}$$

16. *(d)* $\frac{9}{2}$

Explanation: We have, $y = \sin x + \tan x$

Differentiate w.r.t. 'x'

$$\frac{dy}{dx} = (\cos x + \sec^2 x)$$

$$\left[\text{As } \frac{d}{dx}(\tan x) = \sec^2 x;\ \ \frac{d}{dx}(\sin x) = \cos x\right]$$

$$\therefore \quad \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{3}} = \left(\cos\frac{\pi}{3} + \sec^2\frac{\pi}{3}\right)$$

$$= \frac{1}{2} + (2)^2 = \frac{1}{2} + 4 = \frac{9}{2}$$

17. *(a)* $\left\{-1, \frac{4}{3}\right\}$

Explanation: Given that

$f(x) = 3x^2 - 1$ and $g(x) = 3 + x$

$f(x) = g(x)$

$\Rightarrow \quad 3x^2 - 1 = 3 + x$

$\Rightarrow \quad 3x^2 - x - 4 = 0$

$\Rightarrow \quad 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow \quad x(3x - 4) + 1(3x - 4) = 0$

$\Rightarrow \quad (x + 1)(3x - 4) = 0$

$\Rightarrow \quad x + 1 = 0$ or $3x - 4 = 0$

$\Rightarrow \quad x = -1$, or $x = \frac{4}{3}$

$\therefore \quad \text{Domain} = \left\{-1, \frac{4}{3}\right\}$

Important

➥ *Domain is the set of values of x while Range is the set of values of y or [f(x)].*

18. *(c) $(-4, \infty)$*

Explanation: $4x + 3 < 5x + 7$

$\Rightarrow \quad 3 - 7 < 5x - 4x$

$\Rightarrow \quad -4 < x$ or, $x > -4$

$\Rightarrow \quad x \in (-4, \infty)$

19. *(b) Both A and R are true but R is not the correct explanation of A.*

Explanation: Given equation $3x - 4y + 10 = 0$ can be written as

$$y = \frac{3}{4}x + \frac{5}{2} \quad \text{...(i)}$$

Comparing eq. (i) with $y = mx + c$, we have slope of the given line as $m = \frac{3}{4}$.

Equation $3x - 4y + 10 = 0$ can be written as

$$3x - 4y = -10 \text{ or } \frac{x}{-\frac{10}{3}} + \frac{y}{\frac{5}{2}} = 1 \quad \text{...(ii)}$$

Comparing eq. (ii) with $\frac{x}{a} + \frac{y}{b} = 1$, we have x-intercept as $a = -\frac{10}{3}$ and y-intercept as $b = \frac{5}{2}$.

20. *(c) A is true but R is false.*

Explanation: Given $\sec\theta + \tan\theta = m, 0 < m < 1$ ...(i)

Also, $\sec^2\theta - \tan^2\theta = 1$ ...(ii)

Dividing (ii) by (i), we get

$$\sec\theta - \tan\theta = \frac{1}{m} \quad \text{...(iii)}$$

Note that $\frac{1}{m} > 1 \quad (\because \; 0 < m < 1)$

Adding (i) and (iii), we get $\sec\theta = \frac{m^2+1}{2m} > 0$

And subtracting (iii) from (i), we get

$$\tan\theta = \frac{m^2-1}{2m} < 0$$

As $\sec\theta > 0$ and $\tan\theta < 0$

$\therefore \theta$ lies in the fourth quadrant.

Also, $\sin\theta = \tan\theta\cos\theta = \frac{\tan\theta}{\sec\theta} = \frac{m^2-1}{m^2+1}$

SECTION - B

21. Given, X = {5, 6, 7, 8}, Y = {7, 8, 9, 10}, Z = {3, 4, 5, 6}

(A) $X \cap Y = \{5, 6, 7, 8\} \cap \{7, 8, 9, 10\}$

$= \{7, 8\}$

$(X \cap Y) \cup Z = \{7, 8\} \cup \{3, 4, 5, 6\}$

$= \{3, 4, 5, 6, 7, 8\}$

(B) $X \cup Y = \{5, 6, 7, 8\} \cup \{7, 8, 9, 10\}$

$= \{5, 6, 7, 8, 9, 10\}$

$(X \cup Y) \cap Z = \{5, 6, 7, 8, 9, 10\} \cap \{3, 4, 5, 6\}$

$= \{5, 6\}$

OR

Given that $\quad A \cup B = A \cap B$ ...(i)

To show $\quad A = B$.

