

LOGARITHMS

I. Logarithm : If a is a positive real number, other than 1 and $a^m = x$, then we write : $m = \log_a x$ and we say that the value of $\log x$ to the base a is m .

Example :

(i) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

(ii) $3^4 = 81 \Rightarrow \log_3 81 = 4$

(i) $2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$

(i) $(.1)^2 = 0.1 \Rightarrow \log_{(.1)} 0.1 = 2$

❖ EXAMPLES ❖

Ex.1 : Evaluate :

(i) $\log_3 27$ (ii) $\log_7 \left(\frac{1}{343} \right)$ (iii) $\log_{100} (0.01)$

Sol. : (i) Let $\log_3 27 = n$
Then, $3^n = 27 = 3^3$
or $n = 3$

$\therefore \log_3 27 = 3$

(ii) Let $\log_7 \left(\frac{1}{343} \right) = n$

Then, $7^n = \frac{1}{343} = \frac{1}{7^3} = 7^{-3}$ or $n = -3$

$\therefore \log_7 \left(\frac{1}{343} \right) = -3$

(iii) Let $\log_{100} (0.01) = n$

Then, $(100)^n = 0.01 = \frac{1}{100} = (100)^{-1}$
or $n = -1$

$\therefore \log_{100} (0.01) = -1$

Ex.2: Evaluate :

(i) $\log_7 1 = 0$ (ii) $\log_{34} 34$ (iii) $36^{\log_6 4}$

Sol.: (i) We know that $\log_a a = 1$, so $\log_7 1 = 0$

(ii) We know that $\log_a a = 1$, so $\log_{34} 34 = 1$

(iii) We know that $a^{\log_a x} = x$.

Now,

$36^{\log_6 4} = (6^2)^{\log_6 4} = 6^{2(\log_6 4)} = 6^{\log_6 (4^2)} = 6^{\log_6 16} = 16$

Ex.3 : If $\log_{\sqrt{8}} x = 3\frac{1}{3}$, find the value of x .

Sol. : $\log_{\sqrt{8}} x = \frac{10}{3} \Leftrightarrow x = (\sqrt{8})^{10/3}$

$x = (2^{3/2})^{10/3} = 2^{\left(\frac{3}{2} \times \frac{10}{3}\right)} = 2^5 = 32$

Ex.4 : Evaluate

(i) $\log_5 3 \times \log_{27} 25$ (ii) $\log_9 27 - \log_{27} 9$

Sol. : (i) $\log_5 3 \times \log_{27} 25 = \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 27}$

$$= \frac{\log 3}{\log 5} \times \frac{\log(5^2)}{\log(3^3)} = \frac{\log 3}{\log 5} \times \frac{2 \log 5}{3 \log 3} = \frac{2}{3}$$

(ii) Let $\log_9 27 = n$

Then, $9^n = 27$

$$\Leftrightarrow 3^{2n} = 3^3$$

$$\Leftrightarrow 2n = 3$$

$$\Leftrightarrow n = \frac{3}{2}$$

Again, let $\log_{27} 9 = m$

Then, $27^m = 9 \Leftrightarrow 3^{3m} = 3^2 \Leftrightarrow 3m = 2 \Leftrightarrow m = \frac{2}{3}$

$$\therefore \log_9 27 - \log_{27} 9 = (n-m) =$$

$$\left(\frac{3}{2} - \frac{2}{3} \right) = \frac{5}{6}$$

Ex.5 : Simplify : $\left(\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} \right)$

Sol. : $\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} = \log \frac{75}{16} - \log \left(\frac{5}{9} \right)^2 + \log \frac{32}{243}$

$$= \log \frac{75}{16} - \log \frac{25}{81} + \log \frac{32}{243}$$

$$= \log \left(\frac{75}{16} \times \frac{32}{243} \times \frac{81}{25} \right) = \log 2$$

