

ICSE Paper 2013

MATHEMATICS

SECTION A [40 Marks]

(Answer **all** questions from this Section.)

Question 1.

(a) Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.

Find the matrix X such that $A + 2X = 2B + C$.

[3]

(b) At what rate % p.a. will a sum of ₹ 4000 yield ₹ 1324 as compound interest in 3 years ?

[3]

(c) The median of the following observations 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean.

[4]

Solution :

(a) Given : $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$$\therefore A + 2X = 2B + C$$

Putting the given values, we get

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2 \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6 + 4 & 4 + 0 \\ 8 + 0 & 0 + 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$X = \frac{1}{2} \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$$

Ans.

(b) Given : Principal = ₹ 4,000, C.I. = ₹ 1,324,
Amount = P + C.I.
= ₹ (4,000 + 1,324) = ₹ 5,324
Time = 3 years

We know that,

$$A = P \left(1 + \frac{r}{100} \right)^T$$

$$5,324 = 4,000 \left(1 + \frac{r}{100} \right)^3$$

$$\frac{5,324}{4,000} = \left(1 + \frac{r}{100} \right)^3$$

$$\frac{1,331}{1,000} = \left(1 + \frac{r}{100}\right)^3$$

$$\left(\frac{11}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3$$

Therefore,

$$1 + \frac{r}{100} = \frac{11}{10}$$

$$\frac{r}{100} = \frac{11}{10} - 1$$

$$\frac{r}{100} = \frac{1}{10}$$

$$r = \frac{100}{10}$$

$$r = 10\%$$

Ans.

- (c) Given observation are 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47 and median = 24.

$$n = 9 \text{ (odd)}$$

$$\text{Median} = \frac{n+1}{2} \text{th term}$$

$$= \frac{9+1}{2} \text{th term}$$

$$24 = 5\text{th term}$$

$$x + 4 = 24$$

$$x = 24 - 4$$

$$x = 20$$

Therefore, 11, 12, 14, $(20 - 2)$, $(20 + 4)$, $(20 + 9)$, 32, 38, 47
 = 11, 12, 14, 18, 24, 29, 32, 38, 47

Now

$$\text{Mean} = \frac{\Sigma x}{n}$$

$$= \frac{11 + 12 + 14 + 18 + 24 + 29 + 32 + 38 + 47}{9}$$

$$= \frac{225}{9} = 25.$$

Ans.

Question 2.

- (a) What number must be added to each of the number 6, 15, 20 and 43 to make them proportional ? [3]
 (b) If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b . [3]
 (c) Draw a histogram from the following frequency distribution and find the mode from the graph : [4]

Class	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	2	5	18	14	8	5

Solution :

(a) Let the number must be added be x , then

$$\text{the new number} = 6 + x, 15 + x, 20 + x, 43 + x$$

∴ These are proportionals.

$$6 + x : 15 + x :: 20 + x : 43 + x$$

or

$$(6 + x)(43 + x) = (15 + x)(20 + x)$$

or

$$258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2$$

or

$$49x - 35x = 300 - 258$$

or

$$14x = 42$$

or

$$x = 3.$$

Ans.

(b) Let $(x - 2)$ is a factor of the given expression.

∴

$$x - 2 = 0$$

$$x = 2$$

Given expression,

$$2x^3 + ax^2 + bx - 14 = 0$$

$$2(2)^3 + a(2)^2 + b(2) - 14 = 0$$

$$16 + 4a + 2b - 14 = 0$$

$$4a + 2b + 2 = 0$$

$$4a + 2b = -2$$

$$2a + b = -1$$

...(i)

and when given expression is divided by $(x - 3)$

$$x - 3 = 0$$

$$x = 3$$

⇒

$$2x^3 + ax^2 + bx - 14 = 52$$

∴

$$2(3)^3 + a(3)^2 + b(3) - 66 = 0$$

$$54 + 9a + 3b - 66 = 0$$

$$9a + 3b = 12$$

$$3a + b = 4$$

...(ii)

Solving equation (i) and (ii),

$$2a + b = -1$$

$$3a + b = 4$$

$$\begin{array}{r} (-) \quad (-) \quad (+) \\ \hline \end{array}$$

$$-a = -5$$

$$a = 5$$

from (ii),

$$3 \times 5 + b = 4$$

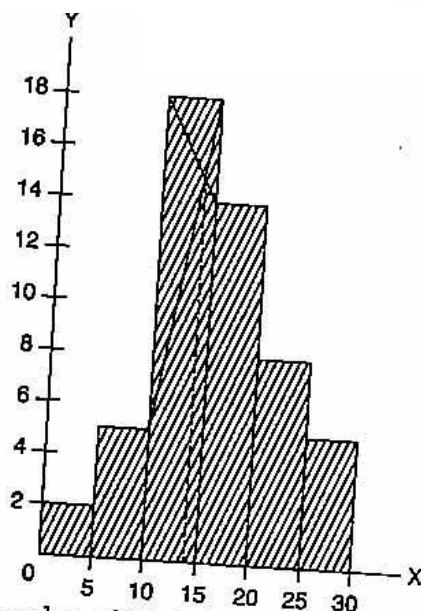
$$b = 4 - 15$$

$$b = -11$$

$$a = 5 \text{ and } b = -11$$

Ans.