Let $a \in A$ be any element.

$\Rightarrow \quad a \in A \cup B \quad [\because A \subseteq A \cup B]$

$\Rightarrow \quad a \in A \cap B \quad$ [Using (i)]

$\Rightarrow a \in A$ and $a \in B \Rightarrow a \in B$

Then, $\quad A \subseteq B$ ...(ii)

Let $a \in B$ be any element

$\Rightarrow \quad a \in A \cup B \quad [\because B \subseteq A \cup B]$

$\Rightarrow \quad a \in A \cap B \quad$ [Using (i)]

$\Rightarrow \quad a \in A$ and $a \in B \Rightarrow a \in A$

Thus, $\quad B \subseteq A$ ...(iii)

From (ii) and (iii), we get

$A = B$.

22. We have ${}^{12}P_{x+1} > 2\,{}^{12}P_x$

$$\Rightarrow \quad \frac{12!}{(11-x)!} > 2\frac{12!}{(12-x)(11-x)!}$$

$$\Rightarrow \quad 1 > 2\frac{1}{(12-x)}$$

$\Rightarrow \quad (12 - x) > 2$

$\Rightarrow \quad x < 10$

$\Rightarrow \quad x \in \{10, 9, 8, 7, 6, 5, 4, 3, 2, 1\}$

23. Given that length of arc, $l = 66$ cm and central angle, $\theta = 30°$

Then, $\theta = 30°$

$= \left(30 \times \frac{\pi}{180}\right)$ radian $\quad [\because 1° = \frac{\pi}{180}$ radian$]$

$= \frac{\neq}{6}$ radian

Hence, radius of circle is

$r = \frac{l}{\theta} = \frac{66 \times 6}{\pi} = \frac{66 \times 6 \times 7}{22} = 126$ cm

24. We have

$3x + 17 \le 2(1 - x)$

$\Rightarrow \quad 3x + 17 \le 2 - 2x$

$\Rightarrow \quad 3x + 2x \le 2 - 17$

$\Rightarrow \quad 5x \le -15$

$\Rightarrow \quad \frac{5x}{5} \le \frac{-15}{5}$

$\Rightarrow \quad x \le -3$

$\Rightarrow \quad x \in (-\infty, -3]$

OR

We have, $-15x > 45$

$\Rightarrow \frac{-15x}{-15} < \frac{45}{-15}$ [On dividing both sides by -15]

$\Rightarrow \quad x < -3$

(A) Given that x is a natural number, *i.e.*, $x \in N$.

Hence, the solution set of given inequality

$= \{x \in N : x < -3\}$ = No Solution.

(B) Given that x is an integer, *i.e.*, $x \in Z$.

Hence, the solution set of given inequality

$= \{x \in Z : x < -3\} = \{\_\_\_, -5, -4\}$.

25. Under this relation R, we obtain 3R2, 5R2, 5R4, 7R2, 7R4 and 7R6.

i.e., $R = \{(3, 2), (5, 2), (5, 4), (7, 2), (7, 4), (7, 6)\}$.