Ex.6 : Find the value of x which satisfies the relation

$$\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$$

Sol. : $\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$

$$\Leftrightarrow \log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + \log_{10} 10$$

$$\Leftrightarrow \log_{10} \{ 3 (4x + 1) \} = \log_{10} [10 (x + 1)]$$

$$\Leftrightarrow 3 (4x + 1) = 10 (x + 1)$$

$$\Leftrightarrow 12x + 3 = 10x + 10$$

$$\Leftrightarrow 2x + 7 \Leftrightarrow x = \frac{7}{2}$$

Ex.7 : Simplify :

$$\left[\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} \right]$$

Sol. : Given expression

$$= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx)$$

$$\left[\because \log_a x = \frac{1}{\log_x a} \right]$$

$$= \log_{xyz}(xy \times yz \times zx) = \log_{xyz}(xyz)^2$$

$$= 2 \log_{xyz}(xyz) = 2 \times 1 = 2$$

Ex.8 : If $\log_{10} 2 = 0.30103$, find the value of $\log_{10} 50$.

Sol. : $\log_{10} 50 = \log_{10} \left(\frac{100}{2} \right) = \log_{10} 100 - \log_{10} 2$
 $= 2 - 0.30103 = 1.69897$

Ex.9 : If $\log 2 = 0.3010$, and $\log 3 = 0.4771$, find the values of:

(i) $\log 25$ (ii) $\log 4.5$

Sol. : (i) $\log 25 = \log \left(\frac{100}{4} \right) = \log 100 - \log 4$
 $= 2 - 2 \log 2$
 $= (2 - 2 \times 0.3010)$
 $= 1.398$

(ii) $\log 4.5 = \log \left(\frac{9}{2} \right)$
 $= \log 9 - \log 2$
 $= 2 \log 3 - \log 2$
 $= (2 \times 0.4771 - 0.3010)$
 $= 0.6532$

Ex.10: If $\log 2 = 0.30103$, find the number of digits in 2^{56}

Sol. : $\log (2^{56}) = 56 \log 2 = (56 \times 0.30103) = 16.85768$
 Its characteristic is 16. Hence, the number of digits in 2^{56} is 17

EXERCISE

Q.1 The value of $\log_2 16$ is
(A) $\frac{1}{8}$ (B) 4 (C) 8 (D) 16

Q.2 The value of $\log_{343} 7$ is
(A) $\frac{1}{3}$ (B) -3 (C) $-\frac{1}{3}$ (D) 3

Q.3 The value of $\log_5 \left(\frac{1}{125} \right)$ is:
(A) 3 (B) -3 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

Q.4 The value of $\log_{\sqrt{2}} 32$ is:
(A) $\frac{5}{2}$ (B) 5 (C) 10 (D) $\frac{1}{10}$

Q.5 The value of $\log_{10} (.0001)$ is
(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) -4 (D) 4

Q.6 The value of $\log_{(.01)} (1000)$ is :
(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

Q.7 The logarithm of 0.0625 to the base 2 is:
(A) -4 (B) -2
(C) 0.25 (D) 0.5

Q.8 If $\log_3 x = -2$, then x is equal to:
(A) -9 (B) -6 (C) -8 (D) $\frac{1}{9}$

Q.9 If $\log_8 x = \frac{2}{3}$, then the value of x is:
(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) 3 (D) 4

Q.10 If $\log_x \left(\frac{9}{16} \right) = -\frac{1}{2}$, then x is equal to:
(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$ (C) 3 (D) $\frac{256}{81}$

Q.11 If $\log_x 4 = 4.0$ then the value of x is:
(A) 1 (B) 4 (C) 16 (D) 32

Q.12 If $\log_{10000} x = -\frac{1}{4}$, then x is equal to:

- (A) $\frac{1}{10}$ (B) $\frac{1}{100}$
(C) $\frac{1}{1000}$ (D) $\frac{1}{10000}$

Q.13 If $\log_x 4 = \frac{1}{4}$, then x is equal to:

- (A) 16 (B) 64 (C) 128 (D) 256

Q.14 If $\log_x (0.1) = -\frac{1}{3}$, then the value of x is:

- (A) 10 (B) 100
(C) 1000 (D) $\frac{1}{1000}$

Q.15 If $\log_{32} x = 0.8$, then x is equal to:

- (A) 25.6 (B) 16 (C) 10 (D) 12.8

Q.16 If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is:

- (A) 2^{10} (B) 2^{100}
(C) 2^{1000} (D) 2^{10000}

Q.17 The value of $\log_{(-1/3)} 81$ is equal to:

- (A) -27 (B) -4
(C) 4 (D) 27

Q.18 The value of $\log_{2\sqrt{3}} (1728)$ is:

- (A) 3 (B) 5
(C) 6 (D) 9

Q.19 $\frac{\log \sqrt{8}}{\log 8}$ is equal to:

- (A) $\frac{1}{\sqrt{8}}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{8}$

Q.20 Which of the following statements is not correct?

