CLASS XI Ch 9 - SEQUENCES & SERIES

EXERCISE 9.2 (NCERT)

Q No 1: Find the sum of all odd integers from 1 to 2001.

Sol: Odd integers from 1 to 2001 are 1, 3, 5, ... 2001.
which are in A.P with
First term $a = 1$ last term $l = 2001$
and common difference $d = a_2 - a_1 = 3 - 1 = 2$.
Now as $l = a + (n-1)d$, wher n is no. of terms.

$\therefore \quad 2001 = 1 + (n-1)2$

$\Rightarrow \quad 2001 - 1 = 2n - 2$

$\Rightarrow \quad 2000 + 2 = 2n$

$\Rightarrow \quad n = \frac{2002}{2} = 1001$

$\therefore$ Required Sum $S_n = \frac{1001}{2}[1 + 2001] \quad \left[\because S_n = \frac{n}{2}[a+l]\right]$

$= \frac{1001}{2}(2002) = 1001 \times 1001 = 1002001.$

Q No 2. Find the sum of all natural nos. lying between 100 and 1000 which are multiples of 5.

Sol. Natural Nos. which are between 100 and 1000 and multiples of 5 are 105, 110, 115, ... 995.

$\therefore \quad S_n = 105 + 110 + \ldots\ldots 995.$

Here $a = 105$, $d = a_2 - a_1 = 110 - 105 = 5$

and $a_n = l = 995$

Now $l = a + (n-1)d.$

$\therefore 995 = 105 + (n-1)5$

$\Rightarrow \quad 995 - 105 = 5n - 5$

$\Rightarrow \quad 890 + 5 = 5n$

$\Rightarrow \quad \frac{895}{5} = n$

$\Rightarrow \quad 179 = n.$

$$\therefore \quad S_n = \frac{n}{2}(a+l)$$

$$\Rightarrow \quad S_{179} = \frac{179}{2}(105+995) = \frac{179}{2}(1100)$$

$$= 179 \times 550 = 98450$$

Q No 3: In an AP, the first term is 2 and the sum of first 5 terms is one-fourth of next five terms. Show that 20^{th} term is -112

Sol. Let d be the common difference of given AP.

According to given condition.

$$(a_1+a_2+a_3+a_4+a_5) = \frac{1}{4}(a_6+a_7+a_8+a_9+a_{10})$$

i.e. $2+(2+d)+(2+2d)+(2+3d)+(2+4d) = \frac{1}{4}\left[(2+5d)+(2+6d)+(2+7d)+(2+8d)+(2+9d)\right]$

$$\Rightarrow \frac{5}{2}\left[2+(2+4d)\right] = \frac{1}{4}\left[\frac{5}{2}(2+5d+2+9d)\right] \quad \left[\text{using } S_n = \frac{n}{2}(a+l)\right]$$

$$\Rightarrow \frac{5}{2}\left[4+4d\right] = \frac{1}{4}\left[\frac{5}{2}(4+14d)\right]$$

$$\Rightarrow 4+4d = \frac{1}{4}(4+14d)$$

$$\Rightarrow 16+16d = 4+14d$$

$$\Rightarrow 16d-14d = 4-16 \Rightarrow 2d = -12 \Rightarrow d = \frac{-12}{2} = -6$$

$$\therefore \quad a_{20} = a+19d. \qquad [\because a_n = a+(n-1)d]$$

$$= 2+19(-6) = 2-114 = -112.$$

Q No 4. How many terms of the A.P. $-6, -\frac{11}{2}, -5 \ldots$ are needed to give the sum -25?

Sol. Given A.P. is $-6, -\frac{11}{2}, -5, \ldots$

Here $a=-6$, $d = -\frac{11}{2}-(-6) = -\frac{11}{2}+6 = \frac{-11+12}{2} = \frac{1}{2}$

Let -25 be the sum of n terms.

$$\therefore \quad S_n = -25.$$

$$\Rightarrow \frac{n}{2}\left[2a+(n-1)d\right] = -25$$

$$\Rightarrow \frac{n}{2}\left[2\times(-6)+(n-1)\frac{1}{2}\right] = -25$$

$$\Rightarrow \frac{n}{2}\left[-12+\frac{n-1}{2}\right] = -25$$

$$\Rightarrow n\left[\frac{-24+n-1}{2}\right] = -25\times 2.$$

$$\Rightarrow n(n-25) = -50\times 2.$$

$$\Rightarrow n^2-25n = -100 \Rightarrow n^2-25n+100 = 0$$

$$\Rightarrow (n-5)(n-20) = 0 \Rightarrow n = 5, 20.$$

Q No 5 In an AP if p^{th} term is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first pq terms is $\frac{1}{2}(pq+1)$ where $p \neq q$.