$\therefore$ Domain (R) = {3, 5, 7} and range (R) = {2, 4, 6}

SECTION - C

26. Let's try to find out $101^{50} - 99^{50}$ in terms of remaining term *i.e.*,

$101^{50} - 99^{50} = (100 + 1)^{50} - (100 - 1)^{50}$

$= (C_0 \cdot 100^{50} + C_1 \cdot 100^{49} + C_2 \cdot 100^{48} + ...)$

$- (C_0 \cdot 100^{50} - C_1 \cdot 100^{49} + C_2 \cdot 100^{48} - ...)$

$= 2(C_1 \cdot 100^{49} + C_3 \cdot 100^{47} + ...)$

$= 2(50 \cdot 100^{49} + C_3 \cdot 100^{47} + ...)$

$= 100^{50} + 2(C_3 \cdot 100^{47} + ...)$

$> 100^{50}$

$\Rightarrow \quad 101^{50} > 99^{50} + 100^{50}$

27. Given equation of circle is

$x^2 + y^2 - 2ax\cos\theta - 2ay\sin\theta = 0$

Which represents a circle (in general form) with centre $(a\cos\theta, a\sin\theta)$

and radius $\sqrt{a^2\cos^2\theta + a^2\sin^2\theta - 0} = a$

Given that the line $x\cos\theta + y\sin\theta - k = 0$ touches the given circle.

So, the perpendicular distance of the line from $(a\cos\theta, a\sin\theta)$ is equal to the radius r of the circle.

$$\therefore r = \left|\frac{(a\cos\theta)\cos\theta + (a\sin\theta)\sin\theta - k}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$$

$\Rightarrow \quad a = |a - k|$

$\Rightarrow \quad \pm a = a - k$

$\Rightarrow \quad k = 0$ or $k = 2a$.

OR

Given that, line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$.

Solving line with parabola, we have

$(mx + 1)^2 = 4x$

$\Rightarrow \quad m^2x^2 + 2mx + 1 = 4x$

$\Rightarrow \quad m^2x^2 + x(2m - 4) + 1 = 0$

Since the line touches the parabola, above equation must have equal roots.

$\therefore \quad$ Discriminant, $D = 0$

$\Rightarrow \quad (2m - 4)^2 - 4m^2 = 0$

$\Rightarrow \quad 4m^2 - 16m + 16 - 4m^2 = 0$

$\Rightarrow \quad 16m = 16$

$\therefore \quad m = 1$

28. Total number of cards = 52.

Number of cards drawn = 1.

(A) Number of favourable cases that the card is a face card =12.

Hence, required probability $= \frac{12}{52} = \frac{3}{13}$.

(B) Number of favourable cases that the card is not a diamond = 39.

Hence, required probability $= \frac{39}{52} = \frac{3}{4}$.

(C) Number of favourable cases that the card is an ace = 4.

Hence, required probability = $\frac{4}{52} = \frac{1}{13}$.

⚠ Caution

➥ *Remember ace is not a face card. The face cards are king, queen and jack only.*

29. We have,

$$\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$$

$$\Rightarrow \frac{\{(1+i)x-2i\}(3-i)+\{(2-3i)y+i\}(3+i)}{(3+i)(3-i)} = i$$

$$\Rightarrow \frac{(3+3i-i-i^2)x-(6i-2i^2)+(6-9i+2i-3i^2)y+(3i+i^2)\}}{3^2-i^2} = i$$

$\Rightarrow$ $(4 + 2i)x - (6i + 2) + (9 - 7i)y + (3i - 1) = i(9 + 1)$

$\Rightarrow$ $4x + 2xi - 6i - 2 + 9y - 7yi + 3i - 1 = 10i$

$$\Rightarrow \frac{(4x+9y-3)+i(2x-7y-3)}{10} = 0+i$$

$$\Rightarrow \frac{4x+9y-3}{10} = 0 \text{ and } \frac{2x-7y-3}{10} = 1$$

$4x + 9y - 3 = 0$ and $2x - 7y - 13 = 0$

$x = 3$ and $y = -1$

OR

$|3^{1/2} - i|^x = 4^x$

$\Rightarrow$ $[(\sqrt{(3^{1/2})^2+(-1)^2})]^x = 4^x$

$\Rightarrow$ $(\sqrt{4})^x = 4^x$

$\Rightarrow$ $4^{x/2} = 4^x$

$\Rightarrow$ $\frac{x}{2} = x$

$\Rightarrow$ $x - \frac{x}{2} = 0$

$\Rightarrow$ $x = 0$

Hence, the given equation does not have a non-zero integral solution.