(c)



From the Histogram the value of Mode is 13.8.

Question 3.

Ans.

- (a) Without using tables evaluate $3 \cos 80^\circ \cdot \operatorname{cosec} 10^\circ + 2 \sin 59^\circ \sec 31^\circ$.
 (b) In the given figure,

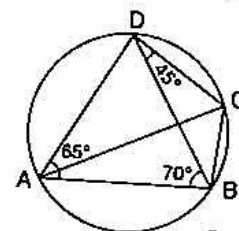
[3]

$$\angle BAD = 65^\circ,$$

$$\angle ABD = 70^\circ,$$

$$\angle BDC = 45^\circ$$

- (i) Prove that AC is a diameter of the circle.
 (ii) Find $\angle ACB$.
 (c) AB is a diameter of a circle with centre C = (-2, 5). If A = (3, -7). Find :
 (i) The length of radius AC
 (ii) The coordinates of B.



[3]

Solution :

[4]

(a) Given :

$$\begin{aligned} & 3 \cos 80^\circ \cdot \operatorname{cosec} 10^\circ + 2 \sin 59^\circ \sec 31^\circ \\ &= 3 \cos 80^\circ \operatorname{cosec} (90^\circ - 80^\circ) + 2 \sin 59^\circ \sec (90^\circ - 59^\circ) \\ &= 3 \cos 80^\circ \sec 80^\circ + 2 \sin 59^\circ \operatorname{cosec} 59^\circ \\ &= 3 \cos 80^\circ \times \frac{1}{\cos 80^\circ} + 2 \sin 59^\circ \times \frac{1}{\sin 59^\circ} \\ &= 3 + 2 = 5. \end{aligned}$$

- (b) Given : $\angle BAD = 65^\circ$, $\angle ABD = 70^\circ$, $\angle BDC = 45^\circ$.
 (i) $\therefore$ ABCD is a cyclic quadrilateral.

Ans.

In $\triangle ABD$,

$$\angle BDA + \angle DAB + \angle ABD = 180^\circ$$

$$\angle BDA = 180^\circ - (65^\circ + 70^\circ)$$

$$= 180^\circ - 135^\circ$$

$$= 45^\circ$$

By using sum property of $\triangle$

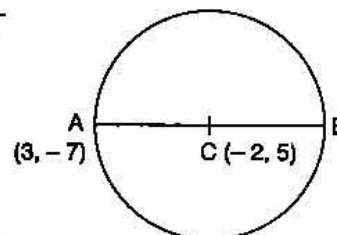
Now from $\triangle ACD$,

$$\begin{aligned}\angle ADC &= \angle ADB + \angle BDC \\ &= 45^\circ + 45^\circ \quad (\because \angle BDA = \angle ADB = 45^\circ) \\ &= 90^\circ\end{aligned}$$

Hence, $\angle D$ makes right angle belongs in semi-circle therefore AC is a diameter of the circle.

- (ii) $\angle ACB = \angle ADB$ (Angles in the same segment of a circle)
 $\therefore \angle ACB = 45^\circ$ ($\because \angle ADB = 45^\circ$) **Ans.**

- (c) (i) The length of radius AC = $\sqrt{(-2-3)^2 + (5+7)^2}$
 $= \sqrt{(-5)^2 + (12)^2}$
 $= \sqrt{25 + 144}$
 $= \sqrt{169}$
 $= 13.$ **Ans.**



- (ii) Let the point of B be (x, y) .
 Given C is the mid-point of AB. Therefore

$$\begin{aligned}-2 &= \frac{3+x}{2} \\ \Rightarrow 3+x &= -4 \\ \Rightarrow x &= -4-3 = -7 \\ \text{and } 5 &= \frac{-7+y}{2} \\ \Rightarrow 10 &= -7+y \\ y &= 17\end{aligned}$$

Hence, the co-ordinate of B $(-7, 17)$.

Ans.

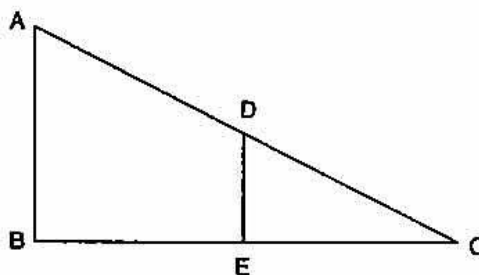
Question 4.

- (a) Solve the following equation and calculate the answer correct to two decimal places :

$$x^2 - 5x - 10 = 0. \quad [3]$$

- (b) In the given figure, AB and DE are perpendicular to BC.

- (i) Prove that $\triangle ABC \sim \triangle DEC$
 (ii) If $AB = 6$ cm, $DE = 4$ cm and $AC = 15$ cm. Calculate CD.
 (iii) Find the ratio of the area of $\triangle ABC$: area of $\triangle DEC$. **[3]**



- (c) Using graph paper, plot the points $A(6, 4)$ and $B(0, 4)$.