Sol: Let a be the first term and d be the common difference of given A.P.

From the given condition

$$a+(p-1)d = \frac{1}{q} \quad \cdots\cdots(1)$$

and $$a+(q-1)d = \frac{1}{p}. \quad \cdots\cdots(2)$$

Subtracting (2) from (1)

$$(p-1-q+1)d = \frac{1}{q}-\frac{1}{p}.$$

$$\Rightarrow (p-q)d = \frac{p-q}{pq}$$

$$\Rightarrow d = \frac{1}{pq}$$

Putting $d = \frac{1}{pq}$ in (1), we get

$$a+(p-1)\frac{1}{pq} = \frac{1}{q}$$

$\Rightarrow \quad a = \frac{1}{q} - \frac{p-1}{pq} = \frac{p-p+1}{pq} = \frac{1}{pq}$

$\therefore \quad S_{pq} = \frac{pq}{2}\left[2 \times \frac{1}{pq} + (pq-1)\cdot\frac{1}{pq}\right] \quad \left[\text{using } S_n = \frac{n}{2}\left[2a+(n-1)\right.\right.$

$= \frac{pq}{2}\left[\frac{2+pq-1}{pq}\right] = \frac{pq+1}{2} = \frac{1}{2}(pq+1)$

Hence the result.

Q.No 6. If the sum of ~~fsxs~~ a certain number of terms of the AP 25, 22, 19, ... is 116, find the last term.

Sol. Here $S_n = 116$, $a = 25$, $d = 22 - 25 = -3$

$n = ?$, $a_n = ?$

Now $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow \quad 116 = \frac{n}{2}\left[2 \times 25 + (n-1)(-3)\right]$

$\Rightarrow \quad 232 = n\left[50 - 3n + 3\right]$

$\Rightarrow \quad 232 = 50n - 3n^2 + 3n$

$\Rightarrow \quad 3n^2 - 53n + 232 = 0$

$\Rightarrow \quad n = \frac{-(-53) \pm \sqrt{(-53)^2 - 4 \times 3 \times 232}}{2 \times 3} \quad \left[\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$

$= \frac{53 \pm \sqrt{2809 - 2784}}{6} = \frac{53 \pm \sqrt{25}}{6}$

$= \frac{53+25}{6}, \frac{53-25}{6}$

$= \frac{58}{6}, \frac{48}{6}$

$= \frac{29}{3}, 8$

$\Rightarrow \quad n = 8 \quad \because \quad n \neq \frac{29}{3}; \quad n \in N.$

$\therefore$ Last term $a_n = a + (n-1)d$

ie $a_8 = 25 + (8-1)(-3) = 25 - 21 = 4$

Sol. Find the sum to n terms of the A.P. whose k^{th} term is $5k+1$

Here k^{th} term $= T_k = 5k+1$

$\therefore$ $T_1 = 5\times1+1 = 6$

$T_2 = 5\times2+1 = 11$

$T_3 = 5\times3+1 = 16$

$T_4 = 5\times4+1 = 21.$

$\therefore$ Series is $6+11+16+21+\ldots\ldots$

Where $a = 6$ and $d = 5$.

$$\therefore \quad S_n = \frac{n}{2}\left[2a+(n-1)d\right] = \frac{n}{2}\left[2\times6+(n-1)5\right]$$

$$= \frac{n}{2}\left[12+5n-5\right] = \frac{n(5n+7)}{2}.$$

QNo.8: If the sum of n terms of an A.P. is $(pn+qn^2)$, where p and q are constants, find the common difference.

Sol. Here $S_n = pn+qn^2$, Put $n=1, 2$

$\therefore$ $S_1 = p+q$

$S_2 = 2p+(2)^2q = 2p+4q.$

Now $T_1 = S_1 = p+q$

$T_2 = S_2 - S_1 = (2p+4q)-(p+q) = p+3q.$

Now $d = T_2 - T_1 = (p+3q)-(p+q) = 2q.$

$\therefore$ $d = 2q.$

QNo 9: The sum of two arithmetic progressions are in the ratio $5n+4 : 9n+6$, find the ratio of their 18^{th} terms.

Sol. Let a_1, a_2 and d_1, d_2 be first terms and common differences of the first and 2nd series respectively.

Now ATQ (According to question)

$$\frac{S_n \text{ of first AP}}{S_n \text{ of Second AP}} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{5n+4}{9n+6}$$

Now Put $\frac{n-1}{2} = 17$ ie $n-1 = 34$ ie $n = 35$

$$\Rightarrow \frac{a_1+17d_1}{a_2+17d_2} = \frac{5\times35+4}{9\times35+6}$$

$$\Rightarrow \frac{18^{th} \text{ term of first AP}}{18^{th} \text{ term of Second AP}} = \frac{179}{321}$$

$\therefore$ Required Ratio is $179:321$.