30. Clearly, $f(x)$ is defined for all x satisfying

$4 - x \geq 0$ and $x^2 - 1 > 0$

$\Rightarrow$ $x - 4 \leq 0$ and $(x - 1)(x + 1) > 0$

$\Rightarrow$ $x \leq 4$ and $x < -1$ or $x > 1$

$\Rightarrow$ $x \in (-\infty, -1) \cup (1, 4]$.

Hence, Domain $(f) = (-\infty, -1) \cup (1, 4]$.

31. We have, $\tan\frac{x}{2} = \frac{m}{n}$

and, $$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2\frac{m}{n}}{1+\frac{m^2}{n^2}} = \frac{2mn}{m^2+n^2}$$

$$\cos x = \sqrt{1-\left(\frac{2mn}{m^2+n^2}\right)^2} = \sqrt{\frac{(m^2+n^2)^2-4m^2n^2}{(m^2+n^2)^2}}$$

$$= \frac{m^2-n^2}{m^2+n^2}$$

$$\therefore m\sin x + n\cos x = m\left(\frac{2mn}{m^2+n^2}\right) + n\left(\frac{m^2-n^2}{m^2+n^2}\right)$$

$$= \frac{2m^2n}{m^2+n^2} + \frac{nm^2}{m^2+n^2} - \frac{n^3}{m^2+n^2} = \frac{3m^2n-n^3}{m^2+n^2}$$

OR

$$\text{L.H.S.} = \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$$

$$= \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{\pi}{8}\right)$$

$$= \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{\pi}{8}\right)$$

$$= 2\left[\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right)\right]$$

$$= 2\left(\left(\frac{1+\cos\frac{\pi}{4}}{2}\right)^2 + \left(\frac{1+\cos\frac{3\pi}{4}}{2}\right)^2\right)$$

$$= \frac{2}{4}\left(\left(1+\frac{1}{\sqrt{2}}\right)^2 + \left(1-\frac{1}{\sqrt{2}}\right)^2\right) = \frac{1}{2}(2+1)$$

$$= \frac{3}{2} = \text{R.H.S}$$

SECTION - D

32. When, $x = 3$, the expression $\lim_{x\to 3}\dfrac{x^3-7x^2+15x-9}{x^4-5x^3+27x-27}$ assumes the form $\dfrac{0}{0}$.

Now, factorising the numerator and denominator, we get

$$\lim_{x\to 3}\frac{x^3-7x^2+15x-9}{x^4-5x^3+27x-27}$$

$$= \lim_{x\to 3}\frac{(x-3)(x^2-4x+3)}{(x-3)(x^3-2x^2-6x+9)}$$

$$= \lim_{x\to 3}\frac{x^2-4x+3}{x^3-2x^2-6x+9} \qquad \left(\text{form}\,\frac{0}{0}\right)$$

$$= \lim_{x\to 3}\frac{(x-3)(x-1)}{(x-3)(x^2+x-3)}$$

$$= \lim_{x\to 3}\frac{x-1}{x^2+x-3} = \frac{3-1}{9+3-3} = \frac{2}{9}$$

33. Let A and B be the events that the number is a multiple of 2 and that of 9 respectively. Multiple of 2 from 1 to 1000 are 2, 4, 6, 8, ..., 1000.

Let n be the number of terms in above series

$\therefore \quad l = 1000$

$\Rightarrow \quad 2 + (n-1)2 = 1000$

$\{l = a + (n-1)d,\ a = 2,\ d = 2\}$

$\Rightarrow \quad 2(n-1) = 998$

$\Rightarrow \quad n-1 = 499$

$\Rightarrow \quad n = 500$

Since, the number of multiples of 2 are 500.

$\therefore \quad P(A) = \dfrac{500}{1000}$

The multiples of 9 are 9,18,27, ..., 999

$l = 999$

$a = 9,\ d = 9$

These are m numbers.