- (i) Reflect A and B in the origin to get the images A' and B' .
 (ii) Write the co-ordinates of A' and B' .
 (iii) State the geometrical name for the figure $ABA'B'$.
 (iv) Find its perimeter. **[4]**

Solution :

(a) Given : $x^2 - 5x - 10 = 0$

Here, $a = 1$, $b = -5$ and $c = -10$ $\therefore$

$$D = b^2 - 4ac$$

$$= (-5)^2 - 4 \times 1 \times -10$$

$$D = 25 + 40 = 65$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{5 \pm \sqrt{65}}{2 \times 1} = \frac{5 \pm 8.06}{2}$$

$$= \frac{5 + 8.06}{2}, \frac{5 - 8.06}{2}$$

$$= \frac{13.06}{2}, -\frac{3.06}{2}$$

$x = 6.53, -1.53$

Ans.(b) (i) From ΔABC and ΔDEC ,

$\angle ABC = \angle DEC = 90^\circ$

(Given)

and

$\angle ACB = \angle DCE = \text{Common}$

 $\therefore$

$\Delta ABC \sim \Delta DEC$

(By AA similarity)

(ii) In ΔABC and ΔDEC ,

$\Delta ABC \sim \Delta DEC$

(proved in (i) part)

$$\therefore \frac{AB}{DE} = \frac{AC}{CD}$$

Given : $AB = 6$ cm, $DE = 4$ cm, $AC = 15$ cm,

$$\therefore \frac{6}{4} = \frac{15}{CD}$$

$$\Rightarrow 6 \times CD = 15 \times 4$$

$$\Rightarrow CD = \frac{60}{6}$$

$$\Rightarrow CD = 10 \text{ cm.}$$

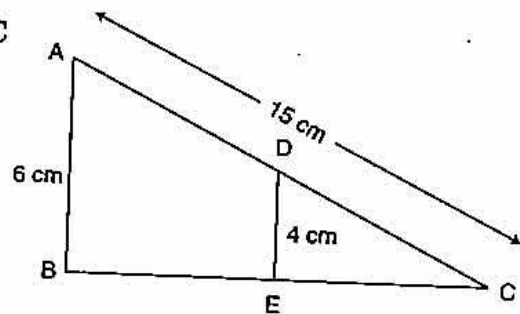
$$(iii) \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEC} = \frac{AB^2}{DE^2}$$

Ans.
($\because \Delta ABC \sim \Delta DEC$)

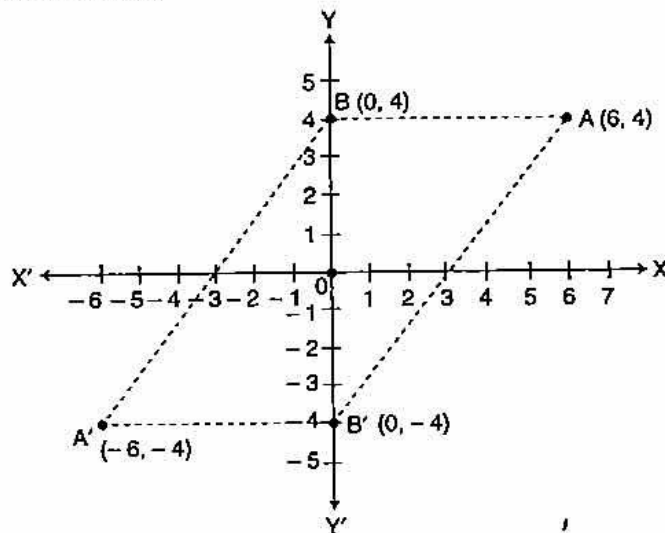
$$= \frac{(6)^2}{(4)^2}$$

$$= \frac{36}{16} = \frac{9}{4}$$

$$\therefore \text{Area of } \Delta ABC : \text{Area of } \Delta DEC = 9 : 4.$$

Ans.

(c) (i) Please See Graph:



- (ii) Reflection of A' and B' in the origin = A' (-6, -4) and B' (0, -4)
 (iii) The geometrical name for the figure AB A' B' is a parallelogram.
 (iv) From the graph, AB = 6 cm, BB' = 8 cm.

In $\triangle ABB'$

$$\begin{aligned}(AB')^2 &= AB^2 + (BB')^2 \\ &= (6)^2 + (8)^2 = 36 + 64 \\ &= 100\end{aligned}$$

$$AB' = 10 = A'B \quad (\text{AB A' B' is a parallelogram})$$

$$\begin{aligned}\text{Perimeter of ABA'B'} &= A'B' + AB' + AB + A'B \\ &= 6 + 10 + 6 + 10 \\ &= 32 \text{ units.}\end{aligned}$$

Ans.

SECTION B [40 Marks]

Answer any four Questions in this Section.

Question 5.

- (a) Solve the following inequation, write the solution set and represent it on the number line :

$$-\frac{x}{3} \leq \frac{x}{2} - 1 \quad \frac{1}{3} < \frac{1}{6}, x \in R \quad [3]$$

- (b) Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets ₹ 8088 from the bank after 3 years, find the value of his monthly instalment.