- (A) $\log_{10} 10 = 1$
(B) $\log (2 + 3) = \log (2 \times 3)$
(C) $\log_{10} 1 = 0$
(D) $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

Q.21 The value of $\log_2 (\log_5 625)$ is :

- (A) 2 (B) 5 (C) 10 (D) 15

Q.22 If $\log_2 [\log_3 (\log_2 x)] = 1$, then x is equal to:

- (A) 0 (B) 12 (C) 128 (D) 512

Q.23 The value of $\log_2 \log_2 \log_3 \log_3 27^3$ is:

- (A) 0 (B) 1 (C) 2 (D) 3

Q.24 If $a^x = b^y$, then:

- (A) $\log \frac{a}{b} = \frac{x}{y}$ (B) $\frac{\log a}{\log b} = \frac{x}{y}$
 (C) $\frac{\log a}{\log b} = \frac{y}{x}$ (D) none of these

Q.25 $\log 360$ is equal to:

- (A) $2 \log 2 + 3 \log 3$
 (B) $3 \log 2 + 2 \log 3$
 (C) $3 \log 2 + 2 \log 3 - \log 5$
 (D) $3 \log 2 + 2 \log 3 + \log 5$

Q.26 The value of $\left(\frac{1}{3} \log_{10} 125 - 2 \log_{10} 4 + \log_{10} 32\right)$ is :

- (A) 0 (B) $\frac{4}{5}$ (C) 1 (D) 2

Q.27 $2 \log_{10} 5 + \log_{10} 8 - \frac{1}{2} \log_{10} 4 = ?$

- (A) 2 (B) 4
 (C) $2 + 2 \log_{10} 2$ (D) $4 - 4 \log_{10} 2$

Q.28 If $\log_a (ab) = x$, then $\log_b (ab)$ is:

- (A) $\frac{1}{x}$ (B) $\frac{x}{x+1}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x-1}$

Q.29 If $\log 2 = x$, $\log 3 = y$ and $\log 7 = z$, then the value of $\log (4\sqrt[3]{63})$ is:

- (A) $2x + \frac{2}{3}y - \frac{1}{3}z$ (B) $2x + \frac{2}{3}y + \frac{1}{3}z$
 (C) $2x - \frac{2}{3}y + \frac{1}{3}z$ (D) $-2x + \frac{2}{3}y + \frac{1}{3}z$

Q.30 If $\log_4 x + \log_2 x = 6$, then x is equal to:

- (A) 2 (B) 4 (C) 8 (D) 16

Q.31 If $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$, then the value of x is:

- (A) 12 (B) 16 (C) 18 (D) 24

Q.32 If $\log_{10} 125 + \log_{10} 8 = x$, then x is equal to:

- (A) $\frac{1}{3}$ (B) .064 (C) -3 (D) 3

Q.33 The value of $(\log_9 27 + \log_8 32)$ is:

- (A) $\frac{7}{2}$ (B) $\frac{19}{6}$ (C) 4 (D) 7

Q.34 $(\log_5 3) \times (\log_3 625)$ equals:

- (A) 1 (B) 2 (C) 3 (D) 4

Q.35 $(\log_5 5) (\log_4 9) (\log_3 2)$ is equal to:

- (A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 5

Q.36 If $\log_{12} 27 = a$, then $\log_6 16$ is:

- (A) $\frac{3-a}{4(3+a)}$ (B) $\frac{3+a}{4(3-a)}$
 (C) $\frac{4(3+a)}{(3-a)}$ (D) $\frac{4(3-a)}{(3+a)}$

Q.37 If $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$, then x is equal to:

