Chapter 13. Inequalities in Triangles

Ex 13.1

Answer 1A.

In the given ΔABC the greatest angle is $\angle B$ and the opposite side to the $\angle B$ is AC.
Hence, the greatest side is AC.
The smallest angle in the ΔABC is $\angle A$ and the opposite side to the $\angle A$ is BC.
Hence, the smallest side is BC.

Answer 1B.

In the given ΔDEF the greatest angle is $\angle F$ and the opposite side to the $\angle F$ is DE.
Hence, the greatest side is DE.
The smallest angle in the ΔDEF is D and the opposite side to the $\angle D$ is EF.
Hence, the smallest side is EF.

Answer 1C.

In ΔXYZ,

$\angle X + \angle Y + \angle Z = 180^\circ$

$76^\circ + 84^\circ + \angle Z = 180^\circ$

$160^\circ + \angle Z = 180^\circ$

$\angle Z = 180^\circ - 160^\circ$

$\angle Z = 20^\circ$

Hence, $\angle X = 76^\circ$, $\angle Y = 84^\circ$, $\angle Z = 20^\circ$

In the given ΔXYZ the greatest angle is $\angle Y$ and the opposite side to the $\angle Y$ is XZ.
Hence, the greatest side is XZ.
The smallest angle in the ΔXYZ is $\angle Z$ and the opposite side to the $\angle Z$ is XY.
Hence, the smallest side is XY.

Answer 2A.

In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$45° + 65° + \angle C = 180°$

$110° + \angle C = 180°$

$\angle C = 180° - 110°$

$\angle C = 70°$

Hence, $\angle A = 45°$, $\angle B = 65°$, $\angle C = 70°$

$45° < 65° < 70°$

Hence, ascending order of the angles in the given triangle is $\angle A < \angle B < \angle C$.

Hence, ascending order of sides in triangle BC, AC, AB.

Answer 2B.

In ΔDEF,

$\angle D + \angle E + \angle F = 180°$

$38° + 58° + \angle F = 180°$

$96° + \angle F = 180°$

$\angle F = 180° - 96°$

$\angle F = 84°$

Hence, $\angle D = 38°$, $\angle E = 58°$, $\angle F = 84°$

$38° < 58° < 84°$

Hence, ascending order of the angles in the given triangle is $\angle D < \angle E < \angle F$.

Hence, ascending order of sides in triangle EF, DF, DE.

Answer 3.

(i) It is known that, in a triangle, the angle opposite to the smallest side is the smallest.

In ΔABC, AC = 4.2cm is the smallest side.

$\therefore \angle B$ is the smallest angle.

(ii) It is known that, in a triangle, the angle opposite to the smallest side is the smallest.

In ΔPQR, QR = 5.4cm is the smallest side.

$\therefore \angle P$ is the smallest angle.

(iii) It is known that, in a triangle, the angle opposite to the smallest side is the smallest.

In ΔXYZ, YZ = 5cm is the smallest side.

$\therefore \angle X$ is the smallest angle.

Answer 4.

In ΔABC,

$BC = AC$ (given)

$\Rightarrow \angle A = \angle B = 35^\circ$

Let $\angle C = x^\circ$

In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$

$35^\circ + 35^\circ + x^\circ = 180^\circ$

$70^\circ + x^\circ = 180^\circ$

$x^\circ = 180^\circ - 70^\circ$

$x^\circ = 110^\circ$

$\angle C = x^\circ = 110^\circ$

Hence, $\angle A = \angle B = 35^\circ$ and $\angle C = 110^\circ$

In ΔABC, the greatest angle is $\angle C$.

As the smallest angles are $\angle A$ and $\angle B$,

smallest sides are BC and AC.

Answer 5.

It is given that $\angle PBC > \angle QCB$ -------(1)

$\angle PBC + \angle ABC = 180^\circ$ [Linear pair angles]

$\Rightarrow \angle PBC = 180 - \angle ABC$

Similarly, $\angle QCB = 180^\circ - \angle ACB$

From (1) and (2)

$180 - \angle ABC > 180 - \angle ACB$

$\Rightarrow -\angle ABC > -\angle ACB$

$\Rightarrow \angle ABC < \angle ACB$ or $\angle ACB > \angle ABC$

It is known that, in a triangle, the greater angel has the longer ide opposite to it.