[3]

- (c) Salman buys 50 shares of face value ₹ 100 available at ₹ 132:

- (i) What is his investment ?
 (ii) If the dividend is 7.5%, what will be his annual income ?
 (iii) If he wants to increase his annual income by ₹ 150, how many extra shares should he buy ?

[4]

Solution :

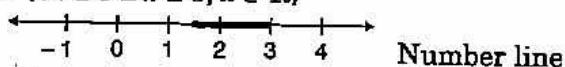
(a) Given : $-\frac{x}{3} \leq \frac{x}{2} - 1 \frac{1}{3} < \frac{1}{6}$

Taking L.C.M. of 3, 2 and 6 is 6.

$$-\frac{x}{3} \times 6 \leq \frac{x}{2} \times 6 - \frac{4}{3} \times 6 < \frac{1}{6} \times 6$$

$$-2x \leq 3x - 8 < 1$$

$$\begin{aligned} \Rightarrow -2x &\leq 3x - 8 & \text{and} & & 3x - 8 < 1 \\ \Rightarrow 8 &\leq 3x + 2x & \Rightarrow & & 3x < 1 + 8 \\ \Rightarrow 8 &\leq 5x & \Rightarrow & & 3x < 9 \\ \Rightarrow \frac{8}{5} &\leq x & \Rightarrow & & x < 3 \end{aligned}$$

 $\therefore$ The solution set is $\{x : 1.6 \leq x < 3, x \in \mathbb{R}\}$ (b) Let the monthly instalment be ₹ x

Given : Maturity amount = ₹ 8,088, Time (n) = 3 years = 3×12 months = 36 months, Rate (R) = 8% p.a.

$$\text{Principle for one month} = P \times \frac{n(n+1)}{2}$$

$$= \frac{x \times 36 \times 37}{2}$$

$$= 18 \times 37x$$

$$\text{Interest} = \frac{18 \times 37x \times 8 \times 1}{100 \times 12}$$

$$\left[\because I = \frac{PRT}{100} \right]$$

$$= \frac{444x}{100}$$

$$\text{Actual sum deposited} = 36x$$

$$\text{Maturity amount} = \text{Interest} + \text{Actual sum deposited}$$

$$8,088 = \frac{444x}{100} + 36x$$

$$8,088 = \frac{4,044x}{100}$$

$$\therefore x = \frac{8,088 \times 100}{4,044} = 200$$

Hence, the monthly instalment be ₹ 200.

Ans.

(c)

$$\text{Number of shares} = 50$$

$$\text{Face value of each share} = ₹ 100$$

$$\text{Market value of each share} = ₹ 132$$

$$\text{Total face value} = ₹ 100 \times 50$$

$$= ₹ 5,000$$

(i)

$$\text{Total investment} = ₹ 132 \times 50$$

$$= ₹ 6,600$$

Ans.

(ii) Rate of dividend = 7.5%
 Annual income = ₹ $\frac{5,000 \times 7.5}{100}$
 = ₹ 375 Ans.

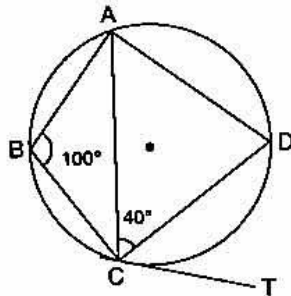
(iii) Let extra share should he buy be x .
 then total number of shares = $50 + x$
 Total face value = ₹ $100 \times (50 + x)$
 Annual income = ₹ $\frac{100 \times (50 + x) \times 7.5}{100}$
 = $(50 + x) \times 7.5$
 $(50 + x) \times 7.5 = 375 + 150$
 $50 + x = \frac{525}{7.5} = 70$
 $x = 70 - 50$
 $x = 20$

Hence, the extra shares should be buy = 20. Ans.

Question 6.

(a) Show that $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$ [3]

(b) In the given circle with centre O , $\angle ABC = 100^\circ$, $\angle ACD = 40^\circ$ and CT is a tangent to the circle at C . Find $\angle ADC$ and $\angle DCT$. [3]



(c) Given below are the entries in a Savings Bank A/c pass book :

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8.	B/F	—	—	₹ 8,500
Feb. 18	To self	₹ 4,000	—	—
April 12	By cash	—	₹ 2,230	—
June 15	To self	₹ 5,000	—	—
July 8	By cash	—	₹ 6,000	—

Calculate the interest for six months from February to July at 6% p.a. [4]

Solution :

(a) L.H.S. = $\sqrt{\frac{1 - \cos A}{1 + \cos A}}$

Multiplying by $\sqrt{1 + \cos A}$ in numerator and denominator

$$= \sqrt{\frac{1 - \cos A}{1 + \cos A}} \times \sqrt{\frac{1 + \cos A}{1 + \cos A}}$$

$$\begin{aligned}
 &= \sqrt{\frac{(1 - \cos A)(1 + \cos A)}{(1 + \cos A)(1 + \cos A)}} \\
 &= \sqrt{\frac{1 - \cos^2 A}{(1 + \cos A)^2}} \\
 &= \sqrt{\frac{\sin^2 A}{(1 + \cos A)^2}} \\
 &= \frac{\sin A}{1 + \cos A} = \text{R.H.S.}
 \end{aligned}$$