Q No 10: If the sum of first p terms of an AP is equal to the sum of the first q terms, then find the sum of first $(p+q)$ terms.

Sol. Let 'a' be the first term and d be the common difference of the AP.

ATQ. $\frac{p}{2}\left[2a+(p-1)d\right] = \frac{q}{2}\left[2a+(q-1)d\right]$

$\therefore 2ap+(p^2-p)d = 2aq+(q^2-q)d$

$\therefore (2ap-2aq)+(p^2-p-q^2+q)d = 0$

$\therefore 2a[p-q]+[(p^2-q^2)-(p-q)]d = 0$

$\therefore 2a(p-q)+[(p-q)(p+q)-(p-q)]d = 0$

$\therefore 2a+(p+q-1)d = 0 \quad \cdots (1)$

Now $S_{p+q} = \frac{p+q}{2}\left[2a+(p+q-1)d\right]$

$= \frac{p+q}{2}\times 0 \quad$ (from (1))

$= 0$

$\therefore S_{p+q} = 0.$

Q No 11 Sum of first p, q and r terms of an AP are a, b, c respectively. Prove that

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0.$$

Sol. Let A be the first term and D be the common diff.

ATQ $\quad a=\frac{p}{2}\left[2A+(p-1)D\right]$

$$\Rightarrow \frac{2a}{p}=2A+(p-1)D \quad \text{---} (1)$$

Similarly $\quad \frac{2b}{q}=2A+(q-1)D \quad \text{----} (2)$

$$\frac{2c}{r}=2A+(r-1)D \quad \text{---} (3)$$

Multiplying (1), (2), (3) by $(q-r),(r-p),(p-q)$ respectively and adding we get

$$\frac{2a}{p}(q-r)+\frac{2b}{q}(r-p)+\frac{2c}{r}(p-q)=2A(q-r+r-p+p-q)$$
$$+D\left[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)\right]$$
$$=2A\times 0+D\left[pq-pr-q+r+qr-qp-r+p+pr-qr-p+q\right]$$
$$=0+D[0]=0$$

$$\therefore \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

Q No 12: The ratio of sums of m and n terms of an AP is $m^2:n^2$. Show that the ratio of m^{th} and n^{th} term is $(2m-1):(2n-1)$

Sol. Let a be the first term and d be the common difference of given AP.

ATQ
$$\frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]}=\frac{m^2}{n^2}.$$

$$\Rightarrow \quad \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}.$$

$$\Rightarrow \quad 2an + (mn-n)d = 2am + (mn-m)d.$$

$$\Rightarrow \quad 2an - 2am = (mn-m)d - (mn-n)d.$$

$$\Rightarrow \quad 2a(n-m) = (n-m)d$$

$$\Rightarrow \quad 2a = d$$

Now $\dfrac{m^{th}\text{ term}}{n^{th}\text{ term}} = \dfrac{a+(m-1)d}{a+(n-1)d} = \dfrac{a+(m-1)2a}{a+(n-1)2a}$ $[\because d = 2a]$

$$= \frac{1+(m-1)2}{1+(n-1)2} = \frac{1+2m-2}{1+2n-2} = \frac{2m-1}{2n-1}$$

$\therefore$ Required ration $= 2m-1 : 2n-1$

Hence the result.

Q No. 13. If the sum of n terms of an AP is $3n^2+5n$ and its m^{th} term is 164, find the value of m.

Sol. Here $S_n = 3n^2+5n$

Now $T_n = S_n - S_{n-1}$

$$= (3n^2+5n) - [3(n-1)^2 + 5(n-1)]$$

$$= 3n^2+5n - [3(n^2+1-2n) + (5n-5)].$$

$$= 3n^2+5n-3n^2-3+6n-5n+5$$

$$= 6n+2$$

Now $T_m = 164$

$$\Rightarrow \quad 6m+2 = 164.$$

$$\Rightarrow \quad 6m = 164-2$$

$$\Rightarrow \quad 6m = 162 \Rightarrow m = \frac{162}{6} = 27.$$

$$\therefore \quad m = 27.$$

Q No. 14. Insert 5 numbers between 8 and 26 such that the resulting sequence is an AP.

Sol. Let A_1, A_2, A_3, A_4, A_5 be the 5 numbers between 8 and 26 so that

$8, A_1, A_2, A_3, A_4, A_5, 26$ are in A.P.

Let d be the common difference of this AP.