- (A) 1 (B) 3 (C) 5 (D) 10

Q.38 If $\log_5 (x^2 + x) - \log_5 (x + 1) = 2$, then the value of x is:

- (A) 5 (B) 10 (C) 25 (D) 32

Q.39 The value of $\left(\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60} \right)$ is:

- (A) 0 (B) 1 (C) 5 (D) 60

Q.40 The value of $(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9)$ is:

- (A) 2 (B) 7 (C) 8 (D) 33

Q.41 The value of $16^{\log_4 5}$ is:

- (A) $\frac{5}{64}$ (B) 5 (C) 16 (D) 25

Q.42 If $\log x + \log y = \log (x + y)$, then

- (A) $x = y$ (B) $xy = 1$

- (C) $y = \frac{x-1}{x}$ (D) $y = \frac{x}{x-1}$

Q.43 If $\log \frac{a}{b} + \log \frac{b}{a} = \log(a + b)$, then:

- (A) $a + b = 1$ (B) $a - b = 1$
 (C) $a = b$ (D) $a^2 - b^2 = 1$

Q.44 $\left[\log \left(\frac{a^2}{bc} \right) + \log \left(\frac{b^2}{ac} \right) + \log \left(\frac{c^2}{ab} \right) \right]$ is equal to:

- (A) 0 (B) 1 (C) 2 (D) abc

Q.45 $(\log_b a \times \log_c b \times \log_a c)$ is equal to:

- (A) 0 (B) 1
 (C) abc (D) $a + b + c$

Q.46 $\left[\frac{1}{(\log_a bc)+1} + \frac{1}{(\log_b ca)+1} + \frac{1}{(\log_c ab)+1} \right]$
 is equal to:

- (A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 3

Q.47 The value of

$$\left[\frac{1}{\log_{(p/q)} x} + \frac{1}{\log_{(q/r)} x} + \frac{1}{\log_{(r/p)} x} \right] \text{ is}$$

(A) 0 (B) 1 (C) 2 (D) 3

Q.48 If $\log_{10} 7 = a$, then $\log_{10} \left(\frac{1}{70} \right)$ is equal to:

- (A) $-(1 + a)$ (B) $(1 + a)^{-1}$
 (C) $\frac{a}{10}$ (D) $\frac{1}{10a}$

Q.49 If $a = b^x$, $b = c^y$ and $c = a^z$, then the value of xyz is equal to:

- (A) -1 (B) 0 (C) 1 (D) abc

Q.50 If $\log 27 = 1.431$, then the value of $\log 9$ is:

- (A) 0.934 (B) 0.945
 (C) 0.954 (D) 0.958

HINTS & SOLUTION

Sol.1 Let $\log_2 16 = n$.
 Then, $2^n = 16 = 2^4$
 $\Rightarrow n = 4$
 $\therefore \log_2 16 = 4$

Sol.2 Let $\log_{343} 7 = n$.
 Then, $(343)^n = 7$
 $\Leftrightarrow (7^3)^n = 7$
 $\Leftrightarrow 3n = 1 \Leftrightarrow n = \frac{1}{3}$
 $\therefore \log_{343} 7 = \frac{1}{3}$

Sol.3 Let $\log_5 \left(\frac{1}{125} \right) = n$ then,
 $5^n = \frac{1}{125}$
 $\Leftrightarrow 5^n = 5^{-3} \Leftrightarrow n = -3$
 $\therefore \log_5 \left(\frac{1}{125} \right) = -3$

Sol.4 Let $\log_{\sqrt{2}} 32 = n$. Then,
 $(\sqrt{2})^n = 32$
 $\Leftrightarrow (2)^{n/2} = 2^5$
 $\Leftrightarrow \frac{n}{2} = 5$
 $\Leftrightarrow n = 10$
 $\therefore \log_{\sqrt{2}} 32 = 10$

Sol.5 Let $\log_{10} (0.0001) = n$
 Then, $10^n = .0001$
 $\Leftrightarrow 10^n = \frac{1}{10000} = \frac{1}{10^4}$
 $\Leftrightarrow 10^n = 10^{-4}$
 $\Leftrightarrow n = -4$
 $\therefore \log_{10} (.0001) = -4$