$\therefore AB > AC$

Answer 6.

Using the exterior angle property in ΔACD, we have

$\angle ACB = \angle CDA + \angle CAD$

$\Rightarrow \angle ACB > \angle CDA$ ------(1)

Now, AB = AC

$\Rightarrow \angle ACB = \angle ABC$ ------(2)

From (1) and (2)

$\angle ABC > \angle CDA$

It is known that, in a triangle, the greater angle has the longer side opposite to it.

Now, In ΔABD, we have $\angle ABC > \angle CDA$

$\therefore AD > AB$

Answer 7.

Given: A Δ ABC in which AD, BE and CF are its medians.

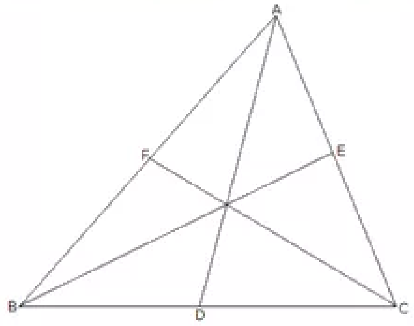
To Prove: We know that the sum of any two dies of a triangle is greater than twice the median bisecting the third side. Therefore,

AD is the median bisecting BC

$\Rightarrow AB + AC > 2\ AD$ ...(i)

BE is the median bisecting AC ...(ii)

And, CF is the median bisecting AB



$\Rightarrow BC + AC > 2EF$ ...(iii)

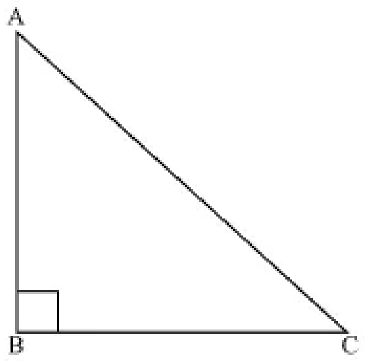
Adding (i), (ii) and (iii), we get

$(AB + AC) + (AB + BC) + (BC + AC) > 2.\ AD + 2.\ BE + 2.CF$

$\Rightarrow 2\ (AB + BC + AC) > 2\ (AD + BE + CF)$

$\Rightarrow AB + BC + AC > AD + BE + CF$

Answer 8.



Let us consider a right angled triangle ABC, right angle at B.

In ΔABC

$\angle A + \angle B + \angle C = 180^\circ$ (angle sum property of a triangle)

$\angle A + 90^\circ + \angle C = 180^\circ$

$\angle A + \angle C = 90^\circ$

Hence, the other two angles have to be acute (i.e. less than 90°).

$\therefore \angle B$ is the largest angle in ΔABC.

$\Rightarrow \angle B > \angle A$ and $\angle B > \angle C$

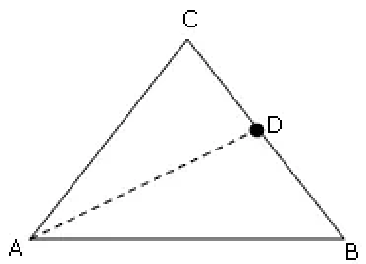
$\Rightarrow AC > BC$ and $AC > AB$

[In any triangle, the side opposite to the larger (greater) angle is longer]

So, AC is the largest side in ΔABC.

But AC is the hypotenuse of ΔABC. Therefore, hypotenuse is the longest side in a right angled triangle.

Answer 9.



Construction: Join AD

In triangle ACD,

AC+CD>AD ... (i)

(Sum of two sides of a triangle greater than the third side)

Similarly, in triangle ADB,

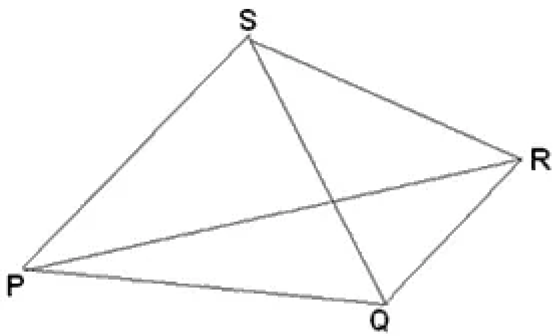
AB+BD>AD ... (ii)

Adding (i) and (ii),

AC+CD+AB+BD>2AD

AB+BC+AC>2AD (Since, CD + BD = BC)

Answer 10.