$\therefore \quad 9 + (m-1)9 = 999$

$\Rightarrow \quad 9(m-1) = 990$

$\Rightarrow \quad m-1 = 110$

$\Rightarrow \quad m = 111$

Since, the number of multiple of 9 are 111

$\therefore \quad P(B) = \dfrac{111}{1000}$

Now, the multiples of 2 and 9 both are 18,36, ..., 990

Let p be the number of terms in the above series.

$\therefore \quad p^{th}$ term $= 990$

$\Rightarrow \quad 18 + (p-1)\,18 = 990$

$\Rightarrow \quad 18\,(p-1) = 972$

$\Rightarrow \quad p-1 = 54$

$\Rightarrow \quad p = 55$

$\therefore \quad P(A \cap B) = \dfrac{55}{1000}$

So, required probability $= P(A) + P(B) - P(A\cap B)$

$$= \frac{500+111-55}{1000} = \frac{556}{1000} = \frac{139}{250}$$

34. Given A is the arithmetic mean and G_1, G_2 be two geometric means between any two numbers.

Let the two numbers be 'a' and 'b'.

The arithmetic mean is given by $A = \dfrac{a+b}{2}$ and the geometric mean is given by $G = \sqrt{ab}$.

We have to insert two geometric means between a and b.

Now, we have the terms a, G_1, G_2, b

G_1 will be the geometric mean of a and G_2 and G_2 will be the geometric mean of G_1 and b.

Hence $G_1 = \sqrt{aG_2}$ and $G_2 = \sqrt{G_1 b}$

Squaring both sides, $G_1 = \sqrt{aG_2}$

$\Rightarrow \quad G_1^{\,2} = aG_2 \qquad$ ...(i)

Put $\quad G_2 = \sqrt{G_1 b}$ in (i)

$\Rightarrow \quad G_1^{\,2} = a\sqrt{G_1 b}$

Squaring on both sides, we get

$G_1^{\,4} = a^2\,(G_1 b)$

$\Rightarrow \quad G_1^{\,3} = a^2 b$

$\Rightarrow \quad G_1 = a^{\frac{2}{3}}\, b^{\frac{1}{3}} \qquad$ ...(ii)

Put value of G_1 in $G_2 = \sqrt{G_1 b}$

$\Rightarrow \quad G_2 = \sqrt{a^{\frac{2}{3}}\, b^{\frac{1}{3}}\, b}$

$$= \left(a^{\frac{2}{3}} b^{\frac{4}{3}}\right)^{\frac{1}{2}}$$

$$= a^{\frac{1}{3}} b^{\frac{2}{3}} \quad \text{...(iii)}$$

Now, we have to prove that $2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$.

Consider $\quad R.H.S = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$

Substitute values of G_1 and G_2 from (ii) and (iii)

$$\Rightarrow \quad RHS = \frac{\left(a^{\frac{2}{3}} b^{\frac{1}{3}}\right)^2}{a^{\frac{1}{3}} b^{\frac{2}{3}}} + \frac{\left(a^{\frac{1}{3}} b^{\frac{2}{3}}\right)^2}{a^{\frac{2}{3}} b^{\frac{1}{3}}}$$

$$= \frac{a^{\frac{4}{3}} b^{\frac{2}{3}}}{a^{\frac{1}{3}} b^{\frac{2}{3}}} + \frac{a^{\frac{2}{3}} b^{\frac{4}{3}}}{a^{\frac{2}{3}} b^{\frac{1}{3}}}$$

$$= a^{\frac{4}{3}-\frac{1}{3}} b^{\frac{2}{3}-\frac{2}{3}} + a^{\frac{2}{3}-\frac{2}{3}} b^{\frac{4}{3}-\frac{1}{3}}$$

$$\Rightarrow \quad R.H.S = 2\frac{a+b}{2}$$

But $\quad A = \frac{a+b}{2}$

Therefore

$\Rightarrow \quad R.H.S = 2A$

Hence, $\quad R.H.S = LHS$

Hence proved.

OR

Let the first term of A.P. be m and common difference be d.

Let the first term of G.P. be l and common ratio be s.