Proved(b) *Given* : $\angle ABC = 100^\circ$

We know that,

$$\angle ABC + \angle ADC = 180^\circ$$

$$\therefore 100^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 180^\circ - 100^\circ$$

$$\angle ADC = 80^\circ$$

(The sum of opposite angles in a cyclic quadrilateral = 180°)Join OA and OC, we have an isosceles $\triangle OAC$,

$$\therefore OA = OC \quad (\text{Radii of a circle})$$

$$\therefore \angle AOC = 2 \times \angle ADC \quad (\text{by theorem})$$

$$\text{or } \angle AOC = 2 \times 80^\circ = 160^\circ$$

In $\triangle AOC$,

$$\angle AOC + \angle OAC + \angle OCA = 180^\circ$$

$$160^\circ + \angle OCA + \angle OCA = 180^\circ \quad [\because \angle OAC = \angle OCA]$$

$$2 \angle OCA = 20^\circ$$

$$\angle OCA = 10^\circ$$

$$\angle OCA + \angle OCD = 40^\circ$$

$$10^\circ + \angle OCD = 40^\circ$$

$$\therefore \angle OCD = 30^\circ$$

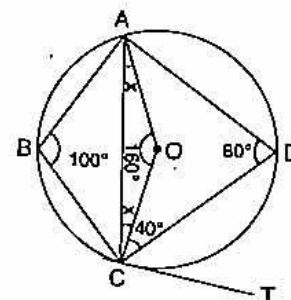
$$\text{Hence, } \angle OCD + \angle DCT = \angle OCT$$

$$\angle OCT = 90^\circ$$

(The tangent at a point to circle is $\perp$ to the radius through the point to contact)

$$30^\circ + \angle DCT = 90^\circ$$

$$\therefore \angle DCT = 60^\circ$$

Ans.

(c)	Date	Particulars	Withdrawals	Deposit	Balance
	Feb. 8	B/F	—	—	₹ 8,500
	Feb. 18	To self	₹ 4,000	—	₹ 4,500
	April 12	By cash	—	₹ 2,230	₹ 6,730
	June 15	To self	₹ 5,000	—	₹ 1,730
	July 8	By cash	—	₹ 6,000	₹ 7,730

Principal for the month of Feb. = ₹ 4,500

Principal for the month of March = ₹ 4,500

Principal for the month of April = ₹ 4,500

Principal for the month of May = ₹ 6,730

Principal for the month of June = ₹ 1,730

Principal for the month of July = ₹ 7,730

Total principal from the month of Feb. to July = ₹ 29,690

$$\text{Time} = \frac{1}{12} \text{ years}$$

Rate of interest = 6%

$$\begin{aligned} \text{Interest} &= \frac{P \times R \times T}{100} \\ &= \frac{29690 \times 6 \times 1}{100 \times 12} \\ &= ₹ 148.45 \end{aligned}$$

Ans.

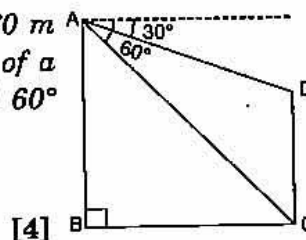
Question 7.

(a) In $\triangle ABC$, $A(3, 5)$, $B(7, 8)$ and $C(1, -10)$. Find the equation of the median through A. [3]

(b) A shopkeeper sells an article at the listed price of ₹ 1,500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of ₹ 36 to the Government, what was the price, inclusive of Tax, at which the shopkeeper purchased the article from the wholesaler? [3]

(c) In the figure given, from the top of a building $AB = 60$ m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :

- The horizontal distance between AB and CD .
- The height of the lamp post.



Solution :

(a) Here D is mid point of BC.

$$\begin{aligned} \therefore \text{The co-ordinate of D} &= \left(\frac{7+1}{2}, \frac{8-10}{2} \right) \\ &= (4, -1) \end{aligned}$$

Now equation of median AD,

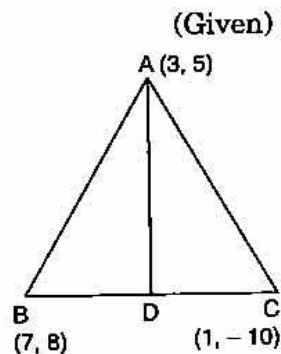
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Here, $x_1 = 3$, $y_1 = 5$, $x_2 = 4$, $y_2 = -1$

$$y - 5 = \frac{-1 - 5}{4 - 3} (x - 3)$$

$$y - 5 = \frac{-6}{1} (x - 3)$$

$$y - 5 = -6x + 18$$



$$y = -6x + 18 + 5$$

$$y = -6x + 23$$

$$6x + y - 23 = 0$$

(b)

Listed price of an article = ₹ 1,500

Rate of VAT = 12%

$$\begin{aligned}\text{VAT on the article} &= \frac{12}{100} \times 1500 \\ &= ₹ 180\end{aligned}$$

Let C.P. of this article be x , then

$$\begin{aligned}\text{VAT} &= \frac{12}{100} \times x \\ &= ₹ \frac{12x}{100}\end{aligned}$$

If the shopkeeper pays a VAT = ₹ 36

Then

$$180 - \frac{12x}{100} = 36$$

$$\frac{18000 - 12x}{100} = 36$$

$$18000 - 12x = 3600$$

 $\therefore$

$$12x = 18000 - 3600 = 14,400$$

$$x = ₹ 1,200$$

 $\therefore$ The price at which the shopkeeper purchased the article inclusive of sales tax

$$= 1,200 + \frac{12}{100} \times 1,200$$

$$= 1,200 + 144$$

$$= ₹ 1,344$$

Ans.