Given: PQRS is a quadrilateral. PR and QS are its diagonals.

To Prove: PQ+QR+SR+PS > PR+QS

Proof: In ΔPQR

PQ+QR>PR (Sum of two sides of triangle is greater than the third side) Similarly, In ΔPSR, PS+SR>PR

In ΔPQS, PS+PQ>QS and in ΔQRS we have QR+SR>QS

Now we have PQ+QR>PR

PS+SR>PR

PS+PQ>QS

QR+SR>QS

After adding above inequalities we get

2(PQ+QR+PS+SR) > 2(PR+QS)

⇒ PQ+QR+PS+SR>PR+QS.

Answer 11.

In Δ AOB, we have

OA + OB > AB ... (i)

In Δ BOC, we have

OB + OC > BC ... (ii)

In Δ COD, we have

OC + OD > CD ... (iii)

In Δ AOD, we have

OA + OD > AD ... (iv)

Adding (i), (ii), (iii) and (iv), we get

2 (OA + OB + OC + OD) > AB + BC + CD + AD

⇒ 2 [(OA + OC) + (OB + OD)] > AB + BC + CD + AD

⇒ 2 (AC + BD) > AB + BC + CD + AD

[∵ OA + OC = AC and OB + OD = BD]

⇒ AB + BC + CD + AD < 2 (AC + BD)

Answer 12.

In triangle APR,

AP+AR>PR ......(i)

In triangle BPQ,

BQ+PB>PQ ......(ii)

In triangle QCR,

QC+CR>QR ......(iii)

Adding (i), (ii) and (iii)

AP+AR+BQ+PB+QC+CR > PR+PQ+QR

(AP+PB)+(BQ+QC)+(CR+AR) > PR+QR+PQ

⇒ AB + BC + AC > PQ + QR + PR.

Answer 13.

In ΔPQT, we have

$PT = PQ \quad \ldots (1)$

In ΔPQR,

$PQ + QR > PR$

$PQ + QR > PT + TR$

$PQ + QR > PQ + TR \quad$ [Using (1)]

$QR > TR$

Hence, proved.

Answer 14.

(i). In ΔABC, we have

$AB + BC > AC \quad \ldots(i)$

In ΔACD, we have

$AD + CD > AC \quad \ldots(ii)$

Adding (i) and (ii), we get

$AB + BC + AD + CD > 2\,AC$

(ii). In ΔACD, we have

$CD + DA > CA$

$\Rightarrow CD + DA + AB > CA + AB$

$\Rightarrow CD + DA + AB > BC \quad [\because AB + AC > BC]$

Answer 15.

a. In the given ΔPQR,

$PS < PR \quad \ldots\ldots \left(\begin{array}{l}\text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.}\end{array}\right)$

$PN < PR \quad \ldots\ldots(i) \quad (\because PN < PS)$

Also,

$RT < PR \quad \ldots\ldots \left(\begin{array}{l}\text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.}\end{array}\right)$

$RN < PR \quad \ldots\ldots(ii) \quad (\because RN < RT)$

Dividing (i) by (ii),

$$\frac{PN}{RN} < \frac{PR}{PR}$$

$$\frac{PN}{RN} < 1$$

$PN < RN$

b. In ΔRTQ,

$\angle RTQ + \angle TQR + \angle TRQ = 180^\circ$

$90^\circ + 60^\circ + \angle TRQ = 180^\circ$

$150^\circ + \angle TRQ = 180^\circ$

$\angle TRQ = 180^\circ - 150^\circ$

$\angle TRQ = 30^\circ$

$\angle TRQ = \angle SRN = 30^\circ \quad \ldots\ldots(iii)$

In ΔNSR,

$\angle RNS + \angle SRN = 90^\circ \quad \ldots\ldots(\because \angle NSR = 90^\circ)$

$\angle RNS + 30^\circ = 90^\circ \quad \ldots\ldots[\text{from (iii)}]$

$\angle RNS = 90^\circ - 30^\circ$

$\angle RNS = 60^\circ \quad \ldots\ldots(iv)$

$\angle SRN < \angle RNS \quad \ldots\ldots(\text{from (iii) and (iv)})$

$SN < SR$

Answer 16.