The n^{th} term of an A.P. is given as $t_n = a + (n-1)d$ where a is the first term and d is the common difference.

The n^{th} term of a G.P. is given by $t_n = ar^{n-1}$ where a is the first term and r is the common ratio.

The p^{th} term (t_p) of both A.P. and G.P. is a

For A.P.

$$t_p = m + (p-1)d$$

$$\Rightarrow \quad a = m + (p-1)d \quad \text{...(1)}$$

For G.P.

$$t_p = ls^{p-1}$$

$$\Rightarrow \quad a = ls^{p-1} \quad \text{...(2)}$$

The q^{th} term (t_q) of both A.P. and G.P. is b

For A.P.

$$t_q = m + (q-1)d$$

$$\Rightarrow \quad b = m + (q-1)d \quad \text{...(3)}$$

For G.P.

$$t_q = ls^{q-1}$$

$$\Rightarrow \quad b = ls^{q-1} \quad \text{...(4)}$$

The r^{th} term (t_r) of both A.P. and G.P. is c

For A.P.

$$t_r = m + (r-1)d$$

$$\Rightarrow \quad c = m + (r-1)d \quad \text{...(5)}$$

For G.P.

$$t_r = ls^{r-1}$$

$$\Rightarrow \quad c = ls^{r-1} \quad \text{...(6)}$$

Let us find $b-c$, $c-a$ and $a-b$

Using (3) and (5)

$$\Rightarrow \quad b - c = (q-r)d \quad \text{...(i)}$$

Using (5) and (1)

$$\Rightarrow \quad c - a = (r-p)d \quad \text{...(ii)}$$

Using (1) and (3)

$$\Rightarrow \quad a - b = (p-q)d \quad \text{...(iii)}$$

We have to prove that $a^{b-c} b^{c-a} \cdot c^{a-b} = 1$

$L.H.S = a^{b-c} b^{c-a} c^{a-b}$

Using (2), (4) and (6)

$$L.H.S = (ls^{p-1})^{b-c} \cdot (ls^{q-1})^{c-a} \cdot (ls^{r-1})^{a-b}$$

$$= \left(\frac{ls^p}{s}\right)^{b-c} \cdot \left(\frac{ls^q}{s}\right)^{c-a} \cdot \left(\frac{ls^r}{s}\right)^{a-b}$$

$$= \frac{l^{b-c} s^{p(b-c)}}{s^{b-c}} \cdot \frac{l^{c-a} s^{q(c-a)}}{s^{c-a}} \cdot \frac{l^{a-b} s^{r(a-b)}}{s^{a-b}}$$

$$= \frac{l^{b-c+c-a+a-b}}{s^{b-c+c-a+a-b}} \cdot s^{p(b-c)} \cdot s^{q(c-a)} \cdot s^{r(a-b)}$$

$$= s^{p(b-c)} \cdot s^{q(c-a)} \cdot s^{r(a-b)}$$

Substituting values of $a-b$, $c-a$ and $b-c$ from (iii), (ii) and (i)

$$= s^{p(q-r)d} \cdot s^{q(r-p)d} \cdot s^{r(p-q)d}$$

$$= s^{pqd-prd} \cdot s^{qrd-pqd} \cdot s^{prd-qrd}$$

$$= s^{pqd-prd+qrd-pqd+prd-qrd}$$

$$= s^0 = 1$$

$\Rightarrow \quad L.H.S = R.H.S$

Hence proved.