Then $26 = T_7$, first term $a = 8$

$\Rightarrow \quad 26 = a + (7-1)d.$

$\Rightarrow \quad 26 = 8 + 6d$

$\Rightarrow \quad 26 - 8 = 6d \quad \Rightarrow d = \frac{18}{6} = 3.$

$\therefore \quad A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 9 = 17$

$A_4 = a + 4d = 8 + 12 = 20$

$A_5 = a + 5d = 8 + 15 = 23.$

$\therefore$ 5 A.M's are 11, 14, 17, 20, 23.

Q No 15 If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the AM between a and b, find the value of n.

Sol. Since AM between a and $b = \frac{a+b}{2}$

$$\therefore \quad \frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}.$$

$$\Rightarrow \quad 2(a^n + b^n) = (a+b)(a^{n-1} + b^{n-1})$$

$$\Rightarrow \quad 2a^n + 2b^n = a^n + ab^{n-1} + ba^{n-1} + b^n$$

$$\Rightarrow \quad a^n - a^{n-1}b = ab^{n-1} - b^n$$

$$\Rightarrow \quad a^{n-1}(a-b) = b^{n-1}(a-b)$$

$$\Rightarrow \quad a^{n-1} = b^{n-1} \qquad [\because a - b \neq 0]$$

$$\Rightarrow \quad \frac{a^{n-1}}{b^{n-1}} = 1$$

$$\Rightarrow \quad \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0 \qquad [\because x^0 = 1 \ \forall x]$$

$$\Rightarrow \quad n - 1 = 0$$

$$\Rightarrow \quad n = 1$$

Q No 16 Between 1 and 31 m numbers have been inserted in such a way that the resulting sequence is an AP. The ratio of 7th and $(m-1)^{th}$ number is 5:9. Find 'm'.

Sol. Let $A_1, A_2, \ldots A_m$ be m arithmetic means between 1 and 31

$\therefore$ $1, A_1, A_2 \ldots A_m, 31$ are in AP.

Let d be the common difference of AP.

$\therefore \quad 31 = T_{m+2}$

$\Rightarrow \quad 31 = 1 + (m+2-1)d. \quad [\because a = 1]$

$\Rightarrow \quad 30 = (m+1)d. \Rightarrow d = \frac{30}{m+1}.$

$A_7 = a + 7d = 1 + 7\left(\frac{30}{m+1}\right) = 1 + \frac{210}{m+1}$

$A_{m-1} = a + (m-1)d = 1 + (m-1)\left(\frac{30}{m+1}\right) = 1 + \frac{30(m-1)}{m+1}$

ATQ $\quad \frac{A_7}{A_{m-1}} = \frac{5}{9}$

$\therefore \quad \frac{1 + \frac{210}{m+1}}{1 + \frac{30(m-1)}{m+1}} = \frac{5}{9}.$

$\Rightarrow \quad \frac{m+1+210}{m+1+30m-30} = \frac{5}{9}$

$\Rightarrow \quad \frac{m+211}{31m-29} = \frac{5}{9} \Rightarrow 9(m+211) = 5(31m-29)$

$\Rightarrow \quad 9m + 1899 = 155m - 145$

$\Rightarrow \quad 146m = 2044$

$\Rightarrow \quad m = \frac{2044}{146} = 14.$

$\therefore \quad m = 14.$

Q No. 17. : A man starts repaying a loan as first installment of Rs 100. If he increases the installment by Rs 5 every month, what amount he will pay in 30th installment.

Sol: Let 'a' be the amount of first instalment d be the increase in instalment and let n be the no. of instalments,

$\therefore$ $a = $ Rs 100, $d = $ Rs 5, $n = 30$.

Required Amount $= T_{30} = a + (30-1)d$

$= 100 + 29 \times 5$

$= 100 + 145 = $ Rs 245

QNo. 18: The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of sides of polygon.

Sol. Let a be the smallest interior angle and d be the difference between two consecutive interior angles of polygon.

$\therefore$ $a = 120^\circ$ and $d = 5^\circ$

Let n be the number of sides of polygon.

Now Sum of interior angles $= n\pi - 2\pi = (n-2)\times\pi = (n-2)180^\circ$ — (1)

Also Sum of interior angles $= \frac{n}{2}[2\times120 + (n-1)5]$

$= \frac{n}{2}[240 + 5n - 5]$ — (2)

From (1) and (2)

$$\frac{n}{2}[240 + 5n - 5] = (n-2)180^\circ$$

$\Rightarrow$ $n[235 + 5n] = (n-2)360^\circ$

$\Rightarrow$ $235n + 5n^2 = 360n - 720$

or $5n^2 - 125n + 720 = 0$

or $n^2 - 25n + 144 = 0$

or $(n-9)(n-16) = 0 \Rightarrow n = 9, 16$

Now When $n = 16$ angle $= 120 + (16-1)5 = 120 + 75 = 195^\circ$

Which is not possible

$\therefore$ $n = 9 \Rightarrow$ No. of sides $= 9$.