In ΔACD,

$AC = CD$ ....(Given)

$\angle CDA = \angle DAC$ ....(ΔACD is isosceles triangle.)

Let $\angle CDA = \angle DAC = x°$

$\angle CDA + \angle DAC + \angle ACD = 180°$

$x° + x° + 105° = 180°$

$2x° + 105° = 180°$

$2x° = 180° - 105°$

$2x° = 75°$

$x = \frac{75°}{2}$

$x = 37.5°$

$\angle C = \angle DAC = x° = 37.5°$......(i)

$\angle DAB = \angle DAC + \angle BAC$

$125° = 37.5° + \angle BAC$ ....from (i)

$125° - 37.5° = \angle BAC$

$87.5° = \angle BAC$

Also, $\angle BCA + \angle ACD = 180°$

$\Rightarrow \angle BCA + 105° = 180°$

$\Rightarrow \angle BCA = 75°$

So, in ΔBAC,

$\angle ACB + \angle BAC + \angle ABC = 180°$

$\Rightarrow 75° + 87.5° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 17.5°$

As $87.5° > 17.5°$

$\angle BAC > \angle ABC$

$\Rightarrow BC > AC$

$\Rightarrow BC > CD$ ....(Since $AC = CD$)

Answer 17A.

In ΔPQS,

$PS < PQ$ ..... $\left(\begin{array}{l}\text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.}\end{array}\right)$

i.e. $PQ > PS$

Also, $QS < QP$ ..... $\left(\begin{array}{l}\text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.}\end{array}\right)$

i.e. $PQ > QS$

Answer 17B.

In ΔPQS,

$PS \perp QR$ ....(Given)

$PS < PR$ .... $\left(\begin{array}{l}\text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.}\end{array}\right)$

i.e. $PR > PS$

Answer 17C.

In ΔPQR,

$PQ + PR > QR$ $\left(\begin{array}{l}\because \text{Sum of two sides of a} \\ \text{triangle is always greater} \\ \text{than third side.}\end{array}\right)$

In ΔPQS,

$PQ + QS > PS$ $\left(\begin{array}{l}\because \text{Sum of two sides of a} \\ \text{triangle is always greater} \\ \text{than third side.}\end{array}\right)$

.....(i)

In ΔPRS,

$PR + SR > PS$ $\left(\begin{array}{l}\because \text{Sum of two sides of a} \\ \text{triangle is always greater} \\ \text{than third side.}\end{array}\right)$

.....(ii)

Adding (i) and (ii),

$PQ + QS + PR + SR > 2PS$

$PQ + (QS + SR) + PR > 2PS$

$PQ + QR + PR > 2PS$

Since $PQ + PR > QR$

$\Rightarrow PQ + QR > 2PS$

Answer 18.

Note : The question is incomplete.
Question should be :
In the given figure, T is a point on the side PR of an **equilateral** triangle PQR.
Show that :
a. PT $<$ QT
b. RT $<$ QT
Solution :
a. In ΔPQR,
$PQ = QR = PR$
$\Rightarrow \angle P = \angle Q = \angle R = 60^\circ$
In ΔPQT,
$\angle PQT < 60^\circ$
$\therefore \angle PQT < \angle P$
$\therefore PT < QT$
b. In ΔTQR,
$\angle TQR < 60^\circ$
$\therefore \angle TQR < \angle R$
$\therefore RT < QT$

Answer 19.

In ΔPQR,

$PQ + PR > QR$ .... $\left(\because \text{Sum of the two sides of a triangle is always greater than the third side.}\right)$ ....(i)

Also, in ΔSQR,

$SQ + SR > QR$ .... $\left(\because \text{Sum of the two sides of a triangle is always greater than the third side.}\right)$ ....(ii)

Dividing (i) by (ii),

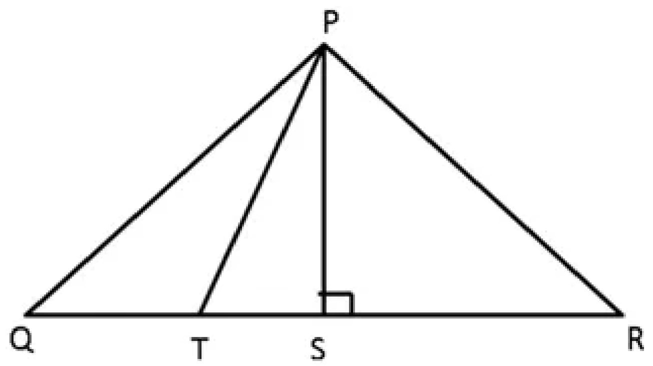
$$\frac{PQ + PR}{SQ + SR} > \frac{QR}{QR}$$

$$\frac{PQ + PR}{SQ + SR} > 1$$

$PQ + PR > SQ + SR$

i.e. $SQ + SR < PQ + PR$

Answer 20.