Sol.6 Let $\log_{(.01)} (1000) = n$
 Then $(.01)^n = (1000)$
 $\Leftrightarrow \left(\frac{1}{100} \right)^n = 10^3$
 $\Leftrightarrow -2n = 3 \Leftrightarrow n = -\frac{3}{2}$

Sol.7 Let $\log_2 0.0625 = n$

Then, $2^n = 0.0625 = \frac{625}{10000}$

$\Leftrightarrow 2^n = \frac{1}{16}$

$\Leftrightarrow 2^n = 2^{-4}$

$\Leftrightarrow n = -4$

$\therefore \log_2 0.0625 = -4$

Sol.8 $\log_3 x = -2$

$\Leftrightarrow x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Sol.9 $\log_8 x = \frac{2}{3}$

$\Leftrightarrow x = 8^{2/3} = (2^3)^{2/3}$

$x = 2^{(3 \times \frac{2}{3})}$

$x = 2^2 = 4$

Sol.10 $\log_x \left(\frac{9}{16} \right) = -\frac{1}{2}$

$\Leftrightarrow x^{-1/2} = \frac{9}{16}$

$\Leftrightarrow \frac{1}{\sqrt{x}} = \frac{9}{16}$

$\Leftrightarrow \sqrt{x} = \frac{16}{9}$

$\Leftrightarrow x = \left(\frac{16}{9} \right)^2 = \frac{256}{81}$

Sol.11 $\log_x 4 = 0.4$

$\Leftrightarrow \log_x 4 = \frac{4}{10} = \frac{2}{5}$

$\Leftrightarrow x^{2/5} = 4$

$\Leftrightarrow x = 4^{5/2} = (2^2)^{5/2}$

$\Leftrightarrow x = 2^{(2 \times \frac{5}{2})} = 2^5$

$\Leftrightarrow x = 32$

Sol.12 $\log_{10000} x = -\frac{1}{4}$

$\Leftrightarrow x = (10000)^{-1/4}$

$x = (10^4)^{-1/4}$

$x = 10^{-1} = \frac{1}{10}$

Sol.13 $\log_x 4 = \frac{1}{4}$

$\Leftrightarrow x^{1/4} = 4$

$\Leftrightarrow x = 4^4 = 256$

Sol.14 $\log_x(0.1) = -\frac{1}{3}$

$$\Leftrightarrow x^{-1/3} = 0.1$$

$$\Leftrightarrow \frac{1}{x^{1/3}} = 0.1$$

$$\Leftrightarrow x^{1/3} = \frac{1}{0.1} = 10$$

$$\Leftrightarrow x = (10)^3 = 1000.$$

Sol.15 $\log_{32} x = 0.8$

$$\Leftrightarrow x = (32)^{0.8}$$

$$x = (2^5)^{4/5} = 2^4 = 16$$

Sol.16 $\log_2 x = 10 \Rightarrow x = 2^{10}$

$$\therefore \log_x y = 100 \Rightarrow y = x^{100} = (2^{10})^{100} \Rightarrow y = 2^{1000}$$

Sol.17 Let $\log_{(-1/3)} 81 = x$. Then,

$$\left(-\frac{1}{3}\right)^x = 81 = 3^4 = (-3)^4 = \left(-\frac{1}{3}\right)^{-4}$$

$$\therefore x = -4 \text{ i.e., } \log_{(-1/3)} 81 = -4$$

Sol.18 Let $\log_{2\sqrt{3}}(1728) = x$

$$\text{Then, } (2\sqrt{3})^x = 1728 = (12)^3 = [(2\sqrt{3})^2]^3$$

$$(2\sqrt{3})^x = (2\sqrt{3})^6$$

$$\therefore x = 6, \text{ i.e., } \log_{2\sqrt{3}}(1728) = 6$$

Sol.19 $\frac{\log \sqrt{8}}{\log 8} = \frac{\log(8)^{1/2}}{\log 8} = \frac{\frac{1}{2} \log 8}{\log 8} = \frac{1}{2}$