(c) Given : $AB = 60$ m $\therefore$ $\therefore$ (i) Now in $\triangle ABC$,

$$\angle PAC = 60^\circ$$

$$\angle PAC = \angle BCA$$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{60}{BC}$$

 $\Rightarrow$

$$\sqrt{3} BC = 60$$

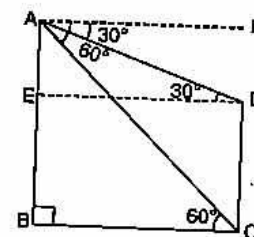
 $\Rightarrow$

$$BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$BC = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

Hence, the horizontal distance between AB and CD = $20\sqrt{3}$ m.(ii) Let $AE = x$ and proved above $BC = 20\sqrt{3}$ m $\therefore$

$$BC = ED = 20\sqrt{3}$$



Ans.

Now in $\triangle AED$,

$$\tan 30^\circ = \frac{AE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$$

$$\Rightarrow \sqrt{3} AE = 20\sqrt{3}$$

$$\Rightarrow AE = 20 \text{ m}$$

$$\text{now } EB = AB - AE$$

$$\therefore EB = 60 - 20 \Rightarrow 40 \text{ m}$$

$$\therefore EB = CD$$

$$\therefore CD = 40 \text{ m}$$

Hence, the height of the lamp post = 40 m.

Ans.

Question 8.

(a) Find x and y if $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ [3]

(b) A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast. [3]

(c) Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots :

$$x^2 + (p-3)x + p = 0 \quad [4]$$

Solution :

(a) Given : $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$$\begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\therefore 5x = 5 \Rightarrow x = 1$$

$$\text{and } 6y = 12 \Rightarrow y = 2$$

Hence, $x = 1$ and $y = 2$

Ans.

(b) Radius of a solid sphere, $r = 15 \text{ cm}$

$$\text{Volume of a solid sphere} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \pi (15)^3 \text{ cm}^3.$$

Now, radius of right circular cone = 2.5 cm

and height, $h = 8 \text{ cm}$.

$$\text{Volume of right circular cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (2.5)^2 \times 8$$

$$\text{The number of cones} = \frac{\text{Volume of a sphere}}{\text{Volume of a cone}}$$

$$= \frac{\frac{4}{3} \pi \times (15)^3}{\frac{1}{3} \pi (2.5)^2 \times 8}$$

$$= \frac{15 \times 15 \times 15}{2.5 \times 2.5 \times 2}$$

$$= 270$$

Ans.

(c) Given equation

$$x^2 + (p-3)x + p = 0$$

$\therefore$ Roots are real and equal, then

$$b^2 - 4ac = 0$$

Here we compare the coefficients of a , b and c with the equation $ax^2 + bx + c = 0$.

$$a = 1, b = p-3 \text{ and } c = p$$

Now putting the values of a , b and c in equation

$$(p-3)^2 - 4 \times 1 \times p = 0$$

$$p^2 + 9 - 6p - 4p = 0$$

$$p^2 + 9 - 10p = 0$$

$$p^2 - 10p + 9 = 0$$

$$p^2 - 9p - p + 9 = 0$$

$$p(p-9) - 1(p-9) = 0$$

$$(p-9)(p-1) = 0$$

$\Rightarrow$

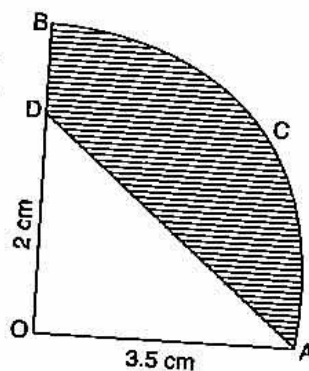
Hence,

$$p = 9 \text{ or } 1$$

Ans.

Question 9.

- (a) In the figure alongside, OAB is a quadrant of a circle. The radius $OA = 3.5$ cm and $OD = 2$ cm. Calculate the area of the shaded portion. (Take $\pi = \frac{22}{7}$) [3]



- (b) A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box. [3]

- (c) Find the mean of the following distribution by step deviation method : [4]

Class Interval	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	10	6	8	12	5	9

Solution :

(a) Radius of quadrant OACB, $r = 3.5$ cm

$$\begin{aligned}\therefore \text{Area of quadrant OACB} &= \frac{1}{4} \pi r^2 \\ &= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 \\ &= 9.625 \text{ cm}^2.\end{aligned}$$

Here,

$$\angle AOD = 90^\circ$$

Then

$$\text{area of } \Delta AOD = \frac{1}{2} \times \text{base} \times \text{height}$$

Base = 3.5 cm and height = 2 cm

$$\therefore = \frac{1}{2} \times 3.5 \times 2 = 3.5 \text{ cm}^2.$$

$$\begin{aligned}\text{Area of shaded portion} &= \text{Area of quadrant} - \text{Area of triangle} \\ &= 9.625 - 3.5 \\ &= 6.125 \text{ cm}^2.\end{aligned}$$

Ans.