35. As origin is the centre of ellipse, so equation of ellipse be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Since this ellipse passes through (–3, 1) and (2, – 2) so,

$$\frac{9}{a^2}+\frac{1}{b^2}=1 \text{ and } \frac{4}{a^2}+\frac{4}{b^2}=1$$

$$\Rightarrow \quad \frac{9}{a^2}+\frac{1}{b^2}=1 \text{ and } \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}$$

Subtracting above two equations we get

$$\frac{8}{a^2}=\frac{3}{4} \Rightarrow a^2=\frac{32}{3}$$

And $\frac{1}{b^2}=1-\frac{9}{a^2}=1-\frac{9}{\frac{32}{3}}=1-\frac{27}{32}=\frac{5}{32}$

$$\Rightarrow \quad b^2=\frac{32}{5}$$

Hence, equation of ellipse is

$$\frac{x^2}{\frac{32}{3}}+\frac{y^2}{\frac{32}{5}}=1 \Rightarrow 3x^2+5y^2=32.$$

OR

Let equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$

Given that, eccentricity, $e=\frac{5}{8}$

Now, the foci of this ellipse are $(\pm ae, 0)$

Distance between foci = 10 (Given)

$\therefore 2ae = 10 \Rightarrow ae = 5$

$\Rightarrow \frac{5}{8}a = 5 \Rightarrow a = 8$

We know that, $b^2 = a^2(1-e^2)$

$\Rightarrow b^2 = 64\left(1-\frac{25}{64}\right) = 64 - 25 = 39$

$\therefore$ Length of latus rectum of ellipse

$$=\frac{2b^2}{a}=2\times\frac{39}{8}=\frac{39}{4}.$$

SECTION - E

36. (A) We have, angle = 240°

Its radian measure $= 240 \times \frac{\pi}{180} = \frac{4\pi}{3}$

(B) In 1 minute *i.e.*, 60 seconds, a wheel makes 360 revolutions.

$\therefore$ In one second, a wheel will make $\frac{360}{60}$ = 6 revolutions.

$\therefore$ Number of radians $= 6 \times 2\pi = 12\pi$

(C) We have

$$(1.2)^c = \left(1.2\times\frac{180}{\pi}\right) = 1.2\times\frac{180}{22}\times 7$$

$= 68.7272° = 68° (0.7272 \times 60)'$

$= 68° 43'(0.63 \times 60)'' = 68° 43' 37''$

OR

We have,

$l = 132$ cm, $\theta = 45° = 45 \times \frac{\pi}{180} = \frac{\pi}{4}$

Now, $\theta = \frac{l}{r} = \frac{132}{r}$

$\Rightarrow \quad \frac{\pi}{4} = \frac{132}{r}$

$\Rightarrow \quad r = \frac{132\times 4}{\pi}$

$\Rightarrow \quad r = \frac{132\times 4}{\frac{22}{7}} = 168$ cm

37. (A) A History book can be selected in 5 ways and a Math book can be selected in 3 ways.

Required number of ways = 5 + 3 = 8

[Using addition Principle]

(B) Now, 2 History books can be chosen in ${}^5P_2$ ways, 1 Maths book can be chosen in ${}^3P_1$ ways and 1 Science book can be chosen in ${}^4P_1$ ways.

$\therefore$ Required number of ways $= {}^5P_2 \times {}^3P_1 \times {}^4P_1 = 240$

(C) Number of ways of arranging History books = 5!

Number of ways of arranging Maths books = 3!

Number of ways of arranging Science books = 4!

$\therefore$ Required number of ways if the books of same subject are put together = 3! ·5! ·4!

OR

The number of selection of books ${}^5P_1 \times {}^3P_2 \times {}^4P_2$ represents the arrangement of 1 History book, 2 Maths books and 2 Science books respectively.

38.

C.I.	Mid-Value x_i	Frequency f_i	f_ix_i	$\|x_i - \bar{x}\|$	$f_i\|x_i - \bar{x}\|$
0–20	10	8	80	33	264
20–40	30	10	300	13	130
40–60	50	13	650	7	91
60–80	70	6	420	27	162
80–100	90	3	270	47	141
		N = 40	Σf_ix_i = 1720		$\Sigma f_i\|x_i - \bar{x}\|$ = 788

(A) Mean $(\bar{x}) = \dfrac{\sum f_i x_i}{N} = \dfrac{1720}{40} = 43$

(B) Mean deviation from mean

$$= \frac{\Sigma f_i |x_i - \bar{x}|}{N} = \frac{788}{40} = 19.7$$

$$\frac{N}{2} = \frac{40}{2} = 20$$