Sol.20 (a) Since $\log_a a = 1$,

$$\text{So } \log_{10} 10 = 1$$

(b) $\log(2+3) = 5$ and

$$\log(2 \times 3) = \log 6 = \log 2 + \log 3$$

$$\therefore \log(2+3) \neq \log(2 \times 3)$$

(c) Since $\log_a 1 = 0$, so $\log_{10} 1 = 0$

(d) $\log(1+2+3) = \log 6 = \log(1 \times 2 \times 3)$

$$= \log 1 + \log 2 + \log 3$$

So, (b) is incorrect

Sol.21 Let $\log_5 625 = x$. Then $5^x = 625 = 5^4$ or $x = 4$

$$\text{Let } \log_2(\log_5 625) = y. \text{ Then, } \log_2 4 = y$$

$$\text{or } 2^y = 4$$

$$\text{or } y = 2$$

$$\therefore \log_2(\log_5 625) = 2$$

Sol.22 $\log_2 [\log_3(\log_2 x)] = 1 = \log_2 2$

$$\Leftrightarrow \log_3(\log_2 x) = 2$$

$$\Leftrightarrow \log_2 x = 3^2 = 9$$

$$\Leftrightarrow x = 2^9 = 512$$

$$\begin{aligned}
 \text{Sol.23 } & \log_2 \log_2 \log_3 (\log_3 27^3) \\
 &= \log_2 \log_2 \log_3 [\log_3 (3^3)^3] = \log_2 \log_2 \log_3 [\log_3 (3)^9] \\
 &= \log_2 \log_2 \log_3 (9 \log_3 3) = \log_2 \log_2 \log_3 9 \\
 &\quad \because \log_3 3 = 1 \\
 &= \log_2 \log_2 [\log_3 (3)^2] = \log_2 \log_2 (2 \log_3 3) \\
 &= \log_2 \log_2 2 = \log_2 1 = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{Sol.24 } & a^x = b^y \Rightarrow \log a^x = \log b^y \\
 & \Rightarrow x \log a = y \log b \Rightarrow \frac{\log a}{\log b} = \frac{y}{x}
 \end{aligned}$$

$$\begin{aligned}
 \text{Sol.25 } & 360 = (2 \times 2 \times 2) \times (3 \times 3) \times 5 \\
 & \text{So, } \log 360 \\
 &= \log (2^3 \times 3^2 \times 5) = \log 2^3 + \log 3^2 + \log 5 \\
 &= 3 \log 2 + 2 \log 3 + \log 5
 \end{aligned}$$

$$\begin{aligned}
 \text{Sol.26 } & \frac{1}{3} \log_{10} 125 - 2 \log_{10} 4 + \log_{10} 32 \\
 &= \log_{10} (125)^{1/3} - \log_{10} (4)^2 + \log_{10} 32 \\
 &= \log_{10} 5 - \log_{10} 16 + \log_{10} 32 \\
 &= \log_{10} \left(\frac{5 \times 32}{16} \right) = \log_{10} 10 = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Sol.27 } & 2 \log_{10} 5 + \log_{10} 8 - \frac{1}{2} \log_{10} 4 \\
 &= \log_{10} (5^2) + \log_{10} 8 - \log_{10} (4^{1/2}) \\
 &= \log_{10} 25 + \log_{10} 8 - \log_{10} 2 = \log_{10} \left(\frac{25 \times 8}{2} \right) \\
 &= \log_{10} 100 = 2
 \end{aligned}$$

$$\text{Sol.28 } \log_a (ab) = x$$

$$\begin{aligned}
 \Leftrightarrow \frac{\log ab}{\log a} &= x & \Leftrightarrow \frac{\log a + \log b}{\log a} &= x \\
 \Leftrightarrow 1 + \frac{\log b}{\log a} &= x & \Leftrightarrow \frac{\log b}{\log a} &= x - 1 \\
 \Leftrightarrow \frac{\log a}{\log b} &= \frac{1}{x - 1} & \Leftrightarrow 1 + \frac{\log a}{\log b} &= 1 + \frac{1}{x - 1} \\
 \Leftrightarrow \frac{\log b + \log a}{\log b} &= \frac{x}{x - 1} \\
 \Leftrightarrow \frac{\log(ab)}{\log b} &= \frac{x}{x - 1} & \Leftrightarrow \log_b (ab) &= \frac{x}{x - 1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Sol.29 } & \log (4\sqrt[3]{63}) = \log 4 + \log (\sqrt[3]{63}) = \log 4 + \log (63)^{1/3} \\
 &= \log (2^2) + \log (7 \times 3^2)^{1/3} \\
 &= 2 \log 2 + \frac{1}{3} \log 7 + \frac{2}{3} \log 3 = 2x + \frac{1}{3}z + \frac{2}{3}y
 \end{aligned}$$