(b) Let the number of black balls be x , then

$$\text{Total number of balls} = 30 + x$$

$$\text{Thus, the probability of black balls} = \frac{x}{30 + x}$$

$$\text{and the probability of white balls} = \frac{30}{30 + x}$$

$$\text{Given : Probability of black ball} = \frac{2}{5} \times \text{probability of white ball}$$

$$\frac{x}{30 + x} = \frac{2}{5} \times \frac{30}{x + 30}$$

$$5x = 60$$

$$x = 12$$

Ans.

Hence, the number of black balls = 12.

(c)

C.I.	Frequency (f_i)	Mid-value (x)	$d_i = \frac{x - a}{h}$	$f_i d_i$
20-30	10	25	-2	-20
30-40	6	35	-1	-6
40-50	8	45	0	0
50-60	12	55	1	12
60-70	5	65	2	10
70-80	9	75	3	27
	$\Sigma f_i = 50$			$\Sigma f_i d_i = 23$

Here, $a = 45$ and $h = 10$

$\therefore$

$$\begin{aligned}\text{Mean} &= a + \frac{\Sigma f_i d_i}{\Sigma f_i} \times h \\ &= 45 + \frac{23}{50} \times 10 \\ &= 45 + 4.6 = 49.6.\end{aligned}$$

Ans.

Question 10.**(a)** Using a ruler and compasses only :

- (i) Construct a triangle ABC with the following data :
 $AB = 3.5$ cm, $BC = 6$ cm and $\angle ABC = 120^\circ$
- (ii) In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC .
- (iii) Measure $\angle BCP$.

[3]**(b)** The mark obtained by 120 students in a test are given below :

Marks	No. of Students
0–10	5
10–20	9
20–30	16
30–40	22
40–50	26
50–60	18
60–70	11
70–80	6
80–90	4
90–100	3

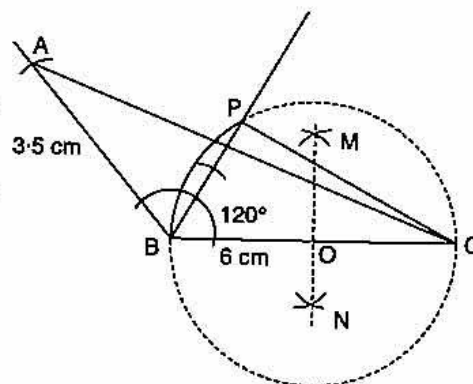
Draw an ogive for the given distribution on a graph sheet.

Using suitable scale for ogive to estimate the following :

- (i) The median.
- (ii) The number of students who obtained more than 75% marks in the test.
- (iii) The number of students who did not pass the test if minimum marks required to pass is 40.

[6]**Solution :****(a) Steps of Construction :**

- (i) Draw a line $BC = 6$ cm.
- (ii) With the help of the point B , draw $\angle ABC = 120^\circ$
- (iii) Taking radius 3.5 cm cut $BA = 3.5$ cm.
- (iv) Join A to C .
- (v) Draw $\perp$ bisector MN of BC .
- (vi) Draw a circle O as centre and OC as radius.
- (vii) Draw angle bisector of $\angle ABC$ which intersects circle at P .
- (viii) Join BP and CP .
- (ix) Now, $\angle BCP = 30^\circ$.

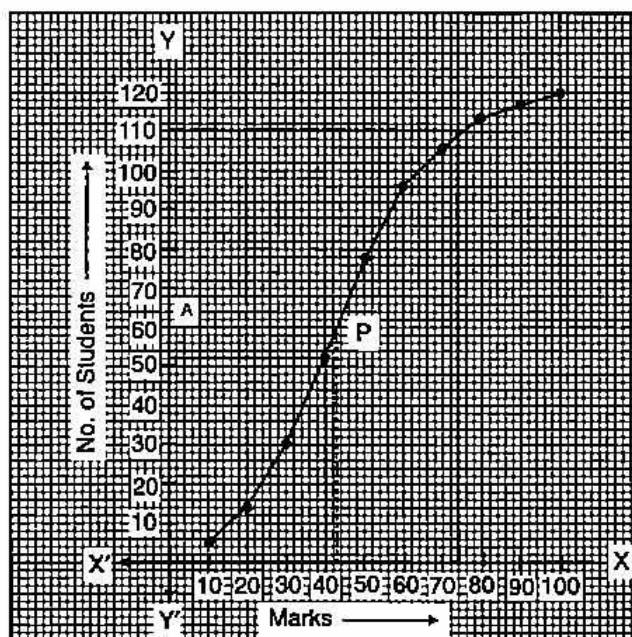


(b)