Let the triangle be ΔPQR.
$PS \perp QR$, the straight line joining vertex P to the line QR.
To prove: $PQ > PT$ and $PR > PT$
In ΔPSQ,
$PS^2 + SQ^2 = PQ^2$ ....(Pythagoras theorem)
$PS^2 = PQ^2 - SQ^2$ ....(i)
In ΔPST,
$PS^2 + ST^2 = PT^2$ ....(Pythagoras theorem)
$PS^2 = PT^2 - ST^2$ ......(ii)
$PQ^2 - SQ^2 = PT^2 - ST^2$ ....[from(i) and(ii)]
$PQ^2 - (ST + TQ)^2 = PT^2 - ST^2$
$PQ^2 - (ST^2 + 2ST \times TQ + TQ^2) = PT^2 - ST^2$
$PQ^2 - ST^2 - 2ST \times TQ - TQ^2 = PT^2 - ST^2$
$PQ^2 - PT^2 = TQ^2 + 2ST \times TQ$
$PQ^2 - PT^2 = TQ \times (2ST + TQ)$
As, $TQ \times (2ST + TQ) > 0$ always.
$PQ^2 - PT^2 > 0$
$PQ^2 > PT^2$
$PQ > PT$
Also, $PQ = PR$
$PR > PT$

Answer 21.

$\angle AEF > \angle ABC$ ...(Exterior angle property)
$\angle AFE = \angle DFC$
$\angle ACB > \angle DFC$...(Exterior angle property)
$\Rightarrow \angle ACB > \angle AFE$
Since $AB = AC$
$\Rightarrow \angle ACB = \angle ABC$
So, $\angle ABC > \angle AFE$
$\Rightarrow \angle AEF > \angle ABC > \angle AFE$
that is, $\angle AEF > \angle AFE$
$\Rightarrow AF > AE$

Answer 22.

In the ΔABE and ΔADE,

$AB = AD$ ....(Given)

$\angle BAE = \angle DAE$ ....(AE is the bisector of $\angle BAC$)

$AE = AE$ ....(Common side)

$\therefore \Delta ABE \cong \Delta ADE$ ....(SAS test)

$\Rightarrow BE = DE$ ....(c.p.c.t.c)

In ΔABD,

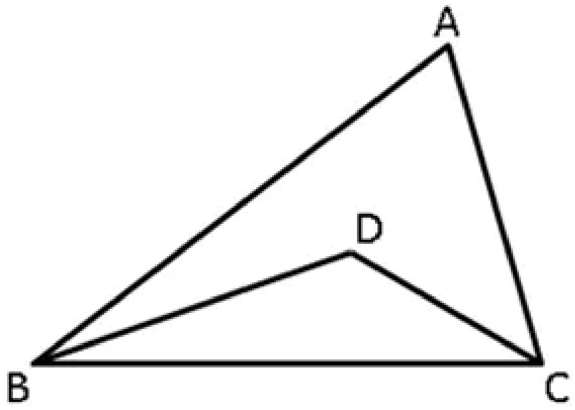
$AB = AD$

$\Rightarrow \angle ABD = \angle ADB$

$\angle ADB > \angle C$ ...(Exterior angle property)

$\Rightarrow \angle ABD > \angle C$

Answer 23.



In the ΔABC,

$AB + AC > BC$ ...... (∵ Sum of the two sides of triangle is always greater than third side.) ....(i)

Also, in the ΔBDC,

$BD + DC > BC$....(ii)

Dividing (i) by (ii),

$$\frac{AB + AC}{BD + DC} > \frac{BC}{BC}$$

$$\frac{AB + AC}{BD + DC} > 1$$

$AB + AC > BD + DC$

i.e. $BD + DC < AB + AC$