Sol.30 $\log_4 x + \log_2 x = 6$

$$\begin{aligned}\Leftrightarrow \frac{\log x}{\log 4} + \frac{\log x}{\log 2} &= 6 \\ \Leftrightarrow \frac{\log x}{2\log 2} + \frac{\log x}{\log 2} &= 6 \\ \Leftrightarrow 3 \log x &= 12 \log 2 \\ \Leftrightarrow \log x &= 4 \log 2 \\ \Leftrightarrow \log x &= \log (2^4) = \log 16 \\ \Leftrightarrow x &= 16\end{aligned}$$

Sol.31 $\log_8 x + \log_8 \left(\frac{1}{6}\right) = \frac{1}{3}$

$$\begin{aligned}\Leftrightarrow \frac{\log x}{\log 8} + \frac{\log \frac{1}{6}}{\log 8} &= \frac{1}{3} \\ \Leftrightarrow \log x + \log \frac{1}{6} &= \frac{1}{3} \log 8 \\ \Leftrightarrow \log x + \log \frac{1}{6} &= \log (8^{1/3}) = \log 2 \\ \Leftrightarrow \log x &= \log 2 - \log \frac{1}{6} = \left(2 \times \frac{6}{1}\right) = \log 12 \\ \therefore x &= 12\end{aligned}$$

Sol.32 $\log_{10} 125 + \log_{10} 8 = x \Rightarrow \log_{10} (125 \times 8) = x$
 $\Rightarrow x = \log_{10} (1000) = \log_{10} (10)^3 = 3 \log_{10} 10 = 3$

Sol.33 Let $\log_9 27 = x$ Then,

$$9^x = 27 \Leftrightarrow (3^2)^x = 3^3 \Leftrightarrow 2x = 3 \Leftrightarrow x = \frac{3}{2}$$

Let $\log_8 32 = y$. Then,

$$8^y = 32 \Leftrightarrow (2^3)^y = 2^5 \Leftrightarrow 3y = 5 \Leftrightarrow y = \frac{5}{3}$$

$$\therefore \log_9 27 + \log_8 32 = \left(\frac{3}{2} + \frac{5}{3}\right) = \frac{19}{6}$$

Sol.34 Given expression = $\left(\frac{\log 3}{\log 5} \times \frac{\log 625}{\log 3}\right)$

$$= \frac{\log 625}{\log 5} = \frac{\log (5^4)}{\log 5} = \frac{4 \log 5}{\log 5} = 4$$

Sol.35 Given expression = $\frac{\log 9}{\log 4} \times \frac{\log 2}{\log 3} [\because \log_5 5 = 1]$

$$= \frac{\log 3^2}{\log 2^2} \times \frac{\log 2}{\log 3} \times \frac{2 \log 3}{2 \log 2} \times \frac{\log 2}{\log 3} = 1$$

Sol.36 $\log_{12} 27 = a \Rightarrow \frac{\log 27}{\log 12} = a$

$$\Rightarrow \frac{\log 3^3}{\log(3 \times 2^2)} = a \Rightarrow \frac{3 \log 3}{\log 3 + 2 \log 2} = a \Rightarrow \frac{\log 3 + 2 \log 2}{3 \log 3} = \frac{1}{a} \Rightarrow \frac{\log 3}{3 \log 3} + \frac{2 \log 2}{3 \log 3} = \frac{1}{a}$$

$$\Rightarrow \frac{2 \log 2}{3 \log 3} = \frac{1}{a} - \frac{1}{3} = \left(\frac{3-a}{3a} \right)$$

$$\Rightarrow \frac{\log 2}{\log 3} = \left(\frac{3-a}{2a} \right) \Rightarrow \log 3 = \left(\frac{2a}{3-a} \right) \log 2$$