Marks	No. of Students (f)	Cumulative Frequency
0–10	5	5
10–20	9	14
20–30	16	30
30–40	22	52
40–50	26	78
50–60	18	96
60–70	11	107
70–80	6	113
80–90	4	117
90–100	3	120
	$n = 120$	

On the graph paper, we plot the following points :

(10, 5), (20, 14), (30, 30), (40, 52), (50, 78), (60, 96), (70, 107), (80, 113), (90, 117), (100, 120).



$$\begin{aligned}
 \text{(i)} \quad \text{Median} &= \left(\frac{n}{2} \right)^{\text{th}} \text{ term} && [\because n = 120, \text{ even}] \\
 &= \frac{120}{2} = 60^{\text{th}} \text{ term}
 \end{aligned}$$

From the graph 60th term = 42

Ans.

$$\begin{aligned}
 \text{(ii)} \quad \text{The number of students who obtained more than 75\% marks in test} \\
 &= 120 - 110 \\
 &= 10.
 \end{aligned}$$

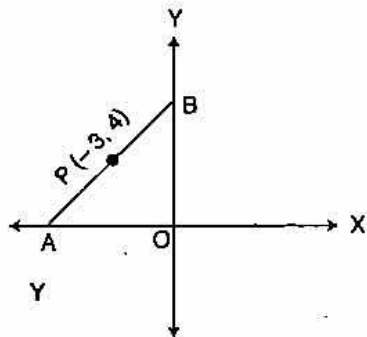
Ans.

$$\begin{aligned}
 \text{(iii)} \quad \text{The number of students who did not pass the test if the minimum pass} \\
 \text{marks 40} &= 52.
 \end{aligned}$$

Ans.

Question 11.

- (a) In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P(-3, 4) on AB divides it in the ratio 2 : 3. Find the coordinates of A and B.



- (b) Using the properties of proportion, solve for x , given

$$\frac{x^4 + 1}{2x^2} = \frac{17}{8} \quad [3]$$

- (c) A shopkeeper purchases a certain number of books for ₹ 960. If the cost per book was ₹ 8 less, the number of books that could be purchased for ₹ 960 would be 4 more. Write an equation, taking the original cost of each book to be ₹ x , and solve it to find the original cost of the books. [4]

Solution :

- (a) Let the co-ordinates of A and B be $(x, 0)$ and $(0, y)$

∴ The co-ordinates of a point P (-3, 4) on AB divides it in the ratio 2 : 3.

i.e.,

$$AP : PB = 2 : 3$$

By using section formula, we get

$$-3 = \frac{2 \times 0 + 3 \times x}{2 + 3} \quad \left[\because x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \right]$$

$$-3 = \frac{3x}{5} \Rightarrow 3x = -15$$

⇒

$$x = -5$$

and

$$4 = \frac{2 \times y + 3 \times 0}{2 + 3} \quad \left[\because y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right]$$

$$4 = \frac{2y}{5} \Rightarrow 2y = 20$$

⇒

$$y = 10$$

Hence, the co-ordinates of A and B are $(-5, 0)$ and $(0, 10)$.

Ans.

- (b) Given : $\frac{x^4 + 1}{2x^2} = \frac{17}{8}$

By using componendo and dividendo, we get

$$\frac{x^4 + 1 + 2x^2}{x^4 + 1 - 2x^2} = \frac{17 + 8}{17 - 8}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \frac{25}{9}$$

$$\left(\frac{x^2+1}{x^2-1}\right) = \left(\frac{5}{3}\right)$$

Taking square root on both sides, we get

$$\frac{x^2+1}{x^2-1} = \frac{5}{3}$$

$$\Rightarrow 5x^2 - 5 = 3x^2 + 3$$

$$\Rightarrow 5x^2 - 3x^2 = 3 + 5$$

$$\Rightarrow 2x^2 = 8 \Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

Ans.

(c) Given the original cost of each book be ₹ x .

$$\text{Total cost} = ₹ 960$$

(Given)

$$\therefore \text{Number of books for 960} = \frac{960}{x}$$

If the cost per book was ₹ 8 less, (i.e., $x - 8$) then

$$\text{Number of books} = \frac{960}{x-8}$$

According to question,

$$\frac{960}{x-8} = \frac{960}{x} + 4$$

$$\frac{960}{x-8} - \frac{960}{x} = 4$$

$$960 \left[\frac{x-x+8}{x(x-8)} \right] = 4$$

$$\frac{8}{x^2-8x} = \frac{1}{240}$$

$$\Rightarrow x^2 - 8x = 1,920$$

$$\Rightarrow x^2 - 8x - 1,920 = 0$$

$$\Rightarrow x^2 - 48x + 40x - 1,920 = 0$$

$$\Rightarrow x(x-48) + 40(x-48) = 0$$

$$\Rightarrow (x-48)(x+40) = 0$$

$$x - 48 = 0$$

$$x = 48$$

$$\text{or } x + 40 = 0$$

$$\text{or } x = -40$$

Ans.

$\therefore -40$ is not possible.

Hence, the original cost of each book = ₹ 48.

Ans.