$$\Rightarrow \log_6 16 = \frac{\log 16}{\log 6} = \frac{\log 2^4}{\log(2 \times 3)}$$

$$= \frac{4 \log 2}{\log 2 + \log 3} = \frac{4 \log 2}{\log 2 \left[1 + \left(\frac{2a}{3-a} \right) \right]}$$

$$= \frac{4}{\left(\frac{3+a}{3-a} \right)} = \frac{4(3-a)}{(3+a)}$$

Sol.37 $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$
 $\Rightarrow \log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + \log_{10} 10$
 $\Rightarrow \log_{10} [5(5x + 1)] = \log_{10} [10(x + 5)]$
 $\Rightarrow 5(5x + 1) = 10(x + 5)$
 $\Rightarrow 5x + 1 = 2x + 10$
 $\Rightarrow 3x = 9$
 $\Rightarrow x = 3$

Sol.38 $\log_5(x^2 + x) - \log_5(x + 1) = 2 \Rightarrow \log_5 \left(\frac{x^2 + x}{x + 1} \right) = 2$

$$\Rightarrow \log_5 \left[\frac{x(x+1)}{x+1} \right] = 2$$

$$\Rightarrow \log_5 x = 2 \Rightarrow x = 5^2 = 25$$

Sol.39 Given expression

$$= \log_{60} 3 + \log_{60} 4 + \log_{60} 5$$

$$= \log_{60} (3 \times 4 \times 5) = \log_{60} 60 = 1$$

Sol.40 Given expression =

$$\left(\frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8} \right)$$

$$= \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2 \log 3}{\log 3} = 2$$

Sol.41 We know that : $a^{\log_a x} = x$.

$$\therefore 16^{\log_4 5} = (4^2)^{\log_4 5} = 4^{2\log_4 5} = 4^{\log_4 (5^2)} = 4^{\log_4 25} = 25$$

Sol.42 $\log x + \log y = \log (x + y) \Rightarrow \log(x + y) = \log (xy)$

$$\Rightarrow x + y = xy \Rightarrow y(x - 1) = x \Rightarrow y = \frac{x}{x-1}$$

Sol.43 $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b)$

$$\Rightarrow \log (a + b) = \log \left(\frac{a}{b} \times \frac{b}{a} \right) = \log 1$$

$$\text{So, } a + b = 1$$

Sol.44 Given expression = $\log \left(\frac{a^2}{bc} \times \frac{b^2}{ac} \times \frac{c^2}{ab} \right) = \log 1 = 0$

Sol.45 Given expression = $\left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a} \right) = 1$

Sol.46 Given expression

$$\begin{aligned} &= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c} \\ &= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)} \\ &= \frac{1}{\log_{abc} a + \log_{abc} b + \log_{abc} c} \\ &= \log_{abc} (abc) = 1 \end{aligned}$$

Sol.47 Given expression

$$\begin{aligned} &= \log_x \left(\frac{p}{q} \right) + \log_x \left(\frac{q}{r} \right) + \log_x \left(\frac{r}{p} \right) \\ &= \log_x \left(\frac{p}{q} \times \frac{q}{r} \times \frac{r}{p} \right) = \log_x 1 = 0 \end{aligned}$$

Sol.48 $\log_{10} \left(\frac{1}{70} \right) = \log_{10} 1 - \log_{10} 70 = -\log_{10} (7 \times 10)$

$$= -(\log_{10} 7 + \log_{10} 10) = -(a + 1)$$

Sol.49 $a = b^x$, $b = c^y$, $c = a^z$

$$\Rightarrow x = \log_b a, y = \log_c b, z = \log_a c$$

$$\Rightarrow xyz = (\log_b a) \times (\log_c b) \times (\log_a c)$$

$$\Rightarrow xyz = \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a} \right) = 1$$

Sol.50 $\log 27 = 1.431 \Rightarrow \log(3^3) = 1.431 \Rightarrow 3 \log 3 = 1.431$
 $\Rightarrow \log 3 = 0.477$
 $\therefore \log 9 = \log(3^2) = 2 \log 3 = (2 \times 0.477) = 0.954$